SCALABLE COMPUTING OF THE MESH SIZE EFFECT ON MODELING DAMAGE MECHANICS IN WOVEN ARMOR COMPOSITES

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ABSTRACT

In this paper the scalability of a computationally intensive, multi-scale theory exhibiting progressive damage in a 3-D woven composite is investigated. The theory is based on evolving some fundamental damage modes in a representative volume element (RVE) of the micro-structure of a 3-D woven composite. The evolving damage modes affect the local stresses in the composite's actual microstructure and eventually the overall stresses in the composite. This situation is considered in the RVE via a transformation field analysis (TFA). Since the model is computationally intensive, it benefited by having its computations made parallel. This was accomplished by coupling the model to LS-DYNA3D, an explicit Lagrangian finite element code. The coupled RVE-TFA-LSDYNA3D code enabled the study of weave-level damage progression in the 3-D woven composites under general impact conditions and could be a useful design tool. In the present work, the scalability of the coupling between the RVE-TFA and LS-DYNA3D codes is examined and results are presented.

1. INTRODUCTION

Light weight armors used in soldier and vehicle protection applications owe their superior performance to the armor configurations which are carefully selected from combinations of material constituents (e.g., fibers, resins, ceramics, composites etc.) and their architectures (e.g. woven or stitched composites, etc). The design process aimed at arriving at these configurations seeks to disperse and attenuate the blast/projectile impact generated high amplitude shock waves to mitigate the energy and momentum transfer into soldiers and their vehicles. The need for increasing the impact resistance requires high strength materials such as ceramics and the need for dissipating impact energy requires resins and elastomers. When two such dissimilar material systems are required to function together, they require yet a third type, in the form of woven composites or encapsulations, to hold them together giving shape and rigidity to the eventual armor structure. Under impacts, the three constituent material systems interact, deform and fracture in a controlled manner while also undergoing myriad damage modes depending on how each material constituent experiences the overall impact.

Other than vast experimental data sets, no procedures are currently available to designers or material scientists that tell them why certain combinations of materials and geometry are better at providing optimal energy and momentum transfer, or how the incipient local damage modes are absorbing the impact. Experiments reveal the dominant damage modes in the armor constituents. Designers explain them in traditional damage mechanics perspective of failures on principal stress planes, but have not transferred such knowledge into the computational tools beyond linear displacement based stress resolution.

Porting the damage mechanics understanding of experimental databases into explicit time integration based non-linear finite element modeling (FEM) has not progressed recently owing to the difficulties in modeling the involved damage modes across the length scales of the material systems. By using averaged properties and thus ignoring length scales, traditional computational approaches often focus only on how the overall impact response is affected by the built-up nature of an armor configuration but not on how the local damage mechanics is enabling such improvement. What is needed then within the FEM is a method to explicitly model the individual damage modes, i.e. damage mechanics handled at least on two length scales: one the familiar global scale of the armor and the other the level of local micros-structure of armor constituents.

One such method reported by Bahei-El-Din, Y.A., Rajendran, A. M. and Zikry, M. A., (2003) uses the

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transformation field analysis (TFA) of a representative volume element (RVE) of a 3-D woven composite to model up to nineteen different modes of damage; each mode consisting of either sliding, splitting, bulk, or tension failures of fiber, matrix and inter-phase materials. While multi-scale theories have been used before, the emphasis has been on the global response, i.e. very fine meshes in the global domain and very coarse meshes in the local micro domain. Even when large local meshes are used, allowing the elements to totally fracture and form debris clouds doesn't yield clues as to the underlying damage modes. By ignoring material fragmentation, the RVE-TFA method takes a mechanics based view of the developing damage modes, i.e. a damage mode once initiated will grow unhindered but controls and modifies the surrounding stress fields. The evolving stress field may further initiate damage elsewhere in the microstructure.

