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SIMULATION AND OPTIMIZATION METHODOLOGIES FOR MILITARY TRANSPORTATION NETWORK ROUTING AND SCHEDULING

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1. INTRODUCTION.

The principal content of this Final Report is the status of our latest project, on

**THE TANKER OPERATIONS REPORTING AND OPTIMIZATION SYSTEM**

The report is presented both as a narrative, and, in an attachment, two PowerPoint versions of this narrative; a brief and a longer one.

In our latest progress report we presented the final versions of three projects, consisting of three doctoral dissertations:

The first dissertation,

**RECOOURSE MODELING FOR THE ARLIFT PROBLEM WITH RANDOM GROUND TIMES,**

represents the continuation and extension of a previous one, which dealt with the embedding of optimization algorithms into those simulations which are utilized by the US Transportation Command and by the Air Mobility Command. It extended the earlier results to include ground time and other types of uncertainties, and provided a stochastic solution to these problems.

The second dissertation,

**APPLICATION OF OPERATIONS RESEARCH TO MEDICAL INFORMATICS AND SYSTEMS,**

represents the culmination and assembly of several results, which we began to work on during a previous research grant (# AFOSR F49620-02-1-0099: Simulation And Optimization Methodologies For Military Transportation Network Routing And Scheduling And For Military Medical Services). Because of the nature of the medical aspects, the necessity of observations for long periods, etc., it took this long to assemble these results. Their applications were in various DOD medical establishments (in particular, at Scott AFB), as well as in civilian locations.

The third dissertation,

**A DECOMPOSITION AND SELECTION METHOD FOR SOLVING VRPSTW AND DEA APPLICATIONS,**
dealt with a combination of the first two. Its principal results were related
to our work with the Transportation Command, involving the
optimization of certain aspects of ground transportation; but, as a
secondary and independent subject, it also involved a project emanating
from the Air Force Command Surgeon's office, and dealing with the
comparative evaluation of all of DOD's medical services.

2. OBJECTIVES.

The purpose of our research was to develop a generic model and
methodology for analyzing and optimizing large-scale air transportation
networks, including both their routing and their scheduling. We were
attempting to achieve this aim in part by studying several specific
eamples of current problems of this type, arising in the operations of the
Air Mobility Command (AMC) at Scott AFB; and in part by developing
further the various paradigms that we had employed successfully in the
past in similar contexts. These include the utilization of the classical
mathematical methodologies of Linear and Integer Programming, time
dependent integer programming, etc. We were continuing to collaborate
with scientists from Scott AFB; indeed, we attempted to improve various
aspects of AMC's current scheduling and routing methodology. We
intended for these aspects to serve as the particular paradigms for the
general model and methodology to be developed. In addition, we
undertook to study the efficiency of certain aspects of the military medical
support services, through simulations, attempts at optimization, and the
comparative evaluation of various medical services provided by the
military; as well as the comparison of military medical services with
current civilian ones. Finally and most recently, we began to turn our
attention, in very specific detail, to the problem of optimizing the
scheduling and assignment of aerial tankers.

3. STATUS OF EFFORT.

During our previous reporting period we succeeded in our attempts to
incorporate optimization into simulation in a novel and seamless way.
Furthermore, we were able to exercise this methodology by using off the
shelf software: namely, PROMODEL for simulation, ILOG for
optimization and EXCEL for data collection and organization.

In particular, the novel way in which we constructed our original routing
and scheduling algorithm can be briefly described as follows:

- Relevant data (i.e., TPFDD’s) are entered into the simulation;
- The simulation (using PROMODEL) begins to run;
- When a choice is available for action, the simulation calls an
  optimization algorithm (in ILOG);
• The results of the optimization are returned to the simulation, which continues until the next choice becomes available;
• Repeat this process until the simulation is completed.

There are several advantages to our methodology. Two of the most important ones are these:

• The methodology is completely transparent to the user, in the sense that all he has to do is run a simulation as usual. This means that since currently routing and scheduling is done purely by (a non-optimized) simulation, users need not be concerned with, nor even become familiar with the underlying optimization mechanism;
• We have shown that considerable time and monetary savings result from employing our methodology.

Upon its completion, we provided three copies of a doctoral dissertation by Brian Albright, entitled

**An Embedded Optimization-Simulation Approach to Dynamic Pickup and Delivery Problems**

This dissertation contained a complete description of the above project.

As the continuation of this project, we extended these results even further, by considering the stochasticity of many elements of routing and scheduling, with the aim of incorporating these considerations in our software. We presented the results of this research in the **first doctoral dissertation** mentioned above.

In the **second of the doctoral dissertations** listed above, we completed our effort with the USAF Command Surgeon’s project, which consisted of the following subjects:

• Diagnosis Prediction Via Neural Networks;
• Simulation and Analysis of the 375th MedGroup Internal Medicine Clinic;
• Simulation and Analysis of the 375th MedGroup Emergency Department;
• Simulation and Analysis of the Missouri Baptist Medical Center Emergency Department;
• The Duke University Resident Scheduling Problem.

As we mentioned above, because of the nature of the medical aspects, the necessity of observations for long periods, etc., it took this long to assemble these results.

The **third doctoral dissertation** consisted of three parts:
  a. Simplification of the NRMO model;
  b. Vehicle Routing Problems with Time Windows;
c. Medical applications, related to the second dissertation above.

We are giving below a sketch of each of these parts.

a. Simplification of the NRMO model

Large-scale military deployment requires transporting equipment and personnel over long distances in appointed time windows, namely airlift problem. Finding the right allocation of given airlift resources to achieve maximum delivery efficiency is one of the primary aims of air mobility analysis. Several models exist in the literature to study this kind of problem. One common weakness of these models is the assumption that no random factors exist in them. Generally, such type of models are over-optimistic and lack robustness when applied to a real airlift system full of random factors, such as weather condition, unexpected required maintenance service, and so on.

