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14. ABSTRACT
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Cognitive CDMA Channelization

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I. Introduction

Recent experimental studies [1] indicate that most of the licensed radio spectrum experiences low utilization. Cognitive radio (CR) [2] emerged as a promising technology that improves spectrum efficiency and utilization by allowing the secondary users/networks to share the spectrum that is licensed by primary users. As licensees, the primary users have always higher priority to use the spectrum [3]. The underlying challenge of this technology is to guarantee the Quality-of-Service (QoS) requirements of the primary system yet to maximize QoS for the secondary users [4]-[7]. Power control for cognitive CDM systems was considered in [8] where

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the proposed method does not take full advantage of signal processing at the receiver end, and thus restricts the throughput of secondary users. Joint beamforming and power allocation algorithms for cognitive radio networks were presented in [9], [10], while auction mechanism for power control were developed in [11].

In this paper, we consider a secondary system with a code division multiplexed (CDM) mode coexisting with a primary CDM system. We study the problem of designing a power and code-channel allocation scheme for the secondary user that maximizes the output SINR of the maximum-SINR linear receiver filter under SINR QoS constraints for all primary users and a peak transmission power constraint for the secondary user. This is a non-convex NP-hard problem. In this paper, we propose a novel, realizable suboptimum solution using semidefinite programming.

The rest of the paper is organized as follows. Section II is devoted to CR system model specifics and the formulation of the optimization problem. In Section III, we present our proposed solution while in Section IV we propose an iterative procedure based on semifinite programming solution, which converges to a suboptimal feasible solution. The performance of the proposed scheme is evaluated through simulations in Section V. A few concluding remarks are drawn in Section VI.

II. System model and problem statement

We consider a primary CDM network with K users and processing gain (code sequence length) L. We also consider a secondary system (with CDM mode of operation) in the uplink spectrum band of the primary system (Fig. 1). For simplicity in presentation, we assume that all signals propagate over a plain (no multipath) additive white Gaussian noise channel. We denote by h_i and q_i, $i = 1, 2, \ldots, K$, the path coefficients from user i to the base station and the secondary receiver, respectively. The path coefficients from the secondary transmitter to the base station and to the secondary receiver are denoted by h_s and q_s, respectively. All path coefficients are modeled as Rayleigh distributed random variables that are independent across user signals and remain constant during several symbol intervals (quasi-static fading). After chip-matched filtering and sampling at the chip rate, the received signal at the primary base station can be represented as

$$\mathbf{r} = \sum_{i=1}^{K} \sqrt{E_i} h_i \mathbf{s}_i b_i + \sqrt{E_s} h_s \mathbf{s}_s b_s + \mathbf{n}_p, \tag{1}$$

while the secondary received signal is

$$\mathbf{y} = \sum_{i=1}^{K} \sqrt{E_i} q_i \mathbf{s}_i b_i + \sqrt{E_s} q_s \mathbf{s}_s b_s + \mathbf{n}_s, \tag{2}$$

where E_i, b_i and $\mathbf{s}_i$ denote the bit energy, the information bit, and the normalized signature vector of the primary user i, respectively; E_s, b_s and $\mathbf{s}_s$ denote the bit energy, the information bit and the normalized signature vector of the secondary user, respectively; $\mathbf{n}_p$ and $\mathbf{n}_s$ represent AWGN $\mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$ at the base station and at the secondary receiver, correspondingly.

The linear filters at the base station and secondary receiver, which exhibit maximum output SINR, can be represented, respectively, as

$$\mathbf{w}_{MSINR,i} = c\mathbf{R}^{-1}\mathbf{s}_i,\ k = 1, 2, \ldots, K$$

$$\mathbf{w}_{MSINR,s} = \tilde{c}\tilde{\mathbf{R}}^{-1}\mathbf{s}_s,$$

where $\mathbf{R} = E\{\mathbf{r}\mathbf{r}^T\}$, $\tilde{\mathbf{R}} = E\{\mathbf{y}\mathbf{y}^T\}$, $c, \tilde{c} > 0$ and $E\{\cdot\}$ denotes statistical expectation. The output SINR of primary user i at the base station and the secondary user at the receiver can be expressed as in (4) and (5) at the top of the following page, where $\mathbf{R}_{/i}$ and $\mathbf{R}_{/s}$ are the "exclude *i*" or "exclude *s*" data autocorrelation matrix, defined as

$$\mathbf{R}_{/i} \triangleq \sum_{k=1, k\neq i}^{K} E_k h_k^2 \mathbf{s}_k \mathbf{s}_k^T + E_s h_s^2 \mathbf{s}_s \mathbf{s}_s^T + \sigma^2 \mathbf{I}$$

$$\mathbf{R}_{/s} \triangleq \sum_{k=1}^{K} E_k q_k^2 \mathbf{s}_k \mathbf{s}_k^T + \sigma^2 \mathbf{I}.$$

