SUBSPACE COMPRESSIVE DETECTION FOR SPARSE SIGNALS

Zhongmin Wang[†], *Gonzalo R. Arce*[†] *and Brian M. Sadler*[*]

[†]Department of Electrical and Computer Engineering
University of Delaware, Newark, DE, USA, 19716
[*] Army Research Laboratory, AMSRD-ARL-CI-CN Adelphi MD 20783
email: zhongmin@udel.edu, arce@ece.udel.edu, bsadler@arl.army.mil

ABSTRACT

The emerging theory of compressed sensing (CS) provides a universal signal detection approach for sparse signals at sub-Nyquist sampling rates. A small number of random projection measurements from the received analog signal would suffice to provide salient information for signal detection. However, the compressive measurements are not efficient at gathering signal energy. In this paper, a set of detectors called subspace compressive detectors are proposed where a more efficient detection scheme can be constructed by exploiting the sparsity model of the underlying signal. Furthermore, we show that the signal sparsity model can be approximately estimated using reconstruction algorithms with very limited random measurements on the training signals. Based on the estimated signal sparsity model, an effective subspace random measurement matrix can be designed for unknown signal detection, which significantly reduces the necessary number of measurements. The performance of the subspace compressive detectors is analyzed. Simulation results show the effectiveness of the proposed subspace compressive detectors.

Index Terms— Subspace, compressed sensing, detection

1. INTRODUCTION

Compressed sensing provides a new framework to jointly measure and compress a sparse signal for sensors that need less sampling resources than traditional approaches. A signal $\mathbf{x} \in \mathcal{R}^N$ is K sparse on some basis $\Psi = [\underline{\psi}_1, \underline{\psi}_2, \dots, \underline{\psi}_N]$ if $\mathbf{x}$ can be represented by a linear combination of K vectors from Ψ with $K \ll N$. Given the sparse signal $\mathbf{x}$, the theory of compressed sensing shows that $\mathbf{x}$ can be recovered from M random measurements with high probability when $M = CK \log N \ll N$, where $C \geq 1$ is the oversampling factor [1], [2]. The measurements are given by $\mathbf{y} = \Phi\mathbf{x}$, where Φ is a $M \times N$ i.i.d. random projection matrix with each entry taken from an i.i.d. random distribution. Note that for illustrative purposes, we represent the signal and the projection waveform in digital form. However, it should be clear that only the projection results y are to be measured and digitalized.

In addition to signal reconstruction, the CS framework can also provide a universal measurement approach for signal detection and classification [3], [4] with a reduced set of measurements. A small number of random measurements suffice to provide sufficient signal statistics for the detection problem. The Matching Pursuit algorithm is proposed in [3] to detect relevant signal components from compressive measurements. Given the compressive measurements, the compressive detector is proposed in [4] for signal detection and estimation without reconstructing the signal.

The i.i.d. random measurement scheme for compressive detection provides universality for signals with different structure. However, since the compressive measurements are not tailored to the underlying signals, the detector is not efficient at gathering signal energy and thus the performance is inferior to traditional detectors when $M < N$. In this paper we show that the performance of detection based on compressive measurements can be significantly improved by exploiting the underlying signal structure, leading to the requirement of far fewer measurements. A subspace compressive measurement matrix can be constructed based on the estimated signal subspace model. The analog projection waveforms generated through the use of this matrix are more efficient at gathering signal energy, in turn, leading to improved detection performance.

The subspace compressive detector is constructed based on the theory of compressive sensing, which also draws on elements of the mixed-signal architecture introduced in [5], [6]. It is different from detection based on classes of liner transforms [7] in that the number of measurements is flexible with more measurements leading to better detection performance. The subspace compressive detector also differs from traditional subspace detector [8] in that no Nyquist sampling of the received signal is required. The analog input signal is projected onto analog projection waveforms and only the small set of projected results are sampled and digitalized. It can be shown that far fewer samples are required for the same detection performance. Compared with compressive detectors in [4], the subspace compressive detector achieves the same detection performance with fewer measurements (larger compression ratio). Although the subspace compressive detector does not provide universality for signal detection w.r.t all signals in the $\mathcal{R}^N$ space, it does provides universality for signal detection w.r.t all signals in a specific subspace.