Stochastic programming is concerned with the study, analysis and solving of mathematical programming problems where some parameters or data are uncertain. A common way of modeling uncertainty in optimization problems is using recourse models, where long-term anticipatory decisions must be taken without full knowledge of random parameters and short-term corrective decisions are available as recourse actions. As part of our research, we introduced the use of recourse modeling technology, and developed two studies on the airlift problem, where the ground times of aircraft are assumed to be random with a finite sample of scenarios, by the application of this modeling technology.

In the first study, we simplify the deterministic NRMO model to best describe our airlift problem. We then modify and extend the simplified model to a two-stage stochastic one by applying recourse modeling technology. In contrast to the corresponding deterministic model, this stochastic one is encouraged to select the efficient routes by anticipating potential bottlenecks in the system, and to prevent unreliable aircraft from using capacity limited airfields. In the second study, we treat the airlift problem as a multistage stochastic decision problem, and represent this decision problem as sequences of two-period simple decision problems, and then implement the 'real' decision process by embedding stochastic optimization models in a simulation. We found that this modeling approach produced more robust results when compared with Albright's EOS modeling approach. We also found that this modeling approach generates very similar cargo delivery efficiency to that of the stochastic NRMO model.

b. Vehicle Routing Problems with Time Windows

The Vehicle Routing Problem with Time Windows (VRPTW) examines certain types of scheduling problems, in which the schedule of the fleet is to be determined, such that all customers are serviced within the customers' time windows, and the operational cost for the fleet is minimized. When considering soft time windows, late services are allowed, but the late times are subject to penalties. For large VRPTW, finding a solution is very hard computationally. Therefore, recent researchers have mainly focused on designing heuristics, which find good and feasible solutions in limited time.

We present a new heuristic called the Decomposition and Selection Method. It solves certain medium sized VRP's with Soft Time Windows. First, we decompose the original problem into several subsets according to the fleet size; then, we relax the nonlinear programs in each subset; later, we select the solutions for those subsets for optimization. Finally, by providing different decomposition plans by rotation, we select from all decomposition plans the best solution with minimum operational cost. By integrating the simulation software ProModel and the
optimization suite ILOG, we make it possible for the whole decomposition and selection process to run continuously with the optimization programs solved in the background.

The selection process can be described as the following: first, initial conditions are set (number of found solutions is 0, initial best solution is a large number (e.g., 1000000.0), initial cost is 0, etc.). Second, search a solution for the shortest path problem (SPP). This solution does not necessarily have to be the optimal solution for the SPP; it is just a feasible solution in ILOG OPL Studio’s searching process. Third, if a feasible solution is found for the SPP, calculate the latency for the resulting route, perform the route selection process (route selection will be discussed later), and then calculate the weighted cost of the selected route. Then, compare current cost with the best cost so far, if current cost is less than the best cost, then let current cost be the best one, and update the route information (route length, latency) for the best route. Moreover, if a next solution can be found for the SPP, and the iteration times is less than certain limit (the limit is 1000 here, actually the iteration time will never exceed 1000 for the SPP problem of this size), then go back to the third step and repeat the same processes. If no more solutions can be found or the number of iteration exceeds the limit, then exit the loop and export the results to plain text data files.

ILOG OPL. Studio solves optimization problems, ProModel is a simulation software, which keeps subsets of optimization problems running continuously and interacts with plain text data.

c. Medical applications
The Scott Air Force Base Emergency Department was one of three major simulation projects for the Air Force. The other two were the BAAA Internal Medicine Clinic and an arbitrary appointment based clinic that is conformable to many different clinic types, and may also be used to simulate field conditions. Additionally, the civilian ED at Missouri Baptist Medical Center was included in the study. The ED simulations were subjected to a novel feedback control system which generates task/time correlations and then uses the information to prioritize medical decision making, so long as that priority does not conflict with physician judgment.

The Duke University project is an example of a solution to the Resource Constrained Mixed Integer Assignment problem, and is mathematically identical to, for example, assignment of piloting crews to aircraft for a Time Phased Force Deployment Document (TPFDD). This consisted of scheduling 57 residents for a four week period to 10 different hospitals under a large number of legal work requirements and staffing necessities. The program was multi-objective and consisted of more than 30,000 variables and 12,000 constraints. Ordinary computing power is insufficient to solve such a problem, and the Constraint Satisfaction with Domain Reduction Techniques Method was employed to find an optimal solution.

These simulations are designed to be adaptable to changes within the system, and provide predictive value to 'what if' scenarios. For example, the Missouri Baptist ED was considering providing in house radiology for most procedures, and outsourcing only CT scans and MRI. The simulation demonstrated minimal improvement in patient throughput, and no revenue increase. The cost of installing the X-ray machines was therefore considered prohibitive.

Additionally, the emergency simulations were tested under a variety of Mass Casualty circumstances designed to emulate various disasters, and terrorist attacks, and strategies for improved triage and team composition were derived.

Using the medical record information provided by the 375th Medical Group at Scott AFB, a neural network was designed to test whether diagnoses could be determined using the information provided in as patient encounter sheet. The information consisted of the patient’s demographics, vital signs, a list of boolean symptom columns, the verbal complaints as recorded by the triage nurse, and the doctor’s diagnosis. This information was sanitized and parsed according to the primary body part of the complaint. The neural network was able to correctly classify the vast majority (82% overall) of the twelve test diagnoses. This could be expanded to provide physician decision support, or be used in the field when doctors are not available.
LAST PROJECT

THE TANKER OPERATIONS REPORTING AND OPTIMIZATION SYSTEM

Section 1: Introduction

Aerial Refueling (AR) is the act of offloading fuel from one aircraft (the tanker) to another aircraft (the receiver) in mid flight. There are two broad categories of AR missions: deployment AR missions and employment AR missions. Deployment AR missions can be loosely described as the AR missions used to escort air combat resources from one theater of operations across an ocean to another theater of operations. Employment AR missions can be loosely described as the AR missions needed to carry out day to day, intra-theater activities such as training, or war fighting.

