Error Rate Improvement in Underwater MIMO Communications Using Sparse Partial Response Equalization

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Abstract*— Point-to-point links using multiple transmitters and receivers or MIMO (multiple input, multiple output) configurations and associated signal processing at the receiver, can provide significant improvements in both data rate and reliability and is a promising technology for enhancing communications in the band-limited and highly-dynamic underwater acoustic (UWA) channel. Although the underwater channel impulse response extent can span tens to hundreds of symbols, it is generally sparse in nature and well suited for sparse partial response equalization (sPRE). This equalization scheme, which is not restricted to MIMO configurations, does not attempt to suppress intersymbol interference completely, rather it retains residual ISI in a controlled manner. This is accomplished by setting a sparse residual impulse response target generally similar in magnitude and time to the dominant, yet also sparse, arrivals within the actual channel impulse response. The resultant partial response equalizer is followed by a complexity-reduction detection scheme known as "belief propagation (BP)" which is an alternative to the optimal Viterbi or MAP (maximum a-posteriori probability) detector. The complexity of the optimal schemes grows exponentially with the total number of taps, regardless of structure; whereas the complexity of BP grows exponentially only with the non-zero taps. Thus the entire receiver structure, sPRE followed by BP, is suitable for the long, sparse channels since it allows more efficient exploitation of the channel structure. The proposed symbol recovery scheme was applied to data collected during a comprehensive multi-institution MIMO Experiment conducted within the Makai Experiment in 2005 off the northwest coast of Kauai, Hawaii. We have demonstrated a reduction in error rates over receiver algorithms using conventional decision feedback equalization techniques due to increased multipath diversity.

I. Introduction

Iterative cancellation of intersymbol interference (ISI), also known as turbo equalization (TE) for coded systems is proposed in [1]. In this scheme a soft input, soft output channel detector (usually the maximum a-posteriori probability (MAP) detector) is used to detect the channel symbols which are then decoded using another MAP decoder for the error correction code. The MAP detector and the MAP decoder then exchange soft (probabilistic) information iteratively to reduce the error rates gradually and obtain significantly improved performance over traditional non-iterative algorithms. However, despite its superior performance, the use of TE is only limited to ISI channels with short memory since the complexity of the MAP detector increases exponentially with the channel memory, making it extremely complex (computationally) for longer ISI channels.

An interesting performance-complexity tradeoff for channels with long memory can be obtained by using a partial response equalizer (PRE) wherein instead of equalizing the channel completely as is done by a linear equalizer or a decision feedback equalizer (DFE), it is equalized to a target impulse response (TIR). In other words instead of removing the ISI completely, the channel energy is concentrated into a smaller number of taps without enhancing the noise significantly. The combination of the original ISI channel and the PRE then resembles an ISI channel with a shorter memory. The PRE is typically followed by the TE scheme which can now be readily implemented due to the smaller memory of the "shortened" channel.

A reduced complexity turbo equalization scheme for multiple input multiple output (MIMO) ISI channels has been proposed in [2], where the authors have designed a MIMO PRE based on the MMSE criterion. The target impulse response is also optimized by maximizing the signal to noise ratio (SNR) at the PRE output under certain constraints on the target tap coefficients. However, there are two key assumptions in the work of [2] and they are: (1) the MIMO ISI channel is quasi-static fading, that is the channel is invariant during the transmission of a code word (frame) and changes independently from one frame to the next, and (2) the channel is known perfectly at the receiver. There is also an implicit assumption that TIR energy is always concentrated in adjacent taps, irrespective of the original channel impulse response structure.

While these assumptions are reasonable for terrestrial wireless environments, they may not apply for certain situations, where the channel is rapidly varying, such as the underwater acoustic (UWA) communication channel. For such channels accurate instantaneous channel estimation or tracking is a computationally intensive and difficult task and adaptive equalization solutions are often preferred. Moreover, the impulse response for the UWA channel is often sparse (arrivals or taps with significant energies are well separated in time), which renders a dense TIR suboptimal.

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14. ABSTRACT

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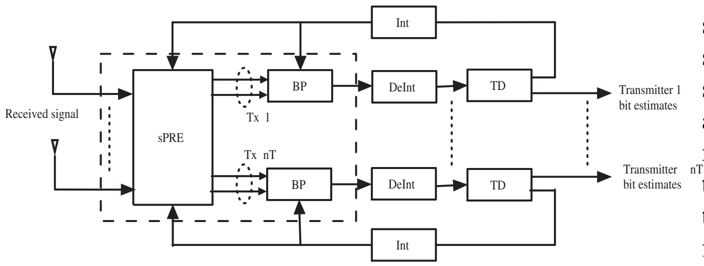

Fig. 1. Iterative sPRE + BP structure.