The sparse signal structure can be estimated with high confidence with relatively small additional cost. It has been shown that the signal sparsity model can be estimated from limited i.i.d. random measurements of the training signals [3],[9]. Furthermore, the detection performance is robust to the estimation error of the signal model. The Basis Pursuit Denoising algorithm [10] is introduced in this paper to estimate the signal structure for its robustness to noise. The estimated signal model is then used to construct a subspace measurement matrix for subsequent signal detection. The setting for the detection problem follows the typical CS setting where only the ba-

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2. DETECTION OF KNOWN SPARSE SIGNAL

Here we assume an explicit subspace linear model for the sparse signal $\mathbf{x}$. The K vectors of Ψ construct a $N \times K$ matrix $\mathbf{H} = [\underline{\psi}_{n_1}, \underline{\psi}_{n_2}, \ldots, \underline{\psi}_{n_K}]$, where $n_i \in \{1, 2, \ldots, N\}$ for $i = 1, \ldots, K$. The signal $\mathbf{x}$ is represented as $\mathbf{x} = \mathbf{H}\underline{\theta}$, where $\underline{\theta}$ is a $K \times 1$ vector with all non-zero entries. $\mathbf{P}_H = \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T$ is the projection matrix for the subspace in which $\mathbf{x}$ lies. In the following, we discuss whether the detection performance can be improved based upon *a priori* knowledge of the signal sparsity model $\mathbf{x} = \mathbf{H}\underline{\theta}$.

2.1. Traditional Detector

Assume the signal $\mathbf{x} \in \mathcal{R}^N$ is known and let $n \sim \mathcal{N}(0, \sigma^2\mathbf{I}_N)$ be i.i.d. Gaussian noise. The detection problem is to distinguish two hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$:

$$\begin{aligned} \mathcal{H}_0 &: \quad \mathbf{y} = \mathbf{n}, \\ \mathcal{H}_1 &: \quad \mathbf{y} = \mathbf{x} + \mathbf{n}. \end{aligned} \tag{1}$$

It is well known that the optimal detector is the matched filter [11]. A sufficient statistic is given by: $t = \langle \mathbf{y}, \mathbf{x} \rangle$. The performance of the matched filter is given by:

$$P_{\mathrm{D}}(\alpha) = Q\left[Q^{-1}(\alpha) - \sqrt{\frac{\mathbf{x}^T\mathbf{x}}{\sigma^2}}\right], \tag{2}$$

where $Q(x) \triangleq (2\pi)^{-\frac{1}{2}} \int_x^\infty e^{-x^2/2} dx$ and P_{D} is the probability of detection. Here, α is set such that the probability of false alarm is $P_{\mathrm{FA}} = \alpha$. The detection performance is unchanged even the explicit signal model $\mathbf{x} = \mathbf{H}\underline{\theta}$ is known.

2.2. Compressive Detector

The detection of transient signals that are wideband and nonstationary requires sampling at the Nyquist rate. The sampling rate can be reduced within the CS framework as follows. Given an $M \times N$ i.i.d. random measurement matrix Φ with $M \leq N$, where the entries of Φ are drawn from an i.i.d. random distribution, the detection problem with compressive measurements for a known signal $\mathbf{x}$ is to distinguish between two hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$:

$$\begin{aligned} \mathcal{H}_0 &: \quad \tilde{\mathbf{y}} = \Phi\mathbf{n}, \\ \mathcal{H}_1 &: \quad \tilde{\mathbf{y}} = \Phi(\mathbf{x} + \mathbf{n}). \end{aligned} \tag{3}$$

The sufficient statistic is given by: $\tilde{t} = \tilde{\mathbf{y}}^T(\Phi\Phi^T)^{-1}\Phi\mathbf{x}$. The performance of the compressive detector is:

$$\tilde{P}_{\mathrm{D}}(\alpha) = Q\left[Q^{-1}(\alpha) - \sqrt{\frac{\mathbf{x}^T\mathbf{P}_\Phi\mathbf{x}}{\sigma^2}}\right], \tag{4}$$

where

$$\mathbf{P}_\Phi = \Phi^T(\Phi\Phi^T)^{-1}\Phi.$$

The detection performance can be approximated as [4]:

$$\tilde{P}_{\mathrm{D}}(\alpha) \approx Q\left[Q^{-1}(\alpha) - \sqrt{M/N}\sqrt{\frac{\mathbf{x}^T\mathbf{x}}{\sigma^2}}\right]. \tag{5}$$

Since $M/N \leq 1$, the compressive detector reduces the number of measurements at the cost of increased miss probability. It is also clear that the detection performance of the compressive detector can not be improved upon using *a priori* knowledge of the signal sparsity model.

