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INITIAL FEASIBILITY STUDY OF THE USE OF POLYURETHANE FOAM IN CONJUNCTION WITH A TENSION STRUCTURE AS A ROOF SYSTEM

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I. BACKGROUND and INFORMATION

The specific purpose of this investigation was to determine an economical way of placing a roof over an octagonal shaped area with a minimum of interior columns. Conventional construction means were considered; however, costs were beyond current funding capability.

The Materials Research Division, Research and Development Department, was funded to determine the feasibility of constructing a tension structure with a spray-applied polyurethane-foam roof.

This investigation deals with the theoretical approach to the development of this tension structure. A one-eighth scale model was constructed to illustrate the various structural elements utilized. Further, an opportunity to observe physical properties of the polyurethane foam under an application similar to actual use is presented.

II. DESCRIPTION

The tension structure considered is constructed of pre-engineered steel pipes, cables, wire mesh, fabric, and a foam-in-place polyurethane roof. More specifically described, the tension structure is supported by a control column with tension cables spanning to a perimeter compression ring which serves as an anchoring point for the tension cables. The compression ring is anchored by pipe columns placed at each corner. Parallel cables placed at regular intervals tie the main tension cables together.

Wire mesh and cloth fabric span the space between cables. Polyurethane spray-foamed over the fabric surface serves as the roof. The geometric configuration is better described by sketches in Appendix A and figures in Appendix B.

III. DISCUSSION

Several assumptions were made at the beginning and throughout the tension structural calculations for simplicity. These assumptions, however, are conservative and in every case result in larger forces than will actually be experienced. The forces determined approximate the actual forces experienced and will not result in excessive oversizing of members. The tension forces in the cables can be varied considerably by varying the cable lengths. As the cable length increases, the tension force decreases rapidly. The deflection curve assumed in this investigation is relatively flat and results in high tension forces. Preformed stainless steel aircraft cable 5/16" in diameter will carry the imposed forces; however, 3/8" cable will provide a more desirable factor of safety. The forces in the compression ring are a direct function of the cable tension. If adequately braced at points of cable anchorage, 3" diameter standard steel pipe will quite adequately carry the design loads. The small forces experienced by the various members of the tension structure are attributed to the low structural dead load. Additionally, tension structures characteristically require smaller members because of their efficient utilization of available members. The total cross sectional area of tension and compression members contributes equally to the load carrying ability while in a bending member the extreme fibers of the members carry the bulk of the load while the bending stresses at the neutral axis are zero.

IV. AREAS OF SPECIAL CONSIDERATION

Two areas which should receive additional consideration prior to construction of a full-size structure are differential thermal expansion and contraction and lift due to wind loads. It is not believed that these areas will pose difficult problems because of the inherent characteristics of the tension structure. The structure's flexibility will tend to compensate for changes in length due to thermal expansion. Simple tie brace in conjunction with the parallel cables will reduce the effects of wind lift.

V. DESCRIPTION OF MODEL

The model base was constructed of two four by eight sheets of plywood placed side by side on 2" by 4" framing. The center column was constructed of 4 1/2" diameter aluminum tubing to represent a special purpose 36" diameter central column. Attention is called to the fact that a 36" diameter column is not required to carry the central column load. An 8" standard steel pipe will quite adequately carry the imposed loads. The compression ring was constructed of 3/4 inch aluminum tubing. The tension cables were constructed of 3/36 steel cables. The vertical columns were constructed of 1/2" diameter aluminum tubing. Fiber glass mesh such as that used for window screens was used to serve as both the wire mesh and cloth fabric. Polyurethane foam of about 2 pcf density was sprayed on the fiberglass screen surface.

Due to the small area being sprayed and a strong wind, there was some surface roughness; however, this excess foam was easily carved away with a sharp knife. The surface roughness appeared out of proportion on the scale model since roughness experienced there was the same or greater than would be expected on the full size structure.

The scale model has not been subjected to load tests other than pressing on it in various locations with the hands. It seems to distribute these loads very well. After coating the surface with an exterior type rubber based latex paint, the model has been and is being subjected to the weather.

NAD Crane's Public Works Officer and his deputy were present during spray application of the polyurethane foam roof. Other engineering personnel from the various departments on station have since observed the scale model. General comments concerning the feasibility of this type of structure have been encouraging.

VI. ECONOMICS

The materials, cable, pipe, fabric, wire mesh, and polyurethane foam required to construct this roof system can be purchased for approximately $3000. The labor cost for construction should be relatively low since the majority of work can be accomplished at ground level. The major cost of this structure would be associated with the concrete floor and footings.

