ABSTRACT

Sensor nodes in wireless sensor networks (WSNs) are often expected to operate on batteries for a long period of time. Battery power-efficiency is a critical factor dictating the lifetime of WSNs. In this paper, we compare two pulse-based modulations, namely pulse position modulation (PPM) and on-off keying (OOK), both of which are suitable for WSNs due to their low complexity transceivers. The comparison is based on a general model that integrates typical WSN transmission and reception modules with realistic non-linear battery models. We analyze and compare the battery power-efficiency of PPM and OOK using coherent detection, and with bit error rate (BER) and cutoff rate criteria. Our results reveal that in sparse WSNs, PPM is more battery-power-efficient. In dense WSNs, OOK outperforms PPM. In addition, the battery power-efficiency of OOK increases as the required cutoff rate decreases.

Index Terms— Battery, power efficiency, WSN, PPM, OOK

1. INTRODUCTION

Battery power-efficiency is a critical factor in wireless sensor networks (WSNs) since sensor nodes are typically powered by non-renewable batteries [1]. Among the existing approaches at the physical layer to improve energy efficiency, a majority (see e.g., [2, 3, 4]) assumes that the batteries are ideal and linear. This assumption implies that the energy required by all the components is equal to the battery power consumption. In fact, however, part of the battery’s capacity (stored energy) may be wasted during its discharge process, especially when the discharge current is large. The battery models extracted from experiments show that the actual battery discharge is a non-linear process [6, 7]. Compared to the conventional low power battery-driven system designs, using those non-linear battery models has the potential to markedly improve the system lifetime [9]. Though most of these approaches deal with hardware and software optimization of a single node [6, 10], or routing of energy-constrained networks [8], realistic battery models were recently employed in the design of physical layer and evaluation of performance for WSNs. In [5], the battery efficiency of the pulse position modulation (PPM) and the frequency shift keying (FSK) have been compared under the average symbol error rate (SER) criterion.

In this paper, we compare the battery power-efficiency of two pulse-based modulations: PPM and on-off keying (OOK). Instead of using the noncoherent receiver in [5], we consider coherent reception for both PPM and OOK. In addition to the error-performance oriented bit-error-rate (BER) criterion, we also adopt the rate-oriented cutoff rate criterion. With the battery criterion, we will exploit the fact that the cutoff rate is maximized by optimizing the transmission probability of ‘1’ in OOK.

To ensure fair comparisons between M-ary PPM and binary OOK, an M-dependent duty-cycle factor is also introduced to equate the bandwidth efficiencies of the two modulation formats. As in [5], we also adopt a realistic empirically derived non-linear battery model and take analog circuit power consumption into consideration. However, different from [5], where the SER upper bound is considered, the exact BER and cutoff rate are employed here.

In the following section, we will introduce the system model. In Section 3, we will specify two system design criteria, BER and cutoff rate. In Section 4, we will give a closed-form expression for the battery power efficiency ratio (defined later). Then, results for both design criteria in sparse WSNs and dense WSNs will be given in Section 5. Finally, we provide numerical results for our analysis in Section 6 and make conclusion remarks in Section 7.

2. SYSTEM MODEL

In this section, we will introduce the system model including the battery model, the pulse-based modulations and the channel model.

2.1. Linear vs. Non-linear Battery Models

With the battery output voltage being a constant V, the average power consumed by a system (e.g., a transmitter or a receiver) is determined by the current i and its pdf (a.k.a. discharge current profile) f(i) as follows: \( P = \int_{I_{\text{min}}}^{I_{\text{max}}} V f(i) \, di \), where \( I_{\text{min}} \) and \( I_{\text{max}} \) are the minimum and maximum currents. In an ideal linear battery model, the battery does not consume additional power; that is, the power pumped out from the battery is the same as the power consumed by the system. Hence, the average actual power consumption (AAPC) of the battery is \( P_0 \).

This, however, is not the case in practice. The additional power loss associated with the battery discharge process can be captured by the following nonlinear model:

\[
P_0 = \int_{I_{\text{min}}}^{I_{\text{max}}} \frac{V i}{\mu(i)} \cdot f(i) \cdot di ,
\]

where \( \mu(i) \in [0.5, 1] \) is the battery efficiency factor [6]. Notice that with \( \mu(i) \) upper bounded by 1, we have \( P_0 \geq P \).