In this paper we develop a sparse partial response equalizer (sPRE) using the RLS algorithm. Unlike the structure of [2], the proposed structure is training based, fully adaptive, and does not require accurate estimates of the UWA channel impulse response. This adaptive sPRE also adapts the TIR tap magnitudes using the maximum-SNR criterion [2]. Most importantly, the sPRE is designed to have an additional degree of freedom, allowing control over the TIR tap placement, without forcing them to be adjacent. This is particularly useful for shallow water acoustic channels where the impulse response is long, yet sparse. For such channels, the sPRE TIR taps can be chosen to be sparse and made to coincide (in position) with those of the original channel. This allows better exploitation of the multipath diversity inherent in ISI channels by retaining energies in the significant channel taps as opposed to a DFE which treats the secondary arrivals as interference and suppresses them. The sPRE block is followed by a low complexity channel detection scheme known a belief propagation (BP) [3] with exponential complexity only in the number of non-zero sPRE TIR taps. For coded transmission, the sPRE+BP structure is followed by a soft input, soft output channel decoder which exchanges probabilistic information with the sPRE+BP block (turbo equalization) and achieves significant improvements in error rates. Finally, the UWA channel suffers from significant doppler shift which, if not compensated for, causes the adaptive equalizer to diverge [4]. Hence, to make the adaptive sPRE design applicable to real UWA channels, a phase locked loop (PLL) working in conjunction with the sPRE has been developed. The joint sPRE-PLL loop adaptively tracks the channel phase and updates the equalizer coefficient similar to the DFE-PLL proposed in [4].

The rest of the paper is organized as follows. Section II describes the transmission scheme and the discrete time channel model. In Section III the sparse PRE algorithm details along with those of the the embedded PLL are derived. Section IV provides numerical results obtained by monte-carlo simulations along with those obtained by using real at-sea data. Finally the paper concludes in Section V.

II. Transmission Scheme and Discrete Time Channel Model

Consider a MIMO system with n_T transmitters and n_R receivers operating over an ISI channel with $D+1$ symbol spaced taps. At the transmitter, the incoming bit stream is spatially multiplexed to form n_T sub-streams. Each sub-stream is encoded using a turbo code and interleaved using a channel interleaver. Each interleaved sub-stream is then modulated, pulse shaped, and transmitted simultaneously over the n_T transmitters. The discrete-time baseband equivalent of the signal received at the j^{th} receive antenna, after low pass filtering and sampling can be expressed by a tapped delay line model as

$$v_j(n) = \sum_{k=1}^{n_T} \left[\sum_{l=0}^{D} h_{kj}^{(l)}(n) d_k(n-l) \right] e^{j\theta_{kj}(n)} + \eta_j(n) \quad (1)$$

where $h_{kj}^{(l)}(n)$ is the l^{th} tap of the $D+1$ tap impulse response vector from the k^{th} transmitter to the j^{th} receiver at time n, and $d_k(n)$ is the symbol transmitted from the k^{th} transmit antenna at time n. The term $h_{kj}^{(l)}(n)$ captures the small scale time variations of the channel whereas the phase rotation due to doppler shift is treated separately using θ_{kj} [4].

III. Sparse Iterative PRE with Embedded PLL

This section develops the training based adaptive, sPRE-PLL using the RLS algorithm. As explained earlier, the motivation behind using a sPRE is to exploit the multipath diversity inherent in ISI channels by retaining controlled residual ISI at the output of the equalizer, and to minimize the TE complexity [1]. The residual ISI structure is optimized, both in magnitude and tap positions, to resemble the original channel and is further mitigated by using a near optimal (low complexity) graph-based detection algorithm known as belief propagation (BP) algorithm. For channel coded systems, the sPRE structure is designed as a soft input, soft output module, allowing us to perform iterative (turbo) equalization [1].

A. Design of sPRE Filter Coefficients

Consider the discrete-time system model of equation 1. The received signal vector at each time is formed by stacking the received samples from each receive antennas as

$$\mathbf{z}(n) = [v_1(n)\ v_2(n)\ \dots\ v_{n_R}(n)]^T \quad (2)$$

and the transmitted symbol vector at time n is denoted by

$$\mathbf{s}(n) = [d_1(n)\ d_2(n)\ \dots\ d_{n_T}(n)]^T \cdot \quad (3)$$

The design goal is two fold: 1) shorten the original dense channel of length $D+1$ to a shorter sparse channel of memory $D_s+1 << D+1$ (partial response equalization); and 2) obtain estimates of the channel phase θ_{kj} from the k^{th} transmitter to the j^{th} receiver and compensate the channel phase explicitly (phase locked loop).

