Research on Nonlinear and Stochastic Dynamics with Defense Applications

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The objectives of project were: (1) inducing chaos in electronic circuits and applications and (2) stochastic resonance and applications. In close collaboration with AFRL scientists, we addressed a number of basic issues associated with our previously developed method of resonant perturbations for inducing chaos. We also explored some fundamental issues in stochastic resonance, focusing on the interplay between phase synchronization and the resonance. We initiated a new area of research: nonlinear dynamics and chaos in micro-electro-mechanical systems. In addition, we obtained results on low-dimensional chaotic systems, chaotic scattering, spatially-extended dynamical systems, and synchronization in complex systems. The outcomes of the project were 38 papers published in, accepted by, or submitted to refereed journals, with a number of collaborative papers with AFRL scientists. One Ph.D. and four Master students graduated under full or partial support from the project, and 14 invited talks on research derived from the project were given.

Nonlinear Dynamics, Inducing Chaos, Electronics Circuits, Stochastic Resonance, MEM systems
Final Report

This report summarizes activities under the Air Force Office of Scientific Research (AFOSR) Grant No. FA9550-06-1-0024 entitled “Research on Nonlinear and Stochastic Dynamics with Defense Applications.” The duration of the project was 1/1/2006-12/31/2008. The report is divided into the following Sections:

1. Objectives
2. Accomplishments and New Findings
3. Personnel Supported and Theses Supervised by PI
4. List of Publications
5. Interactions/Transitions
6. Past Honors
1 Objectives

- Inducing chaos in electronic circuits and applications;
- Stochastic resonance and applications.

2 Accomplishments and New Findings

2.1 Inducing chaos in electronic circuits


We demonstrated previously that resonant perturbations with time-varying frequency and phase can induce chaotic attractors in stable nonlinear circuits. A theoretical issue concerns the effect of resonant-frequency mismatch. We have found that resonant-frequency mismatch can result in attractors that are nonchaotic but are geometrically strange in the sense that they possess no positive Lyapunov exponent but their information dimensions measured using finite numerics assume fractional values. The transition to such "pseudo-strange" attractors as a system parameter changes can be understood analytically using the framework of nonstationary dynamical systems.

A question of practical interest is whether the resonant-perturbation approach can be effective for inducing chaos in MOSFET (Metal-Oxide-Semiconductor Field-Effect Transistor) circuits, the basic components of modern electronic devices. We have extended our work to this type of generic circuits. In particular, we consider situations where chaos is desired, under the following three constraints: (1) the circuit operates in a stable periodic regime, (2) no parameters or state variables of the circuit are directly accessible to adjustment, and (3) the circuit equations are not available. We have constructed a nonlinear MOSFET-based electronic circuit, utilized phase-locked loops to realize resonant perturbations with both frequency and phase match, and demonstrated experimentally that chaos can be induced in the circuit.

2.2 Stochastic resonance

There has been ample experimental evidence that a variety of biological systems use the mechanism of stochastic resonance for tasks such as prey capture and sensory information processing. Traditional quantities for the characterization of stochastic resonance, such as the signal-to-noise ratio, possess a low noise sensitivity in the sense that they vary slowly about the optimal noise level. In order to tune to this level for improved system performance in a noisy environment, a high sensitivity to noise is required. We have shown that, when the resonance is understood as manifestation of phase synchronization, the average synchronization time between the input and the output signal has an extremely high noise sensitivity in that it exhibits a cusp-like behavior about the optimal noise level. We have worked out an analytic theory for this behavior and obtained the first experimental evidence by using a bistable microelectronic-circuit system.

In real-world signal-processing applications based on stochastic resonance, sensitivity to frequency variations is of interest. We have investigated the dependence of the averaged phase-synchronization time on the frequency of the input signal and found that, for typical nonlinear oscillator systems, there can be a frequency regime where the time exhibits significant sensitivity to frequency variations. The finding is substantiated by an analytic theory, numerical support, and experimental evidence from a class of nonlinear switch circuits.

We have also investigated a class of nonlinear wave equations subject to periodic forcing and noise, and addressed the issue of energy optimization. Numerically, we have used the pseudo-spectral method to solve the nonlinear stochastic partial differential equation and compute the energy of the system as a function of the driving amplitude in the presence of noise. In the fairly general setting where the system possesses two coexisting states, one with low and another with high energy, noise can induce intermittent switchings between the two states. A striking finding is that, for fixed noise, the system energy can be optimized by the external driving in the form of resonance. The phenomenon can be explained by the Langevin dynamics of particle motion in a double-well potential system with symmetry breaking. Applicable situations where the finding is relevant include small-size devices such as microelectromechanical (MEM) resonators and to waves in fluid and plasma.