Our research was focused on employment AR missions that take place over the continental United States (CONUS). In particular, we were interested in developing the tools needed to measure, and then minimize the mismatch between the distribution of tankers across CONUS, and the distribution of CONUS employment AR missions.

Ultimately our goal was to add to the theoretical foundations used by military analyst, who plan the tanker basing / tanker utilization components of a military campaign, and by the military operators, who adjust campaign plans as the reality of their situation changes. We were also interested in developing software tools that will help planners and operators make better informed decisions in a shorter amount of time.

Figure 1: Employment Air Refueling missions take place inside well defined areas of air space known as AR Tracks and AR Anchors. Hundreds of tracks and anchors are carved out of CONUS airspace, and I have observed that more than 281 of these are used on a regular basis.
In my analysis I have aggregated the 281 most frequently used AR tracks and anchors into the 78 AR Zones, and 6 geographic regions shown above.

**Section 2: Major Accomplishments**

We started working on this project in May of 2005 and by May of 2006 we had a list of very grand ideas. It is exciting to note that some of these ideas are slowly becoming a reality. We have organized all of the CONUS Tanker Ops data available to us into an Excel database. This database is query-able in the sense that meaningful reports can be (and have been) produced using excel pivot tables. And we have a small but growing suite of Visual Basic and Mat Lab scripts that operate on this database to produce charts and maps. Moreover, AMC/A9 has used this database, and the small suite of tools we created to produce reports in support of current base realignment studies.

**Section 3: Time Line of my (Allen McCoy) progress**

In May of 2005 I started working with Peter Szabo of the Analyses and Assessments Division (AMC/A9) on an analysis of air refueling training missions that take place over the continental United States (CONUS). In particular we were interested in measuring and then minimizing the mismatch between the distribution of KC-135 tankers across 30 tanker bases and the geographic distribution of the employment AR missions they were asked to support. This problem is known as the Tanker Basing Demand Mismatch problem. Its solution is the primary goal of my research.

In September of 2005 I presented Mr. Szabo with three things: (1) the Tanker Base Activity Report; (2) a proposal to organize tanker operations data into a relational database; and (3) a preliminary strategy on how to minimize the tanker basing/demand mismatch. The Tanker Base Activity Report presented a novel way of looking at tanker operations data and was very well liked inside of AMC/A9. Consequently the idea of organizing tanker operations data into a database that could facilitate the production of reports like the Tanker Base Activity Report was also well received. As a result of this I was encouraged to continue working on this project the following summer.
When I returned to AMC/A9 in May of 2006 it quickly became clear that not having a security clearance would be a major impediment to getting anything done. With out a clearance, it would be hard for me to gain access to subject matter experts, computer systems, and data. With a clearance, I would be able to work in the AMC/A9 office along side Mr. Szabo and other AMC/A9 personnel. Moreover I would have no problems accessing the data and computer systems I needed to do my work. Consequently when I was presented with an opportunity to get a clearance, I was encouraged to take it. Specifically, in order to get my clearance, I needed to accept a part time job with the AMC/A9 contractor that supported the flagship air mobility simulation tool AMOS. While the responsibilities and obligations of this change in status have been a bit of an encumbrance, this change in status has had its benefits.

Unfortunately I didn’t actually get my clearance until August of 2006 and much of my own work was put on hold until I did. While I worked as much as I could during the 2006/2007 academic school year, my research efforts didn’t resume in earnest until I finished my last final in May of 2007. I then worked 4 to 5 days a week from the AMC/A9 offices until the end of October of 2007. Though I have had to split my time evenly between AMOS tasks, and research, all of my work has been focused on learning about, and trying to model air refueling operations.

The following sections discuss some of the work I have completed since May of 2007:

Section 4: The Revised Tanker Operations Database
Section 5: The 2007 Tanker Demand Mismatch Report
Section 6: Navigation and Fuel Consumption
Section 7: Minimizing the Tanker Basing / Demand Mismatch - Theory
Section 8: Location-Allocation formulation of the Tanker Basing/Demand Mismatch Problem.
Section 4: The Revised Tanker Operations Database

In its current form, the Tanker Operations Database consists of a single Excel file, split across 9 spreadsheets which are linked together with Excel lookup commands.

A. The Horse Blanket Worksheet

The Horse Blanket worksheet contains a historic record of the AR tasks that were planned and closely approximates what actually happened.

We obtain this data from the Tanker Airlift Control Center (TACC) which collects it from the individual tanker units. In its raw form, it exists as a collection of Excel files and is divided across a set of folders. There is one folder for each quarter of activity. In each folder there is exactly one file for each Tanker unit which details the AR tasks assigned to that unit during the corresponding quarter. The original data found in these files contains the following information:

- The date and time of each AR task
- The sequence and priority number of each AR task
- The receiver unit that generated each AR task and its major command
- The type and number of receiver aircraft needing fuel in each AR task
- The AR track and altitude where each AR task is to occur
- The amount of time and fuel needed for each AR task
- The number of tankers needed for each AR task.
- And details concerning navigation along the AR track.

