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Design of Experiments: A Tutorial

If the title was revised please list the original title above and the revised title here:

PRESENTED III:

WORKING GROUP: 33
COMPOSITE GROUP:
SPECIAL SESSION 1:
SPECIAL SESSION 2:
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Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18
Design of Experiments

A Tutorial

Paul J. Bross
Operations Research Principal
Center for Innovation
June 2007
Tutorial Composition

- Basic Concepts
- Break
- Advanced Concepts
- Break
- Detailed Examples
- Wrap-Up
Design of Experiments

Basic Concepts
• What is DoE?
• Purposes of Experimenting
• Experimentation Strategies
• Basic Principles
• Nuisance Factors
• Design Steps
• Major Guidelines
• Simple Comparison Experiments
• Single Factor Experiments
• Latin Squares
What is DoE?

- **Experiment**: a test or series of tests where the experimenter makes purposeful changes to input variables of a process or system so that we can observe or identify the reasons for changes in the output responses.

- **Design of Experiments**: is concerned with the planning and conduct of experiments to analyze the resulting data so that we obtain valid and objective conclusions.
• 1771 – *Course of Experimental Agriculture*, Arthur Young
  – One of the earliest direct experimental scientific documents
  – Insisted on split-field trials
  – Required repeated trials in different fields

• 1919 – R.A. Fisher started work as a statistician at Rothamsted Agricultural Experimental Station
  – Randomization of trials
  – Creation of the technique “Analysis of Variance”

• Today…. 
Code of Best Practices (COBP)

- **Code of Best Practice for Experimentation**, CCRP, 2002
- **Campaigns of Experimentation: Pathways to Innovation and Transformation**, Alberts & Hayes, 2005

- These documents identify 3 types of experiments:
  - Discovery
  - Hypothesis
  - Demonstration

- This tutorial focuses on aspects of the first two types
Types of Experiments

• **Discovery**
  – Designed to generate new ideas or approaches
  – Usually involve “hands-on” activities
  – May involve systems or processes that are not well understood or refined

• **Hypothesis**
  – Closer to the traditional academic approach
  – Seek to falsify specific hypotheses
  – Used often in the attempt to “prove” a theory, idea, or approach
Why Experiment?

- Determine which variables are the most influential in a process or system
- Determine where to set the inputs so the output is always near the desired state
- Determine where to set the inputs so the output variability is minimized
- Determine where to set the inputs so the influence of uncontrollable factors is minimized (robust design)
Experimentation Strategies

• **Best Guess**
  – **PRO:** Works reasonably well when used by SMEs with solid foundational knowledge on known issues
  – **CONs:**
    ▪ If it fails, need to guess again…and again…until….
    ▪ If get acceptable results first time, may stop without discovering “better”

• **One Factor at a Time**
  – **PRO:** Straight-forward, easily understood
  – **CONs:**
    ▪ Impossible to address interactions
    ▪ Tends to “over collect” data, not efficient sample sizes

• **Factorial**
  – **PROs:**
    ▪ Full evaluation of individual and interaction effects
    ▪ Most efficient design with respect to sample sizes
  – **CON:** More complex to explain to untrained audiences
Basic Principles

• **Replication**
  – Permits estimation of experimental error
  – Permits more precise estimates of the sample statistics
  – Not to be confused with repeated measures

• **Randomization**
  – Insures that observations or errors are more likely to be independent
  – Helps “average out” effects of extraneous factors
  – Special designs when complete randomization not feasible

• **Blocking**
  – Designed to improve precision of comparisons
  – Used to reduce or eliminate nuisance factors
Nuisance Factors

• Definition: A nuisance factor is a “design factor that *probably* has an effect on the response but we are not interested in that effect” [Montgomery, p126, emphasis added]

• Nuisance Factors, Types ⇒ Cures
  – Known and controllable ⇒ Use blocking to systematically eliminate the effect
  – Known but uncontrollable ⇒ If it can be measured, use Analysis of Covariance (ANCOVA)
  – Unknown and uncontrollable ⇒ Randomization is the insurance
Design Steps

• Recognition and statement of the problem in *nonstatistical* language
• Selection of factors, levels, ranges
• Selection of response variables
• Choice of experimental design
• Performance of the experiment
• Statistical analysis of the data
• Conclusions and recommendations
Major Guidelines

• Use team’s **non-statistical** knowledge of the problem to:
  – Choose factors
  – Determine proper levels
  – Decide number of replications
  – Interpret results