2.3 Nonlinear dynamics in MEM systems


Small-sized systems such as MEM resonators have become common in many fields of science and engineering. These systems have a relatively simple structure but they show surprisingly rich nonlinear dynamical behaviors such as bistability, chaos, and energy-localized oscillations. For a MEM resonator, the combination of mechanical nonlinearity and electrical driving force can lead to bistability. In such a case, the system exhibits two coexisting stable oscillatory states (attractors): one with low and another with high energy. Under the influence of noise, with high probability the system can be perturbed into the low-energy state. We have proposed a robust control scheme to place the system in the high-energy
state. Our idea is not to pull the system out of the bistable regime but instead to take advantage of the nonlinear dynamics to achieve high-energy output. In particular, our control scheme consists of two steps: bifurcation control that temporarily drive the system to a regime with only one attractor, one that is the continuation of the high-energy attractor in the bistable regime, and ramping parameter control that restores the bistability while maintaining the system in the high-energy attractor. We have derived an analytic theory to guide the control, provided numerical examples, and suggested a practical method to realize the control experimentally. Our result may find potential usage in devices based on MEM resonators where high output energy is desired.

For a resonator in an electrostatic MEM system, nonlinear coupling between applied electrostatic force and the mechanical motion of the resonator can lead to chaotic oscillations. Better performance of the device can be achieved when the oscillations are periodic with large amplitude. We have investigated the nonlinear dynamics of a system of deformable, doubly clamped beam, the core in many MEM resonators, and proposed a control strategy to convert chaos into periodic motions with enhanced output energy. Chaos control can thus lead to high performance of MEM devices.

Experimental evidence of intrinsic localized modes (ILMs) in MEM oscillator arrays has been reported recently. We have carried out a detailed analysis of a new mechanism for ILMs in typical experimental settings, i.e., spatiotemporal chaos is ubiquitous and it provides a natural platform for actual realization of various ILMs through frequency control. We have found that unstable periodic orbits corresponding to ILMs are pivotal for spatiotemporal chaos to arise and these orbits are the keys to stabilizing ILMs from spatiotemporal chaos by frequency modulation. Based on these results, we have articulated a scheme to induce ILMs at arbitrary bi-element cells in driven micro-oscillator arrays. The idea is to locate the particular oscillator in the array that one wishes to drive to an oscillating state with significantly higher amplitude than the average, and then apply small adjustments to the electrical signal that drives the whole array system. Our scheme is thus global, which is advantageous in micro-systems as local pinning may be difficult to implement at small scales. The control strategy can be potentially useful for DoD applications to disable MEM systems embedded in adversarial devices.

2.4 Chaotic scattering


Chaotic scattering in open Hamiltonian systems under weak dissipation is not only of fundamental interest but also important for problems such as advection and transport of inertial particles in fluid flows. Previous work using discrete maps demonstrated that nonhyperbolic chaotic scattering is structurally unstable in the sense that the algebraic decay of particles from the scattering region immediately becomes exponential as dissipation is introduced, no matter how weak. We have extended the result to continuous-time Hamiltonian systems by using the Hénon-Heiles system as a prototype model. Further, we have gone beyond to investigate the basin structure associated with scattering dynamics. A surprising finding is that, in the common case where multiple destinations exist for scattering trajectories, Wada basin boundaries (a type of fractal boundaries where any boundary point contains in its arbitrarily small neighborhood points that belong to at least three basins) are common and they appear to be structurally stable under weak dissipation even when other characteristics of the nonhyperbolic
scattering dynamics are not. We have provided numerical evidence and a geometric theory for the structural stability of the complex basin topology.

We have also investigated how the fractal dimension of the set of singularities in scattering function varies as the system becomes progressively more dissipative. A crossover phenomenon has been uncovered where the dimension decreases relatively more rapidly as a dissipation parameter is increased from zero and then exhibits a much slower rate of decrease. We have provided a heuristic theory and numerical support from both discrete-time and continuous-time scattering systems to establish the generality of this phenomenon. Our result is expected to be important for physical phenomena such as the advection of inertial particles in open chaotic flows, amongs others.

When noise is present in a scattering system, particles tend to escape faster from the scattering region as compared with the noiseless case. For chaotic scattering, noise can render particle decay exponential, and the decay rate typically increases with the noise intensity. We have recently uncovered a scaling law between the exponential decay rate and the noise intensity. The finding has been substantiated by a heuristic argument and numerical results from both discrete-time and continuous-time models.

