

Lighting and astronomy

Christian B. Luginbuhl, Constance E. Walker, and Richard J. Wainscoat

The rapid growth of light pollution threatens the future of astronomical observation. Detailed modeling of how light from the ground propagates through the atmosphere suggests ways to limit the damage.

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The sky is fading. Prime sites for astronomical observatories are rare. A stable, clear, dry atmosphere is crucial. Yet many of the best sites worldwide are slowly losing their view of the most distant astronomical objects as more and more stray light appears in the last microseconds of what may have been a 10-billion-year journey. That intruding light comes from outdoor lighting used for roadways, parking lots, advertising, and decoration and from automobile headlights. It gets into the sky, and ultimately into telescopes, either directly from light fixtures or through reflection off the ground or other surfaces, followed by scattering from molecules and aerosols in the atmosphere. Largely because of that stray light, or sky glow, new giant telescopes are being built in the most remote corners of the planet. Yet even those sites are now threatened by artificial light from communities that may be located hundreds of kilometers away, as shown in figure 1.

The projected growth of outdoor lighting, illustrated in figure 2, paints a discouraging picture. Whereas the US population is growing at an average rate of less than 1.5% per year, the amount of artificial light is increasing at an annual rate of 6%. Increases in population, standards of living, and isolation from the natural nighttime environment combine to lead communities and individuals to increase not only the number of situations in which outdoor lighting is deemed necessary but also the amount of light used for many applications. Witness the amount of light commonly seen at service stations today compared with that of only a few decades ago: Lighting levels comparable to or even brighter than those recommended for indoor office work are common. Parking lots are often illuminated 5 to 10 times as brightly as they were 20 years ago. Bright light is becoming a form of advertising.

Figure 1. The sky over Mauna Kea, Hawaii, is affected by outdoor lighting in communities from Waimea 30 km away to Honolulu 300 km away. (Photo by Richard Wainscoat.)

Although lighting is rarely installed with the purpose of brightening the sky, even in the best circumstances some fraction of outdoor lighting propagates upward by reflection from the illuminated area. And actual outdoor night lighting rarely represents the best circumstances. Inefficient, careless practices and poorly designed fixtures dramatically increase sky glow through direct upward emission, wasted light falling into areas that do not need illumination, and over-illumination.

Falling back

Astronomers have long been retreating from encroaching lights. As planning began in the 1930s for a new 5-m telescope, planners realized that the Mount Wilson Observatory, home of the then largest telescope (at 2.5 m), already suffered from too much light pollution. A new site on Palomar Mountain was chosen, farther from the lights of the Los Angeles area.

The retreat continued. When the National Observatory was searching for a site in the early 1950s, another remote area was chosen, on Kitt Peak in Arizona, 80 km from Tucson and 170 km from Phoenix, whose populations at the time



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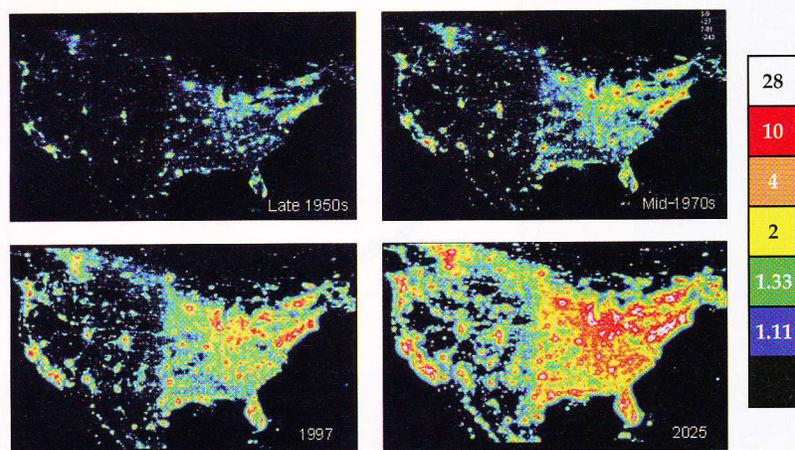


Figure 2. Sky glow over time is estimated by extrapolating 1997 satellite measurements forward and backward using a 6% annual growth rate. That average rate, determined from ground-based measurements of sky brightness, is applied equally to all points; no attempt has been made to model actual growth rates for different regions. The scale shows the brightness normalized by the natural condition; for example, the boundary between blue and green is 33% brighter than the natural sky. (Adapted from ref. 9.)

were about 125 000 and 330 000, respectively. From the 1960s to the present, new telescopes have been built on ever more remote sites in Chile, Arizona, Hawaii, and the Canary Islands. Although the most remote sites currently suffer insignificant light pollution, the prospects for the future are uncertain. There are few high-quality sites for further retreat. The choice of Kitt Peak for the National Observatory was based in part on a general confidence that the desert conditions would limit the growth of southern Arizona communities, but today the Tucson and Phoenix metropolitan areas have populations exceeding 1 million and 4 million, respectively. And populations are rapidly growing near most other observatory sites.