• Keep the design and analysis as simple as possible

• Recognize the difference between practical and statistical significance

• Be prepared to iterate – commit no more than 25% of available resources to first series
• **Goal:**
  - Compare two or more means; variances; probabilities
  - Compare A versus B: [better or worse] – paired comparison is a special case of randomized block design

• **Major Considerations**
  - Sample size
  - Distributional knowledge: Normal, $\chi^2$, F … etc.
  - Structure of the statistical hypothesis
    - One-tailed
    - Two-tailed tests
Single Factor Experiments

- One Factor – Multiple Levels
- “One-level-at-a-time” analysis isn’t efficient
  - Consider one factor with five levels
  - Pair-wise comparison requires 10 pairs \[ \binom{5}{2} = 10 \]
  - If each comparison has \( \alpha = 0.05 \), then
    \[
    \text{Probability(correct assessment)} = (1-\alpha)^{10} = 0.60
    \]

- Technique of Choice – ANOVA
  - Tests hypothesis \( H_0: \mu_1 = \mu_2 = \mu_3 = \ldots \mu_n \)
  - Assumptions
    - Error term is Normal \((0,\sigma^2)\) \(\Rightarrow\) test residuals to confirm
    - Conditions properly randomized
    - Results are independent; errors are independent
  - If reject \( H_0 \) (\text{i.e., failed the test}) then use Newman-Keuls Range Test or Duncan’s Multiple Range Test to determine the specifics
  - Note – there are non-parametric tests in lieu of ANOVA if assumptions are not met (\text{e.g.} Kruskal-Wallis Test)
- Single Factor – Unit Days of Supply
- Levels – 5, 10, 15, 20, 25
• **Latin Square**: An arrangement of conditions such that each combination occurs only once in each row and column of the test matrix.

\[
\begin{array}{cccc}
A & B & C & D \\
B & C & D & A \\
C & D & A & B \\
D & A & B & C \\
\end{array}
\]

• **Graeco-Latin Square**: The superposition of two Latin Squares such that each paired-combination occurs only once in each row and column.

Orthogonal Latin Square
• Conduct a test of new intelligence fusion procedures using four analyst teams examining four scenarios. Each fusion process will take one day of test activity to fully work the process.
  – Day 1 ⇒ Orientation Day for participants; assign teams (A,B,C,D)
  – Days 2 through 5 ⇒ Test days

<table>
<thead>
<tr>
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<th>Tues</th>
<th>Wed</th>
<th>Thurs</th>
<th>Fri</th>
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<tr>
<td>Scenario 2</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>A</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>C</td>
<td>D</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>D</td>
<td>A</td>
<td>B</td>
<td>C</td>
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</tbody>
</table>

• Do it again, 3 months later with different teams (α,β,χ,δ)

<table>
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<th>Wed</th>
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<td>γ</td>
<td>δ</td>
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<td>Scenario 4</td>
<td>δ</td>
<td>α</td>
<td>β</td>
<td>γ</td>
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• Combine analytical results

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<td>D δ</td>
<td>A α</td>
<td>B β</td>
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<tr>
<td>Scenario 4</td>
<td>D δ</td>
<td>A α</td>
<td>B β</td>
<td>C χ</td>
</tr>
</tbody>
</table>
Design of Experiments

Advanced Concepts
Segment Agenda

• Advanced DoE
  – Factorials
    ▪ Full
    ▪ Fractional
    ▪ Other Types
  – Complex Designs
• Definition: An experiment in which for each completed trial or replication of the experiment all possible combinations of the levels of the factors are investigated.

• Design Notation
  – General Notation for 2-level experiment \( \Rightarrow 2^k \) where \( k \) = number of factors
    ▪ 3 factors 2 levels each = \( 2^3 \) design
  – Factors and Levels \( \Rightarrow \) example for 3 factors, 2 levels
    ▪ Aa Bb Cc
    ▪ \( A^+A^- B^+B^- C^+C^- \)
    ▪ \( (1) \ a \ b \ c \)
**Full Factorial Design**

- **All combinations are examined**
  - Example $2^3$ design = 8 experimental settings:
    
    $A^+B^+C^+ B^-C^+ A^+B^-C^+ A^+B-C^+ A-B+C^+ A-B+C^- A-B-C^+ A-B-C^-$

- **Effects Evaluated**
  - Main effects of single factors: $A$, $B$, $C$
  - Second Order (2-factor) interactions: $AB$, $AC$, $BC$
  - Third Order (3-factor) interactions: $ABC$
  - In general, a $2^k$ design evaluates all $1, 2, \ldots k-1, k$-factor effects