In a cognitive radio setup, the secondary transmitter has to guarantee the QoS of all primary users. Hence, our objective is to find the transmission bit energy E_s and the real-valued signature $\mathbf{s}_s$ that maximize $SINR_s$

$$SINR_i = \frac{E\{|\mathbf{w}_{MSINR,i}^T(\sqrt{E_i}h_ib_i\mathbf{s}_i)|^2\}}{E\{|\mathbf{w}_{MSINR,i}^T(\sum_{k=1,k\neq i}^{K}\sqrt{E_k}h_k\mathbf{s}_kb_k + \sqrt{E_s}h_s\mathbf{s}_sb_s + \mathbf{n}_p)|^2\}} = E_ih_i^2\mathbf{s}_i^T\mathbf{R}_{/i}^{-1}\mathbf{s}_i, \tag{4}$$

$$SINR_s = \frac{E\{|\mathbf{w}_{MSINR,s}^T(\sqrt{E_s}q_sb_s\mathbf{s}_s)|^2\}}{E\{|\mathbf{w}_{MSINR,s}^T(\sum_{k=1}^{K}\sqrt{E_k}q_k\mathbf{s}_kb_k + \mathbf{n}_s)|^2\}} = E_sq_s^2\mathbf{s}_s^T\mathbf{R}_{/s}^{-1}\mathbf{s}_s. \tag{5}$$

---

under the constraints that $SINR_i$, $i = 1, 2, \ldots, K$, are above a certain threshold α, i.e.

$$\max_{E_s,\mathbf{s}_s} \; E_s\mathbf{s}_s^T\mathbf{R}_{/s}^{-1}\,\mathbf{s}_s \tag{3}$$

$$subject\ to \quad E_ih_i^2\mathbf{s}_i^T\mathbf{R}_{/i}^{-1}\,\mathbf{s}_i \geq \alpha, \quad i = 1, 2, \ldots, K$$

$$\mathbf{s}_s^T\mathbf{s}_s = 1,\ E_s \leq E_{max}$$

where E_{max} denotes the maximum bit energy for the secondary user. The optimization problem in (3) is a NP-hard nonconvex optimization problem. In the next section, we propose a novel realizable suboptimum solution to this problem.

III. Proposed scheme

Using the matrix inversion lemma, we can express $\mathbf{s}_i^T\mathbf{R}_{/i}^{-1}\mathbf{s}_i$ as

$$\mathbf{s}_i^T\mathbf{R}_{/i}^{-1}\mathbf{s}_i = \frac{\mathbf{s}_i^T\mathbf{R}^{-1}\mathbf{s}_i}{1 - E_ih_i^2\mathbf{s}_i^T\mathbf{R}^{-1}\mathbf{s}_i}, \quad i = 1, 2, \ldots, k. \tag{6}$$

Then (4) and (6) imply that

$$\mathbf{s}_i^T\mathbf{R}^{-1}\mathbf{s}_i \geq \frac{\alpha}{E_ih_i^2 + \alpha E_ih_i^2} \stackrel{\triangle}{=} \gamma_i, \quad i = 1, 2, \ldots, K. \tag{7}$$

Thus, the optimization problem in (3) can be written as follows:

$$\max_{E_s,\mathbf{s}_s} \; E_s\mathbf{s}_s^T\mathbf{R}_{/s}^{-1}\,\mathbf{s}_s \tag{8}$$

$$subject\ to \quad \mathbf{s}_i^T\mathbf{R}^{-1}\mathbf{s}_i \geq \gamma_i, \quad i = 1, 2, \ldots, K$$

$$\mathbf{s}_s^T\mathbf{s}_s = 1,\ E_s \leq E_{max}.$$

Using, once again, the matrix inversion lemma on $\mathbf{R}^{-1}$, i.e.

$$\mathbf{R}^{-1} = \mathbf{R}_{p+n}^{-1} - \frac{E_s h_s^2 \mathbf{R}_{p+n}^{-1} \mathbf{s}_s \mathbf{s}_s^T \mathbf{R}_{p+n}^{-1}}{1 + E_s h_s^2 \mathbf{s}_s^T \mathbf{R}_{p+n}^{-1} \mathbf{s}_s}, \tag{9}$$

and then combining (7) and (9), we can express the optimization constraints in (3) as explicit functions of the code sequence of the secondary user $\mathbf{s}_s$, i.e.

$$\mathbf{s}_i^T \mathbf{R}_{p+n}^{-1} \mathbf{s}_i \geq \frac{E_s h_s^2 \mathbf{s}_i \mathbf{R}_{p+n}^{-1} \mathbf{s}_s \mathbf{s}_s^T \mathbf{R}_{p+n}^{-1} \mathbf{s}_i}{1 + E_s h_s^2 \mathbf{s}_s^T \mathbf{R}_{p+n}^{-1} \mathbf{s}_s} + \gamma_i, \; i = 1, 2, \ldots, K. \tag{10}$$

In (9), $\mathbf{R}_{p+n}$ is the autocorrelation matrix of the primary signals and noise at the base station, i.e.

$$\begin{aligned} \mathbf{R}_{p+n} &\triangleq E\{(\sum_{i=1}^{K} \sqrt{E_i} h_i \mathbf{s}_i b_i + \mathbf{n}_p)(\sum_{i=1}^{K} \sqrt{E_i} h_i \mathbf{s}_i b_i + \mathbf{n}_p)^T\} \\ &= \sum_{i=1}^{K} E_i h_i^2 \mathbf{s}_i \mathbf{s}_i^T + \sigma^2 \mathbf{I}. \end{aligned}$$