When these data sets are imported into the database, we augment each record with several fields to make Pivot Table generation much easier for end users. These fields include: the tanker unit that was assigned to the AR task (we add this column as the individual Horse Blanket files are being merged into one master document); the base from which that tanker unit operates (lookup on tanker unit); the type of tanker that the tanker unit was known to fly during the time period of the task (lookup on tanker unit); and a set of 5 columns that connects the AR task to the different ways we have partitioned US airspace.

B. AR Track to ARR DAT ID Worksheet

The AR Track to ARR DAT ID worksheet is the essential connection between the activities recorded in the Horse Blanket, and the geographic location of those activities.

This worksheet is needed because there are two sets of identification codes for AR Tracks and Anchors. One set comes from the Horse Blanket data (AR TRACK). A second set comes from the AP1B, the primary source for AR route data (ARR DAT ID). Frequently the AR Track code in the Horse Blanket data does not match up exactly with an AR Data ID from the AP1B and human judgment is required to bridge the gap.
The AR_TRACK to ARR_DAT_ID worksheet is the record of this human judgment. It partitions the AR_TRACK IDs found in the Horse Blanket Data into sets which are identified by ARR_DAT_IDs found in the AP1B data.

C. The AR Route Data Worksheet

The AR Route Data worksheet maintains a record of nearly 281 different AR routes. Each AR route has a unique identifier (the ARR_DAT_ID), and is defined by up to six geographic coordinates. Most of the data in this table was obtained from the AP1B, however some data was obtained from navigation charts, and subject matter experts who know where to dig to find such things.

In addition to the fields needed to identify, and define an AR route, each AR Route Data record contains a field that partitions the set of 281 different ARR_DAT_IDs into 124 sets. This partitioning comes about from the fact that many AR routes will have the same exact ground projection, but will have different elevations, and or directions of travel. Normally such routes will have the same root name but will have different suffixes (EG AR006NW & AR006SE).

Each of the 124 sets is identified by a different ARR_SHORT_ID and is characterized on the map by the convex hull around all of the geographic coordinates which can be mapped to it.

![Map of AR Routes](image)

Figure 3: Many AR Routes will have the same exact ground projection, but will have different elevations, and or directions of travel. Most of these ground projections are shown above.

D. The ARR SHORT ID to ARR SGRP ID Worksheet

As mentioned above the ARR_SHORT_ID partitions the set of 281 different ARR_DAT_IDs into 124 sets. This is an overwhelming number of distinct geographic regions to deal with. Moreover the ground projections of the AR zones defined by the ARR_SHORT_ID still overlap in very
significant ways. Therefore, in order to look at fewer, and more distinct geographic regions, further refinement is needed. The ARR_SHORT_ID to ARR_SGRP_ID worksheet partitions the set of 124 ARR_SHORT_IDs into 79 sets. Each set is identified by an ARR_SGRP_ID and characterized by the convex hull of all of the geographic coordinates that can be mapped to it. (Note: SGRP stands for "small group")

Figure 4: The ARR_SHORT_ID to ARR_SGRP_ID table aggregates 124 different ARR_SHORT_IDs into groups of AR Zones that have: the same ground projection; ground projections that significantly overlapped; and or parallel ground projections that are in very close proximity.

E. ARR SGRP ID to ARR RGN ID Worksheet

The continental United States can be divided into 6 different geographical regions, the north-west, north-central, and north-east regions, and the south-west, south-central and south-east regions. The ARR_SGRP_ID to ARR_RGN_ID worksheet assigns each ARR_SGRP_ID to one of these six geographic regions.

Figure 5 Each of the AR Zones defined by the ARR_SGRP_ID is assigned to one of the six geographic regions of the Continental United States.
F. The Tanker Bases Worksheet

The Tanker Base Worksheet contains basic permanent information about tanker bases such as their ICAO, type, name, City, state, coordinates, and region. Currently there are 36 tanker bases in our data set. Several of them do not lie inside the continental United States. These bases are given the RG100 region code.

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<td>Destin</td>
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<td>Japan</td>
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<td>127.77</td>
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G. The Tanker Wings to Base Worksheet

We are currently tracking the activity of 43 different Tanker Wings. The Tanker Wings to Base worksheet maps each wing to the base from which that wing operates.

H. The Squadrons To Wings Worksheet

We are currently tracking 51 different Air Refueling Squadrons (ARS). The Squadrons to Wings Worksheet maps each squadron to its parent wing.

I. Squadron Tanker Counts

We obtain the tanker counts for KC-135 Squadrons through the KC-135 force structure data sheets. A new sheet is produced each year, and contains four quarters of data. The Squadron Tanker Counts worksheet is
designed to maintain this data in such a way that end users will have the greatest flexibility in making pivot tables from this data.
Section 5: The 2007 Tanker Basing Demand Mismatch Report

The 2007 Tanker Basing / Demand Mismatch report was requested by Pete Szabo and was used to support a current base realignment study. The main feature of the report was a set of slides such as the one shown below. This one slide summarizes 4 tables of data produced by executing different excel pivot table commands against the Tanker Operations Database described in the previous section.

![Distribution of Tankers & Track Time Between Tanker Bases & Regions](image)

**Figure 6** There is one map for each of the four fiscal years of data contained in the updated Tanker operations Database. Each map presents one record from each of 4 different tables in the following way: Each dot represents a tanker base. (1) the size of a dot corresponds to the average number of tankers assigned to that base over the course of the year; (2) the color of the dot corresponds to the percentage of AR Track hours assigned to that base over the course of the year. Each Geographic Region is assigned two numbers: (3) the top number corresponds to the percent of AR track hours demanded from the geographic region; (4) the bottom number corresponds to the percent of the tankers assigned to that region.