- **Advantages over “one-factor-at-a-time”**
  - More efficient in time, resources, sample size
  - Addresses interactions
  - Provides insight over a *range* of experimental conditions
Factorial Efficiency – Graphically (1)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
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<tr>
<td>(1)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
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<td></td>
<td>-</td>
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Legend:
- : negative
+ : positive

Factors:
- A
- B
- C

Interactions:
- ABC
- AB
- AC
- BC
- AC
- BC

(1) indicates the reference point.
Main Effect A

\[ \text{Main Effect A} = \frac{1}{4n} \times \text{[blue square - red square]} \]
Main Effect B

\[ = \frac{1}{4n} \times [\text{blue square} - \text{red square}] \]
Main Effect C

\[ = \frac{1}{4n} \times \text{blue square} \cdot \text{red square} \]
Effect AB

\[ = \frac{1}{4n} \times [\text{blue plane} - \text{red plane}] \]
Effect AC

\[ = \frac{1}{4n} \times \text{[blue plane]} \times \text{[red plane]} \]
Effect BC

\[= \frac{1}{4n} \times [\text{blue plane}] \quad \text{-red plane}\]
Effect ABC

\[ = (1/4n) \times \text{[blue tetrahedron]} - \text{red tetrahedron} \]
**Factorial Statistics (1)**

- Standard ANOVA table

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<tr>
<th>Source</th>
<th>Sum of Squares</th>
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<tr>
<td>C</td>
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<td>30040938.28</td>
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**Factorial Statistics (2)**

**• Additional Statistics**

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<td>C.V.</td>
<td>2.394776115</td>
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<tr>
<td>PRESS</td>
<td>1742804.889</td>
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**PRESS = Prediction Error Sum of Squares**
- A measure of how well the model will “predict” new data
- Smaller is better but can only be used in a comparative sense

**R-Squared**
- Measures the proportion of total variability explained by the model
- Value: 0.98156674

**Adj R-Squared**
- An estimate of the signal-to-noise ratio in the data
- An indicator if Response Surface Methods (RSM) are applicable
- Value: 0.97619037

**Pred R-Squared**
- Value: 0.96722976

**Adeq Precision**
- Value: 42.0620068
- Measures the signal-to-noise ratio in the data
- Values >4 are good

**Coefficient of Variation**
- Measures the proportion of total variability explained by the model
- Value: 2.394776115

**Mean of the main measure**
- Value: 8439.46875
• Final Model looks like a regression equation

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<tr>
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<td>* E</td>
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<td>* C * E</td>
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<td>235.84375</td>
<td>* A * C * E</td>
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• Tests of Significance
  – Overall model response
  – Individual coefficients

• Diagnostic tests
  – Residuals
  – Outliers
  – Lack of Fit
Fractional Factorial Designs

(1)

• A way to reduce a huge full factorial to something manageable
  – Considerations
    ▪ Required time, resources
    ▪ Complexity of set-up for experiments
  – Major use is in screening experiments where the knowledge of basic effects is not well known
  – If $2^k$ is very large, may need to run reduced experiment

• Justification
  – Sparsity of Effects – in general, even complex systems are usually driven by a few main effects and low-level interactions
  – Projection Property – fractional factorial designs can be “projected” into larger designs in the subset of significant factors
  – Sequential Experimentation – can combine runs of 2 or more fractional designs into larger designs
Fractional Factorial Designs (2)

• **Issue:**
  - Confounding of Effects (also called “aliasing”) \(\Rightarrow\) reduced experiments do *not* evaluate all levels of the factors and their interactions
  - Some mixture of effects is “confounded” and not identifiable

• **Challenge:**
  - To select the best combination of test elements that stands a reasonable chance of revealing the true effects
  - Alias the (most likely) insignificant or unwanted factors

• **Symbology**
  - \(2^{k-p}\) designs
**Fractional Factorial Designs**

(3)

- **Resolution** → a measure of confounding
  - **Resolution III**
    - No main effect aliased with any other main effect
    - Main effects are aliased with 2-factor interactions
    - 2-factor interactions may be aliased with each other
  - **Resolution IV**
    - No main effect aliased with any other main effect
    - No main effect aliased with 2-factor interactions
    - 2-factor interactions may be aliased with each other
  - **Resolution V**
    - No main effect aliased with any other main effect
    - No main effect aliased with 2-factor interactions
    - No 2-factor interactions may be aliased with each other
    - 2-factor interactions are aliased with 3-factor interactions
# Resolution Trade-offs