For notational simplicity, let us define the $L \times L$ matrix

$$\mathbf{B}_i \triangleq h_s^2 \mathbf{R}_{p+n}^{-1} \mathbf{s}_i \mathbf{s}_i^T \mathbf{R}_{p+n}^{-1} - \beta_i h_s^2 \mathbf{R}_{p+n}^{-1}$$

where $\beta_i \triangleq \mathbf{s}_i^T \mathbf{R}_{p+n}^{-1} \mathbf{s}_i - \gamma_i$. Then, the optimization problem (8) can be written as

$$\begin{aligned} &\max_{\mathbf{x}} \; \mathbf{x}^T \mathbf{R}_{/s}^{-1} \mathbf{x} \qquad (11) \\ &\textit{subject to} \quad \mathbf{x}^T \mathbf{B}_i \mathbf{x} - \beta_i \leq 0, \quad i = 1, 2, \ldots, K \\ &\qquad \mathbf{x}^T \mathbf{x} \leq E_{max} \end{aligned}$$

where $\mathbf{x}$ is the transmitted signal vector of the secondary user, i.e. $\mathbf{x} = \sqrt{E_s} \mathbf{s}_s$. We note that $\mathbf{B}_i$, $i = 1, 2, \ldots, K$, is not positive semidefinite, and the problem in (11) is a nonconvex quadratically constrained quadratic program (nonconvex QCQP).

We observe that if we use the property $\mathit{Tr}\{\mathbf{AB}\} = \mathit{Tr}(\mathbf{BA})$, we are able to represent the objective function in (11) as

$$\mathbf{x}^T \mathbf{R}_{/s}^{-1} \mathbf{x} = \mathit{Tr}\{\mathbf{R}_{/s}^{-1} \mathbf{X}\}, \tag{12}$$

where $\mathbf{X} = \mathbf{x}\mathbf{x}^T$. Thus, the optimization problem in (11) can take the following form:

$$\begin{aligned} \max_{\mathbf{X}} \ & Tr\{\mathbf{R}_{/s}^{-1}\mathbf{X}\} \\ subject\ to \ \ & Tr\{\mathbf{B}_i\mathbf{X}\} \leq \beta_i, \quad i = 1, 2, \dots, K \\ & Tr\{\mathbf{X}\} \leq E_{max},\ \mathbf{X} \succeq \mathbf{0},\ rank(\mathbf{X}) = 1, \end{aligned} \tag{13}$$

where $\mathbf{X} \succeq \mathbf{0}$ denotes that the matrix $\mathbf{X}$ is positive semidefinite. To solve this problem, we propose to relax the rank constraint in (13) and proceed by solving the following problem instead

$$\begin{aligned} \max_{\mathbf{X}} \ & Tr\{\mathbf{R}_{/s}^{-1}\mathbf{X}\} \\ subject\ to \ \ & Tr\{\mathbf{B}_i\mathbf{X}\} \leq \beta_i, \quad i = 1, 2, \dots, K \\ & Tr\{\mathbf{X}\} \leq E_{max},\ \mathbf{X} \succeq \mathbf{0}. \end{aligned} \tag{14}$$

Then, (14) can be solved using semidefinite programming. Strictly speaking, we can solve (14) in polynomial time only within an error ϵ from the optimum solution. More specifically, let $f_o \stackrel{\triangle}{=} Tr\{\mathbf{R}_{/s}^{-1}\mathbf{X}\}|_{\mathbf{X}=\mathbf{X}_o}$ where $\mathbf{X}_o$ is the optimum point, i.e. f_o is the optimal value of the objective function in (14). Then for any given $\epsilon > 0$, semidefinite programming guarantees that we can converge in polynomial time (polynomial in the input size L and in $\log 1/\epsilon$) to a solution that lies in $(f_o - \epsilon, f_o)$. In this paper, for the semidefinite programming problem in (14), we propose to use a primal-dual interior-point method [12], [13]. In particular, we consider the problem in (14) as the primal optimization problem, we create a differently parameterized equivalent dual problem and then solve both problems iteratively in a coupled fashion. Then, each iterations can be implemented in $O(L^3)$ and the algorithm converges after a small number of iteration to the matrix $\mathbf{X}^*$ that makes the objective function $Tr\{\mathbf{R}_{/s}^{-1}\mathbf{X}\}$ attain a value within $(f_o - \epsilon, f_o)$. The proposed method is outlined in the appendix. We note that relaxing the rank constraint of the NP-hard problem in (13) leads to the optimization problem in (14) that can be solved in polynomial time (by semidefinite programming methods) as described in the appendix. However, because of the constraint relaxation itself, the objective function evaluated at the convergence point of the proposed method described above, $Tr\{\mathbf{R}_{/s}^{-1}\mathbf{X}\}$, is just a lower bound on the optimal value of the objective function in (13). To compute a good feasible solution for our problem in (11), we propose to proceed with following randomization procedure.

