Assessing Scour Model Performance with Experimental Data

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Abstract—Equations by Soulsby (Dynamics of Marine Sands, Thomas Teleford: London, 1997) and Whitehouse (Scour at Marine Structures, Thomas Teleford: London, 1998) have been applied to predicting mine burial by scour in tidal estuaries. The main equation, which has the functional form of \[1 - \exp(-t/T)\], computes the depth of the scour pit with time under steady state conditions. This theory may be applied to changing conditions by using the RMS values of the frictional stress at the bed and assuming a quasi-steady state of the RMS values over a small time period. From 1999 – 2002, the Naval Research Laboratory conducted scour burial experiments using instrumented mines that measure mine motion (heading, roll, and pitch) and percent burial (surface area covered with sediment). Using oceanographic and sediment data obtained during these experiments, this study examines how well the predictions match the mine burial measurements.

I. INTRODUCTION

The Navy has recently shown increased interest in predicting mine burial, both because of mining activities in the Persian Gulf and because of the state of prediction has arguably been poor [1]. For sandy environments, mine burial often occurs from scour - the erosion of sediment around an obstacle due to intensified shear stresses and vortices caused by the object’s presence. (Other burial processes can include sand ridge migration, liquefaction, and global sediment transport; impact burial usually is insignificant for sandy bottoms.) The most commonly used scour mine burial models, until recently, were based on equations presented in a 1963 report by Carsten and Martens [2]. Their measured rates of burial are based on scaled laboratory experiments, which may not represent in situ conditions where the mines are much larger than mean grain size or rippled bed forms.

A newer and more promising scour burial model has recently been identified by Friedrichs [3]. The model comes from a set of equations developed by HR Wallingford Ltd. (a United Kingdom civil engineering firm) and presented in books by Soulsby [4] and Whitehouse [5]. It was developed to predict scour in steady state conditions, but has been adapted to changing environments. This adaptation is made by assuming a quasi-steady state of root-mean-squared bottom orbital velocities and/or bottom currents over a small time period and evaluating model output in a time-stepped manner.

Tests of this model have been performed using data collected from recent in situ experiments with an instrumented mine capable of recording burial over a period of a few months. Data from one of these deployments was used by Friedrichs [3] to call attention to the potential usefulness of the model. Comparison with data from two other deployments has shown similarly positive results. This paper will present the theory for this model, followed by comparison of the model output with experimental data as applied to burial of a cylindrical free-body in shallow sandy oceanic environments.

II. THEORY

The depth of the scour pit, \(S(t)\), for steady state flow after an amount of time, \(t\), has been determined [6] empirically to be

\[
S(t) = S_\infty \left(1 - \exp\left(-\frac{t}{T}\right)\right)^p,
\]

where \(S_\infty\) is the scour pit depth after an infinite amount of time, \(T\) governs the rate of scour pit growth, and \(p\) is determined by the geometry of the scouring body. For a 5:1 cylinder, \(p = 0.6\) [7]. The other two variables \((S_\infty, T)\) are calculated from oceanographic conditions and physical properties of the sediment. The next two subsections describe the sequence of calculations required to obtain \(S_\infty\) and \(T\) at each time step, using the statistics of the oceanographic conditions for that time period.

A. Calculating \(S_\infty\)

1) Basic Relation

Whitehouse [7] gives the value of \(S_\infty\) as a relationship between the velocity at the sediment bed, \(U\), relative to the velocity required for the sediment to become mobile, \(U_{cr}\), (the critical velocity) in the following manner

\[
S_\infty = \begin{cases} 
0, & U < 0.75U_{cr} \\
1.15D_0(2U - 1.5U_{cr})/U_{cr}, & 0.75U_{cr} < U < 1.25U_{cr} \\
1.15D_0, & U > 1.25U_{cr} 
\end{cases}
\]

where \(D_0\) is the diameter of the mine.

It is more convenient, however, to rewrite (2) so that the magnitude of the stress at the sediment bed, \(\tau = \rho_w C_r U^2\), is compared to the critical stress, \(\tau_{cr} = \rho_w C_r U_{cr}^2\), required for the sediment to become mobile. (In these relations, \(\rho_w\) is the density of water at the sediment interface and \(C_r\) is a dimensionless mechanical resistance constant related to the roughness of the seafloor and strength of the flow.) The reasons for this change are that the critical stress has been deduced from laboratory experiments, the stress at the seabed...
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**Supplementary Notes**


**Abstract**

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can be calculated from currents and oceanographic conditions, and the critical velocity has to be obtained from knowledge of \( C_r \), which is difficult to assess for \( U = U_c \).