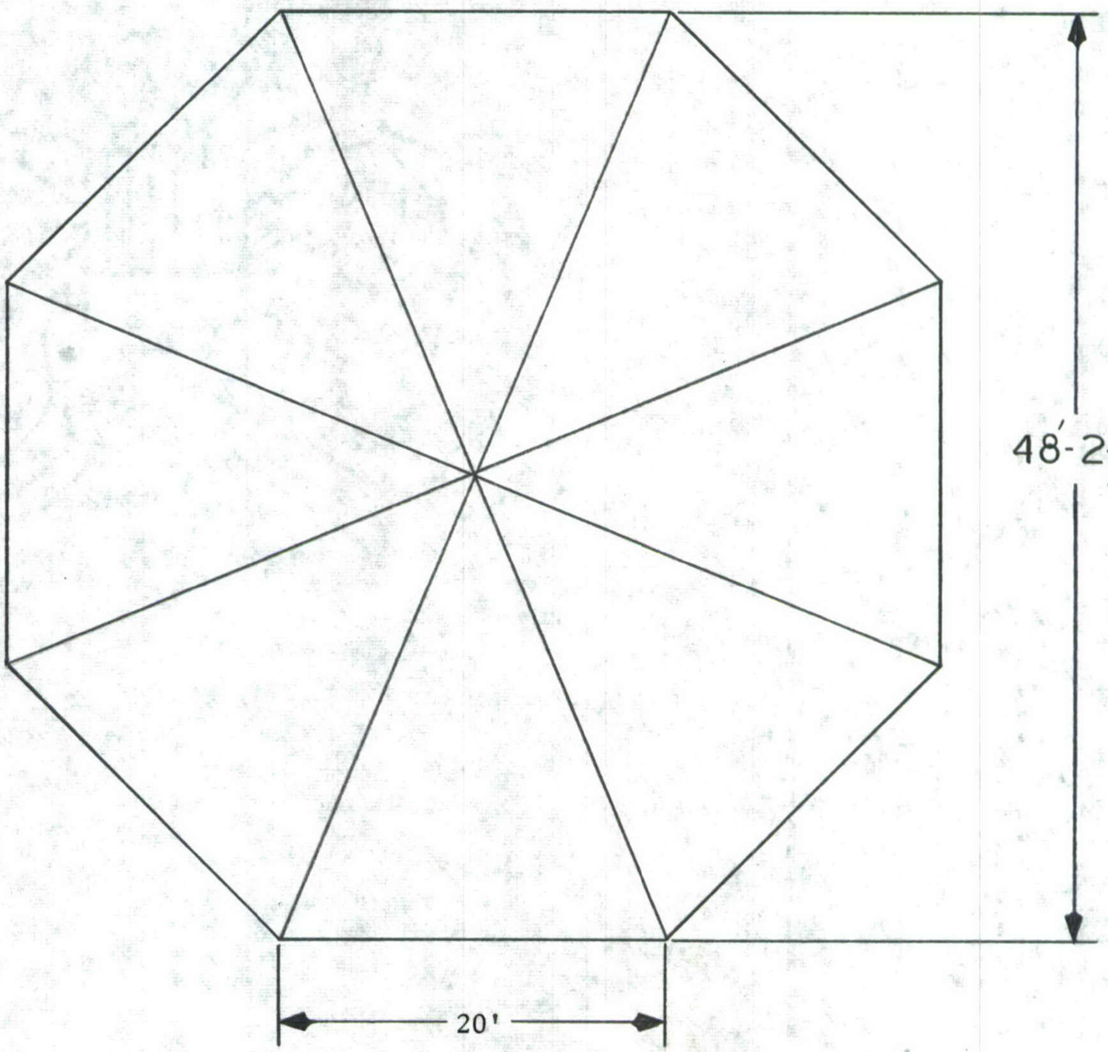
VII. CONCLUSION

In view of the information presented herein, it is concluded that construction of a full size prototype tension structure is economically and structurally feasible.