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<td>1/4 Fract.</td>
<td>1/8 Fract.</td>
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- **Green** = Resolution V
- **Yellow** = Resolution IV
- **Red** = Resolution III
# Half Replicate/Folding

<table>
<thead>
<tr>
<th>STD</th>
<th>RUN</th>
<th>A</th>
<th>B</th>
<th>C</th>
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</tbody>
</table>
Other Design Variations (1)

- Three levels for k-factors ($3^k$) designs
- Fractional 3-level designs ($3^{k-p}$)
- Adding Center runs to
  - Get estimates of process variability
  - Gain familiarity with the process
  - Identify system performance limits
- Mixture Designs – where one or more factors are constrained to add to something
  - Usually have constraints like: $x_1 + x_2 + x_3 + \ldots + x_p = 1$
  - Example: A mixture of contributing probabilities
- Nested and Split-Plot designs for experiments with random factors
Other Design Variations (2)

• **Irregular Fraction**
  – Usually a Resolution V design for 4 to 9 factors where each factor is varied over only 2 levels
  – Two-factor interactions aliased with three-factor and higher
  – Excellent to reduce number of runs and still get clean results

• **General Factorial**
  – For 1 to 12 factors where each factor may have a different number of levels
  – All factors treated as categorical

• **D-Optimal**
  – A special design for categorical factors based on a analyst-specified model
  – Design will be a subset of the possible combinations
  – Generated to minimize the error associated with the model coefficients
Other Design Variations (2)

• **Plackett-Burman**
  - Specialized design for 2 to 31 factors where each factor is varied over only 2 levels
  - Use only if you can reasonably assume NO two-factor interactions; otherwise, use fractional factorial designs

• **Taguchi OA**
  - Saturated orthogonal arrays – all main effects and NO interactions
  - Special attention must be paid to the alias structure for proper interpretation at both the design phase (prior to runs) and during final analysis
Design of Experiments

Practical Examples
Steps in DoE

• Design the experiment
• Evaluate the design
  – Model specification
  – Power calculations \((1-\beta)\)
  – Graphical examination of the standard error of the design
• Conduct the experiment and collect data
• Analyze the results
  – Examine data for transformation suggestions
  – Compute the effects
  – Perform ANOVA
  – Critical!!! – ALWAYS check the diagnostics
  – Examine graphical findings
  – Finalize the analysis
Diagnostics

• Diagnostic steps are the most often omitted – to the analyst’s potential embarrassment

• Which of these ANOVA tables are to be believed?

### Table A

<table>
<thead>
<tr>
<th>Term</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob &gt; F</th>
<th>% Contribution</th>
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<tbody>
<tr>
<td>Intercept</td>
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<td>18.92</td>
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### Table B

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<th>Prob &gt; F</th>
<th>% Contribution</th>
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</thead>
<tbody>
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<td></td>
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<td>1.70</td>
<td>0.11</td>
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</tbody>
</table>
Normality Check

A

Normal Plot of Residuals

Normal % Probability

Studentized Residuals

B

Normal Plot of Residuals

Normal % Probability

Studentized Residuals
Residuals Check

A

Residuals vs. Predicted

B

Residuals vs. Predicted
Residuals vs. Run

A

Residuals vs. Run

B

Residuals vs. Run

Studentized Residuals

Run Number

Studentized Residuals

Run Number
Residuals vs. Settings

A

Residuals vs. Settings

B

Residuals vs. Settings
Outlier T Check

A

Outlier T

B

Outlier T
Cook’s Distance

A

Cook’s Distance

B

Cook’s Distance
Leverage
Power Transform Recommendation

A

Box-Cox Plot for Power Transforms

- Lambda = 1
- Best = -1.43
- Low C.I. = -2.21
- High C.I. = -0.84
- Recommend transform: Power (Lambda = -1.43)

B

Box-Cox Plot for Power Transforms

- Lambda = 1
- Best = 0.54
- Low C.I. = -1.12
- High C.I. = 2.11
- Recommend transform: None (Lambda = 1)
Single Factor Plot

A

One Factor Plot

Cargo Vehicles

A: SD

B

One Factor Plot

Cargo Vehicles

A: SD
Remember!!