Many people suggest that the next stage of the retreat will follow the *Hubble Space Telescope* into orbit, or even to the Moon. (See the article by Paul Lowman Jr, *PHYSICS TODAY*, November 2006, page 50.) But the enormous—yes, astronomical—costs associated with building and maintaining space-based facilities mean that ground-based telescopes must continue to provide the data for the vast majority of observational astronomical research. *Hubble* catches the eye and imagination of the public, but it catches a very small percentage of the photons that lead to discoveries in astronomy.

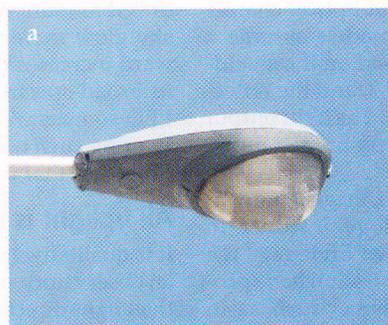
Running the numbers

Although no distinct thresholds of observational capability are crossed as the sky is brightened by artificial lighting, the

effectiveness of telescopes measuring faint sources gradually deteriorates. At the limit in which the source under study is negligibly brighter than the background against which it is observed, a 10% increase in that background means that astronomers need 10% more time to observe the same object with the same signal-to-noise ratio. If sky glow continues to increase, the faintest sources will eventually become unobservable within practical time constraints. During the dark phases of the lunar cycle, the sky over Palomar Observatory is now more than 50% brighter than it would be with no artificial light sources. The effectiveness of the 5-m telescope is thereby reduced to that of a 4-m telescope. (Even with no artificial lighting or moonlight, the sky is not perfectly dark. Natural sky glow in the visible spectrum results primarily from sunlight scattered by dust in the solar system and emission from upper-atmosphere oxygen atoms that were excited by daytime sunlight.)

Quantitative treatment of the relationship between lights and the sky glow they produce began in 1965 when Merle Walker, driven by increasing light pollution over the Lick Observatory on Mount Hamilton, California, undertook an effort to find a new site for observation of very faint objects. Seeking to identify a site with not only good current conditions but also the expectation that encroaching development would not unacceptably brighten the night sky in the foreseeable future, Walker developed a crude estimate of the

Figure 3. (a) A typical light fixture that allows some emission above the horizontal. **(b)** The angular intensity distribution used by Roy Garstang to represent light propagating upward into the sky. The green line represents light emitted directly from fixtures such as the one in panel a, the red line represents light emitted downward and reflected off the ground, and the black line is the sum of the two. (Adapted from ref. 5.)



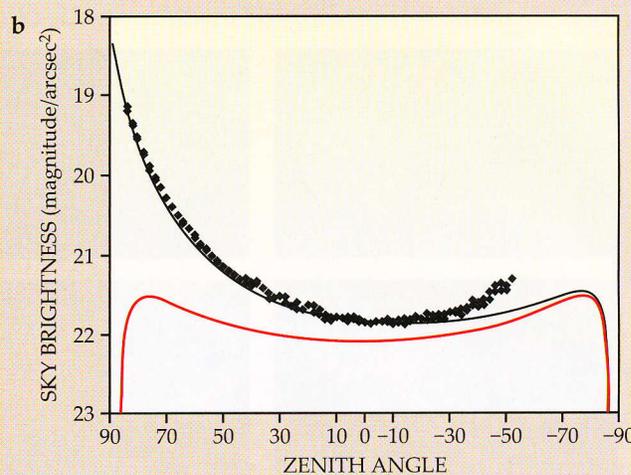
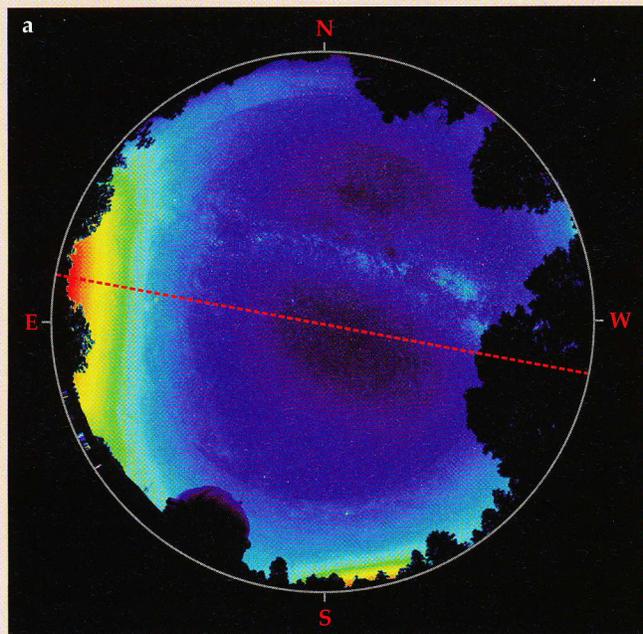