Let $\mathbf{X}^*$ be the convergence point of our proposed method described in the appendix. If we select $\mathbf{x}$ as a Gaussian random vector with $\mathbf{0}$ mean and correlation matrix $\mathbf{X}^*$, i.e. $\mathbf{x} \sim f_{\mathcal{N}}(\mathbf{x}) \stackrel{\triangle}{=} \mathcal{N}(\mathbf{0}, \mathbf{X}^*)$, then $\mathbf{x}$ is the optimum solution of the nonconvex QCQP in (11) "on average" over all possible distribution of $\mathbf{x}$, i.e.

$$
\begin{aligned}
f_{\mathcal{N}}(\mathbf{x}) = arg \max_{f(\mathbf{x})} \;\; & E\{\mathbf{x}^T\mathbf{R}_{/s}^{-1}\,\mathbf{x}\} \\
subject\ to \quad & E\{\mathbf{x}^T\mathbf{B}_i\mathbf{x}\} \leq \beta_i, \quad i = 1, 2, \ldots, K \\
& E\{\mathbf{x}^T\mathbf{x}\} \leq E_{max},
\end{aligned}
\tag{15}
$$

where $f(\mathbf{x})$ denotes the probability density function of $\mathbf{x}$. Implementation wise, a "good" feasible vector can be obtained by sampling $\mathbf{x}$ a sufficient number of times and then, among the feasible solutions (i.e. the ones that satisfy the constraints in (15)) we simply choose the vector that maximizes the objective function $\mathbf{x}^T\mathbf{R}_{/s}^{-1}\,\mathbf{x}$. However, global optimality can be guaranteed only if x is sampled an infinite number of times, which is not realistic. Our extensive studies indicate that if we sample $\mathbf{x}$ until we obtain the first feasible point, then we can use this vector as the initial point of an iterative procedure that converges to a good feasible solution. The iterative procedure is outlined below and its performance is evaluated by simulations in the next section. First we express $\mathbf{R}_{/s}$ as

$$
\mathbf{R}_{/s} = \mathbf{S}\Sigma\,\mathbf{S}^T + \sigma^2\mathbf{I}, \tag{16}
$$

where $\mathbf{S} \stackrel{\triangle}{=} [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K]$ denotes the matrix that has the signatures of primary users as columns, and $\Sigma = diag(E_1q_1^2, E_2q_2^2, \ldots, E_Kq_K^2)$. Using the matrix inversion lemma, $\mathbf{R}_{/s}$ can be expanded as

$$
\mathbf{R}_{/s}^{-1} = \frac{1}{\sigma^2}\mathbf{I} - \frac{1}{\sigma^4}\mathbf{S}(\Sigma^{-1} + \mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T. \tag{17}
$$

Substitution of (17) into (11) leads to maximization of the following objective function

$$\frac{1}{\sigma^2}\mathbf{x}^T\mathbf{x} - \frac{1}{\sigma^4}\mathbf{x}^T\mathbf{Q}\,\mathbf{x}, \tag{18}$$

where the matrix $\mathbf{Q} \overset{\triangle}{=} \mathbf{S}(\Sigma^{-1} + \mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T$. In (18), the first term $\frac{1}{\sigma^2}\mathbf{x}^T\mathbf{x}$ is a convex function while the second term $-\frac{1}{\sigma^4}\mathbf{x}^T\mathbf{Q}\,\mathbf{x}$ is a concave function (the latter implies that $\frac{1}{\sigma^4}\mathbf{x}^T\mathbf{Q}\,\mathbf{x}$ is convex). Based on the first-order conditions of convex functions [14], we have

$$\mathbf{x}^T\mathbf{x} \geq 2\mathbf{x}^{(0)T}\mathbf{x} - \mathbf{x}^{(0)T}\mathbf{x}^{(0)}, \tag{19}$$

where $\mathbf{x}^{(0)}$ denotes an initial feasible vector. Then we combine (18) and (19) and form an optimization problem that maximizes the following concave function

$$\frac{2}{\sigma^2}\mathbf{x}^{(0)T}\mathbf{x} - \frac{1}{\sigma^4}\mathbf{x}^T\mathbf{Q}\,\mathbf{x} - \frac{1}{\sigma^2}\mathbf{x}^{(0)T}\mathbf{x}^{(0)} \tag{20}$$

that leads to a suboptimum solution for our original problem in (11). To solve (20) we restrict all nonconvex constraints into convex sets (linearization). In particular, we consider the nonconvex constraints

$$\mathbf{x}^T\mathbf{B}_i\mathbf{x} - \beta_i \leq 0, \quad i \in \mathcal{I}_{nc}, \tag{21}$$

where $\mathcal{I}_{nc}$ denotes the set of all indices where $\mathbf{x}^T\mathbf{B}_i\mathbf{x}$ is a nonconvex function. Then we decompose the matrix $\mathbf{B}_i$ into its positive and negative parts as

$$\mathbf{B}_i = \mathbf{B}_i^+ - \mathbf{B}_i^- \tag{22}$$

where $\mathbf{B}_i^+ = h_s^2\mathbf{R}_{p+n}^{-1}\mathbf{s}_i\mathbf{s}_i^T\mathbf{R}_{p+n}^{-1}$ and $\mathbf{B}_i^- = \beta_i h_s^2\mathbf{R}_{p+n}^{-1}$ are all positive semidefinite. Therefore, the original constraints (21) can be written as

$$\mathbf{x}^T\mathbf{B}_i^+\,\mathbf{x} - \beta_i \leq \mathbf{x}^T\mathbf{B}_i^-\,\mathbf{x}, \quad i \in \mathcal{I}_{nc} \tag{23}$$

where both sides of the inequality are convex quadratic functions. Linearization of the right-hand side of (23) around the vector $\mathbf{x}^{(0)}$ leads to

$$\mathbf{x}^T\mathbf{B}_i^+\,\mathbf{x} - \beta_i \leq \mathbf{x}^{(0)T}\mathbf{B}_i^-\,\mathbf{x}^{(0)} + 2\mathbf{x}^{(0)T}\mathbf{B}_i^-\,(\mathbf{x} - \mathbf{x}^{(0)}),\ i \in \mathcal{I}_{nc}. \tag{24}$$