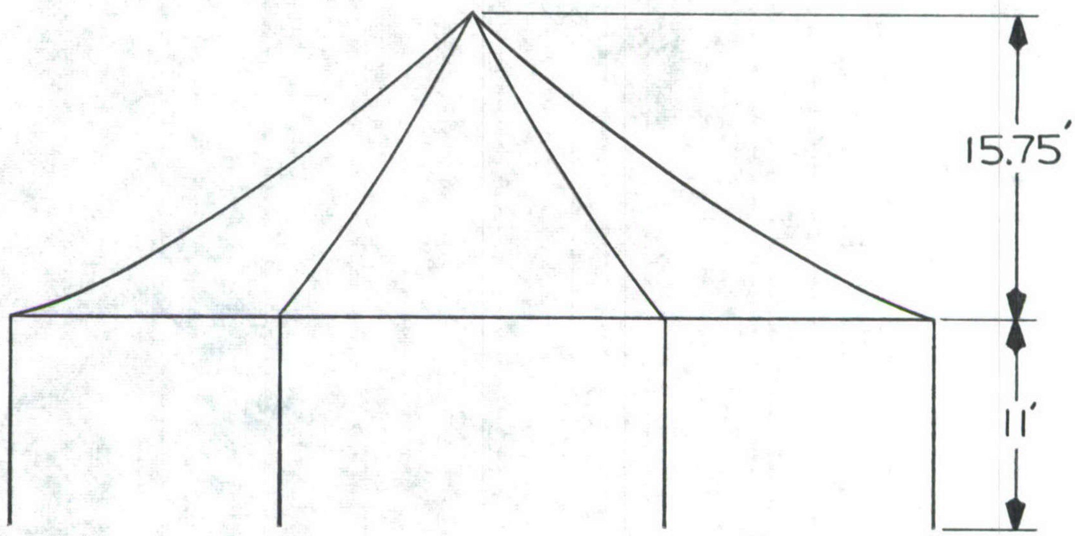
APPENDIX A

TENSION STRUCTURE CALCULATIONS

1. Description: The structure under consideration has an octagonal shaped floor plan, with a spray-on polyurethane foam roof supported by a tension structure. The geometric configuration of the subject structure is illustrated below:



ROOF PLAN



ELEVATION

II. Design Considerations:

1. The dead load is uniformly distributed and sustained entirely by the cables.

2. The live loads, wind and snow, are 20 psf and 30 psf respectively and act on projected plans perpendicular to their respective direction of action.

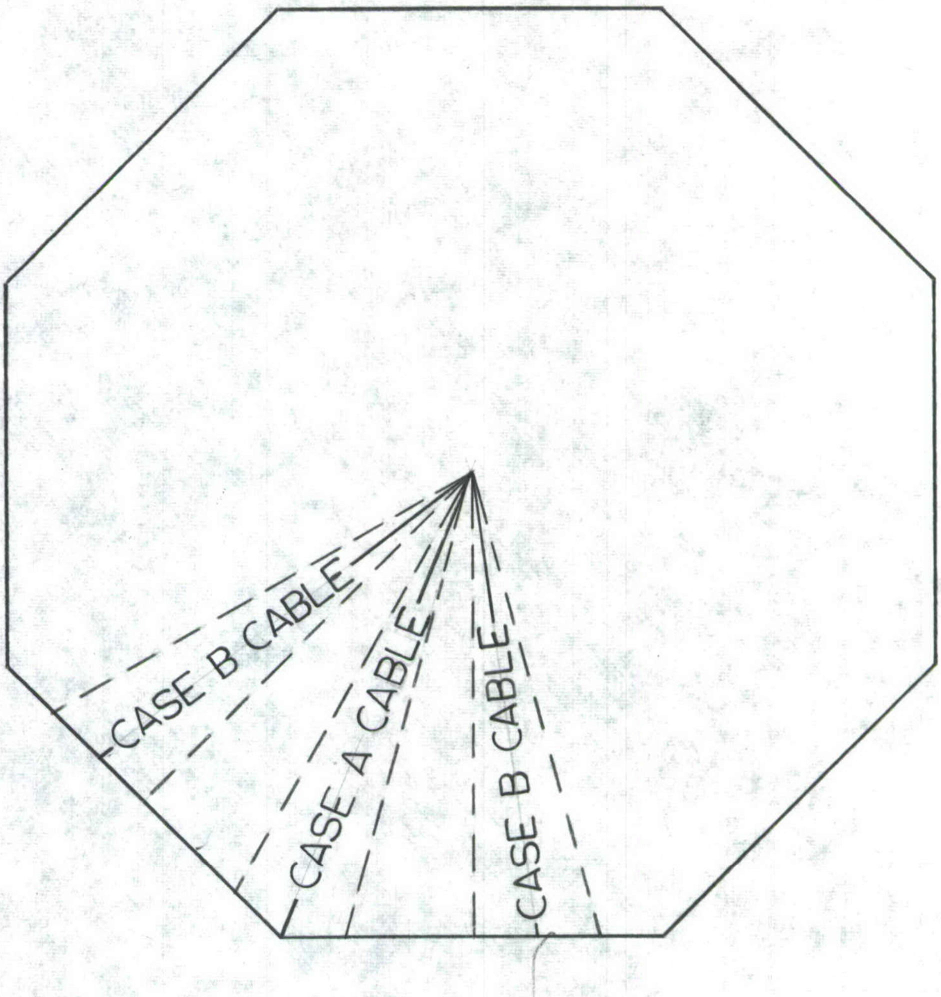
3. The cable supports are immovable.

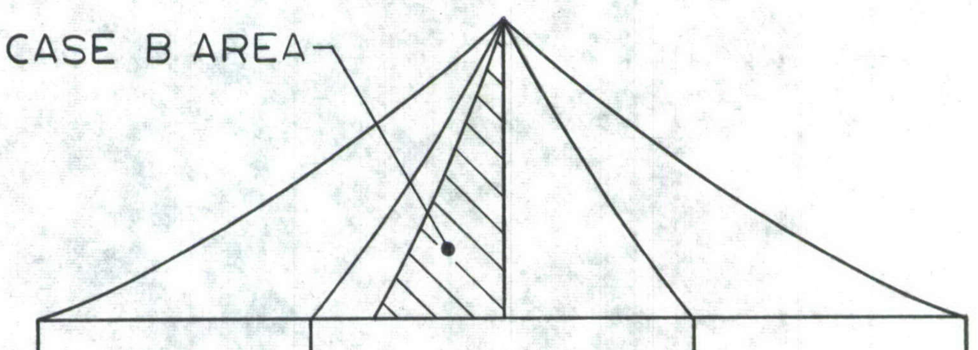
4. The cables are perfectly flexible.