The adaptive sPRE consists of a bank of n_{Rs} finite impulse response (FIR) filters as shown in Figure 2. The function of each filter is to collapse the original channel of memory D into a much shorter channel of memory $D_s + 1 << D + 1$. Moreover in the sPRE structure we have the flexibility of choosing the number of equalizer outputs as $n_{Rs} \geq n_T$ [2].

For our application we will typically choose $n_{Rs} = Ln_T$, $L \geq 1$. Thus, corresponding to each transmitter we have L sPRE outputs.

Each FIR filter of the sPRE consists of a non-causal part of length N_1 and a causal part of length N_2. Using the received signal vector $\mathbf{z}(n)$ in equation 2 the equalizer input vector $\mathbf{y}(n)$ is formed as

$$\mathbf{y}(n) = \left[\mathbf{z}^T(n+N_1)\ \ldots\ \mathbf{z}^T(n)\ \ldots\ \mathbf{z}^T(n-N_1)\right]^T \qquad (4)$$

where $N = N_1 + N_2 + 1$. Suppose $\hat{\theta}_{ij}$ is the phase estimate of the link from the i^{th} transmitter to the j^{th} receiver at time n. Using this information, the phase compensator matrix for the i^{th} transmit antenna is formed as

$$\Lambda_i(n) = \text{diag}\left[e^{-j\hat{\theta}_{i1}}, \cdots, e^{-j\hat{\theta}_{in_R}} \cdots e^{-j\hat{\theta}_{i1}}, \cdots, e^{-j\hat{\theta}_{in_R}}\right] \cdot (5)$$

The phase compensated signal vector for the i^{th} transmitter is then given by

$$\mathbf{v}_i(n) = \Lambda_i(n)\mathbf{y}(n), \quad 1 \leq i \leq n_T\cdot \qquad (6)$$

As explained earlier, each transmit antenna has L FIR filters. Denoting the l^{th} filter for the i^{th} transmit antenna as $\mathbf{w}(i,l)$, the corresponding sPRE output is given by

$$\tilde{y}_n(i,l) = \mathbf{w}_n(i,l)\mathbf{v}_i(n)\cdot \qquad (7)$$

The aim of the sPRE design can now be more formally stated as

- adaptively compute $\mathbf{w}_n(i,l)$ such that the equalizer output vector $\tilde{y}_n(i,l)$ is equalized to a TIR;
- optimally design the TIR.

The shortened TIR vectors are denoted as $\mathbf{h}_n^{(s)}(i,l)$ (row vector of length $D_s + 1$) which represents the residual ISI at the l^{th} output of the i^{th} transmitter. Moreover, if the UWA channel has a sparse structure (significant energy only in a few well-separated taps), we propose to design the TIR to be sparse as well, with the non-zero taps made to coincide with the dominant taps of the original channel. Note that an accurate estimate of the original channel is not needed for this purpose. Only a crude measurement of the power-delay profile, obtained by channel probing suffices to choose the delays in the TIR. Let the position of the non-zeros TIR taps be denoted by the vector $[\tau_0\ \tau_1 \cdots \tau_{D_s}]$ (i.e., the i^{th} entry of the TIR vector corresponds to the delay τ_i). Thus the output of the sPRE corresponding to the i^{th} transmitter consist of residual ISI from the symbols $d_i(n-\tau_0)$, $d_i(n-\tau_1)$ up to $d_i(n-\tau_{D_s})$.

Let us denote the shortened data vector for transmitter i as

$$\mathbf{x}_i^{(s)}(n) = [d_i(n-\tau_0), \cdots, d_i(n-\tau_{D_s})]^T\cdot \qquad (8)$$

Thus the sPRE output for the i^{th} transmitter has residual ISI from the symbols contained in $\mathbf{x}_i^{(s)}(n)$. Using the above notations, the output of the sPRE can be written as

$$\tilde{y}_n(i,l) = \mathbf{h}_n^{(s)}(i,l)\mathbf{x}_i^{(s)}(n) + \eta'(n)\cdot \qquad (9)$$

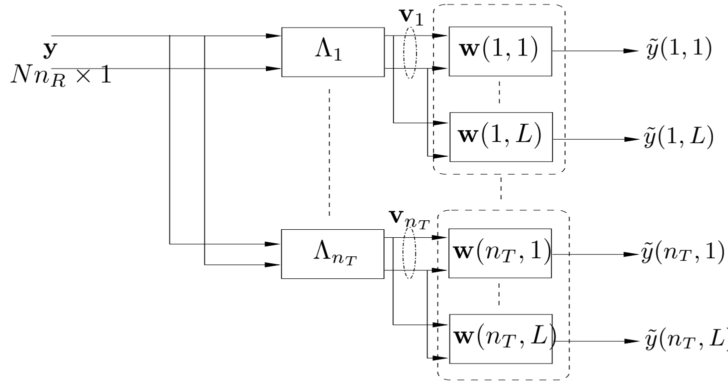

Fig. 2. Adaptive sPRE structure.