In [6], two experiment-based formulas were provided to describe the relationship between the battery efficiency factor \( \mu(i) \) and the discharge current \( i \):

\[
\mu(i) = 1 - \omega i + \nu i^2 ,
\]

where \( \omega \) and \( \nu \) are both positive constants. These formulas show that the battery efficiency factor \( \mu(i) \) is a monotonically decreasing function of \( i \). To distinguish different scenarios, we will use superscripts \( o \) and \( p \) to denote OOK and PPM; and subscripts \( t \) and \( r \) to denote quantities associated with transmitter and receiver.

Under the same bandwidth efficiency (rate-bandwidth ratio) and BER/cutoff rate constraints, we will compare the average battery power consumptions for PPM \( (P_0^p) \) and OOK \( (P_0^o) \) in terms of their battery power efficiency ratio (BPER) defined as follows:

\[
\rho := 10 \log \left( \frac{P_0^p}{P_0^o} \right) ,
\]
Battery Power Efficiency of PPM and OOK in Wireless Sensor Networks

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With this definition, PPM is more battery power-efficient if \( \rho < 0 \) and OOK is more battery power-efficient otherwise. Given batteries with identical capacity \( C_0 \), the BPER can be translated into the ratio of the battery lifetimes of two nodes employing PPM and OOK: 
\[
10 \log \left( \frac{T_p^d}{T_o^d} \right) = -\rho.
\]
To ensure a fair comparison, we will consider PPM and OOK schemes with the same bandwidth efficiency.

2.2. PPM and OOK Modulations

For \( M \)-ary PPM, the symbol period is \( T_p = MT_p \) with \( T_p \) being the pulse duration. Accordingly, the transmission bandwidth \( B \) is approximately \( 1/T_p = M/T_p^d \). As a result, the bandwidth efficiency for \( M \)-ary PPM is \( B_p := \frac{T_o^d}{T_p^d} = \frac{\log_2 M}{\log_2 B} \approx \frac{\log_2 M}{M} \), where \( B_p \) is the PPM data rate (bit/sec). OOK modulation, on the other hand, has a fixed cardinality of 2. Its bandwidth efficiency is then given by \( B_o := \frac{T_o}{T_p} = \frac{1}{T_p} \approx \frac{1}{M} \), where \( B_o \) is the OOK data rate (bit/sec). Notice that, in order to have the same transmission bandwidth, PPM and OOK pulses have the same duration of \( T_p \).

In order to ensure \( B_o = B_p \), OOK signals need to be duty-cycled with factor
\[
\phi(M) := T_p/T_p^d = \log_2 M/M.
\]
Thus, for different modulation size \( M \) of PPM, one should use different duty cycle \( \phi(M) \) of OOK. Clearly, \( \phi(M) \leq 0.5 \), where the equality holds if and only if \( M = 2 \).

2.3. Channel Model

The channel model we consider here is a \( R^{th} \)-power path-loss channel with additive white Gaussian noise (AWGN). The channel gain factor \( G(d) \) depends on the transceiver distance \( d \) and is given by
\[
G(d) = E_s/E_d = M_{ic}^{M_{ci}} G_1,
\]
where \( E_s \) and \( E_d \) are the transmitted and received energy per symbol, \( M_{ic} \) is the link margin and \( G_1 \) is the gain factor at \( d = 1, K \geq 2 \).

3. SYSTEM DESIGN CRITERIA

Based on the battery and channel models, and the duty-cycled modulations, we will next introduce the comparison criteria.

3.1. Criterion I: BER

In this case, the battery power efficiencies of PPM and OOK will be compared under the same BER requirement. Specifically, for a prescribed BER value, we first determine the required energy per symbol at the receiver for \( M \)-ary PPM, namely \( E_p^{d}(M) \), and the required energy corresponding to symbol ‘1’ at the receiver for OOK, namely \( E_o^{d} \). With the channel model in (5), the required transmitted energy per symbol for \( M \)-ary PPM and OOK is given by:
\[
E_p^{d}(M, d) = E_p^{d}(M)G(d) \quad \text{and} \quad E_o^{d}(d) = E_o^d G(d).
\]
Accordingly, the average transmitted energy per bit for \( M \)-ary PPM and OOK are, respectively:
\[
E_p^{d}(M, d) = \frac{E_p^{d}(M,d)}{\log_2 M} \quad \text{and} \quad E_o^{d}(q,d) = q \cdot E_o^{d}(d),
\]
where \( q \) is the probability that symbol ‘1’ is transmitted. These energies can then be used to derive the discharge current profiles, and accordingly the battery power efficiency.