The earlier focus in the RVE-TFA work was on obtaining the local microscopic stresses and relating them to overall stress increments. The multi-scale nature of RVE-TFA enabled RVE meshes to be separate from the global meshes, and thus lead to the consideration of larger local RVE meshes for a more realistic modeling of the local composite weave details, and the associated stress and strain fields, (Valisetty et al., 2005). But because the local RVE-TFA analysis are conducted at each and every global time step, the attendant RVE computations become time consuming placing a premium on the RVE mesh size. Although some convergence was shown for the overall stresses (Valisetty et al., 2007), the role of the RVE mesh size, i.e. how much it helps to have a larger mesh vs. a smaller one, needed to be examined from the perspective of how the local RVE stress distributions are affected by the locally spreading micro-damage. This issue was studied in detail and the progression of damage modes under the applied overall strain increments was reported in 2006 ASC. By detailing the effect of local micro mesh size on the overall response as well as on evolutions of the local damage modes, the numerical underpinning for the RVE-TFA was computationally demonstrated (Valisetty, 2008).

In the present work, the RVE-TFA model was introduced into LS-DYNA3D, an explicit non-linear dynamic finite element code, as a user defined material subroutine. The ability of this subroutine to model the effect of the progressions of a select number of damage modes on the overall stress-strain behaviors was demonstrated in a report (2006 ASC). The results demonstrating the scalable nature of the insertion of the RVE-TFA model into the LS-DYNA3D finite element code are presented in this work.

2. RVE-TFA COUPLING TO LS-DYNA3D

RVE-TFA is added as a user defined material subroutine to parallel LS-DYNA3D. The computations of the global mesh are handled by LS-DYNA3D and are spread among processors using global domain decomposition. At integration points where RVE-TFA code is used, LS-DYNA3D supplies the current global strain increments to the RVE-TFA user defined material subroutines, and in turn the subroutines perform the TFA analyses and supply back the global stress increments to LS-DYNA3D.

The coupling of RVE-TFA to LS-DYNA3D is accomplished such that computations pertaining to the global mesh are performed by LS-DYNA3D remain parallel, while the computations in the RVE-TFA code are done in a serial manner by the processors on which the RVE-TFA user defined material sub-routines are called. The data required by the RVE-TFA user material subroutines are read by the processors from within these subroutines and are held in local arrays. The processors take turns reading the data, and by keeping the data in local arrays, the global data arrays that LS-DYNA3D uses for its part of parallel global mesh computations are not disturbed, and the scalability of the eventually coupled RVE-TFA-LS-DYNA3D computations are held at the level of the LS-DYNA3D global computations. After describing the RVE-TFA inputs and the nature of the data that this code requires from one time step to the next one, results are presented to demonstrate the scalability of the coupled code.

The coupling of RVE-TFA to LS-DYNA3D rests on the fact that an RVE can be used to compute the global stresses as sums of elastic stresses computed directly from the global strain increments that are known at the integration points of a global mesh and some self-equilibrating local transformation fields which are significant within the RVE. Each of the transformation fields are associated with the progression of a damage mode within the RVE; the magnitude of each one of such fields are determined to keep the local stresses at levels to maintain orderly progressions of the damage modes as per some pre-selected damage criteria; and the task of keeping the local RVE stresses at progressive damage levels is achieved by adjusting the local stresses to restore local equilibrium of the RVE. The TFA, or the transformation field analysis, refers to the last step of local RVE stress adjustment. The multi-scale nature of RVE-TFA approach is self-evident from the facts that different RVE's can be considered simultaneously at different integration points of a global mesh and calculations performed in one RVE are independent of the calculations performed in other RVE's.

While the computations that occur within the LS-DYNA3D follow the well known explicit time integration schemes, contact algorithms, etc., the key to the success of the RVE-TFA coupling to LSDYNA-3D is the recognition of the nature of the computations that occur in the RVE-TFA code, the data required for conducting such computations, and how that data is managed and updated between the time integrations that occur in LS-DYNA3D code. While the end results of TFA are the global stress increments, these do come about after posing and solving the TFA problem with the task of keeping the locally developing damage modes under progressive control from one global time step to the next one. Thus there is a presumption of progressive damage growth. This aspect is also described next together with the information on how the data necessary for tracking these damage modes is handled; because this aspect is the key to achieving the scalable, coupling of the RVE-TFA to LS-DYNA3-D.

2.1. LOCAL STRESS EQUATIONS

The theoretical development of RVE-TFA is described in literature. From among the equations of the theory, only those equations are presented below that help the description of the data that is necessary for computing the local stresses, the transformation fields that correct the local stresses, and the data to keep track of the growth of the local damage modes.