The stresses are evaluated in terms of the non-dimensional Shields parameter, \( \theta \),

\[
\theta = \frac{\tau}{(\rho_s - \rho_w)gdS_0}, \tag{3}
\]

where \( \rho_s \) is the average bulk density of the sediment, \( g \) is gravitational acceleration, and \( d_{so} \) is the median diameter of the sediment particles. For \( \tau = \tau_{cr} \), the critical Shield’s parameter is \( \theta_{cr} \). It can be seen from the above relationships between stress and velocity and stress and the Shield’s parameter that

\[
\theta_{cr} = \frac{1}{2} \frac{\tau_{cr}}{U_{cr}^2}.
\]

Rewriting (2) in terms of the Shield’s parameters, one obtains

\[
S_{\alpha} = \begin{cases} 
0, & \frac{\sqrt{\theta/\theta_{cr} < 0.75}}{1.5} \sqrt{\theta/\theta_{cr} < 1.25}, \\
1.15D_0 \left(2\sqrt{\theta/\theta_{cr} < 1.5}\right), & \frac{\sqrt{\theta/\theta_{cr} > 1.25}}{1.15D_0}.
\end{cases} \tag{4}
\]

The critical Shield’s parameter has been evaluated empirically to be [8]

\[
\theta_{cr} = \frac{0.3}{1 + 1.2D_0} + 0.055\left(1 - \exp(-0.02D_0)\right) \tag{5}
\]

where

\[
D_0 = d_{50} \left((s-1)g/\nu^2\right)^{1/3}. \tag{6}
\]

\( s = \rho_s/\rho_w \), and \( \nu \) is the kinematic viscosity of seawater.

2) Obtaining \( \theta \) from Oceanographic Conditions

The last remaining unknown in (4) is \( \theta \). Soulsby [9] provides the following methodology for obtaining total stress, \( \tau \), which gives \( \theta \) by (3). The total stress is obtained from the stresses induced by wave motion, \( \tau_w \), and by current, \( \tau_c \). The following equation relates these stresses to the total stress at the seabed,

\[
\tau = \left[(\tau_m + \tau_w \cos \phi)^2 + (\tau_w \sin \phi)^2\right]^{0.5}, \tag{7}
\]

where \( \phi \) is the angle between the current stress and wave stress vectors and

\[
\tau_m = \tau_c \left[1 + 1.2\left(\frac{\tau_w}{\tau_c + \tau_w}\right)^{3.2}\right]. \tag{8}
\]

The quantity \( \tau_m \) is the mean shear stress induced by waves and currents. (Equation (5) accounts for nonlinear interactions between \( \tau_w \) and \( \tau_c \); \( \tau_m \) is in the same vector direction as \( \tau_c \).) In (5), \( \tau_c \) is calculated from the assumption

\[
\tau_c = \rho_w C_D U^2, \tag{9}
\]

where \( U \) is the depth averaged current velocity evaluated from

\[
U(z) = \begin{cases} 
(z/0.32h)^{1/7} U, & 0 < z < 0.5h, \\
1.07U, & 0.5h < z < h.
\end{cases} \tag{10}
\]

and \( C_D \) is the drag coefficient given by

\[
C_D = \left[\frac{0.4}{1 + \ln(12h/d_{50})}\right]^2, \tag{11}
\]

where \( h \) is water depth. The other stress, \( \tau_w \), is assumed to have a form identical to (9). Depth averaged current velocity is replaced with the orbital velocity, \( U_b \), and the drag coefficient is replaced by a wave friction factor, \( 0.5f_w \), so that

\[
\tau_w = 0.5\rho_w f_w U_b^2, \tag{12}
\]

where

\[
f_w = 1.39 \cdot \left(6U_b T_w / \pi d_{50}\right)^{-0.52}, \tag{13}
\]

and \( T_w \) is the wave period.

B. Obtaining \( T \)

Once \( S_{\alpha} \) is evaluated for the current time step from (2), \( T \) is the only remaining variable in (1) needing to be evaluated. An empirical relation for \( T \) has been found to be [6, 10]

\[
T = T^* \left((s-1)g^3d_{50}^3\right)^{-1/2} D^2, \tag{14}
\]

where

\[
T^* = A \theta_{\infty}^B, \tag{15}
\]

\( A = 0.095 \) and \( B = -2.02 \) for a free cylinder [7]. The \( \infty \) subscript on the Shield’s parameter means that this quantity is evaluated for when the mine is absent. Placing \( \theta \) in for \( \theta_{\infty} \) in (15) and putting the result into (14) produces

\[
T = A \theta^B \left((s-1)g^3d_{50}^3\right)^{-1/2} D^2. \tag{16}
\]
C. Time stepping $S(t)$ to calculate mine burial and re-exposure