5. The points on the cable are assumed to move along fixed vertical lines when undergoing elastic deflection.

III. Determine Design Loads

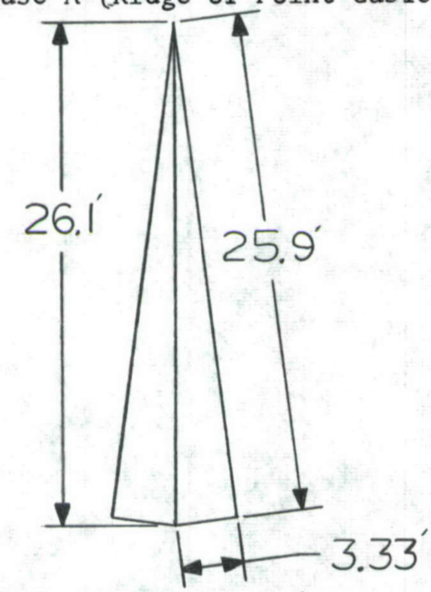
Sketch of Loading Plans





III. <u>Determine Design Loads Cont.</u>

Case A (Ridge or Point Cable)

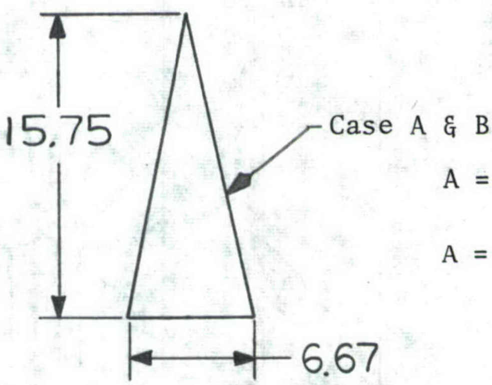


$A = \frac{2(25.9 \times 3.33)}{2}$

$A = 86.2\ Ft^2$

$A = 86.2\ Ft^2$

Horizontal Projected Area



$A = \frac{15.75 \times 6.67}{2}$

$A = 52.5\ ft^2$

Vertical Projected Plane

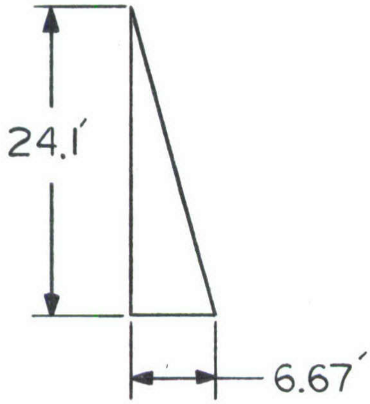
Vertical Load:

Dead Load	1 psf x 86.2 Ft^2 =	86.2 lbs.
Live Load	30 psf x 86.2 Ft^2 =	2586.0 lbs.
Total Vertical Load	=	2672.2 lbs.

Horizontal Load:

Wind Load 20 psf x 52.5 Ft^2 = 1050 lbs.