Fig. (1) shows a typical RVE used for a 3-D woven composite. Overall dimensions of the RVE are based on the dimensions apparent in the weave's micrograph. The number of sub-volumes in an RVE can be arbitrary but it is convenient to have each RVE sub-volume belonging to a distinct materials: fiber bundle, matrix, or interface.
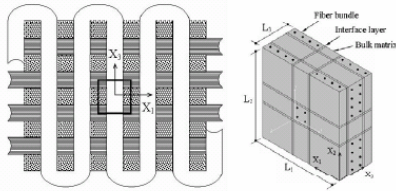

Fig. 1. A Schematic of a 3-D woven composite and corresponding RVE

Original formulation of the TFA is due to Dvorak (1992) who later used the method for inelastic composite materials in Dvorak et al. (1994). Dvorak and Zhang (2001) used the TFA to analyze damage evolution in two-phase composites. Bahei-El-Din et al. (2004) extended the TFA method to the study of plate impact problems of 3-D woven composites by considering a full complement of damage modes including matrix and fiber cracking, fiber sliding, interface failure and debonding.

A brief summary of the relevant equations (Bahei-El-Din and Boutros, 2003) of TFA is presented below using a notation of boldface lower case letters for representing the (6x1) stress/strain vectors, and boldface upper case letters for the corresponding stiffness/compliance matrices. The global stresses/strains, $\overline{\sigma}$ and $\overline{\varepsilon}$, at an (integration) point where an RVE is used, are weighted volume sums of local sub-volume stresses and strains. The local sub-volumes are assumed to undergo elastic deformation. Deviations from this mode of deformation are treated by adding transformation fields to account for the progressing damage modes. A part of the local sub-volume stresses/strains can be related to the global applied loads following the treatment of elastic composite aggregates by Hill (1963, 1965), and the rest to a superposition of all the transformation fields originating in all local sub-volumes within the RVE, Dvorak (1992) as

$$\varepsilon_r = \mathbf{A}_r\,\overline{\varepsilon} + \sum_{s=1}^{Q^*}\mathbf{D}_{rs}\mu_s \,, \, \sigma_r = \mathbf{B}_r\,\overline{\sigma} + \sum_{s=1}^{Q^*}\mathbf{F}_{rs}\lambda_s \;\; r=1,\ldots,Q \quad (1)$$

$$\varepsilon_r = \mathbf{M}_r\sigma_r + \mu_r \,, \; \sigma_r = \mathbf{L}_r\varepsilon_r + \lambda_r \quad (2)$$

$$\overline{\sigma} = \sum_{r=1}^{Q} c_r\sigma_r \,, \quad (3)$$

where Q is the number of sub-volumes in the RVE, Q^* is the number of ongoing damage modes in the RVE, c_r the sub-volume fractions, $\mathbf{A}_r$ and $\mathbf{B}_r$ the stress/strain concentration factors, $\mathbf{D}_{rs}$ and and $\mathbf{F}_{rs}$ the transformation influence factors for the rth local sub-volume, $\mathbf{L}_r$ and $\mathbf{M}_r$ elastic stiffness and flexibility matrices and μ_r and $\lambda_r = -\mathbf{L}_r\,\mu_r$ are transformation strains and stresses.

All the factors, $\mathbf{A}_r$, $\mathbf{B}_r$, $\mathbf{D}_{rs}$, $\mathbf{F}_{rs}$, $\mathbf{L}_r$ and $\mathbf{M}_r$ depend upon the RVE local geometry and properties of the local materials and are determined from an elastic analysis of the RVE mesh and are supplied as input to the RVE-TFA-LS-DYNA3D. The stress concentration factors are computed as statically equivalent to the global stresses available on the RVE. For example, the kth column, k=1,…,6, of the stress concentration factor, $\mathbf{B}_r$, r = 1, 2,…,Q, is computed by giving a unit value to the kth stress component and 0 to the rest of the 6x1 overall stress vector, $\overline{\sigma}$. This is done in the usual finite element sense by assuming a linearly varying local displacement field in the RVE, and by securing the RVE against rigid body deformation, (Dvorak and Teply, 1985).