Now that (1) can be evaluated, it needs to be re-expressed in a time-stepped fashion to make it applicable to quasi steady-state conditions. The procedure developed by Friedrichs [3, 11] will be used for this purpose. At the $j^{th}$ time step, the quasi steady-state starts at $t = t_j$ and ends at $t = t_{j+1}$, where $\Delta t$ is the size of the time step. At $t = t_j$, the values for $S(t)$, $T$ and $S_{\infty}$ are $S(t_j)$, $T(t_j)$ and $S_{\infty}(t_j)$. Thus, (1) becomes

$$S(t_{j+1}) = S_{\infty}(t_j) \left[ 1 - \exp \left( - \frac{t_{j} + \Delta t}{T(t_j)} \right)^p \right]. \quad (17)$$

The appropriate value to use for $t$ is the time it would take for $S(t) = S(t_j)$ under a steady state condition with zero initial scour, $T = T(t_j)$ and $S_{\infty} = S_{\infty}(t_j)$.

$$t = -T(t_j) \ln \left[ 1 - \frac{S(t_j)}{S_{\infty}(t_j)} \right]^p \quad (18)$$

This condition calculates the process along the appropriate portion of the $S(t)$ curve. Equation (18), however, assumes that $S(t_j) < S_{\infty}(t_j)$. ($S(t_j) \geq S_{\infty}(t_j)$ gives a complex time, which is physically unmeaningful.) This condition will be false when the scouring current is weakening because $S(t_j)$ was calculated at the end of the previous time step while $S_{\infty}(t_j)$ is applicable to the present time step. In this instance, $S(t_{j+1}) = S_{\infty}(t_j)$ is set. Hence, (18) is rewritten as follows

$$S(t_{j+1}) = \begin{cases} S_{\infty}(t_j) \left[ 1 - \exp \left( - \frac{t_{j} + \Delta t}{T(t_j)} \right)^p \right], & S(t_j) < S_{\infty}(t_j) \\ S_{\infty}(t_j), & S(t_j) \geq S_{\infty}(t_j) \end{cases} \quad (19)$$

To predict the amount of mine burial, $B_D$, it is assumed that the mine immediately settles to the bottom of the scour pit as the pit grows. Thus, $B_D(t_j)$ is the maximum value of $S(t < t_j)$, which will be denoted $\max \{ S(t < t_j) \}$, if no infilling occurs. The depth of the scour hole, however, depends on infill once the currents subside. The function $S(t)$ gives the instantaneous scour depth at time $t = t_j$. When $0 < S(t_j) < B_D$, then the scour process is re-exposing the mine. Thus, the burial of the mine under this assumption is given by

$$B_D(t_j) = \max \{ S(t_j \leq t) \} - \alpha S(t_j) \quad (20)$$

where $\alpha$ is the efficiency of the scour process to expose the mine, assumed to be 0.6 [3].

III. COMPARISON WITH EXPERIMENTS

Three experiments, one at Scripps Institute of Oceanography in 1999 and two at Martha’s Vineyard Coastal Observatory (MVCO) in 2001 and 2002, have used an instrumented mine (Fig. 1) to measure burial by scour and other post-impact processes. Environmental and geotechnical data required to run the scour model were obtained during these experiments. Both sites had fine sand bottoms (median grain sizes: Scripps, 190 microns; MVCO, 180 microns). This section presents results from these measurements and model predictions. The specifics of the equipment used and the numerical methods employed will be discussed first, followed by measurements and modeling results.

A. Experimental Set-Up

The mine was designed by NRL and OMNI technologies of New Orleans based on an instrumented mine developed by Ingo Stender of Forschungsanstalt der Bundeswehr für Wasserschall- und Geophysik (FWG) in Kiel, Germany [12]. Heading (+/- 1°) is measured with three solid-state compasses and roll and pitch (+/- 1°) of the mine are measured with a three-axis accelerometer. Burial is measured by three rings of paired optical sensors externally mounted at 15° intervals around the mine. Transmitting optical sensors are LED’s and receiving sensors are phototransistors. Burial is measured by blockage between these 72 pairs of sensors. The mine is made of aluminum and is 1.5 m long and 0.47 m in diameter. The weight in air is 619 kg and in water is 357 kg, but these weights are adjustable. For the Scripps Pier experiment, data was stored on a hard drive inside the mine. During the MVCO experiments, measurements were made every 2-5 minutes and transmitted to a shore-side computer.