CASE B



$$A = \frac{6.67 \times 24.1}{2}$$

$$A = 80.3 \text{ Ft}^2$$

Horizontal Area

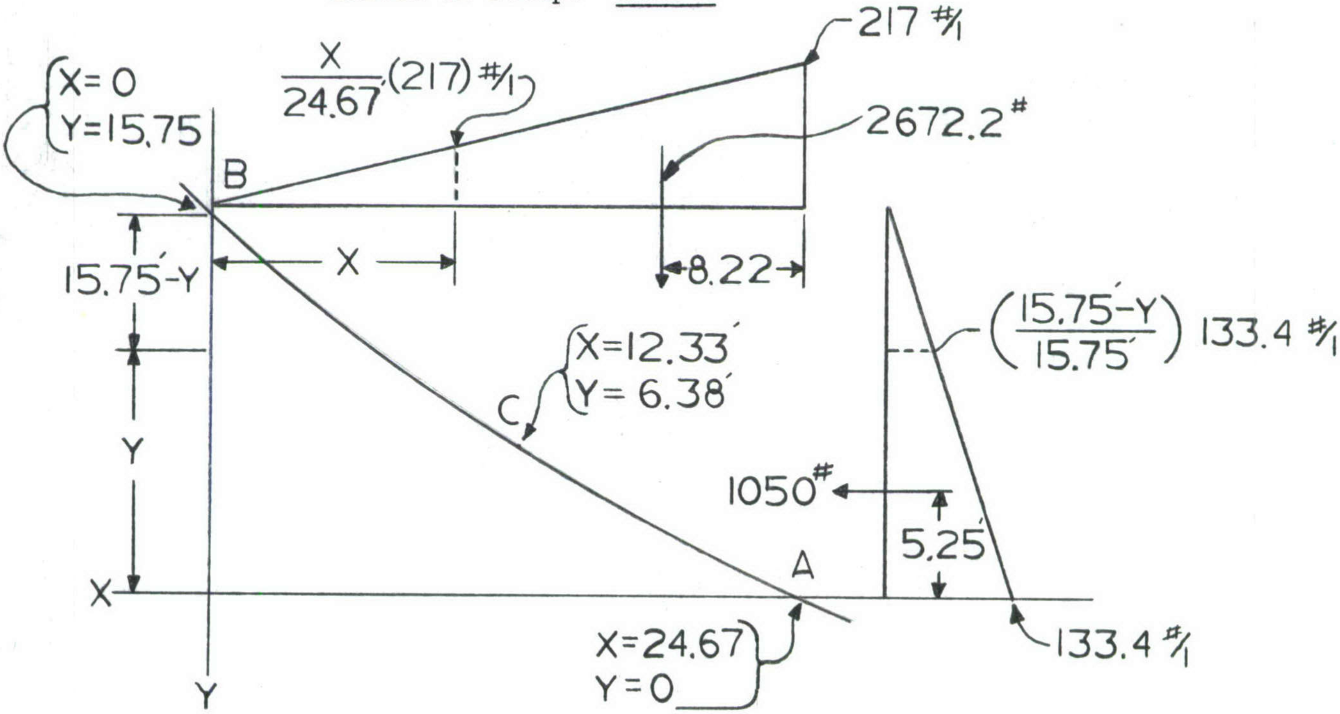
Vertical Area - Same as Case A

Vertical Load (1 psf DL + 30 psf LL) 80.3 Ft^2 = 2489

Wind Load Same as Case A

IV. Determine Cable Reactions:

Sketch of Setup. CASE A



Discussion:

The tension cable supported at the upper end by the central column and the lower end by the compression ring constitutes a structure which is statistically indeterminate in the first degree. This means that some elastic property in addition to the equations of static equilibrium must be considered in determining cable reactions. In this solution the cable is assumed perfectly flexible. In line with this assumption the bending moment at every point along the deflected curve of the cable will be equal to zero. There is a small error in this assumption due to the ability of the cable to resist bending. It can easily be proven that errors in resultant reactions due to this assumption are less than one-tenth of one percent and for all practical purposes can be neglected.

CASE A. Write Equation for the bending moment in the cable

$$M = B_v x - B_h(15.75-y) - \frac{217x^3}{6x24.67} - \frac{(15.75-y)^3\ 133.4}{6 \times 15.75}$$

The Cable lengths will be designed so the deflection curve of the cable passes through Point C.

Solve for B_v and B_h by using known values of x and y.

Eq. #1 =

$$0 = 12.33\ B_v - 9.37B_h - B_h(15.76-6.38) - \frac{217(12.33)^3}{6 \times 24.67} - \frac{(15.75-6.38)^3 133.4}{6 \times 15.75}$$

$$0 = 12.33\ B_v - 9.37Bh - 2748 - 1161$$

$$12.33B_v = 9.37B_h + 3909$$

$$B_v = \frac{9.37B_h}{12.33} + \frac{3909}{12.33}$$

$$B_v = .7599B_h + 317$$

IV. Determine Cable Reactions Cont.

Solve for B_v and B_h

Eq. #2 -

$$M = B_v\,24.67 - B_h(15.75-0) - \frac{217(24.67)^3}{6 \times 24.67} - \frac{133.4(15.75-0)^3}{6 \times 15.75}$$