In order to apply the RLS algorithm to compute $\mathbf{w}_n(i,l)$ and $\mathbf{h}_n^{(s)}(i,l)$, we define the error signal at the $(i,l)^{th}$ output of the sPRE as

$$\begin{aligned} e_n(i,l) &= \mathbf{h}_n^{(s)}(i,l)\mathbf{x}_i^{(s)}(n) - \tilde{y}_n(i,l) \\ &= \mathbf{h}_n^{(s)}(i,l)\mathbf{x}_i^{(s)}(n) - \mathbf{w}_n(i,l)\mathbf{v}_i(n) \qquad (10) \end{aligned}$$

and form the error vector as $\mathbf{e}_n = [(e_n(1,1), \cdots, e_n(1,L)) \cdots (e_n(n_T,1), \cdots, e_n(n_T,L))]^T$. The cost function to be minimized is defined as

$$\varepsilon(n) = \sum_{k=1}^{n} \lambda^{n-k}\mathbf{e}_k^H\mathbf{e}_k\cdot \qquad (11)$$

The optimum solution in the least squares sense is given by

$$\mathbf{w}_n^H(i,l) = \mathbf{\Phi}_{\boldsymbol{i}}^{-1}(n)\ \boldsymbol{\theta}_{\boldsymbol{i}}(n)\ \mathbf{h}_n^{(s)H}(i,l), \quad 1 \leq i \leq n_T, 1 \leq l \leq L \ (12)$$

where

$$\mathbf{\Phi}_{\boldsymbol{i}}(n) = \sum_{k=1}^{n} \lambda^{n-k}\mathbf{v}_i(k)\mathbf{v}_i^H(k) \qquad (13)$$

$$\boldsymbol{\theta}_{\boldsymbol{i}}(n) = \sum_{k=1}^{n} \lambda^{n-k}\mathbf{v}_i(k)\mathbf{x}_{\mathbf{i}}^{\mathbf{(s)H}}(k) \qquad (14)$$

are the (time averaged) output correlation matrix and the input-output cross correlation matrix respectively. Denoting $\mathbf{\Phi}_{\boldsymbol{i}}{}^{-1}(n) = \mathbf{P}_i(n)$ for simplicity, we have

$$\begin{aligned} \mathbf{w}_n^H(i,l) &= \mathbf{P}_i(n)\ \boldsymbol{\theta}_{\boldsymbol{i}}(n)\ \mathbf{h}_n^{(s)H}(i,l) \\ &= \mathbf{c}_i(n)\ \mathbf{h}_n^{(s)H}(i,l) \quad 1 \leq i \leq n_T, 1 \leq l \leq L \ (15) \end{aligned}$$

where $\mathbf{c}_i(n) = \mathbf{P}_i(n)\ \boldsymbol{\theta}_{\boldsymbol{i}}(n)$. The recursion for $\mathbf{c}_i(n)$ is given by

$$\mathbf{c}_i(n) = \mathbf{c}_i(n-1) + \mathbf{K}_i(n)\boldsymbol{\alpha}_i^H(n), \qquad (16)$$

where $\boldsymbol{\alpha}_i(n)$ is the "a-priori" error vector [5] and $\mathbf{K}_i(n)$ is the RLS gain defined as

$$\boldsymbol{\alpha}_i(n) = \mathbf{x}_i^{(s)}(n) - \mathbf{c}_i^H(n-1)\mathbf{v}_i(n) \qquad (17)$$

$$\mathbf{K}_i(n) = \frac{\mathbf{P}_i(n-1)\mathbf{v}_i(n)}{\lambda_i + \mathbf{v}_i^H(n)\mathbf{P}_i(n-1)\mathbf{v}_i(n)}\cdot \qquad (18)$$

Using equations 13, 14, and the matrix inversion lemma [5], the inverse correlation matrix $\mathbf{P}_i(n)$ can be updated as

$$\mathbf{P}_i(n) = \left[\mathbf{I} - \mathbf{K}_i(n)\mathbf{v}_i^H(n)\right]\mathbf{P}_i(n-1)/\lambda_i\cdot \qquad (19)$$

The shortened data vector $\mathbf{x}_i^{(s)}(n)$ is known during the training mode and is replaced with

$$\tilde{\mathbf{x}}_i^{(s)}(n) = \arg\min_{\mathbf{x}} ||\mathbf{H}_{n-1}^{(s)}(i)\mathbf{x} - \tilde{\mathbf{y}}_n(i)||^2 \quad (20)$$

where

$$\begin{aligned} \mathbf{H}_n^{(s)}(i) &= [\mathbf{h}_n^{(s)}(i,1)^T, \cdots, \mathbf{h}_n^{(s)}(i,L)^T]^T, \\ \tilde{\mathbf{y}}_n(i) &= [\tilde{y}_n(i,1), \cdots, \tilde{y}_n(i,L)]^T \end{aligned} \quad (21)$$

during the decision directed mode. The proposed equalizer is thus versatile since it can always be trained to retain ISI of any desired structure at the equalizer output.