2.3. Subspace Compressive Detector

The measurement scheme of the compressive detection in Sec. 2.2 does not exploit the inherent structure of the sparse signal. In the following, a subspace compressive detector is proposed where the measurement matrix Φ can be tailored to the signal structure so that fewer measurements and better detection performance can be achieved. Later we show that the signal structure can be estimated using reconstruction algorithms.

If $\mathbf{H}$ and $\underline{\theta}$ are known, the proposed subspace measurement matrix is given by:

$$\bar{\Phi} = \mathbf{G}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T, \tag{6}$$

where $\mathbf{G}$ is an $M \times K$ i.i.d. random matrix with $M \leq K$. The detection problem is to distinguish between two hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$:

$$\begin{aligned} \mathcal{H}_0 &: \quad \bar{\mathbf{y}} = \bar{\Phi}\mathbf{n}, \\ \mathcal{H}_1 &: \quad \bar{\mathbf{y}} = \bar{\Phi}(\mathbf{x} + \mathbf{n}). \end{aligned} \tag{7}$$

It is easy to show that the sufficient statistic is given by:

$$\bar{t} = \bar{\mathbf{y}}^T(\bar{\Phi}\bar{\Phi}^T)^{-1}\bar{\Phi}\mathbf{x} = \bar{\mathbf{y}}^T[\mathbf{G}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{G}^T]^{-1}\mathbf{G}\underline{\theta}. \tag{8}$$

The detector performance is given by:

$$\bar{P}_{\mathrm{D}}(\alpha) = Q\left[Q^{-1}(\alpha) - \sqrt{\frac{\mathbf{x}^T\mathbf{P}_{\bar{\Phi}}\mathbf{x}}{\sigma^2}}\right], \tag{9}$$

where

$$\mathbf{P}_{\bar{\Phi}} = \bar{\Phi}^T(\bar{\Phi}\bar{\Phi}^T)^{-1}\bar{\Phi}.$$

Furthermore, we have:

$$\begin{aligned} \mathbf{x}^T\mathbf{P}_{\bar{\Phi}}\mathbf{x} &= \mathbf{x}^T\bar{\Phi}^T(\bar{\Phi}\bar{\Phi}^T)^{-1}\bar{\Phi}\mathbf{x} \\ &= \underline{\theta}^T\mathbf{G}^T[\mathbf{G}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{G}^T]^{-1}\mathbf{G}\underline{\theta}. \end{aligned}$$

When $M = K$, $\mathbf{G}$ is invertible with probability 1, which leads to: $\mathbf{x}^T\mathbf{P}_{\bar{\Phi}}\mathbf{x} = \underline{\theta}^T\mathbf{H}^T\mathbf{H}\underline{\theta} = \mathbf{x}^T\mathbf{x}$. Compared with (2), there is no performance loss if $M = K$. Note that $\mathbf{G}\mathbf{G}^T \approx K\mathbf{I}_{M\times M}$, $\mathbf{G}^T\mathbf{G} \approx M\mathbf{I}_{K\times K}$. If Ψ is an orthogonal basis, it is easy to show that: $\mathbf{x}^T\mathbf{P}_{\bar{\Phi}}\mathbf{x} \approx (M/K)\mathbf{x}^T\mathbf{x}$. Then, with $M \leq K$, the detector performance can be approximated as:

$$\bar{P}_{\mathrm{D}}(\alpha) \approx Q\left[Q^{-1}(\alpha) - \sqrt{M/K}\sqrt{\frac{\mathbf{x}^T\mathbf{x}}{\sigma^2}}\right]. \tag{10}$$

The approximation of $\bar{P}_{\mathrm{D}}(\alpha)$ by (10) is also true when ψ_is in Ψ are only approximately orthogonal to each other. Compared with (5), the proposed detector provides better detection performance with the same number of measurements. Note that only $M < K$ measurements are required. Much like the compressive detector provides a universal detection scheme for signals in the N-dimensional space, the subspace compressive detector exploiting the linear signal model provides a universal detection scheme for all signals in the same subspace. The introduction of the random matrix makes the detector robust to magnitude variations over $\underline{\theta}$.