$$0 = 24.67B_v - 15.75B_h - 22{,}011 - 5515$$

$$0 = 24.67B_v - 15.75B_h - 27{,}506$$

$$0 = 24.67(.7599B_h + 317) - 15.75B_h - 27{,}506$$

$$0 = 18.746B_h + 7820 - 15.75B_h - 27{,}506$$

$$0 = 2.99B_h - 19{,}686$$

$$B_h = \frac{19{,}686}{2.99} = \underline{6585\#}$$

$$B_v = 7599(6584) + 317$$

$$B_v = 5003 + 317$$

$$B_v = \underline{5320}$$

Solve for A_v and A_h by solving static equations

Sum of $F_v = 0$

↑	↓
5320	2672
	A
5320 =	A_v + 2672

Sum of $F_h = 0$

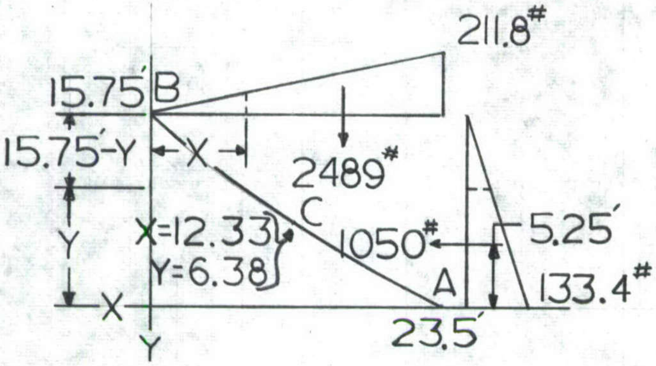
→	←
A_h	6584#
	1050
A_h	7634

$$A_v = 5320 - 2672$$

$$A_v = \underline{2648}$$

$$A_h = \underline{7634}$$

SKETCH OF SETUP <u>CASE B</u>



Bending Moment Equation

$$M = xB_v - (15.75-y)B_h - \frac{211.8x^3}{6 \times 23.5} - \frac{133.4(15.75-y)^3}{6 \times 15.75}$$

Cable length will be designed so the deflection curve of the cable passes through Point C.

Solve for B_v and B_h by using known values of x and y in bending moment equation.

<u>Eq. #1.</u>

$$M = 12.25B_v - (15.75-6.38)B_h - \frac{211.8(12.25)^3}{6 \times 23.5} - \frac{133.4(15.75-6.38)^3}{6 \times 15.75}$$

$$0 = 12.25B_v - 9.37B_h - 2761 - 1161$$

$$12.25B_v = 9.37B_h + 3922$$

$$B_v = \frac{9.37B_h}{12.25} + \frac{3922}{12.25}$$

<u>Eq. #1</u> . $B_v = .765B_h + 320$

<u>Eq. #2.</u> $M = 23.5B_v - 15.75B_h - \frac{211.8(23.5)^3}{6 \times 23.5} - \frac{133.4(15.75)^3}{6 \times 15.75}$

$$0 = 23.5B_v - 15.75B_h - 19{,}494 - 5515$$

$0 = 23.5\ (.765B_h + 320) - 15.75B_h - 25{,}009$

$0 = 17.98B_h - 15.75B_h + 7520 - 25{,}009$

$2.23B_h = 17{,}489$

$B_h = \underline{7842}$

Eq. #1

$B_v = .765(7842) + 320$

$B_v = 5999 + 320$

$B_v = \underline{6319}$

Solve for A_h and A_v by the equations of statics

Sum of $F_v = 0$		Sum of $F_h = 0$	
↓ A_v	↑ $B_v = 6319$	→ A_h	← $B_h = 7842$
2489			1050
$A_v + 2489$	6319	A_h	8892

$A_v = 6319 - 2489$

$A_v = 3830$

$A_h = 8892$

V. Determine Equation for Cable Curve

CASE A

$$M = B_v x - B_h(15.75-y) - \frac{217x^3}{6 \times 24.67} - \frac{133.4(15.75-y)^3}{6 \times 15.75}$$

$$0 = 5320x - 6584(15.75-y) - 1.47x^3 - 1.41(15.75-y)^3$$

$$1.41(15.75-y)^3 + 6584(15.75-y) = 5320x - 1.47x^3$$

CASE B

$$0 = 6319x - 7842(15.75-y) - 1.50x^3 - 1.41(15.75-y)^3$$

$$1.41(15.75-y)^3 + 7842(15.75-y) = 6319x - 1.50x^3$$

VI. Determine Tension in Cables

CASE A

$T_A^2 = A_v^2 + A_h^2$

$T_A^2 = (2648)^2 + (7634)^2$

$T_A = \underline{8080\#}$

$T_B^2 = B_h^2 + B_v^2$

$T_B^2 = (6584)^2 + (5320)^2$

$T_B = \underline{8464\#}$

CASE B

$T_A^2 = A_v^2 + A_h^2$

$T_A^2 = (3830)^2 + (8892)^2$

$T_A = \underline{9682\#}$

$T_B^2 = B_h^2 + B_v^2$

$T_B = \underline{10,070\#}$

VII. Determine Slope of Cables at Points A and B

CASE A

$\tan\theta_A = \frac{2648}{7634} = .347 \qquad \theta_A = 19.13°$

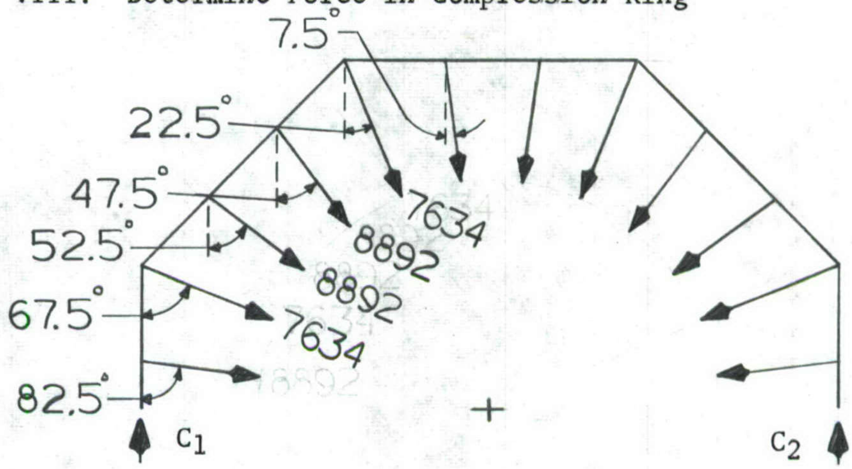
$\tan\theta_B = \frac{6584}{5320} = 1.238 \qquad \theta_B = 51.1°$