Figure 4. (a) A false-color “fish-eye” view of the sky over the US Naval Observatory Flagstaff Station. Sky glow from Flagstaff 8 km away and from the Phoenix metropolitan area 150 km away is visible in the eastern and southern skies. (East and west appear reversed because the observer is looking up.) **(b)** Sky brightness along the red dotted line in panel a. The black diamonds are measured values. The red line is the natural brightness that would be expected at solar minimum with no artificial light sources or moonlight. The black line is calculated from a model that accounts for the light output from Flagstaff as determined by a lighting survey. Observations in the zenith-angle ranges of -30° to -55° fall in the Milky Way, which is neither removed from the measurements nor included in the model. (Adapted from ref. 5.)

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distance a site must be from a city to keep the sky glow below a 10% increase in the natural condition at the zenith. He extended his work in 1977 to develop a general empirical relation, now called Walker’s law, between sky brightness, population, and distance.¹ The law states that the sky-glow intensity from a light source is approximately proportional to the distance raised to the -2.5 power. The intensity falls off more quickly than the inverse square primarily because of atmospheric absorption.

A more comprehensive approach came in 1986, when Roy Garstang of the Joint Institute for Laboratory Astrophysics (now JILA) in Boulder, Colorado, published models that treated the scattering of light off molecules and aerosols in the atmosphere, including the variation in the density of molecules and aerosols with altitude.² The models also accounted for Earth’s curvature. They have become the standard in the field and have successfully reproduced the variation of sky glow with position in the sky and distance from light sources. What they have not done is relate the sky glow produced by cities to the way lighting is actually used on the ground, such as the number, brightness, and optical characteristics of lighting fixtures. Instead, the models assume a particular angular distribution function based on the shielding of typical light fixtures, shown in figure 3, and the reflection off surfaces of light directed downward. The light output of cities is then empirically adjusted based on limited measures of sky glow.

Using Garstang’s models, Pierantonio Cinzano (now at the Light Pollution Science and Technology Institute in Thiene, Italy), his coworkers, and Christopher Elvidge of the National Oceanic and Atmospheric Administration have produced maps of artificial night-sky brightness,³ such as in figure 2; they created the maps by using US Defense Meteorological Satellite Program measurements of light emitted by towns and cities.

Recent work by one of us (Luginbuhl) connects ground-based surveys of lighting amounts and fixture types with Garstang’s models to compare the predictions with detailed measurements of sky glow.⁴ With the addition of a treatment for the partial blocking of light emissions due to objects near the ground,⁵ excellent cross-the-sky agreement between model predictions and measures of sky glow has been obtained, as shown in figure 4. It is now possible to understand the effects of different upward angular intensity distributions and spectral characteristics of artificial lighting.