3. DETECTION WITH UNKNOWN PARAMETERS

Very often at the time of detection, the signal is unknown or the signal has unknown parameters. In the following, it is assumed that the signal sparsity model $\underline{\theta}$ is unknown but $\mathbf{H}$ is available. The development of a general likelihood ratio test (GLRT) detector with a signal linear model that samples the received signal at the Nyquist rate can be found in [11]. In the following, a detector that uses subspace compressive measurements is discussed.

3.1. Compressive GLRT Detector

For a compressive detector with i.i.d. random measurements, we usually assume that $M \geq K$, or the detection performance is not acceptable. In this case, the unknown parameters $\underline{\theta}$ in the signal sparsity model can be estimated. With the same hypothesis testing problem as in (3) and with the *a priori* information of $\mathbf{H}$, it can be shown that the compressive GLRT detector for the unknown signal is to decide $\mathcal{H}_1$ if the sufficient statistic satisfies:

$$\tilde{T}'(\mathbf{x}) = \tilde{\mathbf{y}}^T\mathbf{C}^{-1}(\mathbf{V}^T\mathbf{C}^{-1}\mathbf{V})^{-1}\mathbf{V}^T\tilde{\mathbf{y}} > \tilde{\gamma}, \qquad (11)$$

where $\mathbf{C} = \sigma^2\Phi\Phi^T$ and $\mathbf{V} = \Phi\mathbf{H}$. In deriving $\tilde{T}'(\mathbf{x})$, the estimate of $\underline{\theta}$, $\hat{\underline{\theta}} = (\mathbf{V}^T\mathbf{C}^{-1}\mathbf{V})^{-1}\mathbf{V}^T\mathbf{C}^{-1}\mathbf{y}$, is used. The detection performance is given by:

$$\begin{aligned}\tilde{P}'_{\text{FA}} &= Q_{\chi^2_K}(\tilde{\gamma})\\ \tilde{P}'_{\text{D}} &= Q_{\chi'^2_K(\tilde{\lambda})}(\tilde{\gamma}),\end{aligned}$$

where $Q_{\chi^2_K}$ is the right-tail probability of a chi-squared random variable with K degrees of freedom; $Q_{\chi'^2_K(\tilde{\lambda})}$ is the right-tail probability for a noncentral chi-squared random variable with K degrees of freedom and noncentrality parameter

$$\tilde{\lambda} = \frac{\underline{\theta}^T(\mathbf{H}^T\Phi^T(\Phi\Phi^T)^{-1}\Phi\mathbf{H})^{-1}\underline{\theta}^T}{\sigma^2}. \qquad (12)$$

It can be shown that with *a priori* information of the subspace $\langle\mathbf{H}\rangle$, the performance of the compressive GLRT detector is improved compared to the case when $\mathbf{H}$ is unknown. However, the improvement is limited. A more efficient detection scheme is needed.

3.2. Subspace Compressive GLRT Detector

As in Sec. 2.3, the subspace measurement matrix $\bar{\Phi}$ is designed according to (6) and only $M < K$ measurements are needed. With the same hypothesis testing problem (7), the likelihood ratio is:

$$L(y) = \frac{p(\bar{\mathbf{y}}; \bar{\Phi}\mathbf{H}\hat{\underline{\theta}}, \mathcal{H}_1)}{p(\bar{\mathbf{y}}; \mathcal{H}_0)}, \qquad (13)$$

where $\bar{\Phi}\mathbf{H}\hat{\underline{\theta}}$ is the MLE of $\bar{\Phi}\mathbf{H}\underline{\theta}$ under $\mathcal{H}_1$. Clearly, with $M < K$, the MLE of $\bar{\Phi}\mathbf{H}\underline{\theta}$ is $\bar{\mathbf{y}}$. The sufficient statistic of the subspace compressive GLRT detector reduces to:

$$\bar{t}' = \frac{\bar{\mathbf{y}}^T(\bar{\Phi}\bar{\Phi}^T)^{-1}\bar{\mathbf{y}}}{\sigma^2} > \bar{\gamma}. \qquad (14)$$