CASE B

$\tan\theta_A = \frac{3830}{8892} = .431 \qquad \theta_A = 23.3°$

$\tan\theta_B = \frac{7842}{6319} = 1.241 \qquad \theta_B = 51.15°$

VIII. Determine Force in Compression Ring



Sum of Forces in y-y direction = 0

↓ ↑

2(8892) Sin 7.5° = 2(8892 x . 1305) = 2321 $C_1 + C_2$

2(7634) Cos 52.5° = 2(7634) x .383) = 5848

2(8892) Cos 52.5° - 2(8892 x .609) = 10,830

2(8892) Cos 47.5° = 2(8892 x .676) = 12,022

2(7634) Cos 22.5° = 2(7634 x .924) = 15,108

2(8892) Cos 7.5° = 2(8892 x .988) = 15,570

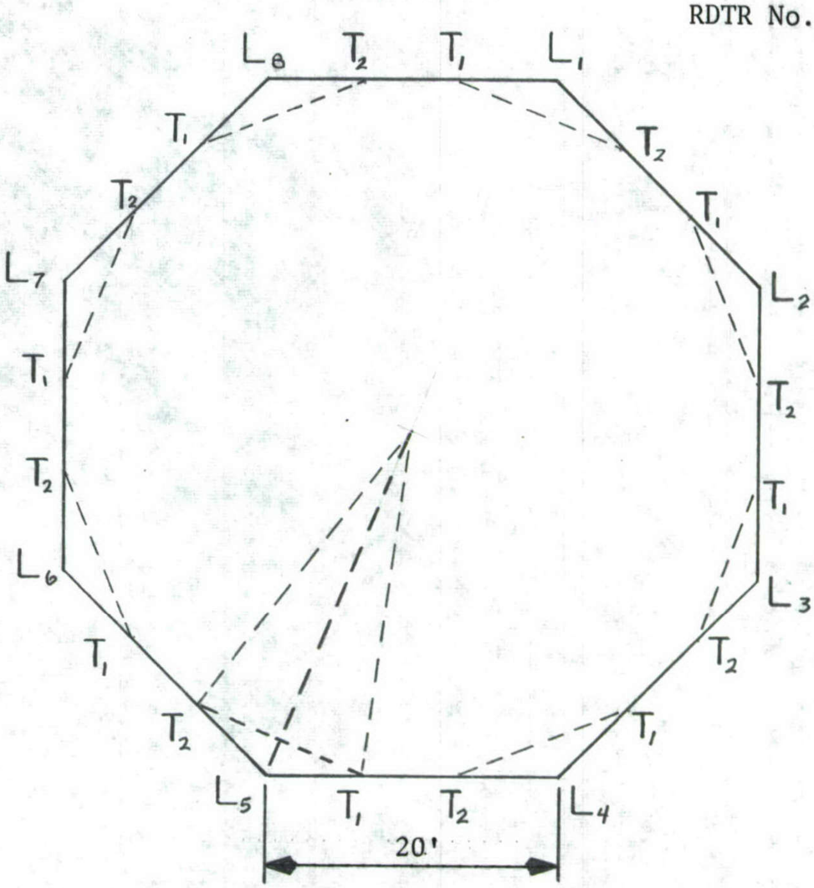
62,699

$2C_1$ = 62,699

C_1 = 31,349#

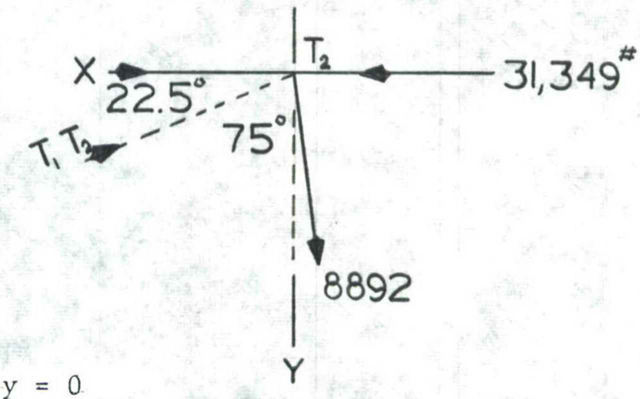
VIII. Compression Ring Design:

Sketch of Compression Ring is presented on the next page



Discussion: The compression ring is composed of 8 individual members similar in load and geometric dimensions. Each of these members, say L_1, L_2 for example, is subject to combined axial and bending load. Since L_1, L_2 is 20' long its diameter must be proportionately large to insure an acceptable L/r ratio. However, if L_1 L_2 was adequately braced at points T_1 and T_2 in order to reduce or eliminate the bending moment, the diameter of L_1 L_2 could be greatly reduced. The compression ring will, therefore, be so designed as to provide horizontal and vertical bracing forces at points T_1 and T_2 equal and opposite to the tension forces imposed by the cables.