A similar procedure is also used for computing the transformation stress influence factors, $\mathbf{F}_{rs}$, by realizing that a kth column, k=1,2,…,6, of $\mathbf{F}_{rs}$, r,s=1,2,…,Q, is a response in the sub-volume V_r after a unit stress, $\lambda_k = 1$, is applied in the sub-volume V_s. A total of 6Q RVE finite element solutions are required to completely evaluate the transformation factors. Since the RVE stiffness matrix remains the same, the only variants are the effective loads on the right hand side.

Upon entering the RVE-TFA user defined material subroutines for the first time, each of the LS-DYNA3D computing processors read the factors, $\mathbf{B}_r$ and $\mathbf{F}_{rs}$, as input. After the input is read, the only data that the subroutines keep track of, from one global time step to the next one, is the data that help them to compute the transformation fields. Naturally such tracking is necessary to bring the effect of local material damage into the local stress computations. This is described next.

2.2. COMPUTATION OF TRANSFORMATION STRESSES

In the RVE-TFA equations (1-3) presented above, the local transformation fields are used as corrections to the local stresses computed from the global strain increments under an elastic material assumption. When the RVE materials' are elastic, there is no need for these transformation fields. When some damage develops in one or more of RVE sub-volumes, the transformation fields are used to offset the local RVE stresses from their elastic values to restore the specific damage criteria, in the sense of radial return plasticity. The fact that stress loss due to spreading damage can be handled in this manner was demonstrated by Bahei-El-Din (1996), Dvorak and Zhang (2001) and Bahei-El-Din and Botrous (2003).

The transformation stress fields are found by recasting Eqs. (1). For an RVE of a composite which is descritized into Q sub-volumes, $V_r, r = 1,2,...,Q,$ the undamaged local elastic stresses are found from the stress concentration factors, $\mathbf{B}_r$, and the applied overall stress as $\mathbf{B}_r \overline{\sigma}$, the first parts on the right hand side of Eq. $(1)_2$. Suppose now there is some progressing damage in one or many sub-volumes, $V_r, r = 1,2,...,Q^*, Q^* \leq Q$ of the RVE. The computed elastic stresses then will exceed the underlying strength values in those sub-volumes. One or several damage modes may be progressing in those sub-volumes. Then to bring the computed elastic stresses to conform with the requirements for an orderly progression of each and every one of the progressing damage modes,

transformation stress fields, equal in number to those causing the progressing damage modes, are introduced; these are the second parts in the right hand side of Eq. $(1)_2$.

Essentially Eqs. (1) are statements that the local stresses, i.e. the sums of the elastic stresses and the transformation stresses, should be set at values as required by the damage criteria for the involved damage modes. This may result in the development of damage in new sub-volumes and as such the process is repeated to compute the local stresses in the presence of all possible damage modes caused under the applied overall strain increment. The transformation fields are evaluated setting the current stresses on the left hand side of Eqs. (1) at values that can be sustained in the presence of progressing damage modes.

To present the equations and later the damage modes and their growth and criteria, local stresses are considered in local coordinate systems with appropriate co-ordinate transformation matrices, $\mathbf{R}_\rho$. The local coordinate systems are not unique; they change depending on sub-volume material orientations and the specific damage criteria. Referring to such local coordinate systems and using stress ratios, $\varphi_k^{(\rho)}$, which are current stresses divided by their elastic undamaged values, the equations for computing transformation stress are presented as (Bahei-El-Din et al., 2004):

$$\sum_{\eta=1}^{Q^*} \mathbf{F}_{\rho\eta} \mathrm{diag}(\alpha_k)\lambda_\eta = -\mathbf{I} - \mathbf{R}_\rho^{-1} \mathrm{diag}(\varphi_k^{(\rho)})\mathbf{R}_\rho)\mathbf{B}_\rho \overline{\sigma}, \rho=1,2,,Q^*,$$

(4)

$$\alpha_k = \begin{cases} 1 \text{ if } 0 \leq \phi < 1 \\ 0 \text{ if } \phi = 1 \end{cases},$$

