TRANSMISSION SUBSPACE TRACKING FOR MULTIPLE-INPUT MULTIPLE-OUTPUT (MIMO) COMMUNICATIONS SYSTEMS

This paper describes the benefits of transmission subspace tracking for multiple input multiple output communications systems, and applies the concepts of previous work on adaptive transmit antenna algorithms to this problem. A specific stochastic gradient technique of subspace or "multi-mode" tracking of the independent modes of the MIMO transfer function and the application to space time coding is considered. The technique provides gains by tracking the active modes when there are more transmit than receive antennas. With the proposed algorithm, the receiver generates binary feedback selecting preferred perturbed weights, which gives the transmitter a gradient estimate useful for subspace tracking. The capability resulting from this approach is shown by simulation.

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Transmission Subspace Tracking for MIMO Communications Systems

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Abstract - This paper describes the benefits of transmission subspace tracking for multiple input multiple output communications systems, and applies the concepts of previous work on adaptive transmit antenna algorithms to this problem. A specific stochastic gradient technique of subspace or “multi-mode” tracking of the independent modes of the MIMO transfer function and the application to space time coding is considered. The technique provides gains by tracking the active modes when there are more transmit than receive antennas. With the proposed algorithm, the receiver generates binary feedback selecting preferred perturbed weights, which gives the transmitter a gradient estimate useful for subspace tracking. The capacity resulting from this approach is shown by simulation.

I. INTRODUCTION

In recent years there have been advances in multiple-input multiple-output (MIMO) communications systems, specifically the concept of “space time coding” (STC) for wireless communications [5][6][11]. The STC algorithms use the independent modes of the MIMO transfer function to effectively attain multiple parallel transmission pipes, giving an increase in the effective available transmission bandwidth and allowing for greater transmission bit rates. The optimal transmission algorithm uses a “water filling” power/rate allocation strategy [12] into these non-zero modes. In frequency division duplex systems only the receiving unit can measure the channel transfer function, and hence feedback is required to provide this information to the transmitting unit and the complexity of water filling may be prohibitive. This paper describes an algorithm for tracking the principal non-zero modes of the MIMO transfer function utilizing feedback, without the additional complexity of doing power/rate allocation into those modes.

Tracking of the principal modes provides benefit when some of the available transmission subspaces have null or near null response and hence deliver no power to the receiver. This condition will arise if the channel response is ill-conditioned due to a poor scattering environment, as can occur even with fading independent across all antennas [7], or if the number of receive antennas is less than the number of transmit antennas due to cost or size constraints. The specific algorithm proposed is a subspace tracking variation on the algorithm previously analyzed in [1][2], which is a single receive antenna subset of the algorithm described in the present paper.

There is a substantial body of literature on the subject of signal processing approaches for transmit adaptation in multiple-input single-output (MISO) environments and for subspace tracking in the context of receive systems, examples being [2][9] and [10][13][14] respectively. The focus of this paper is a signal processing approach to tracking the transfer function principal subspace, without consideration of the coding techniques applied once those modes are determined. Any of a number of standard coding techniques can be applied, and the...
odd

\[ r = U^H r = A_{1,2} V^H t + U^H n \]
\[ = A_{1,2} V^H t + \bar{n} \]

Since V and U are unitary matrices, \( r \) and \( \bar{n} \) have the same equal diagonal second moments as their source vectors \( t \) and \( n \), and the independent channels from \( t \) to \( r \) provide the capacity "multiplexing gain".

\[ C_{\text{mode}} = \log \left( \frac{1 + p^{(r)}}{2N_r \sigma^2 \bar{H}^H \bar{H}} \right) \]

where \( (\lambda_k > 0) \) represents the numerical value of the boolean: 0 if false, 1 if true.

Applying (5) and (6) with this modified modulation scheme with \( N_r > N_r \), we find

\[ C_{\text{mode}} = \log \left( \frac{1 + p^{(r)}}{2N_r \sigma^2 \bar{H}^H \bar{H}} \right) \]

Hence, a capacity improvement of \( 10 \log_{10}(N_r/N_r) \) dB arises from this multi-mode tracking without full water filling. The object of the algorithm presented in this paper is to track those non-zero modes, i.e. the subspace orthogonal to the nullspace of \( \bar{R} \).