![Fig. 1. The NRL instrumented mine deployed at Martha’s Vineyard Coastal Observatory](image-url)
B. Numerical Techniques

The scour model was coded to run on a PC in MATLAB. The code used was adapted from the original version written by Friedrichs [3, 11]. Time series inputs required for the models are orbital velocity and wave period; currents are assumed to cause an insignificant effect. For the Scripps Pier deployment, orbital velocity was calculated from wave height and period data using linear wave theory [8]. The MVCO model runs used velocity data and wave period measured by ADCP and ADV sensors. In addition, a sector-scan sonar (SSS) was deployed at MVCO for continuous recording of acoustic imagery of the mine and surrounding seabed [13].

The time step was chosen to be 20 minutes. If data was missing at appropriate intervals, spline interpolation was used to fill in missing values. For the MVCO runs, two sets of calculations were made. One used wave period data to calculate the wave friction factor given by (13). The other run used a constant wave friction factor of 0.008. The Scripps Pier model run used the calculated wave friction factor only. In all cases, burial was assumed to be no less than 10%.

C. Scripps Results

The instrumented mine was deployed off the Scripps research pier in 8 meters of water from 25 July to 19 September, 1999 [14]. Wave energy, although low, was sufficient to induce scour. Figure 2 (top part) shows the time series data for mean wave height and the deduced mean orbital velocity. The bottom shows the burial data obtained by the average blockage of light from the 3 sets of rings.

Both prediction and data show that burial occurs just after a peak in the orbital velocity. The theory assumes that the pit immediately refills after peaks in the orbital velocities, whereas data suggest that burial occurs at a much slower rate, often over a period of 3-10 days. Nevertheless, the overall level of burial appears to roughly match the experimental data.

A few issues became apparent during this deployment. The model assumes that the mine scours equally about the surface and will sink levelly into the sand. Bed armoring, however, was evident under the end at ring 3 resulting from winnowing of sand and scavenging of cobblesized gravel. This bed armoring made further scour improbable on that end and skewed the overall results toward less burial. In addition, onshore migration of sand after the first week increased burial. The model does not account for these processes.

D. Martha’s Vineyard Result

Two mine deployments were performed at MVCO, which is located offshore of the southern coastline of Martha’s Vineyard. The depth at the mine was 12 meters. The first deployment (Fig. 3) took place from 5 December 2001 to 22 January 2002 [13]. The second (Fig. 4) took place from 5 April to 28 May 2002.

Mine burial predictions for each deployment follow experimental results reasonably; an exception is notable at the end of the first MVCO deployment where more re-exposure is predicted than is observed. From an examination of the SSS imagery, this re-exposure appears to have been caused by 10 cm changes in bed elevation.

The first deployment was the more energetic of the two, with complete burial achieved during a stormy event that occurred between 7 January and 11 January. Complete burial was confirmed by the SSS and diver observations. The mine only buried to 60% during the second deployment but problems with interpreting the data from ring 3 may have compromised the results and the burial may be more than reported in Fig. 4. Unlike the Scripps data, however, mine burial occurs much more rapidly, agreeing better in this regard with the model output.

IV. DISCUSSION

These experiments show that the HR Wallingford equations appear to be promising for predicting the scour of a free body cylinder in sand with grain sizes on the order of 200 microns. They also show that other burial processes can significantly influence overall mine burial. These include changes in beach slope (onshore sediment transport), bed armoring at the Scripps pier and migrating sand ridges at MVCO.

Additional experiments on coarse sand and on sediments with mixtures of sand and mud are needed to validate the model. Additional observations by acoustic methods (SSS) are needed to gain greater insight into the development and size of scour pit and rates of infilling. In addition, work is required to assess the applicability to a wider variety of scenarios. These deployments only tested the applicability of a cylinder on fine sands. Experiments on coarser sands also need to be run and analyzed. In addition, the geometrically determined parameters, $p$ from (1) and $A$ and $B$ from (16), are given only for 5:1 cylinders, pipelines and pilings. Work is required to determine what these parameters may be for other mine geometries. The usefulness of including the tidal currents also needs to be addressed.
Fig. 3. Mine burial data and model output for the first deployment at MVCO, Winter 2001-2002. Top: Oceanographic data. Bottom: mine burial data and model output.

Fig. 4. Mine burial data and model output for the second deployment at MVCO, Spring 2002. Top: Oceanographic data. Bottom: mine burial data and model output.

V. SUMMARY

The HR Wallingford equations have been applied to predicting cylindrical mine burial in fine sands and tested against data from three experiments. In these experiments, the predictions and data generally match. Problems have been identified for future work on the model’s development. In its present state, it appears mature enough for use as a prediction tool for cylinders in fine sands.

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REFERENCES


[9] Reference [3], Ch. 4-6.