Remarks are due here on the selection of \( q \) for OOK. The BER is given by \( P_e = Q(\sqrt{E_o^d/2N_0}) \), which depends on the received signal-to-noise ratio (SNR), but is independent of \( q \). However, treating the OOK transceiver pair as a binary symmetric channel (BSC) with transition probabilities \( P(1|0) = P(0|1) = P_e \), we notice that the average mutual information is maximized when \( q = 0.5 \). Hence, this value will be used for the BER criterion.

With \( q = 0.5 \), and with coherent detection for both \( M \)-ary PPM and OOK, we have established the following results with respect to the relationship among the required transmitted energies.

**Lemma 1:** With coherent detection and equal BER, and when \( q = 0.5 \) for OOK, we have
\[
\theta := \frac{E_p^{d}(M,d)}{E_o^{d}(d)} < 1, \forall M \leq 4.
\]

Proof: The BER of 2-PPM is given by \( P_e = Q(\sqrt{E_o^d(2)/2N_0}) \). It then follows that \( E_p^{d}(2) = 2E_o^{d}(2) \), and accordingly, \( E_p^{d}(2,d) = E_o^{d}(0.5,d) = 0.5E_o^{d}(d) \) [c.f. (5), (6)]. In addition, \( E_p^{d}(M,d) \) decreases as \( M \) increases, since PPM is an orthogonal modulation. As a result, we have \( E_p^{d}(M,d) \log_2 M E_o^{d}(M,d) < E_o^{d}(d) \), \( \forall M \leq 4 \).

Interestingly, for 2-PPM and OOK, the BER criterion is equivalent to the average mutual information criterion. We assume equiprobable OOK, \( q=1/2 \), since it maximizes mutual information.

3.2. Criterion II: Cutoff Rate

The cutoff rate \( R_c \) is the data rate below which the average BER of randomly selected codes approaches 0 when the code length approaches infinity (see e.g., [11, 14]). We consider a discrete-input continuous-output (soft decision) channel model, which is well known to outperform a hard decision channel. The normalized cutoff rate is
\[
R_c = \log_2 \left[ \frac{M}{1 + (M-1)\exp\left(-E_p^{d}(M)/2N_0\right)} \right] / \log_2 M,
\]
for \( M \)-ary PPM [14], and
\[
R_c = -\log_2 \left[ q^2 + (1-q)^2 + 2q(1-q)\exp\left(-E_o^{d}/4N_0\right) \right].
\]
for OOK [13]. Notice that, as \( E_o^{d} = E_o^{d}(q,d)/q/G(d) \) [c.f. (6)], \( q = 1/2 \) is not guaranteed to maximize the cutoff rate \( R_c \) for a fixed average energy per bit \( E_o^{d}(q,d) \). In other words, for any given cutoff rate \( R_c \), there exists an optimum \( q \in [0,0.5] \) that minimizes the average energy per bit \( E_o^{d}(q,d) \), as suggested in [12, 13].

4. NETWORK NODE AAPP

Notice that both the PPM and the duty-cycled OOK may have active and idle periods during the transmitting and receiving modes. Only during the active periods, the transmission power and the circuit power consumption are non-zero. As a result, the discharge current pdfs for both modulations should have the following form:
\[
f(i) = C \cdot \delta(i-I) + (1-C) \cdot \delta(i),
\]
where \( C \in (0,1] \) is a modulation-dependent constant, \( I \) is the discharge current during the active period and \( \delta(\cdot) \) is the Dirac delta function. The pdf \( f(i) \) can be fully described by \( C \) and \( I \).

For PPM, the transmitter is ‘on’ over one pulse duration \( T_p \), out of each symbol duration \( T_p^d = MT_p \). As a result, the constant \( C \) in (10) for PPM is \( C_p^d = M^{-1} \). For OOK, the transmitter is duty-cycled yielding \( C_o^d = q \log_2(M)/M \).