III. ALGORITHM DESCRIPTION

A. Algorithm Operation

The algorithm goal is to use feedback from the receiver to assist the transmitter in tracking the dominant modes of the channel matrix and to accomplish transmission according to (8). This is accomplished first by defining a code stream vector \( s \) of dimension \( N_{\text{STC}} \times 1 \). \( N_{\text{STC}} \) is less than or equal to the overall rank of \( \bar{H} \), as determined by some combination of capacity requirements, number of transmit and receive antennas, and perhaps measurement by the receiver determining the channel's usable rank, as this might be less than \( \min(N_r, N_r) \) in many environments. The code stream \( s \) is then mapped to the transmission antennas by a set of \( N_{\text{STC}} \) transmission weight vectors contained in the \( N_r \times N_{\text{STC}} \) matrix \( W \).

The system incorporates pilot sequence transmissions with each parallel coded traffic transmission, shown in Figure 1. The weight set applied to the pilot is even and odd time multiplexed between perturbed matrices so that the receiver can generate a channel estimate for demodulation and at the same time generate a single feedback bit selecting the preferred perturbed pilot weight matrix. Considering sampling at the modulation symbol (or chip) rate with Nyquist filtering the transmission \( t \) is

\[ t = \left[ \begin{array}{c} s_{\text{pilot}}^{(r)} \\ s_{\text{traffic}}^{(r)} \end{array} \right] = \left[ \begin{array}{c} W_{\text{pilot}}^{(r)} \bar{H}_{\text{pilot}} \end{array} \right] \bar{H}_{\text{traffic}} \left[ \begin{array}{c} i \\ \sum_{j=0}^{M_{\text{traffic}}-1} \end{array} \right] s_{\text{traffic}}^{(r)} + \left[ \begin{array}{c} W_{\text{traffic}}^{(r)} \bar{H}_{\text{traffic}} \end{array} \right] \left[ \begin{array}{c} i \\ \sum_{j=0}^{M_{\text{traffic}}-1} \end{array} \right] s_{\text{traffic}}^{(r)} \]

where \( M_{\text{traffic}} \) is the number of modulation samples per perturbation slot and \( M_{\text{traffic}} \) is the number of perturbation slots per measurement/feedback interval. With a perturbation parameter \( \beta_i \) and a perturbation matrix \( P \) the even/odd time
J is the expected value of the weight change prior to orthonormalization, accomplishing the desired subspace tracking, and the gradient of received power, denoted $\gamma$.

**B. Convergence Performance**

The inverse cost function of the adaptive system is the total received power using the Frobenius norm.

$$
\begin{align*}
\text{J}(k) &= \| H(k) \|_F^2 = \text{tr}(W^H(k)RW(k)) \\
\end{align*}
$$

It is shown in [14] that the maximization of the quantity (17) accomplishes the desired subspace tracking, and the gradient of $J$ is

$$
\nabla W J(k) = 2RW(k)
$$

Extending a result from [3] (Append. A), if $P$ is comprised of i.i.d. random complex Gaussians with variance twice unity, the expected value of the weight change prior to orthonormalization is the scaled normalized gradient of $J$ with respect to $W$. Hence, assuming that $\beta_i$ is small enough for $1^{\text{st}}$ order approximation of the result, neglecting estimation error in the receiver, and assuming reliable feedback, the weight matrix update is

$$
\begin{align*}
W(k+1) &= Q(W(k+1)) \\
&= \frac{1}{\beta_i} \left[ \frac{2}{\pi} \text{tr}(W(k)RW(k)) + \beta_i E(k) \right]
\end{align*}
$$

where the prime represents the update just prior to orthonormalization and $E$ is a zero mean error matrix. $E$ has some autocorrelation, due to the normalized gradient, which is extracted from $\beta_i P$ to leave $E$. The eigenmodal representation in the update prior to orthonormalization is then a diagonal modification plus noise from $E$.

$$
V''W(k+1) = \left( I + \beta_i \frac{2}{\pi} \frac{\Lambda}{\| A V'' W \|^2} \right) V''W(k) + \beta_i V''E(k)
$$

As discussed in [14], a gradient update with the form of (20) will cause the weight matrix to converge in expected value to the principal subspaces of $R$. Considering that the direction of the first column of $W$ is unchanged by the Gram-Schmidt procedure, it is clear from the principle of matrix power iteration that this column of $W$ will track towards the principal eigenvector of $R$, the second column of $W$ will track towards the second principal eigenvector, etc.