Mr. Szabo requested this report in late August. I completed it by mid October (2007). The bulk of that time was spent modifying and updating the database, writing the Mat Lab and visual basic scripts needed to produce the maps, and formatting the final layout in Power Point. Once the database maintenance and programming tasks were completed, the report itself only took a day to generate. When the database and map generating codes are packaged into a single user friendly application, a report like this one shouldn’t take more than a few hours to create.

Though I have only discussed this report in brief here, the entire 39 page can be made available upon request.
Section 6: Navigation and Fuel Consumption

Most military OR models don't need to explicitly deal with fuel consumption or navigation. Even when the availability of fuel resources are regarded as binding constraints, detailed information about a distribution channel's actual path is not materially relevant because fuel consumption can be characterized by the length of the path and the fuel economy of the vehicle used to traverse it. Moreover fuel is usually only a resource.

Models of AR operations deal with fuel as a commodity. And because "commodity" fuel is consumed in the process of delivering AR services, accuracy with regard to the amount of fuel needed complete an AR objective is important. That being said, variations in the direction and intensity of the winds along a flight path can have an impact on the fuel economy of tankers. And one time assessments of the fuel costs and delivery capacities of tankers traversing standard air-lanes can be rendered inaccurate by changes in wind conditions. Consequently, models of AR operations that do not provide a way for analysts to

Figure 7: Wind must be an exogenous model input provided by end users. Though CONUS winds at operational altitudes flow rather consistently from east to west, there are still significant variations in intensity as well as the north/south component of the wind's direction. Over long distances, these variations can affect fuel consumption calculations. Moreover, in different parts of the world the wind patterns can be completely different. The CONUS maps were obtained from the NOAA website. The European map was obtained from http://euro.wx.propilots.net/.
account for wind direction and intensity lack credibility in the eyes of operators. In fact, one of the biggest reservations AMC/A9 analyst have expressed about using some of the more recent academic AR Employment and Deployment models is that they do not allow for wind, nor do they provide analysts an easy way to insert allowances for wind into their decision making processes.

Wind can be incorporated into an AR model through flight planning modules that are executed during pre-processing steps, and or between simulation events. Per the suggestion of analysts inside of AMC/A9 I obtained some of the formulas needed to develop these modules from the Ed Williams Aviation Forumulary website (http://williams.best.vwh.net/). And I have obtained some of the knowledge needed to implement these formulas through my work on AMOS related tasks.

While I am starting to understand many of the rules and formulas that go into flight planning, I have not yet produced flight planning modules that can be used for planning purposes. I know this with some certainty because my models do not yet yield results that are comparable with those produced by Falcon View. This is the standard software package used by operators and analysts to plan one or two flights at a time and is certified by the Air Force for planning real missions. Ideally I will find a way to re-use much of the effort that went into developing Falcon View, either by tapping directly into Falcon View function libraries or by creating my own functions with Falcon View documentation as my guide.

Section 7: Minimizing the Tanker Basing / Demand Mismatch - Theory

Reallocating tanker resources in peace time is expensive, disruptive, and complicated by political wrangling. Discovering that tankers have been miss-allocated in the middle of a battle is a much worse prospect. Thus a particular allocation of tanker resources to airfields (hence forth a tanker basing strategy) should be expected to persist with minor modification for “a long period of time”. Clearly the definition of “a long period of time” will depend on the situation being modeled. In the case of combat operations it could be months, or years depending on the nature of the operations. In the case of CONUS peace time operations it could mean decades. In either case, a particular tanker basing strategy will be expected to support thousands of AR operations and possibly hundreds of different battle scenarios.

<table>
<thead>
<tr>
<th>YEAR</th>
<th>Theater</th>
<th>Operation</th>
<th>Days</th>
<th>No. AR Sorties</th>
</tr>
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<td>Desert Storm</td>
<td>43</td>
<td>15,434</td>
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<tr>
<td>1998</td>
<td>Kosovo</td>
<td>Allied Force</td>
<td>78</td>
<td>5,000 +</td>
</tr>
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<td>2003</td>
<td>Iraq</td>
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<td>29</td>
<td>6,193</td>
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<td>1991-2003</td>
<td>Iraq</td>
<td>Northern Watch</td>
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<td></td>
</tr>
<tr>
<td>1992-2003</td>
<td>Iraq</td>
<td>Southern Watch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001-2007+</td>
<td>Afghanistan</td>
<td>Operation Enduring Freedom</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003-2007+</td>
<td>Iraq</td>
<td>Operation Iraqi Freedom</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 8: The first three lines of data were obtained from Major Mark MacDonald’s 2005 AFIT OR master’s thesis: Handbook For Tanker Employment Modeling.
Complicating matters further, analysts don’t know which of the hundreds of different scenarios they need to plan for. At most analysts have a probabilistic model of the receiver’s mission which can be used to predict how many tanker resources will be needed, where they will be needed, and how frequently they will be needed in those locations. Consequently, I have suggested in previous reports that the time and location of employment AR missions should be viewed as random variables, and that a simulation model like the one outlined below should be used to find basing strategies with minimum expected costs.

But this is not the only approach to choosing basing strategies under uncertainty. In fact, while sitting through the 2007 AFOSR Optimization and Discrete Mathematics Program Review, I recall Gerald Brown cautioning everyone present against considering and minimizing average costs alone. He suggested that we should also be concerned with minimizing the costs of worst case scenarios. Applied to our present situation his words mean that we need to develop a model that finds tanker basing strategies which not only work well on the norm but are also sufficiently robust that they continue to work in extreme scenarios.

However, before we can employ either of these strategies we need a way to choose and evaluate a particular tanker basing strategy. In the previous reports I did not discuss how I would generate new basing strategies, nor did I discuss how these strategies would be evaluated other than suggesting that the situation seemed like it could be modeled as a Vehicle Routing problem.