In (24), the right-hand side is an affine lower bound on the original function $\mathbf{x}^T\mathbf{B}_i^-\mathbf{x}$. It is thus implied that the resulting constraints are convex and more conservative than the original ones, hence the feasible set of the linearized problem is a convex subset of the original feasible set. Thus, by linearizing the concave parts of all constraints, we obtain a set of convex constraints that are tighter than the original nonconvex ones. Now, the original optimization problem takes the form

$$\begin{aligned} &\max_{\mathbf{x}} \ \frac{2}{\sigma^2}\mathbf{x}^{(0)T}\mathbf{x} - \frac{1}{\sigma^4}\mathbf{x}^T\mathbf{Q}\mathbf{x} - \frac{1}{\sigma^2}\mathbf{x}^{(0)T}\mathbf{x}^{(0)} \\ &subjec\,to \ \mathbf{x}^T\mathbf{B}_i^+\mathbf{x} - \mathbf{x}^{(0)T}\mathbf{B}_i^-(2\mathbf{x}-\mathbf{x}^{(0)}) - \beta_i \le 0, \ i \in \mathcal{I}_{nc} \\ &\mathbf{x}^T\mathbf{B}_i\mathbf{x} - \beta_i \le 0, \quad i \in \overline{\mathcal{I}}_{nc} \\ &\mathbf{x}^T\mathbf{x} \le E_{max}. \end{aligned} \tag{25}$$

The problem in (25) is a convex QCQP problem and can be solved efficiently by standard convex system solvers [15] to produce a new feasible vector $\mathbf{x}^{(1)}$ (the objective function in (11) evaluated at $\mathbf{x}^{(1)}$ takes a value that is larger than the value that is based on $\mathbf{x}^{(0)}$). Repeating the linearization method, we can obtain a sequence of feasible vectors with non-decreasing values of the objective function in (11). This procedure converges after few iterations. Our proposed scheme for power and code allocation for the secondary user is outlined in Fig. 2.

V. SIMULATION STUDIES

We consider a primary DS-CDMA system with K synchronous users and a pair of secondary transmitter and receiver with CDM mode, and the system processing gain is $L = 16$. At the base station, the transmitted SNRs of K users are all equal to $10dB$ while the maximum transmitted SNRs for the secondary user is set to $8dB$. All signatures for primary users are generated from the optimum binary signature set, which achieves the Karystinos-Pados bound with the length L. The SINR threshold for primary users is set to $3dB$. The channel coefficients h_n and q_n, $n = 1, 2, \ldots, K$, are considered as complex Gaussian random variables with mean 0 and variance 1, while h_s and q_s are set to 0.9. In the randomized procedure, the Gaussian variable is sampled 1000 times, and the first and best feasible vectors are denoted by $\mathbf{x}^{(0)}$ and $\mathbf{x}_{max}$, respectively. In the linearization procedure, we pick $\mathbf{x}^{(0)}$ as the initial vector, and the output convergence vector is denoted by $\mathbf{x}_{out}$.

In Fig. 3, we plot secondary transmission percentage of the SDP solution $\mathbf{X}^*$ with rank 1 and rank more than 1 as a function of the number of primary users. We observe that the total secondary transmission percentage decrease as the number of primary users increases. Most secondary transmissions can be realized with the SDP solution $\mathbf{X}^*$ with rank 1, which means that the optimal vector $\mathbf{x}$ can be directly generated from the SDP solution $\mathbf{X}^*$ without the randomized and linearization procedure. The secondary transmissions achieved by the linearization procedure are below 5%, especially almost zero when the number of primary users is greater than 19. It means that most secondary transmissions can be realized without the linearzation procedure, then the large complexity of linearization is avoided.

For comparison purposes, we evaluate the SINR loss, $SINR(\mathbf{x}_{R-OPT}) - SINR(\mathbf{x})$, of $\mathbf{x} = \mathbf{x}^{(0)}$, $\mathbf{x}_{max}$ and $\mathbf{x}_{out}$, with respect to the SINR of the optimal real vector $\mathbf{x}_{R-OPT} = \sqrt{E_{max}}\mathbf{q}_1$, where $\mathbf{q}_1$ denotes the eigenvector of $\mathbf{R}_{/s}^{-1}$ with the largest eigenvalue.

In Fig. 4, we plot the SINR loss of $\mathbf{x}^{(0)}$, $\mathbf{x}_{max}$ and $\mathbf{x}_{out}$ as a function of the number of primary users. Under a limited number of samples, the linearized output vector $\mathbf{x}_{out}$ has a less SINR loss than the "best" vector $\mathbf{x}_{max}$. We observe that the SINR losses of both $\mathbf{x}_{max}$ and $\mathbf{x}_{out}$ are much better than $\mathbf{x}^{(0)}$. The linearization procedure improved the SINR performance from the initial point, and meanwhile realized the derandomization.

V. CONCLUSION

In this paper, we developed a power and code allocation for a secondary user that coexists with a primary CDM network. First, we formulated the task as a NP-hard nonconvex constraint optimization problem (3). Then relaxation of the rank-1 constraint led to a problem that can be solved by semidefinite programming.