The detection performance is given by:

$$\begin{aligned}\bar{P}'_{\text{FA}} &= Q_{\chi^2_M}(\bar{\gamma})\\ \bar{P}'_{\text{D}} &= Q_{\chi'^2_M(\bar{\lambda})}(\bar{\gamma}),\end{aligned}$$

where the noncentrality parameter $\bar{\lambda}$ is given by:

$$\bar{\lambda} = \frac{\theta^T\mathbf{G}^T\left[\mathbf{G}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{G}^T\right]^{-1}\mathbf{G}\underline{\theta}}{\sigma^2}. \qquad (15)$$

It can be shown that for $\mathbf{x}^T\mathbf{P}_{\bar{\Phi}}\mathbf{x}/\sigma^2 \ll M$, the detection performance can be approximated as:

$$\begin{aligned}\bar{P}'_{\text{D}}(\alpha) &\approx Q\left[Q^{-1}(\alpha) - \frac{\mathbf{x}^T\mathbf{P}_{\bar{\Phi}}\mathbf{x}}{\sigma^2}\sqrt{\frac{1}{2M}}\right], \qquad (16)\\ &\approx Q\left[Q^{-1}(\alpha) - \sqrt{\frac{M}{2}}\frac{\mathbf{x}^T\mathbf{x}}{K\sigma^2}\right], \qquad (17)\end{aligned}$$

when Ψ is (approximately) orthogonal. Compared with (10), there is performance loss due to unknown $\underline{\theta}$ when $\mathbf{x}^T\mathbf{x}/\sigma^2 \leq 2M$. However, with the same number of measurements, the subspace compressive GLRT detector outperforms the compressive GLRT detector that uses random compressive measurements, as will be illustrated in the simulations.

4. ESTIMATION OF SIGNAL SPARSITY MODEL

In the traditional CS setting, the signal x is assumed to be K sparse on a known basis Ψ. But the location of the K relevant vectors of x in Ψ is unknown, which leads to the matrix H being unavailable at the time of detection. However, if training signals can be provided and the compressive measurements of the training signals are available, then based on the random measurements of these signals, reconstruction algorithms such as Matching Pursuit and Basis Pursuit Denosing can be employed to estimate the locations of the K relevant vectors in Ψ. Although these algorithms were initially proposed for sparse signal reconstruction, reconstruction of the sparse signal with high precision is not possible with very limited i.i.d. random measurements from the training signals. Instead, we expect to use these algorithms to get approximate information about the signal subspace. Far fewer measurements are required for signal structure estimation than for exact sparse signal reconstruction [3]. For the purpose of detection, approximate subspace information can sufficiently lead to great improvement on detection performance.

Considering the sparse nature of the underlying signal and the wideband noise effect, the Basis Pursuit Denoising (BPDN) algorithm is introduced in this paper to identify the sparse signal from the noisy measureemnts by solving the following problem:

$$\min_{\underline{\theta}} \ \|\Phi\Psi\underline{\theta} - \mathbf{y}_n\|_2^2 + \lambda\|\underline{\theta}\|_1 \quad \text{subject to } \mathbf{y}_n = \Phi(\Psi\underline{\theta} + \mathbf{n}), \quad (18)$$

where $\lambda \leq \|2(\Phi\Psi)^T\mathbf{y}\|_\infty$ [12]. With increased computational complexity, BPDN can give better estimation for the sparse signal subspace than Matching Pursuit.

5. SIMULATIONS

In this section, the performance of the proposed subspace compressive detectors are evaluated through several simulations. Only the detection of sparse signals with unknown parameters are simulated. For all the simulations, the sparse signal $\mathbf{x}$ is given by: $\mathbf{x} = \mathbf{H}\underline{\theta}$, where $\mathbf{H} \in \mathcal{R}^{N\times K}$, $\theta \in \mathcal{R}^{K\times 1}$, and $K \ll N$.