Free Body Diagram of Point T_1 or T_2



Sum of $F_{y-y} = 0$

↓ 8892 Cos 7.5° = 8803 ↑ T_1T_2 Sin 22.5°

$$T_1T_2 = \frac{8803}{.383} = 22,985\#$$

Sum of $F_{x-x} = 0$

→ U_1T_2 ← 31,349

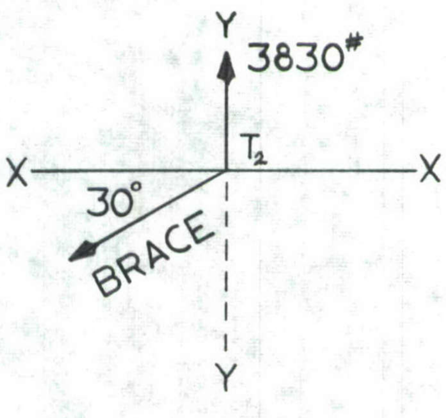
22,985(.924)

8892(.1305)

U_1T_2 = 31,347 - 21238 - 1160

U_1T_2 = 8951# compression

Determine Force in Vertical Brace Members



Sum of Fyy = 0

↑ 3830 ↓ Brace Force Sin 30°

$$\text{Brace Force} = \frac{3830}{\sin 30^\circ} = 7660\#\ \text{Tension}$$

IX. Determine Force in Center Column

Sum of $F_v = 0$

↓

$16 \times 6319\# = 101,104$ ↑ F_c

$8 \times 5320\# = \underline{42,560}$

$143,664\#$

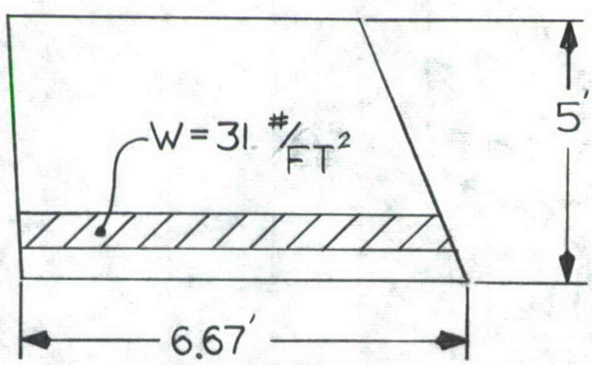
$F_c = \underline{143,664}$

X. Foam Roof Considerations:

The foam roof is considered sufficiently strong to transmit the live loads to the respective cables through the bending action of the foam. This assumption leads to a foam thickness requirement of approximately 6" of 2 lb. density foam as illustrated in the following calculations.

Roof Calculations:

SKETCH:



Determine foam thickness for a 1' wide section assuming 85% of the load is carried in the long direction. Foam slab is continuous so the Maximum Moment is equal approximately to $\frac{Wl^2}{12}$

Calculations

$$6 = \frac{Mc}{I} .85 \qquad I = \frac{bh^3}{12}$$

$$I/c = \frac{M}{6} .85 \qquad I/c = \frac{bh^2}{6} = \frac{bh^2}{6}$$

$$\frac{bh^2}{6} = \frac{Wl^2}{12} \times 12.85$$

$$\frac{h^2 = 6 \times 31(6.67)^2 \times 12}{12 \times 12} .85$$

$$h^2 = \frac{15.5 \times 6.67 \times 6.67}{20} .85$$

$$h^2 = 29.3$$

$$h = 5.4''$$

In the Author's opinion the foam thickness requirement can be greatly reduced by considering the composite action of the foam, fabric, and wire mesh. However, design calculations will not be presented until further experimental data can be completed.

APPENDIX B

Figure 1. Elevation View showing Polyurethane foam roof system on scale model.

Figure 2. Illustrates the tension cables, foam roof system, and center column of scale model.

FIGURE 1. ELEVATION VIEW SHOWING POLYURETHANE FOAM ROOF SYSTEM ON SCALE MODEL.

FIGURE 2. ILLUSTRATES THE TENSION CABLES, FOAM ROOF SYSTEM, AND CENTER COLUMN OF SCALE MODEL.

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13. ABSTRACT

A method of covering an octagon shaped area with a polyurethane roof supported on tension cables is discussed herein. The proposed roof structure is comprised primarily of a center supporting column, tension cables, compression ring, perimeter columns, and foam roof system. It was concluded from the material covered in this report that construction of this type of roof system is economically and structurally feasible.