*1) **Soft Input, Soft Output sPRE Module**:* For coded systems, the non-adaptive partial response equalizer proposed in [2] is typically followed by the well known turbo equalization (TE) scheme [1]. The PRE itself however operates only once and is not a part of the TE process. But for our application where the partial response equalization is performed adaptively, the convergence of the iterative algorithm can be significantly improved if the sPRE structure is able to utilize soft information. With this motivation, the vector slicer operation of equation 20 is modified, enabling it to process additional soft information.

At every time instant, the vector slicer of equation 20 provides an estimate of the shortened data vector $\mathbf{x}_i^{(s)}(n)$ as defined in equation 8. For simplicity let us assume that BPSK modulation is used. Thus the entries of $\mathbf{x}_i^{(s)}(n)$ consists of equiprobable ± 1 symbols. Note that the proposed technique is quite general and can be easily extended to any $M-$ary modulation scheme. Also, for notational convenience let us denote the k^{th} entry of $\mathbf{x}_i^{(s)}(n)$ as x_k. In the presence of an outer decoder, the a-priori information about x_k is available as $L_a(x_k) = \log[p(x_k = +1)/p(x_k = -1)]$.

Using this additional information the operation of the vector slicer (equation 20) can be modified as follows. The log likelihood ratio (extrinsic information) of x_k is computed as

$$\begin{aligned} L_e(x_k) &= \log \frac{p(\tilde{\mathbf{y}}_n(i)|x_k = +1)}{p(\tilde{\mathbf{y}}_n(i)|x_k = -1)} \\ &= \log \frac{\sum_{x_k=+1} \exp\left[-\frac{\|\tilde{\mathbf{y}}_n(i) - \mathbf{H}_\mathbf{n}^{(\mathbf{s})}(i)\mathbf{x}_i^{(s)}(n)\|^2}{2\sigma^2} + \sum_{j\neq k} \frac{x_j L_a(x_j)}{2}\right]}{\sum_{x_k=-1} \exp\left[-\frac{\|\tilde{\mathbf{y}}_n(i) - \mathbf{H}_\mathbf{n}^{(\mathbf{s})}(i)\mathbf{x}_i^{(s)}(n)\|^2}{2\sigma^2} + \sum_{j\neq k} \frac{x_j L_a(x_j)}{2}\right]} \end{aligned}$$

After computing the extrinsic information for $\forall x_k$, the hard symbol decision for x_k is made as $\hat{x}_k = \text{sign}\left[L_e(x_k) + L_a(x_k)\right]$ to form the shortened data vector $\mathbf{x}_i^{(s)}(n)$. Note that without the a-priori information ($L_a(x_k) = 0$) the above procedure reduces to the simple vector slicer of equation 20.

B. Design of Optimal Target Impulse Response

We now compute the optimal TIR vectors $\mathbf{h}_n^{(s)}(i,l)$. Using the notation of equation 10, the error vector for transmitter i is defined as

$$\mathbf{e}_n(i) = [e_n(i,1), \cdots, e_n(i,L)]^T = \mathbf{H}_n^{(s)}(i)\mathbf{x}_i^{(s)}(n) - \tilde{\mathbf{y}}_n(i) \quad (22)$$

where $\mathbf{H}_n^{(s)}(i)$ and $\tilde{\mathbf{y}}_n(i)$ are defined in equation 21. The covariance matrix of $\mathbf{e}_n(i)$ is given by

$$\begin{aligned} \mathbf{R}_i &= E\left[\mathbf{e}_n(i)\mathbf{e}_n(i)^H\right] \\ &= \mathbf{H}_n^{(s)}(i)\left[\mathbf{R}_{\mathbf{x_i^{(s)}}\mathbf{x_i^{(s)}}} - \mathbf{R}^{\mathbf{H}}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}\mathbf{R}^{-\mathbf{1}}_{\mathbf{v_i}\mathbf{v_i}}\mathbf{R}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}\right]\mathbf{H}_n^{(s)}(i)^H \end{aligned} \quad (23)$$