Since then I have found that the Tanker Basing / Demand Mismatch problem can best be described as either a Stochastic Location-Routing Problem, or a
Stochastic Location-Allocation Problem. The main difference between the two is that the former assumes tankers have the capacity to serve multiple AR zones before returning to their home airfields. Meanwhile, the latter assumes that tankers only have the ability to fly to and service one AR zone per flight (Min Et Al. 1998).

So which is it? There is at least one special case when it might be particularly advantageous for tankers to connect with receivers at multiple AR zones. This is the case when tankers returning to base “topped off” tankers heading into the field. There are two places where I have seen this tactic referred to in literature. Lt. Col. Don Anderson, of AMC/A9 wrote a lengthy paper on his experience using this tactic in the field. And Major Mark MacDonald mentions how this tactic was used during the first Gulf War, and the Kosovo action to continuously top off a tanker with no specific AR assignments.

Lt Col. Anderson found that significant gains in efficiency could be made if tankers returning to base “topped off” tankers heading into the field as their paths crossed. These gains included not only the use of less fuel, but the use of fewer tankers, and fewer crew hours to execute the same requirements. That being said the operation, on which Lt. Col Anderson wrote, required tankers to fly long distances, through relatively uncongested air space. Moreover, the tankers being topped off were headed to a specific refueling task and not to a stand by / emergency capacity orbit. Finally, the receivers that required fuel in this operation were large aircraft and required large offloads.

The situation described by Major Mark MacDonald is very different. The “reliability tanker” has no specific AR task. Its job is to “serve as a fuel depository for overweight tankers returning to base” and it is supposed to step in whenever there are tanker maintenance issues, weather related tanker delays, and or emergency “pop-up” receivers (MacDonald, 2005). In his research, Major Mark MacDonald found that the bottle neck in the first Gulf War, and later in the Kosovo action was not the ability to get, and maintain large quantities of fuel in the air. The bottle neck was safely providing an adequate number of refueling points over a compact and congested airspace, to satisfy the needs of many small receivers. So while reliability tankers helped keep fuel in the air, they were an expensive use of a refueling point, wasted valuable crew time, and over used airframes. For these reasons, Maj. MacDonald reports that “tanker planners effectively quelled” the “push” to use reliability tankers during the second Gulf War.

“Their position was that while the reliability tanker may provide flexibility, it was one less refueling point that could be utilized for planned missions. Given the size and pacing of the planned air campaign, they felt it was an inappropriate use of needed assets. Instead, tanker sorties were planned to 80% capacity leaving 20% of airborne fuel flexible to cover mission changes and contingencies.”

In light of Maj. MacDonald’s research on the matter, and the fact that compact areas of operation are more the norm than the exception, I have decided that it may be more appropriate to view the this problem as a Location-Allocation Problem. The rest of this document sketches out a draft formulation of this problem as applied to tanker basing.
Section 8: Location-Allocation formulation of the Tanker Basing/Demand Mismatch Problem.

Indices:

<table>
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<th>Index</th>
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<tr>
<td>( h \in {1,\ldots,500} )</td>
<td>Tanker tail numbers</td>
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<td>Ramp fuel weights (light, medium, heavy)</td>
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<td>( j \in {1,\ldots,32} )</td>
<td>Airfield IDs</td>
</tr>
<tr>
<td>( k \in {1,\ldots,78} )</td>
<td>AR zone IDs</td>
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Decision Variables:

1. Minutes of Orbit Time

\( O_{hik} \in [0,\infty) \) is the amount of time that tanker \( h \), with ramp fuel \( i \), flying from airfield \( j \), is required to orbit AR zone \( k \). This amount is measured in minutes.

2. Pounds of Fuel Offloaded

\( F_{hik} \in [0,\infty) \) is the amount of fuel that tanker \( h \), with ramp fuel \( i \), flying from airfield \( j \), is required to offload while orbiting AR zone \( k \). This amount is measured in thousands of fluid pounds.

3. A Binary Variable

A resource in our model is the composite choice of a tanker, ramp fuel weight, airfield, and AR zone. The goal of our model is to choose between the nearly 4 million different combinations of tankers, ramp fuel weights, airfields, and AR zones to fulfill the demand requirements described later. We will use the binary variable \( T_{hik} \) to indicate that a one of these combinations has been chosen. Here are a few concrete examples of what \( T_{hik} \) means for different values of \( (h,i,j,k) \):

\[
T_{6,3,5,18} = 1
\]
Tanker 6, with a medium fuel load, departed from airfield 5, and orbited AR zone 18.

\[
\sum_{i=1}^{3} \sum_{j=1}^{32} \sum_{k=1}^{78} T_{hik} = 0
\]
Tanker 5 was not used.

\[
\sum_{k=1}^{78} \sum_{j=1}^{32} \sum_{i=1}^{3} \sum_{h=1}^{500} T_{hijk} = 10
\] \( \Leftarrow \) 10 tankers, with various ramp fuel weights, departed from various airfields to fulfill receiver requirements at AR zone 4.

Objective Function:

23
Our objective is to minimize total fuel consumption. Fuel is consumed in one of three ways: it is burned in transit, it is burned while orbiting, and it is offloaded into receivers.

1. Fuel consumed in transit

It would be nice if we could model transit fuel costs as a convex function of orbit time; fuel offloaded; one way distance; and ramp fuel. Unfortunately such a function is not available at this time. Consequently, we need to compute a table of values for each choice of \((h, i, j, k)\) from the set of different tanker, ramp fuel, airfield, AR zone combinations, and design the objective function so that we incur the fixed cost associated with a particular \((h, i, j, k)\) whenever \(r_{hijk} = 1\).