(5)

where diag(x) is a $(6Q^* \times 6Q^*)$ matrix. The stress ratios, $\varphi_k^{(\rho)}$, k=1,2,...,6, $\rho=1,2,...,Q^*$, $Q^* \leq Q$, imply pre- and post-damage loading as follows: $\varphi_k^{(\rho)} = 1$ for a stress component that has yet not violated the onset of damage, $\varphi_k^{(\rho)} = 0$ indicates complete unloading, and $0 \leq \varphi_k^{(\rho)} \leq 1$ indicates progressing damage with partial property decay. If $\varphi_k^{(\rho)} < 1$ for all k=1,2,…,6 and for all Q^* sub-volumes, then there are $6Q^*$ equations in Eqs. (4) for evaluating the transformation fields λ_r which number to $6Q^*$. If some of the local stress components are unaffected by the underlying damage criteria, the corresponding stress ratios would equal unity. In such cases $\alpha_k = 0$, and the corresponding transformation stresses will not be needed and enough equations will drop out from Eqs. (4). The

equations available after this consideration are solved for the transformation stresses that are remaining.

While the $6Q^* \times 6Q^*$ matrix in the right hand side of Eqs. (5) does not often change, the left hand side is updated at every time step in response to the progression of the damage modes. As described next, this damage modes' progression is iteratively computed with the knowledge of local stresses at the beginning of the current time step as well as the knowledge of how these local stresses are causing materials in the RVE sub-volumes to be either on loading, unloading and reloading paths. Since, in coupling the RVE-TFA code to LS-DYNA3D as user defined material subroutine, only the overall strain increments and stress increments are passed between the codes, the data necessary in the form of stresses from the previous time step and material loading/unloading state are written to, and read from, the external files.

2.3. DAMAGE MODES AND PROGRESSION

Although any microstructure can be considered with the aid of a separate local RVE mesh, the one considered in this study is suitable for modeling 3-D woven composites. In this section, the damage modes that are generic to this type of composite are described and particulars are presented with regards to their numerical implementation in coupling the two RVE-TFA LS-DYNA3D codes. Described next are the numerical data required for computing the damage progression and its effect on local stresses.

2.3.1 Damage modes

The RVE-TFA code (Bahei-El-Din et al, 2004) considered damages of fiber bundles, interface layers, and matrix but specializes the progression of these damages on planes specific to 3-D woven fabrics. Specific damage modes are considered in each material system. Four damage modes are considered in the fiber bundles as follows: longitudinal fiber rupture, transverse inter-bundle fiber sliding, longitudinal inter-bundle fiber sliding, and transverse inter-bundle fiber splitting. Two damage modes are considered in the interface layers as follows: shear sliding and peeling. The bulk matrix damage is assumed to be terminal, i.e. without any recovery.

Some of these damage modes interact with each other and while others do not. For example, $\hat{\sigma}_{22}, \hat{\sigma}_{33}, \hat{\sigma}_{23}$, the stresses resolved in a co-ordinate frame aligned with the axis of a fiber bundle, affect fiber bundle sliding and splitting. Therefore, these modes are deemed interactive. On the other hand, the two interface layer damage modes do not interact, and these are deemed independent. The

stress ratios, φ_k, which are introduced to keep track of the damage progression, become unique and independent when the corresponding damage modes do not interact; while the stress ratios involved in the interacting damage modes are not unique. For such interacting modes, the stress ratio which controls the weakest of the interacting damage modes is selected.

2.3.2 Material strength loss modeling

For all the damage modes considered, the stress and strain allowable curves are assumed using scalars quantities, (s,e), that can be computed with the knowledge of the local stresses and the damage modes. These curves are initially assumed to be linear, until the ultimate stresses and the corresponding elastic strain limits are reached (s_{ult}, e_{el}). They are later assumed to show softening. The softening branches are assumed to take a linear or a nonlinear form, defined symbolically as $s=g(e-e_{el})$, where $e_{el} \leq e \leq e_{ult}$, and e_{ult} are the ultimate strains. Unloading/reloading follows a linear path between the origin and the (s,e) curves.

2.3.3 Damage Progression

The afore-mentioned post-elastic material behavior of the RVE sub-volumes is implemented with the aid of numerical loading, i.e by keeping track of stresses at the beginning of the current time steps, and how the current stresses are causing loading, unloading or re-loading w.r.t all the active damage modes.