To do that, we proposed a primal-dual interior-point method that leads to a matrix solution (rank ≥ 1). A subsequent randomization procedure made it possible to obtain a "good", on average, feasible solution after searching over several feasible sampled solutions. To reduce the computational complexity associated with the randomization procedure, we developed an alternative method that iteratively solves the original nonconvex optimization problem by restricting all nonconvex constraints into convex sets and by initializing itself at any feasible solution of the randomized procedure (e.g. the first feasible sampled solution encountered in the randomization process).

APPENDIX

Interior point algorithm

(i) Formulate the pair of primal and dual SDP problems:

Primal

$$\max_{\mathbf{X}} \; Tr\{\mathbf{R}_{/s}^{-1}\mathbf{X}\}$$

$$subject\ to \quad Tr\{\mathbf{B}_i\mathbf{X}\} \leq \beta_i, \quad i = 1, 2, \ldots, K$$

$$Tr\{\mathbf{X}\} \leq E_{max}, \; \mathbf{X} \succeq \mathbf{0}$$

Dual

$$\max_{\mathbf{y},\mathbf{Z}} \; \mathbf{b}^T\mathbf{y}$$

$$subject\ to \quad \sum_{i=1}^{K} \mathbf{y}_i\mathbf{B}_i + \mathbf{y}_{K+1}\mathbf{I} = \mathbf{R}_{/s}^{-1} + \mathbf{Z}$$

$$\mathbf{y} \geq \mathbf{0}, \; \mathbf{Z} \succeq 0.$$

(ii) Choose $0 \leq \delta < 1$ and define $\mu = \sigma \frac{Tr\{\mathbf{XZ}\}}{L}$.

(iii) Determine $\Delta\mathbf{X}$, $\Delta\mathbf{y}$ and $\Delta\mathbf{Z}$ by the method in [13].

(iv) Using $\mathbf{XZ}$ method, replace ΔX by $\frac{1}{2}(\Delta X + \Delta X^T)$.

(v) Choose steplengths α, β and update the iterates by

$$\mathbf{X} \leftarrow \mathbf{X} + \alpha\Delta\mathbf{X}$$

$$\mathbf{y} \leftarrow \mathbf{y} + \beta\Delta\mathbf{y}$$

$$\mathbf{Z} \leftarrow \mathbf{Z} + \alpha\Delta\mathbf{Z}$$

(vi) Repeat (ii)-(v) until $\mathbf{X}$ is feasible for the primal, $(\mathbf{y}, \mathbf{Z})$ is feasible for the dual, and the primal and dual objective values agree to a specified number.

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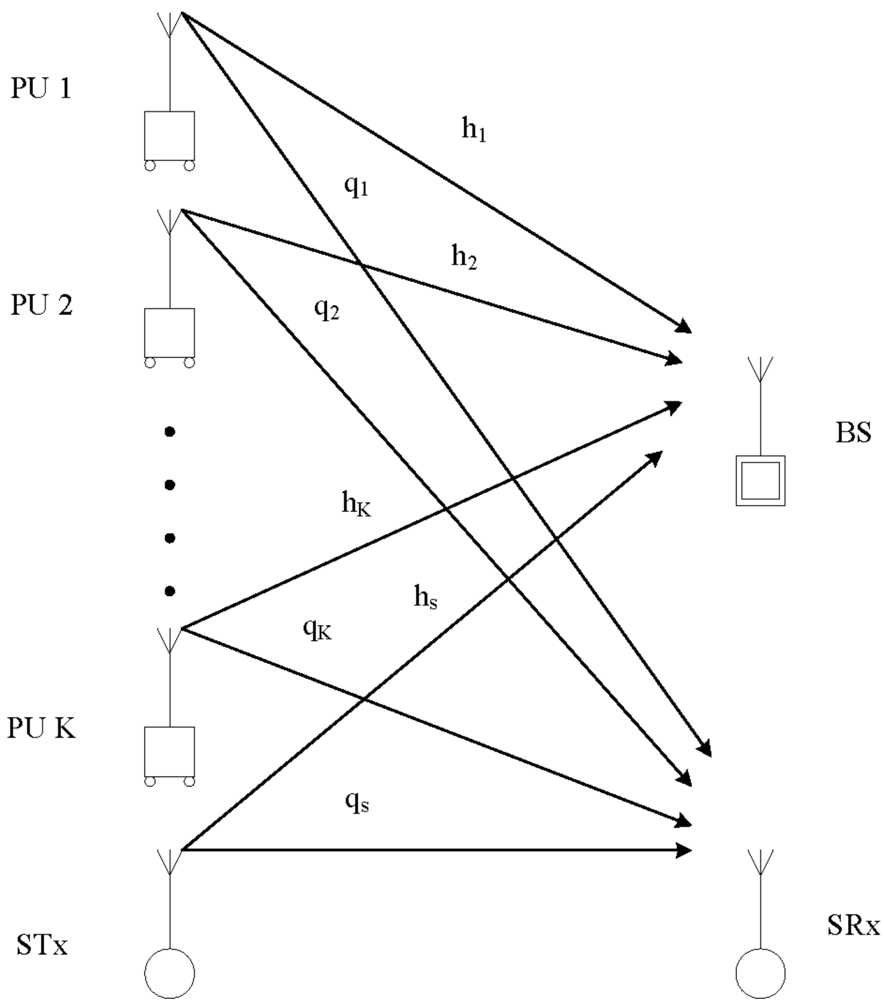