ALWAYS perform the diagnostic tests!
Measuring the Effect of C3I on Combat: Methodology and Results

An Example of the Application of Design of Experiments Concepts and Techniques
Initial Guidance

• Evaluate the impact of the representations of C3I on combat outcomes in a campaign-level force-on-force model.
  – Perform sensitivity analyses across all areas of C3I, utilizing the existing test scenario.
  – Determine which C3I-related input data have the most impact on combat outcomes.
Approach

- Select specific C3I functions to be examined.
- Design the Experiment.
- Prepare model software and scenario.
- Execute model runs.
- Analyze the output and report findings.
Excluded Elements

The following items are not part of the study as they are either not controllable from a military sense or they represent different tactics or behaviors, which are not of interest for this study:

- Weather
- Intelligence Ratings
- Force Structure
- ISR Collection Plans
• Bundled multiple factors into 3 categories to describe C3I functionality in terms of:
  – Timeliness
  – Quantity
  – Quality
• Each candidate factor could influence combat outcome either:
  – By itself,
  – In concert with another factor,
  – In opposition to another factor.

• Previous research in this area has shown serious non-linear effects.
Three-factor design with interaction and non-linear terms.

\[
CO = \beta_0 + \beta_1 T + \beta_2 Q_T + \beta_3 Q_L + \beta_{11} T^2 + \beta_{22} Q_T^2 + \beta_{33} Q_L^2 + \beta_{12} T Q_T + \beta_{13} T Q_L + \beta_{23} Q_T Q_L + \beta_{123} T Q_T Q_L + \varepsilon
\]

where:
- \( CO \) = Combat Outcome
- \( T \) = Timeliness
- \( Q_T \) = Quantity
- \( Q_L \) = Quality
- \( \beta_i \) = an unknown value to be estimated
- \( \varepsilon \) = the error term
Measurement Points

Face-Centered Central Composite Design

Quantity

Quality

Timeliness

Low

High

Low

High

-1

+1

-1

+1
The FC-CCD yields the following design matrix:

\[
CO = \begin{bmatrix}
-1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\
-1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 \\
-1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & -1 & +1 & 0 \\
0 & 0 & -1 & +1 & 0 & 0 & 0 \\
-1 & +1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}^T
\]

Corner Points

Face Points
Each parameter was chosen so that 3 settings were possible to match the FC-CCD requirements:

- High (meaning improved or enhanced performance)
- Center (baseline)
- Low (meaning reduced or degraded performance)
# Timeliness (T) Settings

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Low (-1)</th>
<th>Center (0)</th>
<th>High (+1)</th>
</tr>
</thead>
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<tr>
<td>Reporter Delay Time (RDT)</td>
<td>8</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Presented Communications Load (PCL)</td>
<td>1.25*(PCL_{\text{Base}})</td>
<td>(PCL_{\text{Base}})</td>
<td>0.75*(PCL_{\text{Base}})</td>
</tr>
<tr>
<td>Maximum Communications Network Capacity (MCNC)</td>
<td>0.75*(MCNC_{\text{Base}})</td>
<td>(MCNC_{\text{Base}})</td>
<td>1.25*(MCNC_{\text{Base}})</td>
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</table>
## Quantity ($Q_T$) Settings

<table>
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<th>Center (0)</th>
<th>High (+1)</th>
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<tbody>
<tr>
<td>IMINT Probability of Detection ($P_{d-IMINT}$)</td>
<td>0.4</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Sensor Footprint (SFP)</td>
<td>0.707*SFP$_{Base}$</td>
<td>SFP$_{Base}$</td>
<td>1.414*SFP$_{Base}$</td>
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<tr>
<td>COMINT Sensor Search Rate (CSSR)</td>
<td>0.5*CSSR$_{Base}$</td>
<td>CSSR$_{Base}$</td>
<td>2*CSSR$_{Base}$</td>
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# Quality ($Q_L$) Settings