For a given strain $e^{(i)}$, at the ith global time step, the elastic stress is computed as $s^{(i)}=(s_{ult}/e_{el})e^{(i)}$, which may or may not fall on the allowable (s,e) curve. The correct stress magnitude, and hence the current stress ratio, $\varphi^{(i)}$, is determined to match the allowable from the (s,e) curve and the values of $s^{(i)}$, $e^{(i)}$, and $\varphi^{(i-1)}$, the stress ratio at the previous step, as follows:

$$
\begin{aligned}
&\text{undamaged state}: \varphi^{(i)}=1, \qquad \text{if } \varphi^{(i-1)}=1, 0 \leq e^{(i)} < e_{el} \\
&\text{damage reversal}: \varphi^{(i)}=\varphi^{(i-1)}, \text{if } \varphi^{(i-1)}<1, 0 \leq e^{(i)} < e_{el} \\
&\text{continued damage}:
\end{aligned} \qquad (6)
$$

$$
\begin{aligned}
&\varphi^{(i)}=\frac{g(e^{(i)}-e_{el})}{s^{(i)}}, \text{if } \frac{g(e^{(i)}-e_{el})}{s^{(i)}} < \varphi^{(i-1)}, e_{el} \leq e^{(i)} < e_{ult} \\
&\text{damage reversal}:
\end{aligned} \qquad (7)
$$

$$
\varphi^{(i)}=\varphi^{(i-1)}, \text{if } \frac{g(e^{(i)}-e_{el})}{s^{(i)}} > \varphi^{(i-1)}, e_{el} \leq e^{(i)} < e_{ult}
$$

The 1st condition in Eq. (6) represents the response of an undamaged material, while the 1st condition in Eq. (7)

describes continued loading of a previously damaged state. The remaining conditions above represent linear unloading/reloading from a damaged state. Hence, damage is recorded during the loading history through the incremental values of the stress ratio $\varphi_k^{(i)}$.

The stress and strain pair at the current loading step $(s^{(i)}, e^{(i)})$, has different interpretations depending on the type of the RVE material and the associated damage mode. In what follows, these quantities are related to the local stress and strain averages for a fiber bundle, the bulk matrix, and the matrix interface layers. It is assumed that the total overall stress $\overline{\sigma}^{(i)}$ at the current loading step is known, either directly by following a defined, stress-controlled loading path, or as the stress $\mathbf{B}_r \mathbf{L} \, \overline{\varepsilon}^{(i)}$ caused in an undamaged RVE by a strain-controlled loading,. The corresponding local stress in a sub-volume $V_\rho, \rho = 1, 2, ..., Q$, of the RVE, can then be found from Eq. $(1)_2$. When this is transformed to the local coordinates of the sub-volume, and the result is substituted into the first term of Eq. $(2)_2$, the local stress, $\hat{\overline{\sigma}}_{rr}$, and strain, $\hat{\overline{\varepsilon}}_{rr}$, described in the local coordinate system of the sub-volume, are found at loading step (i) as

$$\hat{}_r^{(i)} = \mathbf{R}_r \mathbf{B}_r \overline{\sigma}^{(i)}, \quad \hat{\varepsilon}_r^{(i)} = \hat{\mathbf{M}}_r \hat{\sigma}_r^{(i)}, \quad r = 1, 2, ..., Q . \tag{8}$$

Matrix $\mathbf{R}_r$ relates the stresses described in the local and the overall coordinate systems, and $\hat{\mathbf{M}}_r$ is the elastic compliance described in the local axes. Expressions for evaluating $s^{(i)}$ in terms of the resolved stress $\hat{\sigma}$ and material properties e_{el}, e_{ult}, s_{ult} for the above listed damage conditions follow composite material mechanics damage concepts. The specific ones used in this study are from Bahei-El-Din et al., 2004.

4. RESULTS

In this section, numerical results are presented to show the scalability of the coupling of the RVE-TFA code to the parallel LS-DYNA3D. For this purpose a global impact problem shown in Fig.2 was considered in which a circular glass composite panel was under impact by several impactors, all traveling at 154 m/s. The global impact response was handled by the parallel LS-DYNA3D, the local composite response was computed by the RVE-TFA code.
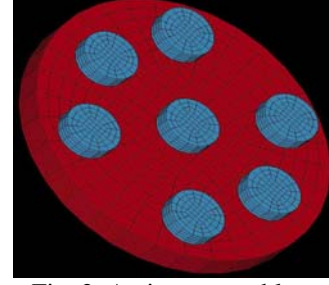