Fig. 1. System model

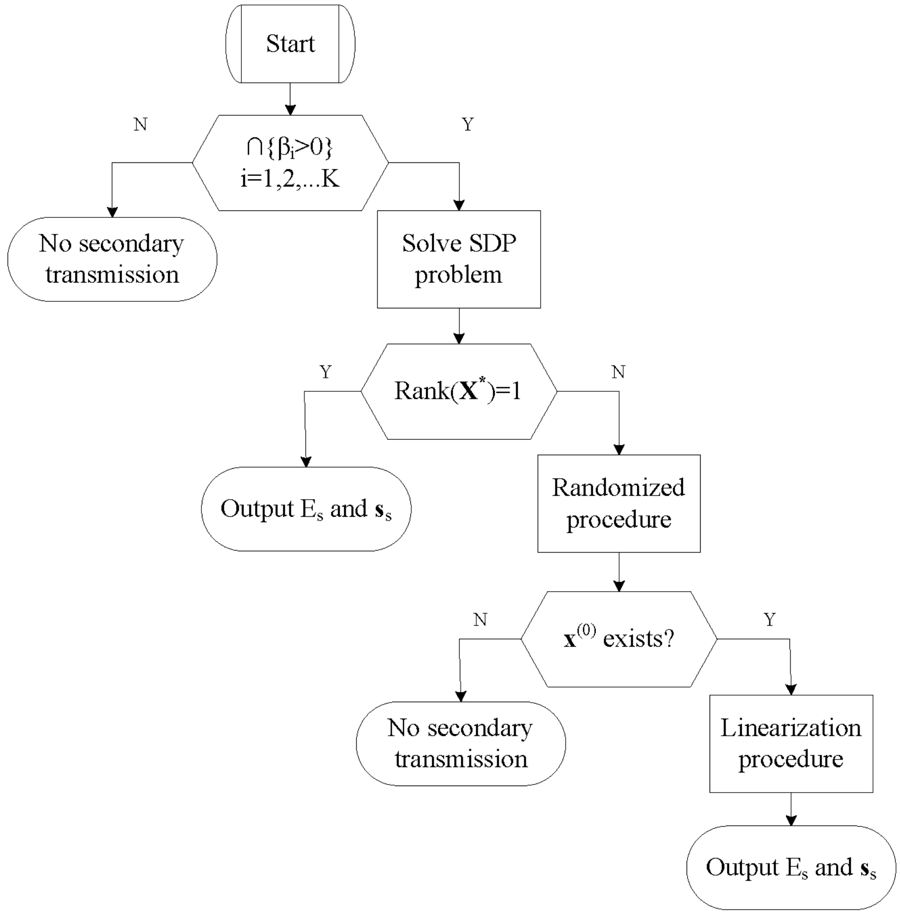

Fig. 2. The flow chart of the proposed power and code allocation for the secondary user