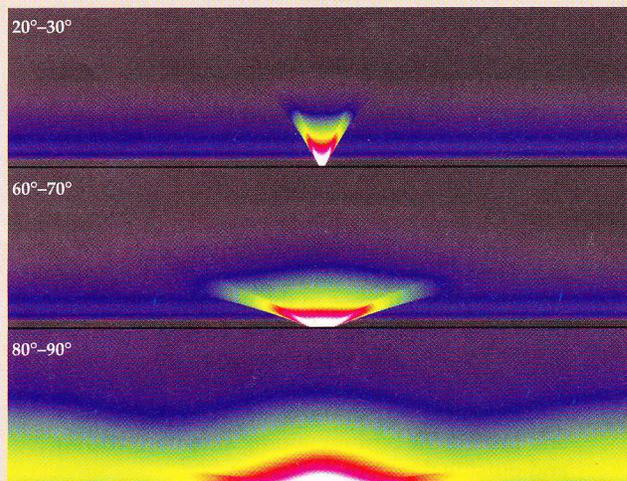
Two topics of current interest in outdoor lighting exemplify the importance of sky-glow models. Members of the lighting profession frequently point out that shielding lighting fixtures incompletely and thereby allowing a few percent of the light output to be directed just above the horizontal will also provide a wider distribution of light in a downward direction. Fixtures could be placed farther apart, and perhaps 10–15% less light could be used to accomplish a given lighting task. At first glance, the tradeoff would appear to be favorable for astronomy. But it raises the question: Does the reduced amount of light from widely spaced fixtures decrease sky glow more than the small amount of light emitted upward increases it? There is also much interest in recent years in broad-spectrum (white) lighting from metal halide and LED sources. What are the implications for astronomy? Answering such questions leads to insight into the nature of the processes that produce light pollution over observatories.

All uplight is not equally polluting

It is qualitatively clear that light directed upward and toward an observatory site has a greater impact on the observatory sky than light directed toward the zenith or away from the observatory. Quantitative analysis, described in box 1, shows that light emission between zenith angles of 60° and 90° (0° to 30° above the horizontal plane) is far more harmful to observatory

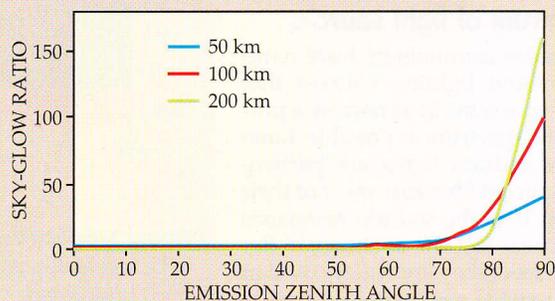
Box 1. Direction matters

To quantify the relationship between upward emission angle and sky glow, we have modeled a series of nine light sources, each emitting light upward into a 10°-wide zone spanning 0°–10°, 10°–20°, . . . , 80°–90° from zenith.¹⁰ The light source's altitude is set at 1 km; the observatory altitude, 3 km; and the ratio of total aerosol scattering to molecular scattering, 3:1. That ratio



corresponds to a low aerosol content and a very clear atmosphere, typical of world-class observatory sites. False-color images of the sky glow from three of the angular segments, as seen at an observatory 50 km away, are shown in the figure at left (adapted from reference 10).

Any light source brightens some parts of the sky more than others, but it is convenient to have a single number to represent the glow over the whole sky. One such measure, representative of the parts of the sky most commonly used in astronomical observation, is a weighted average of the sky glow at the zenith and at four points with zenith angle 60°—one toward the light source and the others at 90° intervals in azimuth—with the zenith assigned twice the weight of the other points. That average sky glow is divided by the sky glow produced by an equal amount of light directed downward and reflected off a surface with 15% reflectivity, a typical value. The resulting sky-glow ratio is shown in the figure at right, as a function of the light



source's zenith angle, for observatories 50, 100, and 200 km from the light source.

Of course, a real light source emits light over a range of angles, both above and below the horizontal. The uplight intensity distribution shown in figure 3, from Roy Garstang's models, can be used to represent the upward emission from real light fixtures. The table below shows the resulting sky-glow ratios for fixtures with 1%, 3%, and 10% direct uplight as measured at observatories from 50 to 200 km away. The 3% figure is representative of fixtures commonly discussed in the tradeoff between uplight and pole spacing, as described in the text. The minimum practical limit for partially shielded fixtures is about 1%. Though fractions lower than 1% can be optically designed, the accumulation of dirt and deterioration in the optical surfaces drives the uplight fraction toward 1% or higher as the fixtures age. And light pollution researchers, starting with Garstang, have found 10% to be representative of the average uplight proportion from all fixtures used for outdoor lighting.