Fig. 2. An impact problem

The composite panel was made of a 3-D woven made of a S2 glass/epoxy system. The composite was evaluated to have the micro-geometry shown in Fig. 2.
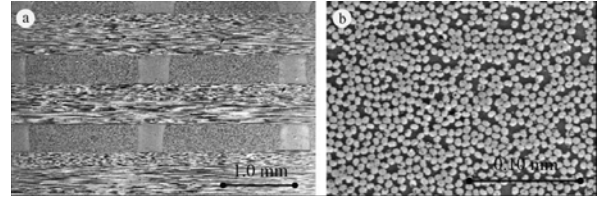

Fig. 3. Cross-sections through (a) the warp fiber bundle and (b) through the weft fiber bundle.

As seen in the micrographs of Fig. (3a), the fiber bundles in the warp and weft directions have a rectangular cross-section with aspect ratio of about 4.0, while the z-fiber bundle has a square cross-section. For the composite, a simple RVE idealization with an approximate material placement of the fiber bundles in the warp, weft and z directions, and the resin and interfacial layers is considered shown in Fig. $(1)_2$. The dimensions of the RVE parallel to the overall in-plane axes were taken as 12mm and 12mm, and in the z-direction as 2.2mm.

For fiber bundles, the following properties are used: E_L=59.7 GPa, E_T=11.7 GPa, ν_{LT}=0.248, ν_{TT}=0.371, G_{LT}=5 GPa, G_{TT}=4.68 GPa, $\sigma_{max\text{-}L}$=3.36 GPa, γ_T=0.0112, $\sigma_{max\text{-}T}$=0.08 GPa, ε_T=0.0068, $\varepsilon_{max\text{-}T}$=0.0068, $\tau_{max\text{-}L}$=0.057 GPa, $\tau_{max\text{-}T}$ = 0.048 GPa, γ_L=0.0114, $\gamma_{max\text{-}L}$=0.0114, $\tau_{max\text{-}T}$=0.048GPa, $\gamma_{max\text{-}T}$=0.0112, where E, G, ν, σ, τ, ε and γ are Young's modulus, shear modulus, Poisson ratio, normal stress, shear stress, normal strain and shear strain, respectively, and subscripts L and T indicate longitudinal and transverse directions. For the matrix, the following properties are used: E=2.9 GPa, =0.3, σ_{max}=0.06 GPa, ε_{max}=0.021, τ_{max}=0.035 and γ_{max} =0.0313.

Having thus developed the RVE of Fig. $(1)_2$, they are used for the present study. To begin the study, several RVE meshes were considered as shown in Fig. (4). Once

again several possibilities existed for increasing the sub-volumes, but the indicated pre-ponderance of sub-volumes in the z-direction was in view of the intended application of RVE-TFA in composite armor impact design studies.
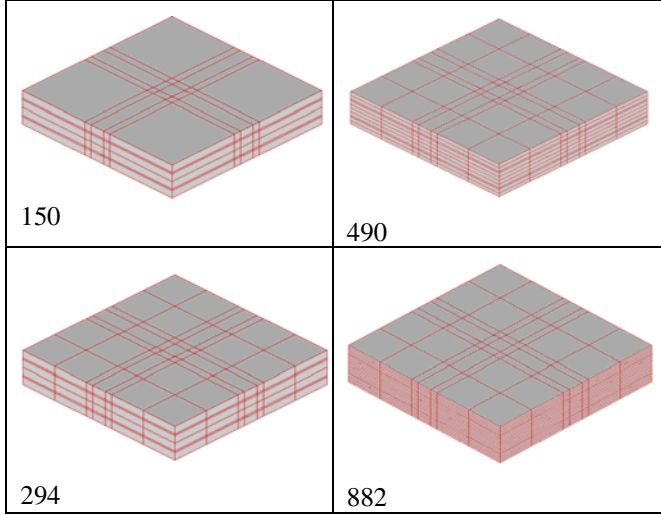