To determine \( I \) in (10), one needs to take into account the transmission power (\( P_e \)) per symbol in order to satisfy the desired BER or cutoff rate requirement, the circuit power consumption \( P_{ct} \), as well as the power consumptions at the power amplifier (PA) and the DC/DC converter. In fact, the latter power consumptions depend on \( P_s \) and \( P_{ct} \). Specifically, denoting the loss coefficient of the PA as \( \alpha \in (0,1) \) and the transfer efficiency of the DC/DC converter as \( \eta \in (0,1) \), we have the discharge current during the active transmitting period as [2, 5]:
\[
I = I_e = (E_s\beta + P_{ct})/(V\eta).
\]
for both PPM and OOK, where $\beta = (1 + \alpha)B$. Hence, we have the transmitter AAPCs for PPM and OOK as [c.f. (1), (10) and (11)]:

$$P^a_{cr} = \frac{E^c_t(M, d) + \rho_{cr}}{\eta_\mu(I^r_c(M, d))}, P^o_{cr} = q \frac{\log_2 M E^c_o(M, d) + \rho_{ct}}{\eta_\mu(I^r_o(d))}. \tag{12}$$

For OOK, we notice that the receiver only needs to be ‘on’ during the pulse duration $T_p$ to check whether or not a signal has been transmitted. Unlike the OOK transmitting mode, however, the OOK receiver is always active during the duty-cycled transmission $T_p = \phi(M)T^o_c$. As a result, the constant $C$ in (10) for OOK is $C^o_c = \phi(M)$. For PPM, as the information transmitted is purely conveyed by the position of the pulse, the receiver has to continuously check the received signal in order to locate the position of the transmitted pulse. Therefore, we have $C$ in (10) for PPM as $C^p_c = 1$.

At the receiving mode, there is no a PA but a low noise amplifier (LNA) with nearly constant power consumption. Notice that there is no transmission power either. Thus, we have

$$I = I_r = P_{cr}/(\eta V), \tag{13}$$

where $P_{cr}$ is the circuit power consumption at the receiver. Therefore, we have the battery AAPCs at the receiver as [c.f. (1), (10) and (13)]:

$$P^p_{cr} = \frac{P_{cr}}{\eta_\mu(I_r)}, P^o_{cr} = \frac{P_{cr}}{\eta_\mu(I_r)} \log_2 M. \tag{14}$$

Finally, suppose that the node operates in a half-duplex communication mode with a portion $\theta$ in the transmitting mode and $(1 - \theta)$ in the receiving mode. Combining (12), (14) and the definition of BPER (3), we have

$$\rho = 10 \log \left\{ \frac{E^c_t(M, d) + \rho_{cr}}{(E^c_o(q, d) + \rho_{ct}) \mu(I_r)} \right\}, \tag{15}$$

where $\rho_{cr} = \frac{(1-\alpha)P_{cr}}{\eta_\mu(I_r)}$.

5. BATTERY POWER EFFICIENCY COMPARISON

In this section, we will compare the battery power efficiencies of PPM and OOK using the BER and cutoff rate criteria.

5.1. BPER Comparison Based on BER

The density of WSN nodes determines the average inter-sensor distance $d$ and thus the transmit power, which is proportional to $d^4$. On the other hand, the circuit power consumption remains approximately constant independent of $d$.

In sparse WSNs with large $d$, the circuit power consumption is much smaller than the transmit power and can be neglected; that is, $P_{ct} = 0$ and $P_{cr} = 0$. The BPER expression in (15) can be simplified to

$$\rho = 10 \log \left( \frac{r_{\mu}(M, d) E^c_t(M, d)}{E^c_o(0.5, d)} \right), \tag{16}$$

where $r_{\mu}(M, d) := \mu(I^r_c(M, d))/\mu(I^r_o(M, d))$. The following results can be obtained.

**Proposition 1:** With the BER criterion and in sparse WSNs, where $d$ is large and the circuit power consumption can be neglected, the following results hold:

1. PPM is always more battery power efficient than OOK for all $M$.
2. The battery power-efficiency advantage of PPM over OOK increases as $d$ increases for $M \leq 4$.