Uplight	Sky-glow ratio		
	50 km	100 km	200 km
0%	1.0	1.0	1.0
1%	1.3	1.6	2.0
3%	1.8	2.7	3.9
10%	3.8	6.7	10.6

skies than light directed toward the zenith, even though on average much of the near-horizontal light is directed away from the observatory. And the sky-glow increase from the near-horizontal rays is 6 to 160 times as great as that of an equal flux directed downward and reflected off the ground.

Because most of the upward light emission from incompletely shielded fixtures is directed just above the horizontal, such fixtures have a disproportionate effect on sky glow. From the table in box 1, a fixture with an unshielded fraction of only 3% produces between 80% and 290% more sky glow than a fully shielded fixture with the same light output, with the worst value occurring for the most distant light sources. Startlingly, for a typical community that emits 10% of its light directly upward, direct uplight causes almost three-fourths of the sky glow at an observatory 50 km away and more than nine-tenths at a site 200 km away. Even though the amount of direct uplight (10%) is similar to the amount of light reflected off the ground ($90\% \times 0.15$ albedo = 13.5%), direct up-

ward emission produces the majority of artificial sky glow.

Those numbers don't account for the blocking of light by vegetation and structures near the ground. In a model of sky glow over the US Naval Observatory near Flagstaff, Arizona, accounting for such blocking reduced the relative impact of upward emissions by 50–60%. Even so, direct uplight still produced much more sky glow than the same amount of light directed downward. Furthermore, the model did not account for the fact that direct upward emission usually arises from fixtures some distance above the ground, such as on buildings or poles, and may therefore be subject to less blocking than light reflected from the ground.

The answer to the lighting professionals' proposition is clear: The detrimental effect on observatory skies of even 3% direct uplight vastly outweighs the benefit of a 10–15% reduction in the total amount of light. Even if fixtures could be kept to just 1% direct uplight, the competing effects might approximately balance only for observatories located near

cities; for the more distant observatories the detrimental effects still dominate.

A spectrum of light sources

For decades astronomers have naturally favored lighting sources that confine emissions to as narrow a portion of the spectrum as possible. Low-pressure sodium lights are particularly preferred,⁶ because most of their emission is in the sodium resonance doublet near 589 nm, as shown in figure 5. Indeed, they are widely used in several regions near major astronomical observatories. High-pressure sodium lamps are second best: They emit mostly in the yellow portion of the spectrum, but through pressure broadening and the inclusion of other compounds in the discharge arc they produce some light in the red and the blue.

About 20 years ago, heavy marketing pressures and improvements in lamp technology led to the more widespread use of broad-spectrum metal halide sources. More recently, white LEDs have begun to emerge as contenders in the outdoor lighting market, as described in box 2. Their greater efficiency makes them especially attractive to municipalities seeking to use economic stimulus money tied to energy savings.

All such broad-spectrum sources interfere with astronomical observation at more wavelengths than do sodium sources, so they leave essentially no unpolluted windows in the visible spectrum. As a further complication, the shorter wavelengths they emit are much more strongly scattered by molecules in the atmosphere. The potential increase in sky glow from such sources is a concern, although the increased scattering leads also to increased attenuation with distance.

Damage control

Walker, in 1973, identified the critical issue of light pollution facing astronomy:⁷

At the time of their founding, the sites of the present major optical astronomical observatories in California and Arizona were among the best in the world. Now, however, work at all of these

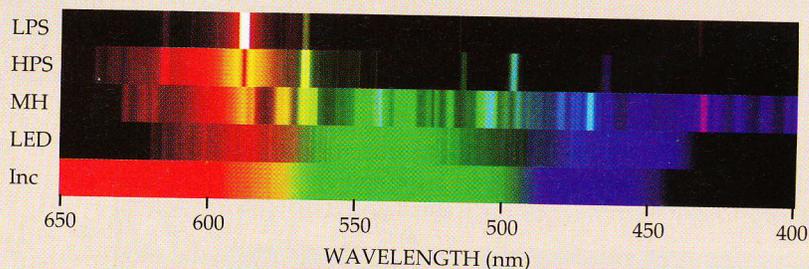


Figure 5. Spectra of representative outdoor lighting sources. Low-pressure sodium (LPS) and high-pressure sodium (HPS) lamps leave large parts of the spectrum relatively unpolluted, whereas the broadband metal halide (MH), white LED, and incandescent (Inc) sources do not.

installations is either presently or potentially limited by the increase in the illumination of the night sky from nearby cities. . . . It is essential that immediate efforts be undertaken to: (1) Control outdoor illumination to lengthen the useful life of existing observatory sites, and (2) Identify and protect the best remaining sites both within and outside the United States.