Fig. 4. Different finite element meshes for the RVE

For computing the composite's global response, the problem was modeled with a global mesh of 5360 nodes and 4320 elements. At integration points of this global mesh, LS-DYNA3D called the coupled RVE-TFA code to compute the local stresses, any local damage, and finally the global stress increments. Since the size of the local RVE mesh can influence the RVE-TFA predictions, and a larger mesh can improve such predictions, the tendency is to use a larger RVE mesh for computing the global stress increments at integration points. But increasing the local RVE mesh sizes increases not only the numerical cost of the local computations and but can also affect the scalability of the overall solution. A recent study (Valisetty 2007) showed that a RVE mesh size of 490 is sufficient to capture all the damage modes that are important in the 3-D woven composites. The purpose of the present study was then to show that RVE mesh sizes up to 490 elements can be used for TFA's in a global impact problem without affecting the scalability of global computations.

Table 1: Mesh size vs. size of the set of the Transformation Influence Factors

RVE mesh Elements	No. of Nodes	Size of all $\mathbf{F}_{rs}$
150	252	809712
294	448	3111696
490	704	8643024
882	1216	28000656

Among the input that the TFA analysis requires, i.e., the $\mathbf{B}_r$ and $\mathbf{F}_{rs}$ factors in Eq. 1, the input of $\mathbf{F}_{rs}$ is voluminous as shown in Table 1. The very first time after entering the user defined RVE-TFA material subroutines, each of the parallel compute processors were made to read this data one after other. After subtracting the wait times for reading this input data, the global solution times for running 70 global time steps were noted and were plotted in Fig. 5. The multi-processor run times for solutions with each of the four RVE mesh sizes were non-dimensionalzed with respect to the corresponding serial, i.e. one processor, run times.
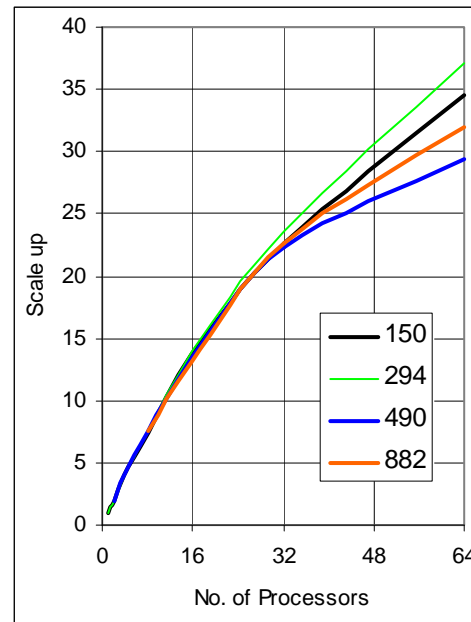

Fig.5 Scalability results

For all the four RVE mesh size, there is good scalability for up to 20 processors. The scalability reduces for runs with higher number of processors, but the drop from ideal scalability is not much. Even this drop can be attributed to the fact that on some processors local damage was not an issue and the respective processors were finishing their local computations earlier. Considering the fact that before this coupling to the parallel LS-DYNA3D, the effort of computing the coupled global-local solutions with one processor runs were taking days for problems with larger global meshes and local RVE meshes of size 490-882, the drop in scalability for larger number of processor runs is not a big numerical cost; the present study should enable undertaking of investigations into salient damage modes and their growth in real impact situations for armor composites.

5. CONCLUSIONS AND FUTURE WORK

In this paper, a computationally intensive RVE-TFA code used to study the progression of damage modes in 3-D woven composite microstructures was coupled, in the manner of a user defined material subroutine to provide overall stress increments to, the parallel LS-DYNA3D a Lagrangian explicit code used in many armor composite impact studies.

The coupling of RVE-TFA to LS-DYNA3D was accomplished such that the computations pertaining to the global mesh are performed by LS-DYNA3D and remain parallel, while the computations of the RVE-TFA code were done in a serial manner by the processors on which the RVE-TFA user defined material sub-routines are called. With the help of a brief summary of equations, the various inputs required for conducting the TFA were identified and a description was provided as to how these inputs were read and maintained by the individual processors. The numerical implementation of the damage modes' progression was described together with the description of the data required for this purpose and how such data was handled by the processors from one global time step to the next.

With the aid of a global impact on a 3-D woven composite, the scalability of the coupling of the two codes was demonstrated. The results of this study should enable the undertaking of detailed studies of damage modes' propagation in 3-D woven composites in future.

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