2.5 Spatially extended dynamical systems


Spatially extended systems exhibit extremely rich dynamical phenomena such as nonlinear waves, synchronization, intermittency, and spatiotemporal chaos or turbulence. These have been studied extensively, due to their relevance to many branches of science and engineering. One important phenomenon that has received relatively less attention is extreme events that occur rarely but with intensities far greater than those associated with typical events in the system. We have demonstrated that rare intense events in spatiotemporal systems modeled by the complex Ginzburg-Landau equation can be significantly suppressed by using only observation-based control. Our control strategy rids of the requirement of precise system model and prediction. Analysis of the control and numerical verification have been obtained. The issue of time delay has also been addressed. We expect our results to provide insight into the challenging problem of harnessing spatially extended complex systems.

Continuum coupled maps were previously proposed as a generic and universal class of models to understand pattern formation in oscillating granular layers. Such models usually involve two features: temporal period doubling in local maps and spatial coupling, and the capability to generate various patterns that bear striking similarities to those observed in real experiments. Two questions are outstanding: (1) how robust are patterns generated by continuum coupled maps, and (2) are there limitations, at the quantitative level, to the continuum coupled-map approach? We have addressed the first question by investigating the effects of noise and spatial inhomogeneity on patterns generated. We have also proposed a measure to characterize the sharpness of the patterns. This allows us to demonstrate that patterns generated by the model are robust to random perturbations in both space and time. For the second question, we have investigated the temporal scaling behavior of the disorder function, which was proposed to characterize experimental patterns in granular layers. We have found that patterns
generated by continuum coupled maps do not exhibit scaling behaviors observed in experiments, suggesting that the coupled map approach, while insightful at a qualitative level, may not yield behaviors that are of importance to pattern characterization at a more quantitative level.

2.6 Low-dimensional dynamical systems


Nonstationary dynamical systems arise in applications, but little was done previously on the characterization of such systems, as most standard notions in nonlinear dynamics such as the Lyapunov exponents and fractal dimensions are developed for stationary dynamical systems. We have proposed a framework to characterize nonstationary dynamical systems. A natural way is to generate and examine ensemble snapshots using a large number of trajectories, which are capable of revealing the underlying fractal properties of the system. By defining the Lyapunov exponents and the fractal dimension based on a proper probability measure from the ensemble snapshots, we have shown that the Kaplan-Yorke formula, which is fundamental in nonlinear dynamics, remains valid most of the time even for nonstationary dynamical systems.

Finite-time Lyapunov exponents of a chaotic attractor typically exhibit fluctuations about their asymptotic values. In computations or in experiments large fluctuations of these exponents are of concern, as they can lead to incorrect estimates of the actual exponents. We have found that in common situations where a chaotic attractor contains two distinct dynamically connected components, such as one after crisis or arising in random dynamical systems, the extreme fluctuations of the finite-time exponents follow a universal, exponential distribution. We have developed a physical analysis based on the random-matrix theory and provided numerical evidence to substantiate the finding.

Multistability has been a phenomenon of continuous interest in nonlinear dynamics. Most existing works focused on smooth dynamical systems. Motivated by the fact that non-smooth dynamical systems can arise commonly in realistic physical and engineering applications such as impact oscillators and switching electronic circuits, we have investigated multistability in such systems. In particular, we have considered a generic class of piecewise smooth dynamical systems expressed in normal form but representative of non-smooth systems in realistic situations, and focused on the weakly dissipative regime and the Hamiltonian limit. We have found that, as the Hamiltonian limit is approached, periodic attractors can be generated through a series of saddle-node bifurcations. A striking phenomenon is that the periods of the newly created attractors follow an arithmetic sequence. This has no counterpart in smooth dynamical systems. We have provided physical analyses, numerical computations, and rigorous mathematical arguments to substantiate the finding.

Detecting weak signals from chaotic time series is a problem of interest in science and engineering. We have investigated and explored a signal detection algorithm for which chaos theory, nonlinear dynamical reconstruction techniques, neural networks, and time-frequency analysis are put together in a synergistic manner. By applying the scheme to numerical simulation and different experimental data sets (Hénon map, chaotic circuit, and \(NH_3\) laser data sets), we have demonstrated that weak signals hidden beneath the noise floor can be detected by using a model-based detector. Particularly, the exact signal frequencies
can be extracted from the time-frequency space. By comparing the model-based method with the standard de-noising wavelet technique as well as supervised principal-component analysis detector, we have also shown that our neural network-based approach is effective for extracting weak signal frequencies from chaotic clutter data.

2.7 Synchronization in complex dynamical systems


A central issue in nonlinear science is synchronization and the effect of noise. We have investigated the interplay between noise and chaotic phase and generalized synchronization. A general phenomenon concerning phase synchronization in coupled chaotic oscillators has been uncovered: the average synchronization time can exhibit a non-monotonic behavior with the amplitude of common noise. In terms of generalized synchronization, we have found that noise can either induce/enhance or destroy the synchronization.