**IV. SIMULATION RESULTS**

**A. Simulation Environment**

The algorithm is simulated as described above with $N_7=8$ and $N_7=8$. In all cases $\beta_i=0.005$, small enough to ensure $1^{\text{st}}$ order gradient extraction without $2^{\text{nd}}$ order effects in the receiver measurement. Channel estimation at the receiver was considered to be perfect for purposes of generating the feedback and capacity calculations, a not unreasonable assumption when the data rate is much larger than the channel fading rate, which allows for a relatively small pilot transmission power. The feedback is received without errors, and $\beta_i$ was varied to find its best value. The channel model is independent Raleigh flat fading with time correlation given by Jakes model and a Doppler frequency of 5Hz.

Two cost metrics were evaluated through the simulation:

$$
\begin{align*}
J_0 &= \frac{E(J(k))}{E(J_{\text{opt}}(k))} \\
J_1 &= \frac{E(J_{\text{opt}}(k))}{J_{\text{opt}}(k)}
\end{align*}
$$

where $J_{\text{opt}}(k)$ is the time varying value of (17) and $J_{\text{opt}}(k)$ is the time varying value of (17) for perfect subspace tracking. In addition, mean capacity values in units of bits/second/Hertz were evaluated according to

$$
\bar{C} = E\left[ \log\left(1 + \frac{p^0\| W^H R W \|^2}{2N_0\sigma^2} \right) \right]
$$

These capacities were evaluated for several conditions for MIMO channels with both 8 and 4 transmit antennas:

- **SISO AWGN**: SISO system, $N_t=N_r=1$, with no fading (AWGN channel, baseline reference)
- **SISO**: $N_t=N_r=1$
- **SIMO**: $N_t=1$, $N_r=2$
- **MIMO WF**: perfect water filling
- **MIMO OST**: optimal subspace tracking (error free tracking of the transmission eigenmodes)
- **MIMO Blk**: blind transmission into all transmit subspaces
- **MIMO GA**: gradient algorithm subspace tracking
Figure 2: Simulated gradient subspace tracking cost functions $J_0$ and $J_1$ vs adaptation $\beta_i$; various feedback rates “FBR”; $N_f=4, N_r=2$

Figure 5: Simulated gradient subspace tracking cost functions $J_0$ and $J_1$ vs adaptation $\beta_i$; various feedback rates “FBR”; $N_f=8, N_r=2$

B. Discussion

The key capacity results are given in Figure 3, Figure 4, Figure 6, and Figure 7, showing the gains available from the use of space time coding in the MIMO environment and the tradeoffs associated with the selection of the strategy. The x-axis energy per bit or energy per Nyquist symbol are shown for a total transmit power and mean single channel gain $|h_0|_2$, so that the benefit of directing the transmission power toward the receiver is visible.

$$|h_0|_2 = \frac{E[|h_0|_2]}{2}$$  (24)