Transit fuel costs can be calculated in advance, if we know the following things:

a. The tanker's performance characteristics
b. Its ramp fuel weight
c. The distance from the airfield of departure to the AR zone it orbits
d. Its orbit time, and the amount of fuel it offloads

The first three values listed above are determined by a choice of \((h, i, j, k)\) but the last two values are decision variables, and are going to be left to the model to determine. Fortunately, given a choice of \((h, i, j, k)\) we can also compute a Maximum Orbit Time / Maximum Fuel Offload Boundary.

![Diagram](image)

*Given tanker performance characteristics, a ramp fuel value, the distance between an airfield and an AR zone, and standard route planning tables, we can calculate the maximum amount of fuel a tanker has to offload into a receiver, or burn while orbiting an AR zone. Using this maximum value, and a burn rate per minute of orbit time, we can calculate a Maximum Orbit Time / Maximum Fuel Offload.*

When we compute the amount of fuel consumed in transit for a particular \((h, i, j, k)\) we will assume that the model only chooses offload/orbit time pairs that lie on the Maximum Orbit Time / Maximum Fuel Offload Boundary. This assumption isn't too harmful to the validity of our results because the model will be restricted to choose offload/orbit times pairs that lie beneath this boundary. Therefore we have at most overestimated the weight of the tanker on the outbound leg of its flight. Moreover, this is
not necessarily a bad mistake to make. Tankers will often take more fuel than they need for the assignment they are given in order to accommodate unscheduled receivers. Ultimately, the formula for calculating the amount of fuel burned in transit is:

\[
\text{Fuel Burned In Transit} = \sum_{h=1}^{500} \sum_{i=1}^{3} \sum_{j=1}^{32} \sum_{k=1}^{78} T_{hij} (TB)_{hijkl}
\]

While we don't need to calculate the coefficients of nearly 4 million boundaries we still have nearly 30 thousand boundaries to consider each time the weather conditions change. With four different tanker types, and three different ramp fuel weights, there are 12 different tanker configurations to consider. And each of these configurations could travel from one of 32 different bases, to one of 78 different AR Zones.

If we don't consider weather, then we only need to consider 240 boundaries. Again there are only 12 different tanker configurations, but when we neglect weather there is a much smaller set of "flight plans". Specifically, because we aggregated AR Tracks together to create AR zones, the margin of error in our distance calculations is at least 100 Nautical Miles. Therefore, we can get away with calculating a generic 100nm, 200nm, up to the generic 2000nm flight plan for each of the 12 tanker configurations. Consequently we only need to calculate 20 boundaries for each of the configurations, or 240 boundaries total to generate our table of \((TB)_{hijkl}\) values.

2. Fuel Consumed while Orbiting

We approximate the amount of fuel consumed while orbiting by multiplying the number of minutes spent orbiting by the constant Orbit Burn Rate associated with the tanker's type. The formula for calculating the total amount of fuel burned while orbiting is given by:

\[
\text{Fuel Consumed While Orbiting} = \sum_{h=1}^{500} \sum_{i=1}^{3} \sum_{j=1}^{32} \sum_{k=1}^{78} (OBR)_{hij} O_{hijkl}
\]
3. Fuel Consumed by Offloading

The amount of fuel consumed by offloading is simply the total amount of fuel offloaded.

\[
\text{Fuel Offloaded} = \sum_{k=1}^{500} \sum_{i=1}^{32} \sum_{j=1}^{78} F_{kij}
\]

4. The total Cost Function

The total cost function is thus the sum of the three component functions:

\[
\text{Total Fuel Cost} = \sum_{k=1}^{500} \sum_{i=1}^{32} \sum_{j=1}^{78} \left[ (OB)_{kij} O_{kij} + F_{kij} + (TB)_{kij} T_{kij} \right]
\]

**Constraints:**

In addition to non-negativity constraints placed on each of the continuous decision variables, and the "binary constraints" placed on the binary decision variables we also need to consider the following:

1. Tanker Utilization Constraints:

Only one configuration of a tanker can be used in any one period. Moreover, a tanker can depart from no more than one base, and orbit no more than one AR zone.

\[
\sum_{i=1}^{32} \sum_{j=1}^{78} T_{kij} \leq 1 \quad \forall \ h \in \{1, ..., 500\}
\]

2. Airfield Utilization Constraints:

There are limitations on the number of tankers that an airfield can launch within a period (either due to ramp space, or other airfield restrictions).

\[
\sum_{k=1}^{500} \sum_{i=1}^{32} \sum_{j=1}^{78} T_{kij} \leq (RS)_j \quad \forall \ j \in \{1, ..., 32\}
\]

3. Maximum Orbit Time / Maximum Fuel Offload Boundaries:

Given the amount of fuel a tanker needs to offload there is a limitation on the amount of time it can orbit an AR zone.
In particular if we know:

a. The performance characteristics of tanker \( h \) with ramp fuel \( i \).

b. The distance between departure base \( j \) and AR zone \( k \).

c. The standard fuel requirements such as takeoff, climb, approach, and reserve fuel

Then we can compute constants: \( a_{hik} \), \( b_{hik} \), and \( (\text{MOMF})_{hik} \) such that:

\[
a_{hik} O_{hik} + b_{hik} F_{hik} \leq (\text{MOMF})_{hik}
\]

defines the Maximum Orbit Time/Maximum Fuel Offload Boundary when tanker \( h \), with ramp fuel \( i \), departs from airfield \( j \), and orbits AR zone \( k \). On the other hand, if tanker \( h \), does not depart from airfield \( j \), with ramp fuel \( i \), to orbit AR zone \( k \), then it cannot orbit, and it can not offload fuel! In this case the Maximum Orbit Time / Maximum Fuel Offload Boundary is simply \( \{0, 0\} \). More importantly, if we do not force orbit times, and offload amounts to zero when a tanker, ramp fuel, airfield, AR zone combination is not used, the model could fulfill AR zone demand with out incurring any transit costs. Therefore we need to revise the above constraint so that \( O_{hik} \) and \( F_{hik} \) are forced to zero when \( T_{hik} \) is set to zero. Thus, the revised Maximum Orbit Time / Maximum Fuel Offload Boundary is given by:

\[
a_{hik} O_{hik} + b_{hik} F_{hik} \leq (\text{MOMF})_{hik} T_{hik}
\]