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<th>Parameters</th>
<th>Low (-1)</th>
<th>Center (0)</th>
<th>High (+1)</th>
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</thead>
<tbody>
<tr>
<td>Probability of Correct Classification for MTI sensors ($P_{CC-MTI}$)</td>
<td>0.75 : 0.25</td>
<td>0.5 : 0.5</td>
<td>0.25 : 0.75</td>
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<tr>
<td>Quality Probability for explicit IMINT search ($P_{Q-IMINT}$)</td>
<td>$P_{\text{Degrate-Q-IMINT}}$</td>
<td>$P_{Q-IMINT}$</td>
<td>$P_{\text{Upgrade-Q-IMINT}}$</td>
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<tr>
<td>Association Threshold (AT)</td>
<td>0.5*AT$_{\text{Base}}$</td>
<td>AT$_{\text{Base}}$</td>
<td>2*AT$_{\text{Base}}$</td>
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</table>
• Quality Classification Probability is actually a distribution, not a single value, where $\sum p_i = 1$

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<th>Quality Level</th>
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<th>2</th>
<th>3</th>
<th>N</th>
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<td>$p_2$</td>
<td>$p_3$</td>
<td>$p_N$</td>
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</table>
• Developed a “Shift” transform that moves probability from one category to another.
# Combat Outcome Measures

<table>
<thead>
<tr>
<th>Combat Outcome</th>
<th>Sources</th>
</tr>
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</table>
| $R_{K-All}$    | Direct Fire KVSB*  
                              | Indirect Fire KVSB  
                              | Air-to-Ground KVSB          |
| $R_{K-DF}$     | Direct Fire KVSB  |
| $R_{K-IF}$     | Indirect Fire KVSB |
| $R_{K-A2G}$    | Air-to-Ground KVSB |

*KVSB = Killer-Victim Scoreboard*
## Run Results Summary

<table>
<thead>
<tr>
<th>Response</th>
<th>Name</th>
<th>Observations</th>
<th>Minimum</th>
<th>Maximum</th>
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</thead>
<tbody>
<tr>
<td>Y1</td>
<td>DF Kills - Red</td>
<td>150</td>
<td>166.696716</td>
<td>543.007375</td>
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<tr>
<td>Y2</td>
<td>IF Kills - Red</td>
<td>150</td>
<td>38.507262</td>
<td>317.413308</td>
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<tr>
<td>Y3</td>
<td>A2G Kills - Red</td>
<td>150</td>
<td>639.270296</td>
<td>1986.671009</td>
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<tr>
<td>Y4</td>
<td>Total Kills - Red</td>
<td>150</td>
<td>1039.098472</td>
<td>2422.615195</td>
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</table>

DF = Direct Fire  
IF = Indirect Fire  
A2G = Air-to-Ground
### ANOVA – Total Kills

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<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob &gt; F</th>
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<td>Model</td>
<td>6,024,287.18</td>
<td>9</td>
<td>669,365.24</td>
<td>20.2233</td>
<td>&lt; 0.0001 *</td>
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<td>3,235,042.05</td>
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<td>3,235,042.05</td>
<td>97.739</td>
<td>&lt; 0.0001 *</td>
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<tr>
<td>QT</td>
<td>659,344.82</td>
<td>1</td>
<td>659,344.82</td>
<td>19.9205</td>
<td>&lt; 0.0001 *</td>
</tr>
<tr>
<td>QL</td>
<td>794,562.33</td>
<td>1</td>
<td>794,562.33</td>
<td>24.0058</td>
<td>&lt; 0.0001 *</td>
</tr>
<tr>
<td>T²</td>
<td>212,422.81</td>
<td>1</td>
<td>212,422.81</td>
<td>6.4178</td>
<td>0.0124 *</td>
</tr>
<tr>
<td>QT²</td>
<td>558,906.79</td>
<td>1</td>
<td>558,906.79</td>
<td>16.886</td>
<td>&lt; 0.0001 *</td>
</tr>
<tr>
<td>QL²</td>
<td>356,637.37</td>
<td>1</td>
<td>356,637.37</td>
<td>10.7749</td>
<td>0.0013 *</td>
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<td>TQT</td>
<td>17,652.64</td>
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<td>17,652.64</td>
<td>0.5333</td>
<td>0.4664</td>
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<td>47,410.91</td>
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<td>47,410.91</td>
<td>1.4324</td>
<td>0.2334</td>
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<tr>
<td>QTQL</td>
<td>199,200.94</td>
<td>1</td>
<td>199,200.94</td>
<td>6.0184</td>
<td>0.0154 *</td>
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<td>Residual</td>
<td>4,633,827.61</td>
<td>140</td>
<td>33,098.77</td>
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<tr>
<td>Lack of Fit</td>
<td>660,829.65</td>
<td>5</td>
<td>132,165.93</td>
<td>4.4909</td>
<td>0.0008 *</td>
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<td>Pure Error</td>
<td>3,972,997.96</td>
<td>135</td>
<td>29,429.61</td>
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<td></td>
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<tr>
<td>Cor Total</td>
<td>10,658,114.79</td>
<td>149</td>
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</tbody>
</table>