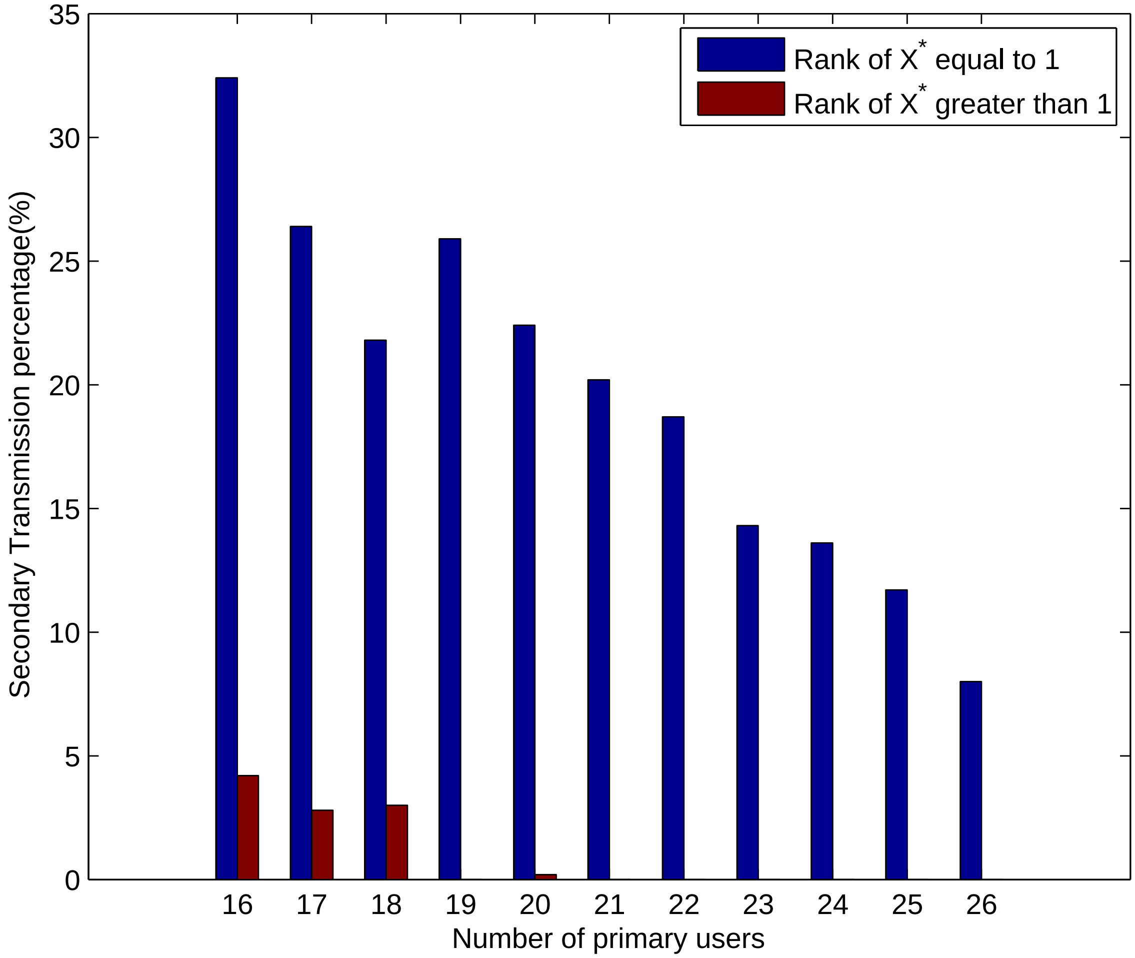

Fig. 3. Secondary transmission percentage of the SDP solution $\mathbf{X}^*$ with rank 1 and rank more than 1 versus of the number of primary users.

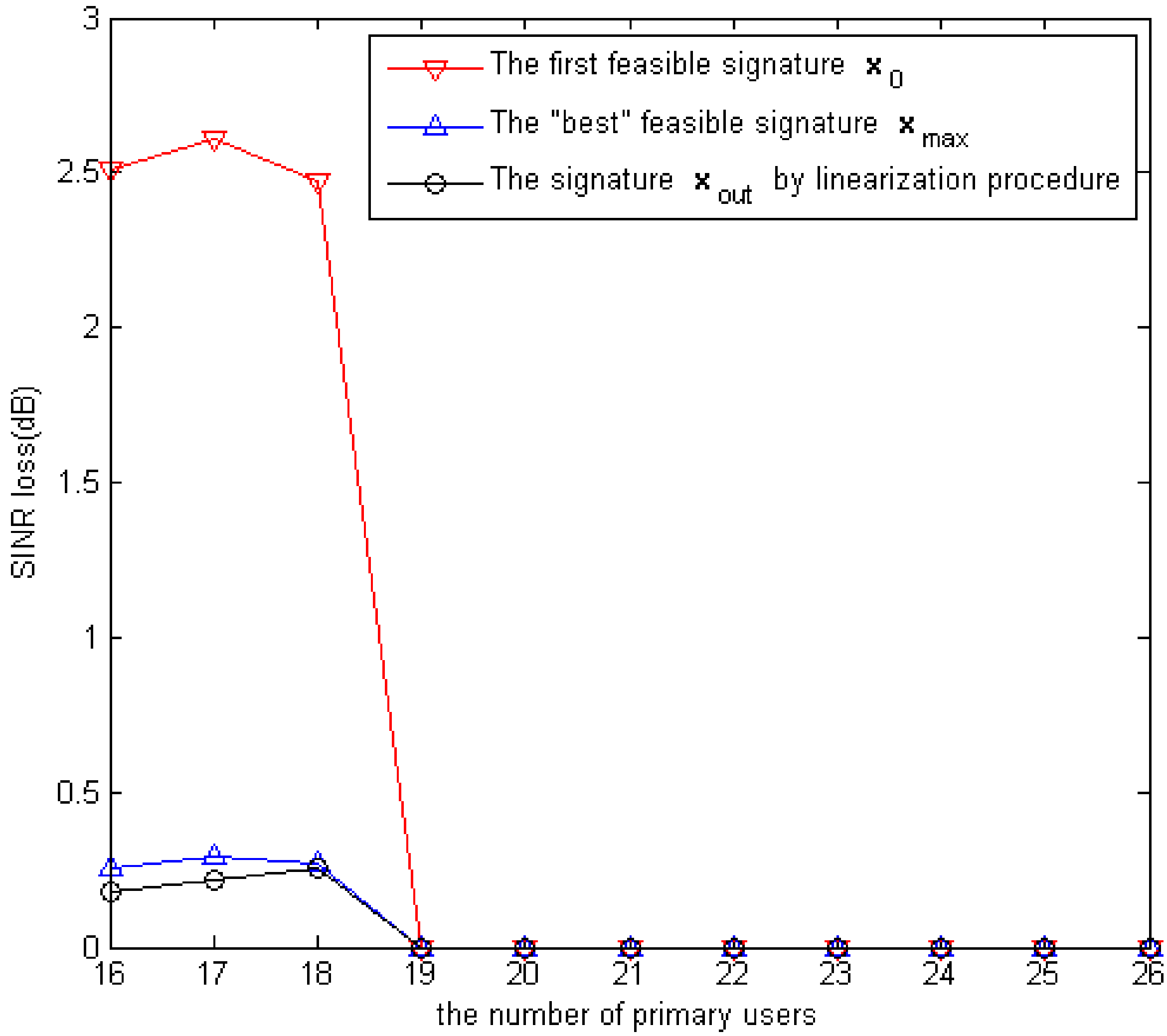

Fig. 4. SINR loss of $\mathbf{x}^{(0)}$, $\mathbf{x}_{max}$ and $\mathbf{x}_{out}$ versus the number of primary users.