In the first simulation, the performance of detecting a signal $\mathbf{x}$ with unknown coefficients $\underline{\theta}$ is evaluated. The SNR for the detection is 15 dB. The subspace matrix $\mathbf{H}$ has dimensionality $N = 2048$ and $K = 200$. It is assumed that $\mathbf{H}$ is known in the signal sparsity model. Each entry of $\mathbf{H}$ and $\underline{\theta}$ is drawn from an i.i.d. normal distribution. The column vectors in $\mathbf{H}$ are approximately orthogonal to each other. The subspace compressive GLRT detector presented in Sec. 3.2 employs subspace random measurements that are tailored to the signal subspace model with the number of measurements $M_1 = 120$. The compressive GLRT detector presented in Sec. 3.1 uses i.i.d. random measurements with the number of measurements $M_2 = 400$, and exploits the signal structure when estimating the unknown coefficients $\underline{\theta}$. Simulation results in Fig. 1 show that the subspace compressive GLRT detector achieves better detection performance with fewer measurements. The compressive detector exploiting the signal structure achieves better performance than the compressive detector that does not have the signal structure

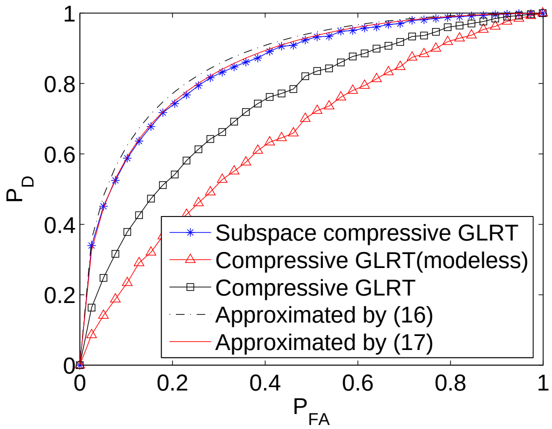

Fig. 1. Probability of detection of subspace compressive GLRT detector.

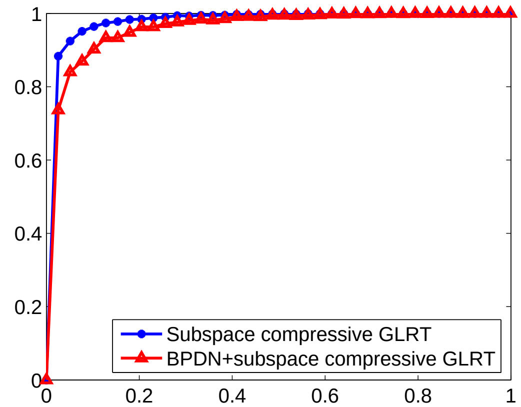

Fig. 2. Performance of BPDN with subspace compressive GLRT detector.

information (modeless). The simulation also shows that the approximations given by (16) and (17) are quite accurate.

In the second simulation, the performance of the subspace compressive GLRT detector that uses i.i.d. random measurements to estimate the signal structure is investigated. The basis Ψ is a $N \times N$ wavelet Daubechies-4 orthonormal basis with $N = 1024$. The signal is $\mathbf{x} = \mathbf{H}\underline{\theta}$ where $\mathbf{H}$ is composed of $K = 50$ randomly selected column vectors from Ψ. Each entry of $\underline{\theta}$ is drawn from an i.i.d. normal distribution. The detection of received signals with SNR= 16 dB is simulated. Signals are present or absent with equal probability. At the time of detection, only Ψ and K are assumed known. For each burst transmission, 200 training signals are first transmitted. For each training signal, the number of measurements equals $4K$. The BPDN algorithm is employed to estimate H from averaged noisy measurements. The estimated H is then used to construct an $M \times N$ subspace measurement matrix with $M = 45$ according to (6). With $10,000$ simulation results at each fixed P_{PA}, the performance of the detector is compared with the subspace compressive GLRT detector where $\mathbf{H}$ is assumed known. It is shown in Fig. 2 that BPDN is effective at estimating the signal structure and the performance loss due to unknown signal structure is small.

6. CONCLUSION

In this paper, we evaluate the performance of a set of subspace compressive detectors that exploit the signal sparsity model explicitly. Furthermore, algorithms are introduced to estimate the signal sparsity model for subsequent unknown signal detection. The results presented in this paper can also be easily extended to the case when the noise variance is unknown.

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