Consider the capacity plots as the system approaches infinitesimal data rate in “power limited” operation in Figure 4 and Figure 7. The reference AWGN SISO system asymptote is the well known -1.6dB $E_b/N_0$ limit. The 1x1 SISO system in fading approaches the same limit, as the infinitesimal data rate gives infinite time diversity to each transmission symbol; for higher data rates we see the performance degradation of the SISO system due to the fading channel. The required $E_b/N_0$ for both 1x2 SIMO and 2x2 blind MIMO systems approach the same asymptote for infinitesimal data rate: -4.6dB, representing a 3dB performance enhancement relative to the AWGN baseline due to the 2 receive antennas. It is interesting to note that in this limit the “multiplexing gain” afforded to the MIMO system by the multiple transmit antennas does not afford any performance improvement with blind transmission; one might
as well transmit with only 1 antenna. The optimal $N_x^2$ subspace tracking MIMO systems provide an additional performance gain of $10 \log_2(\frac{N_x^2}{N_y})$ dB relative to the 1x2 SIMO or $N_x^2$ blind MIMO systems, for gains of 6dB and 9dB over the AWGN reference for $N_x^2=4$ and $N_x^2=8$ respectively. Finally, the perfect water filling system can squeeze out 1.95dB for 4x2 MIMO or 1.41dB for 8x2 MIMO beyond what the optimal subspace tracking implementation gives.

In Figure 3 and Figure 6 the capacities as a function of transmission power are shown. In the "bandwidth limited" condition the performance of all of the MIMO algorithms provide 2bit/3dB of capacity improvement as the power is increased, as is expected for the two mode MIMO environment, while the SIMO and SISO algorithms show 1bit/3dB with only one transmission mode. In this region the optimal subspace tracking algorithm provides the same performance as perfect water filling, and continues to give a $10 \log_2(\frac{N_x^2}{N_y})$ dB performance gain over blind MIMO transmission.

Together, the capacity plots show the gains available by subspace tracking. In order to obtain substantial gains from a MIMO system operating in the power limited portion of the capacity curve where $N_x \neq N_y$ some form of transmission adaptation is clearly required, as the blind space time coding transmission does not offer substantial gain in this operating region. Since many practical systems will be interference limited, and additional power transmitted for each information bit transferred is interference to other users, there is strong motivation for operating in or near the power limited rather than the bandwidth limited portion of the capacity curve. Hence, we see the motivation for adopting some form of transmission adaptation scheme, with subspace tracking forming perhaps the simplest general class of appropriate adaptation.

The simulation results of Figure 2 and Figure 5 show the performance of the proposed gradient algorithm for various parameters. There is a strong dependency on both the selection of $\beta_2$ and of the feedback rate. Increasing the feedback rate allows for faster adaptation with a smaller $\beta_2$ and significantly improves the performance of the gradient subspace tracking algorithm. For the simulated fading rate of 5Hz, it is clear that the receive unit to transmit unit sign feedback bit rate must be at least on the order of 1kHz, and approaching 10kHz provides near optimum performance for the 4x2 MIMO case. The realized mean capacity for the gradient algorithm with the best values of $\beta_2$ for arc also shown in Figure 3, Figure 4, Figure 6, and Figure 7, with feedback rates of 8kfps and 2kfps ($N_y=8$) and 1kfps ($N_y=4$). These show that the loss captured in the performance metrics $J_0$ and $J_1$ are reasonable approximations of the actual loss of capacity relative to perfect subspace tracking.

Obtaining reasonably high feedback bit rates, on the order of 1000x the fading rate, appears to be a requirement for good operation of the gradient feedback algorithm. This is not likely to be a large cost in overall bandwidth resource, since the data rates on the forward link would likely be far larger, but it also implies a small channel estimation time for each decision, which is important when imperfect channel estimation problems are considered. While the algorithm performs largely as expected and desired, the practical realization of a system of the type proposed in this paper requires further research. The gains which are possible from the algorithm are clearly substantial.

V. CONCLUSION

The desirability of subspace tracking algorithms for low rank MIMO systems has been identified and verified through analysis and simulation. For systems with more transmit than receive antennas the gain available from multi-mode tracking is $10 \log_2(\frac{N_x^2}{N_y})$ dB over the application of blind, non-adaptive, space time coding techniques. It has been shown that blind space time coding techniques give very little gain if the system is operating in the "power limited" region of the capacity curve, which is a desirable operation region from the standpoint of minimizing interference. Finally, a specific gradient algorithm incorporating feedback from the receiver to track transmission weights to the principal channel transfer modes has been presented and shown to be capable of providing performance near that of perfect subspace tracking.

REFERENCES