Finally note that there will be a Maximum Orbit Time / Maximum Fuel Offload Boundary for every combination of tanker, ramp fuel, airfield, and AR zone. That is to say, there will be approximately 3,744,000 such constraints: one for every element:

\[
(h, i, j, k) \in \{1, \ldots, 500\} \times \{1, \ldots, 3\} \times \{1, \ldots, 32\} \times \{1, \ldots, 78\}.
\]

4. Minimum Fuel Offload Time Boundaries

Tankers do not start offloading fuel as soon as they arrive at an AR zone. Moreover, fuel is not offloaded continuously from tankers onto receivers. Typically tankers will have to wait for receivers to arrive. And, even when receivers are stacked two or three deep waiting their turn, there is still an amount of time needed to clear a full receiver from the boom, and connect with the an empty one before fuel can start flowing again. Finally, the rate at which fuel flows between tankers and receivers depends both on the tanker, and the receiver.

Unless we formulate a different and possibly much more complex model, we can not include all of these details and have made the following simplifying assumptions:

a. Waiting time between receivers is negligible (i.e. there is no loitering).

b. Connection and disconnection times are negligible.
c. Fuel passes from tankers to receivers at a constant average rate of transfer.

That being said, we can correct some of the error these assumptions introduce into our results by tailoring a Minimum Fuel Offload Time Boundary to each Tanker Type, AR zone pair. Doing this should at least account for the different performance characteristics of tankers, and different demand characteristics of AR zones.

![Diagram showing Minimum and Maximum Orbit Time Boundaries with Fuel Offloaded and Orbit Time axes.]

Specifically if we know the performance characteristics of a tanker $h$, and the demand characteristics of an AR zone $k$ then we can find constants $c_m$, $d_m$, and $(MFOT)_m$ such that:

$$c_m \sum_{i=1}^{32} \sum_{j=1}^{32} O_{hijk} + d_m \sum_{i=1}^{32} \sum_{j=1}^{32} F_{hijk} \geq (MFOT)_m$$

defines the Minimum Fuel Offload Time Boundary. Note in this formulation we are assuming that there will be exactly one ramp weight, airfield combination such that $O_{hijk}$ and $F_{hijk}$ are positive. This is a bad assumption.

While we can assume that there will be no more than one ramp weight, airfield combination, per choice of tanker, and AR zone, such that $O_{hijk}$ and $F_{hijk}$ are positive (the tanker utilization, and Maximum Orbit Time/Maximum Offload Boundary constraints take care of this), we can not assume that at there will be at least one ramp weight, airfield combination, per choice of tanker and AR zone, such that $O_{hijk}$ and $F_{hijk}$ are positive. In fact most for most choices of tanker index $h$, and AR Zone $k$ we expect:

$$c_m \sum_{i=1}^{32} \sum_{j=1}^{32} O_{hijk} + d_m \sum_{i=1}^{32} \sum_{j=1}^{32} F_{hijk} = 0$$

However, observe that we expect the above equation to hold if and only if tanker $h$ is NOT assigned to orbit, and offload fuel at AR zone $k$. In other words the set of constraints proposed above are inconsistent with the rest of the model if and only if:

$$\sum_{i=1}^{32} \sum_{j=1}^{32} T_{hijk} = 0$$
Therefore there the solution to our problem is to revise these constraints so that they are relevant if and only if tanker $h$ is assigned to orbit and offload fuel at AR zone $k$. This can be done if we use the following formulation:

$$c_{hk} \sum_{i=1}^{32} \sum_{j=1}^{32} O_{hkij} + d_{hk} \sum_{i=1}^{32} \sum_{j=1}^{32} F_{hkij} -(MFOT)_{hk} \sum_{i=1}^{32} \sum_{j=1}^{32} T_{hkij} \geq 0$$

Finally note that this set of constraints will contain a element for every $(h,k) \in \{1,...,500\} \times \{1,...,78\}$. Consequently there will be approximately 39,000 such constraints.

5. Demand Constraints

Tankers need to be dispatched from bases to AR zones in such a way that they can provide the orbit Time and pounds of fuel requested at each AR zone. Thus:

$$\sum_{k=1}^{500} \sum_{i=1}^{32} \sum_{j=1}^{32} O_{hkij} \geq (OD) \quad \forall k \in \{1,...,78\}$$

$$\sum_{k=1}^{500} \sum_{i=1}^{32} \sum_{j=1}^{32} F_{hkij} \geq (OD) \quad \forall k \in \{1,...,78\}$$

Section 9: Bibliography


4. ACCOMPLISHMENTS/NEW FINDINGS

See point 3. above

5. PERSONNEL ASSOCIATED WITH THIS RESEARCH

Faculty:

Professor Ervin Y. Rodin (PI)

Graduate Students:

T. Eugene Day
Allen McCoy
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6. PUBLICATIONS

None so far; several submitted for publication.

7. INTERACTIONS/TRANSITIONS
Joint development with AMC/XPY, US TRANSCOM, Scott AFB Hospital, Duke University Medical School and Missouri Baptist Hospital.

8. ATTACHMENTS

PowerPoint presentation