α = 0.05
Fitted Model – Total Kills

\[
CO = \left( 1595.89 + 179.86T + 81.20Q_T + 89.14Q_L + \\
+ 90.89T^2 + 147.43Q_T^2 - 117.77Q_L^2 \\
- 14.85TQ_T - 24.34TQ_L - 49.90Q_TQ_L \right)
\]
• Evaluate the formal design by examining the following parameters:
  – Calculate power of the tests
  – Perturbation plots
  – Contour plots
  – Standard error graphs
## Power of the Design

<table>
<thead>
<tr>
<th>Term</th>
<th>StdErr**</th>
<th>Power at 5% alpha level for effect of:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1/2 Std. Dev. 1</td>
</tr>
<tr>
<td>T</td>
<td>0.1</td>
<td>69.90%</td>
</tr>
<tr>
<td>QT</td>
<td>0.1</td>
<td>69.90%</td>
</tr>
<tr>
<td>QL</td>
<td>0.1</td>
<td>69.90%</td>
</tr>
<tr>
<td>T²</td>
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<td>71.20%</td>
</tr>
<tr>
<td>QT²</td>
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<td>QL²</td>
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<td>71.20%</td>
</tr>
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<td>TQT</td>
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<td>60.30%</td>
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<tr>
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<td>0.1118034</td>
<td>60.30%</td>
</tr>
<tr>
<td>QTQL</td>
<td>0.1118034</td>
<td>60.30%</td>
</tr>
</tbody>
</table>

**Basis Std. Dev. = 1.0**
Perturbation Plot

- Plot of Standard Error of Design
  - Shows error of the estimates increases at the edge of the design space
  - All factors overlap: they have the same standard error
  - Conclusions based on extreme values may be subject to major qualification

- DESIGN-EXPERT Plot
  - StdErr of Design
  - Actual Factors
    A: Timeliness = 0.00
    B: Quantity = 0.00
    C: Quality = 0.00

- Perturbation
  - Deviation from Reference Point
  - StdErr of Design
    - A: 0.235702
    - B: 0.218557
    - C: 0.201411

- Conclusions based on extreme values may be subject to major qualification
Contour Plot

- Plot of Standard Error of Design
  - 2-Factor view for a constant setting of the 3\textsuperscript{rd} factor
  - Tight contours indicate steepness of response
  - More difficult to read than a 3-D plot
3-D Standard Error Plot

- Plot of Standard Error of Design
  - 3-D, 2-Factor view for a constant setting of the 3\textsuperscript{rd} factor
  - Corresponding contour plot is shown on the base
  - Depth of shading indicates steepness of slope
Diagnostic Tests

• Examine data output with:
  – Normal plot of the residuals
  – Residuals vs. predicted error
  – Residuals vs. run
  – Residuals vs. Timeliness
  – Residuals vs. Quantity
  – Residuals vs. Quality

• Conduct outlier investigation

• Conduct transform analysis
Residual Plot

- Desired – data points fall on a straight line
- Actual – does not show any serious abnormality
- Results – OK
- Residual Analysis
  - Desired – no apparent pattern in the observed data
  - Actual – no pattern in the observed data
  - Results – OK
Residual Analysis

- Desired – no apparent pattern in the observed data
- Actual – no pattern in the observed data
- Results – OK
Residuals vs. Timeliness

Residual Analysis

- Desired – no apparent pattern in the observed data
- Actual – no pattern in the observed data
- A slight expansion as settings shift from Low to High but not strong enough to invalidate results
- Results – OK
Residual Analysis

- Desired – no apparent pattern in the observed data
- Actual – no pattern in the observed data
- Results – OK
Residual Analysis

- Desired – no apparent pattern in the observed data
- Actual – no pattern in the observed data
- Results – OK
Outlier Analysis

- Desired – no apparent pattern in the observed data
- Actual – no pattern in the observed data
- Results – OK
Outlier Analysis

- Desired – strong clustering near the zero point
- Actual – strong clustering near the zero point
- Results – OK
Outlier Investigation (3)

- Outlier Analysis
  - Desired – strong clustering near the zero point
  - Actual – strong clustering near the zero point
  - Results – OK