We have developed a general method to analyze synchrony from multichannel time series, based on a matrix whose elements are various times for pairs of channels to maintain temporal synchronization in their phases. Monitoring of the properties of the matrix provides an effective way to assess changes in synchrony. The method has been validated by a control model of coupled nonlinear oscillators and tested using experimental data. The method is quite general and it can be applied to large-scale sensor data analysis in DoD applications and biomedical signal processing (e.g., multi-channel EEG or ECoG analysis).

We have also investigated, quite extensively, synchronization in complex networks of nonlinear oscillators. For example, there has been mounting evidence that many types of technological, social, and biological networks possess a complex clustered topology. We have focused on one fundamental question: under what condition can the network synchronizability be optimized? In particular, since the two basic parameters characterizing a complex clustered network are the probabilities of inter-cluster and intra-cluster connections, we have investigated, in the corresponding two-dimensional parameter plane, regions where the network can be best synchronized. Our study has yielded a quite surprising
finding: a complex clustered network is most synchronizable when the two probabilities match each other approximately. Mismatch, for instance caused by an overwhelming increase in the number of intra-cluster links, can counterintuitively suppress or even destroy synchronization, even though such an increase tends to reduce the average network distance. This phenomenon provides possible principles for optimizing synchronization in complex clustered networks. In a series of papers on this topic, we have provided extensive numerical results and analytic theories to establish the generality of the synchronization-optimization phenomenon in different settings of complex dynamical systems.

An outstanding problem in nonlinear science is the effect of colored noise on various dynamical properties. We have developed and validated an algorithm for integrating stochastic differential equations under green noise. Utilizing it and the standard methods for computing dynamical systems under red and white noise enables us to address the problem of synchronization among chaotic oscillators in the presence of common colored noise. We have found that colored noise can induce synchronization, but the onset of synchronization, as characterized by the value of the critical noise amplitude above which synchronization occurs, can be different for noise of different colors. A formula relating the critical noise amplitudes among red, green, and white noise has been uncovered, which holds for both complete and phase synchronization. The formula suggests practical strategies for controlling the degree of synchronization by noise, e.g., utilizing noise filters to suppress synchronization.

3 Personnel Supported and Theses Supervised by PI

3.1 Personnel Supported

The following people received salaries from the AFOSR Project in various time periods.

- Faculty (partial summer salary):
  Ying-Cheng Lai (PI), Professor of Electrical Engineering, Professor of Physics

- Post-Doctoral Fellows (full-time or part-time appointments)
  1. Kwangho Park (1/1/06-12/31/07)
  2. Liang Huang (11/1/08-12/31/08)

- Graduate Students (part-time appointments)
  1. Liang Huang, Ph.D. student in Electrical Engineering
  2. Qingfei Chen, Ph.D. student in Electrical Engineering
  3. Lakshmi Rajagopalan, MS student in Electrical Engineering
  4. Srinivasan Gopal, MS student in Electrical Engineering
  5. Aditya Rao, MS student in Electrical Engineering
  6. Abhijeeth P. Aarey, MS student in Electrical Engineering
  7. Akshay M. Vijaykumar, MS student in Electrical Engineering
  8. Uzair Mohammed, MS student in Electrical Engineering

- Visitors partially supported by the project
  1. Ruth Serquina, Visiting Fulbright Scholar (from the Philippines), 8/15/06 - 4/30/07.
  2. Tamás Tél, Visiting Professor (from Hungary), 1/06/08-1/24/08.
3.2 Theses supervised by PI

1. Liang Huang, Ph.D. in Electrical Engineering, ASU, December 2008; Dynamics and security of complex clustered network systems (received the Palais' Outstanding Doctoral Student award for 2008-2009, ASU).


4 List of Publications


5 Interactions/Transitions

Collaboration with AFRL scientists:

- Dr. John Gaudet and Dr. Michael Harrison, AFRL at Kirtland AFB, on *inducing chaos in electronic circuits*.
- Dr. David Dietz, AFRL at Kirtland AFB, on *nonlinear dynamics and chaos in MEM systems*.
- Dr. Vassilios Kovanis, AFRL at Wright Patterson AFB, on *compressive sensing and complex dynamical systems*.

Invited talks on topics derived from the project

1. “Inducing chaos by resonant perturbations: theory and experiments,” Seminar, Department of Physics, National University of Singapore, February 8, 2006.
5. “Phase synchronization and applications to stochastic resonance and biomedical signal analysis,” Biological Physics Seminar, ASU, September 6, 2006.


6 Past Honors


2. Election as a Fellow of the American Physical Society, 1999. Citation: For his many contributions to the fundamentals of nonlinear dynamics and chaos.