where the ensemble averages $\mathbf{R}_{\mathbf{x_i^{(s)}}\mathbf{x^{(s)}}}$, $\mathbf{R}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}(n)$, and $\mathbf{R}_{\mathbf{v_i}\mathbf{v_i}}(n)$ are defined as

$$\begin{aligned} \mathbf{R}_{\mathbf{x_i^{(s)}}\mathbf{x_i^{(s)}}} &= E\left[\mathbf{x}_i^{(s)}\mathbf{x}_i^{(s)H}\right] = \mathbf{I}_{D_s+1} & (24) \\ \mathbf{R}_{\mathbf{v_i}\mathbf{v_i}}(n) &= E\left[\mathbf{v_i}(n)\mathbf{v_i}^H(n)\right] & (25) \\ \mathbf{R}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}(n) &= E\left[\mathbf{v_i}(n)\mathbf{x_i^{(s)H}}(n)\right]\cdot & (26) \end{aligned}$$

Note that, while $\mathbf{R}_{\mathbf{v_i}\mathbf{v_i}}(n)$ and $\mathbf{R}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}(n)$, in general, are different from the time averaged quantities $\mathbf{\Phi}_i(n)$ and $\boldsymbol{\theta}_i(n)$ (equations 13, 14), $\mathbf{R}^{-\mathbf{1}}_{\mathbf{v_i}\mathbf{v_i}}(n)\mathbf{R}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}(n)$ can be well approximated by $\boldsymbol{\Phi_i^{-1}(n)\theta_i}(n)$, if we assume that the transmitted data vectors evolve as an ergodic process. Thus we write

$$\mathbf{R}^{-\mathbf{1}}_{\mathbf{v_i}\mathbf{v_i}}(n)\mathbf{R}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}(n) = \mathbf{P}_i(n)\boldsymbol{\theta}_i(n) = \mathbf{c}_i(n)\cdot \quad (27)$$

Equation 23 can then be written as

$$\mathbf{R}_i = \mathbf{H}_n^{(s)}(i)\left[\mathbf{I} - \mathbf{R}^{\mathbf{H}}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}\mathbf{c}_i(n)\right]\mathbf{H}_n^{(s)}(i)^H = \mathbf{H}_n^{(s)}(i)\hat{\mathbf{R}}_i\mathbf{H}_n^{(s)}(i)^H\cdot$$

The recursion for $\mathbf{R}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}$ can be shown to be

$$\begin{aligned} \mathbf{R}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}(n) &= E\left[\mathbf{v_i}(n)\mathbf{x_i^{(s)H}}(n)\right] \\ &= \frac{n-1}{n}\lambda\, \mathbf{R}_{\mathbf{v_i}\mathbf{x_i^{(s)}}}(n-1) + \frac{1}{n}\mathbf{v_i}(n)\mathbf{x_i^{(s)H}}(n)\cdot \end{aligned}$$

$\mathbf{H}_n^{(s)}(i)$ is then chosen by maximizing trace$[\mathbf{R}_i^{-1}]$. This optimization can be achieved by choosing the rows of $\mathbf{H}_n^{(s)}(i)$ to be the eigenvectors of $\hat{\mathbf{R}}_i$ which diagonalizes $\mathbf{R}_i$. Thus we compute

$$\mathbf{h}_n^{(s)}(i,l) = \mathbf{U}_l^H\cdot \quad (28)$$

where $\mathbf{U}_l$ is the eigenvector corresponding to the l^{th} largest eigenvalue of $\hat{\mathbf{R}}_i$. Finally, we substitute equations 28 and 16 in equation 15 to obtain the optimum sPRE coefficient vector $\mathbf{w}_n^H(i,l)$.

C. Design of PLL

Recall from equation 10 that the error at the $(i,l)^{th}$ sPRE output is given by

$$e_n(i,l) = \mathbf{h}_n^{(s)}(i,l)\mathbf{x}_i^{(s)}(n) - \mathbf{w}_n(i,l)\left[\Lambda_i(n)\mathbf{y}(n)\right]$$

where $\Lambda_i(n)$ is defined in equation 5. The channel phase estimate $\hat{\theta}_{ij}(n)$, $(1 \le i \le n_T,\ 1 \le j \le n_R)$ is obtained by minimizing the cost function

$$\varepsilon_\theta = E[\mathbf{e}^H(n)\mathbf{e}(n)] = \sum_{i=1}^{n_T}\sum_{l=1}^{L} E|e_n(i,l)|^2\cdot \quad (29)$$

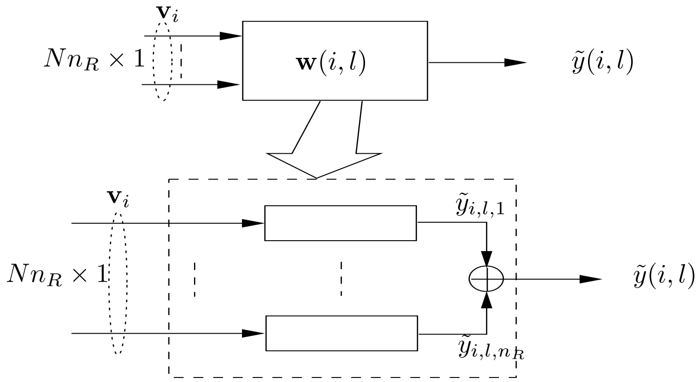

Fig. 3. Breakdown of each PRE block.