**Proof:** From Lemma 1, we know that $E^c_t(M, d) < E^c_o(0.5, d)$ for $d \leq 4$. As a result, we would be the same situation as for the BER criterion. In turn reduces the circuit power consumption of OOK, and makes OOK more battery power-efficient than PPM (see (15)).

5.2. BPER Comparison based on Cutoff Rate

As mentioned in Section 3, the cutoff rate $R^\mu$ for PPM is maximized with equi-probable inputs, while $R^o$ for OOK is maximized by choosing different $q \in \{0, 0.5\}$ values at different $R^\mu$. Exploiting this, we have the following results:

**Proposition 3:** With the cutoff rate criterion, the following results hold:

1. When $R \rightarrow 1$, the results in Propositions 1 and 2 still hold.
2. As $R$ decreases, OOK becomes more battery power-efficient.

**Proof:** When $R \rightarrow 1$, we have $q \rightarrow 0.5$ and the required energy per bit for the optimized $q$ approaches the one for $q = 0.5$ [12]. Thus, it turns out to be the same situation as for the BER criterion. As $R$ decreases, $q$ also decreases [12]. This in turn reduces the circuit power consumption of OOK, and makes OOK more battery power-efficient than PPM (see (15)).
In this paper, we have investigated the battery power-efficiency of two pulse-based modulations, namely PPM and OOK, for a half-duplexing WSN node. With the BER and cutoff rate criteria, our system model and the BPER comparisons accounted for the transmit power, the analog circuit power consumption and the battery non-linearity. Both the analysis and simulations show that PPM is more battery power-efficient in sparse WSNs or with high cutoff rate requirement; while the duty-cycled OOK is more efficient in dense WSNs or with low cutoff rate requirement.

8. REFERENCES


6. SIMULATION RESULTS

To verify our analysis, we present quantitative results for Propositions 1-3. The parameters used in the simulations are shown in Table 1 [2]. Parameters α, β, η and μ(,) are obtained from the PA and battery models, and do not depend upon the modulation. Given distance d, we obtain pathloss G(d) from (5). For a given BER, we calculate the required SNR using the BER expression for OOK and PPM, and hence $\xi_b$. For a given cutoff rate, we can obtain the transmission energy from (8) and (9). We next compute $I_2$ and $I_\omega$ from (11) and (13). As noted earlier, when (9) is used, we get a range of possible $(q, \xi_b^c(q, d))$ values to use. We use the pair with the minimum $\xi_b^c(q, d)$.

Fig. 1 shows the BPER versus distance based on the BER criterion. The plot shows that $\mu > 0$ (OOK more efficient) at small d (dense WSNs) and $\mu < 0$ (PPM more efficient) at large d (sparse WSNs). These observations agree very well with Propositions 1 and 2. At large d, the BPER based on the sparse WSN assumption is also plotted with dashed curves. As shown in Fig. 1, for small-to-medium $M$, these are close to the curves with the circuit power consumption accounted for. This confirms the validity of neglecting $P_{cr}$ and $P_m$ in sparse WSNs. However, the approximation is not very accurate when $M$ is large. Intuitively, as $M$ increases, the duty-cycle factor $\phi(M)$ of OOK decreases, reducing the circuit power consumption at the OOK receiver. This effect, however, is ignored by the sparse WSN assumption. Fig. 2 depicts the BPER based on the cutoff rate criterion. The curves corresponding to a large $R (0.99)$ are almost the same as the ones in Fig. 1. On the other hand, the BPER at small $R (0.1)$ gives very different indications. These verify Proposition 3.

7. CONCLUSIONS

Fig. 1. BPER vs. distance with $K = 3$, based on the BER criterion. Dashed curves: sparse WSN assumption.

Table 1. Simulation parameters

<table>
<thead>
<tr>
<th>$I_{max}$ (A)</th>
<th>$P_{cr}$ (mW)</th>
<th>$G_1$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>52.5</td>
<td>27</td>
</tr>
<tr>
<td>3.7</td>
<td>105.8</td>
<td>40</td>
</tr>
</tbody>
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Fig. 2. BPER vs. distance with $K = 3$, based on the cutoff rate criterion.