Today, his words are as true for the remotest observatory sites as they were for California and Arizona 36 years ago.

Astronomers' efforts to address the issue have been ongoing since the late 1950s and are now having some effect. The effort has been aided in recent years by a broadening coalition of interests concerned about the many detrimental effects of artificial light at night: energy waste, poor visibility due to glare, disturbance of biological systems,⁸ and loss of starry skies for casual stargazers. A comprehensive study of lighting in Flagstaff⁹ shows that the growth rate of light pollution per person added to the population has been cut approximately in half since 1989, when a stringent outdoor lighting code was adopted that limits the total amount of light permitted. Sky glow continues to increase, but at a slower pace.

Lighting designers and manufacturers are increasingly aware of the many harmful effects of light pollution. Through

Box 2. LEDs for outdoor lighting

The typical white LED used for outdoor lighting is made from a blue LED that emits light at about 450 nm and a phosphor that converts some of the blue light to green and red. White LEDs are typically characterized by their correlated color temperature (CCT), the temperature of the blackbody radiator that most closely resembles the appearance of the LED light.

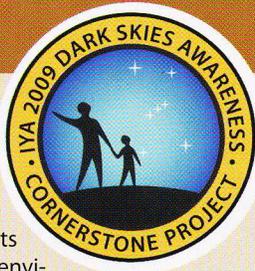
The blue emission is particularly harmful to astronomers and to the environment. Rayleigh scattering, responsible for the daytime blue sky, has a λ^{-4} wavelength dependence: The 450-nm emission is nearly three times as strongly scattered as is the astronomers' preferred low-pressure sodium emission at 589 nm. Furthermore, wavelengths shorter than 500 nm interfere more strongly with circadian rhythms and melatonin production in humans and other animals. The higher the CCT of a white LED, the more strongly its light is scattered and the larger

the perturbation to biological systems.

Fortunately for astronomers, few people like the appearance of the high-CCT LEDs (5000–6000 K, or daylight color), with many describing them as looking like welding torches. Even so, some municipalities prefer them because they are somewhat more efficient. Other communities, such as Anchorage, Alaska, have specified that white LEDs should not have a CCT higher than that of moonlight (4200 K), but even that approach does not properly account for the fact that the LEDs are more damaging than moonlight due to the blue peak, which is not present in moonlight, and because the Moon is below the horizon during half of the nighttime. Low-CCT white LEDs (3000 K or lower, the color of typical incandescent lamps) are the least harmful to the environment and astronomical observation.

Box 3. The International Year of Astronomy and Dark Skies Awareness

Dark Skies Awareness is a Global Cornerstone Project of the United Nations-sanctioned 2009 International Year of Astronomy. Its goal is to raise the level of public knowledge about adverse effects of excess artificial lighting on local environments and to make more people aware of the ongoing loss of a dark night sky for much of the world's population. Toward that end, a range of programs and resource materials has been developed. One such program is GLOBE at Night, an international citizen-science event that takes place every March to encourage everyone—students, educators, dark-sky advocates, and the general public—to measure the darkness of their local skies and contribute their observations online to a world map. Everyone is invited to participate in GLOBE at Night and the other Dark Skies Awareness programs offered as potential local solutions to a global problem. To learn more, visit <http://www.darkskiesawareness.org>.



extensive educational efforts led by the International Dark-Sky Association (<http://www.darksky.org>) and other similar organizations throughout the world, a greater selection of fully shielded lighting fixtures is becoming available. Trained lighting professionals are using more fully shielded fixtures, at least in areas where the sensitivity to light-pollution issues is high due to heightened environmental sensitivity or the presence of observatories.

Unfortunately, in most areas insufficient awareness of the problems that can arise from lighting at night still leads to poor control of upward emission and lighting amounts. In many places, particularly in small towns and rural areas, the majority of outdoor lighting is not designed by lighting professionals. And outdoor lighting is used for more situations and in greater amounts than it used to be. The best hope for progress is through continuing education, as described in box 3, about the value of a starry sky—a value not just for astronomy and science but for everyone. Nobody ever seems to make the mistake of thinking that Yellowstone National Park and the Grand Canyon are protected just for geologists and rock hounds. Does the vista of a star-filled night sky matter only to astronomers?

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