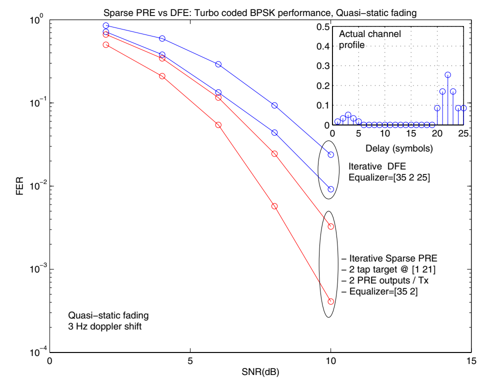

Fig. 4. Simulated frame error rate comparison for 1 transmit, 1 receive antenna.

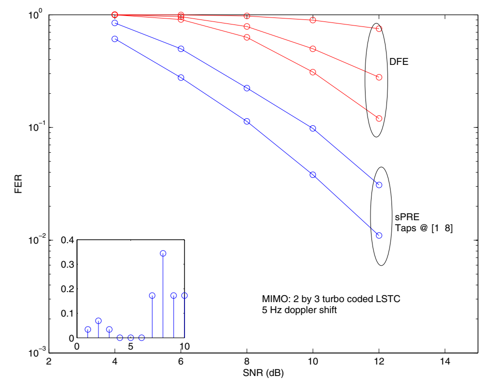

Fig. 5. Simulated frame error rate comparison for 2 transmit, 3 receive antenna. Turbo coded LSTC on each transmitter.

The gradient of ε_θ can be written as

$$\frac{\partial \varepsilon_\theta}{\partial \theta_{ij}} = -E \sum_{l=1}^{L} \Im\left\{e_n^*(i,l)\tilde{y}(i,l,j)\right\} \tag{30}$$

where $\sum_{j=1}^{n_R} \tilde{y}(i,l,j) = \tilde{y}(i,l)$ as shown in Figure 3. The phase update is then computed as (similar to [4])

$$\hat{\theta}_{ij}(n+1) = \hat{\theta}_{ij}(n) + K_{f1}\beta_{ij}(n) + K_{f2}\sum_{m=0}^{n}\beta_{ij}(m), \tag{31}$$

where $\beta_{ij}(n)$ is the instantaneous gradient estimate given by $\beta_{ij}(n) = \sum_{l=1}^{L} \Im\left\{e_n^*(i,l)\tilde{y}(i,l,j)\right\}$.

IV. Results

A. Simulation Results

Numerical results based on monte-carlo simulations are provided next. We compare the performance of the iterative DFE [6] and its MIMO version [7], [8] with the proposed sPRE, under identical conditions. First we consider the single input, single output (SISO) case where at the transmitter, the incoming bit stream is encoded using a rate $1/2$ turbo code with constituent codes having generator polynomials as $[1, 5/7]_{\text{octal}}$. The resultant codeword is then interleaved and BPSK modulated. We consider the quasi-static fading channel case (where the channel assumes a particular realization for a block of symbols and changes independently in the next block). The block length is chosen to be 1000 symbols, which is approximately equivalent to the channel coherence time observed using real data. Moreover, to accurately simulate a shallow water acoustic channel, the original channel impulse response is chosen to be sparse and spans 25 symbols as shown in the sub-figure of Figure 4. At the receiver the sPRE is implemented with a 2 tap TIR, and the taps are positioned at symbols $\tau_0 = 1$ and $\tau_1 = 21$ to approximately coincide with the dominant arrivals of the original channel. The simulated frame error rate (FER) is shown in Figure 4. Note that with optimally chosen tap positions the sPRE provides significant performance improvement over the iterative DFE. This is due to the fact that the controlled, residual ISI present at the sPRE output allows better exploitation of the multipath diversity inherent in ISI channels. The DFE on the other hand tries to suppress the ISI it rather than exploit it. The increased diversity order for the sPRE can thus be observed by noting the rate at which the FER falls with respect to the SNR.

Next we consider the MIMO case with 2 transmit and 3 receive antennas. At the transmitter the incoming bit stream is multiplexed on to the two transmitters and independently encoded using the previously described turbo code. Figure 5 shows the FER performance for both sPRE and DFE. We observe that for this case, in the high SNR regime, there is considerable improvement obtained using the sPRE. This is because for a 2 by 3 system there is large diversity order already present in the system. The sPRE adds to that by exploiting an additional degree of freedom (multipath diversity) from the channel, resulting in very high overall diversity order. This, coupled with iterative equalization results in significant performance improvement over DFE.

B. Experimental Results using Real Data

In this sub-section, results are presented using real data collected during a multi-institutional experiment conducted off the coast of Kauai, Hawaii in September 2005. For this case we present the decoding results for turbo coded SISO packets. Figure 6 shows the convergence behavior of the DFE and the sPRE in the high frequency regime (near 50 kHz).

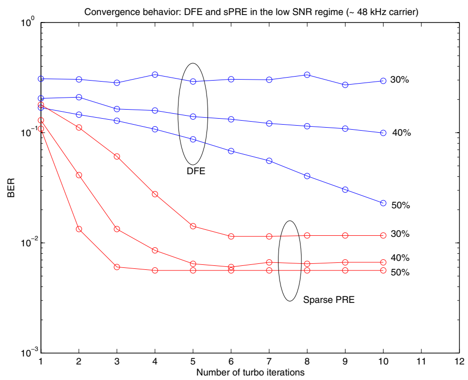

Fig. 6. Turbo equalizer convergence behavior comparison using real data in the low SNR regime (48 kHz carrier) for different training lengths. sPRE tap position: [1,6].

TABLE I

DECODING RESULT USING REAL DATA FOR TURBO CODED SINGLE TRANSMITTER DATA PACKET TRANSMITTED AT 40.5 KHZ CARRIER. MULTIPLE NUMBERS IN A COLUMN INDICATES MULTIPLE TURBO ITERATIONS.

% training	DFE # bit errors/4800	sPRE # bit errors/4800 2 tap TIR placed @ [1 6]
25	219, 0	0, 0
20	228, 0	0, 0
15	691, 358, 0	0, 0
10	510, 76, 0	0, 0
2	failed, 50 % errors	868, 0
1	failed, 50 % errors	604, 0
0.4	failed, 50 % errors	774, 0

The operating SNR in this regime is significantly low due to frequency dependent attenuation. For the sPRE, the tap positions are determined by examining the channel power-delay profile estimates obtained using periodically transmitted channel probes. The performance improvement observed by simulations is evident with real data also, wherein Figure 6 shows that the sPRE converges much faster (lower decoding latency) and with significantly less training than the DFE (higher overall throughput).

Next, we present the decoding results for the medium-to-low SNR regime (near 40 kHz) in Table I. The channel profile for this case exhibited a dominant secondary arrival at a delay of 6 symbols and a weaker third arrival near the 20^{th} symbol. The sPRE for this case is implemented with a TIR placed at $[1, 6]$. A 3-tap TIR placed at $[1, 6, 20]$ was also tried but since it did not provide significant gains over the 2-tap TIR, its results are omitted. From Table I we observe that the sPRE is able to decode successfully with considerably less training than the DFE. Whereas the DFE required nearly 10% training, the sPRE is able to clear the packet of errors even at 0.4% training. Moreover, for higher training lengths ($> 15\%$) we observe that even though both schemes provide zero errors, the sPRE does so after the very first turbo iteration, while the DFE requires to iterate two or three times before it can successfully clear all the errors.

V. CONCLUSIONS

In this paper we have proposed an adaptive and iterative MIMO sparse partial response equalization (sPRE) scheme for shallow water UWA channels, along with a jointly optimized phase tracking loop. The proposed scheme exploits the unique long and sparse structure of the UWA channel and attempts to extract multipath diversity by retaining controlled residual ISI at the output of the equalizer. The residual ISI structure is chosen to be sparse, to approximately coincide with the structure of the original channel. The proposed structure is also designed to be soft input, soft output. The residual ISI is further mitigated by using a low complexity, near optimal detection algorithm known as the "belief propagation" (BP) scheme. The sPRE+BP scheme is followed by the well known turbo equalization block in order to reduce the error rates gradually. The proposed scheme is compared with the canonical iterative MIMO DFE via simulations and also using real shallow water data. It is found that for channels with long and sparse structure, the sPRE significantly outperforms the DFE in terms of error rates. Moreover, it is shown using real data that the sPRE requires much fewer training symbols than the DFE and also converges much faster (fewer turbo iterations) compared to the DFE, making it an attractive low latency, high throughput alternative to the DFE for shallow water acousticng channels.

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