DESIGN-EXPERT Plot
Total Kills - Red

Leverage vs. Run

Leverage

[Graph showing leverage vs. run numbers]

Run Number

Leverage

-1.00
-0.83
-0.67
-0.50
-0.33
-0.17
0.00
Predicted vs. Actual

- Outlier Analysis
  - Desired – no apparent pattern in the observed data
  - Actual – no pattern in the observed data
  - Results – OK

DESIGN-EXPERT Plot
Total Kills - Red
Transform Analysis
- Desired – no transform
- Actual – Log transform recommended
- Results – transform not pursued due to constraints:
  - Time available
  - Quality of data

Box-Cox Plot for Power Transforms

Lambda
Current = 1
Best = -0.03
Low C.I. = -0.74
High C.I. = 0.7

Recommend transform Log
(Lambda = 0)
Evaluation Results

• Model has statistical power:
  – For Type I error of 5%, Type II error is less than 0.1%.

• Diagnostics acceptable:
  – No problems based on residual analysis.
  – No problems based on outlier analysis.
  – Data transform suggested but not deemed essential for this task.
• **Review the results in terms of:**
  - ANOVA Table
  - Perturbation plots
  - Single factor response
  - Interaction response
  - Contour plots
  - 3-D surface plots
  - Cube plot
## ANOVA – Total Kills

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob &gt; F</th>
<th>α = 0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>6,024,287.18</td>
<td>9</td>
<td>669,365.24</td>
<td>20.2233</td>
<td>&lt; 0.0001</td>
<td>*</td>
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<tr>
<td>T</td>
<td>3,235,042.05</td>
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<td>3,235,042.05</td>
<td>97.739</td>
<td>&lt; 0.0001</td>
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<tr>
<td>QT</td>
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<td>659,344.82</td>
<td>19.9205</td>
<td>&lt; 0.0001</td>
<td>*</td>
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<tr>
<td>QL</td>
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<td>794,562.33</td>
<td>24.0058</td>
<td>&lt; 0.0001</td>
<td>*</td>
</tr>
<tr>
<td>T²</td>
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<td>6.4178</td>
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<td>QL²</td>
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<td>356,637.37</td>
<td>10.7749</td>
<td>0.0013</td>
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<td>TQL</td>
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<td>47,410.91</td>
<td>1.4324</td>
<td>0.2334</td>
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<tr>
<td>QTQL</td>
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<td>6.0184</td>
<td>0.0154</td>
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<td>Residual</td>
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<td>140</td>
<td>33,098.77</td>
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<tr>
<td>Lack of Fit</td>
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<td>Pure Error</td>
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<td>29,429.61</td>
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<tr>
<td>Cor Total</td>
<td>10,658,114.79</td>
<td>149</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fitted Model – Total Kills

\[
CO = \left( 1595.89 + 179.86T + 81.20Q_T + 89.14Q_L + 90.89T^2 + 147.43Q_T^2 - 117.77Q_L^2 - 14.85TQ_T - 24.34TQ_L - 49.90Q_TQ_L \right)
\]
Perturbation Plot

- Single Factor Analysis
  - Shows curvature for each factor at the Center point
  - Provides visual confirmation of the ANOVA statistics
  - “Opposing” shift for Quality (C) reflects value of the squared term in the fitted equation

Design-Expert Plot

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>Deviation from Reference Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.000</td>
<td>Total Kills - Red</td>
</tr>
<tr>
<td>-0.500</td>
<td></td>
</tr>
<tr>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>0.500</td>
<td></td>
</tr>
<tr>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

Actual Factors
A: Timeliness = 0.00
B: Quantity = 0.00
C: Quality = 0.00

Total Kills - Red

1039.0985
1384.9777
1730.8568
2076.7360
2422.6152
Single Factor - Timeliness

Curvature in each of the panels shows the single factor response.

No significant effect would be a straight line with slope = 0.
Single Factor - Quantity
Single Factor - Quality
Interaction: $T$ vs. $Q_T$

- Curvature in each of the panels shows the response for constant Quality.
- Upper (Red) line shows Quantity = High.
- No significant effect would be over-lapping straight lines with slopes = 0.
Interaction: T vs. $Q_L$
Interaction: $Q_T$ vs. $Q_L$
Curvature in each of the panels shows the response for constant Quality.

Closeness of contour indicates relative steepness of slope.
Contours: T vs. Q_L
Contours: $Q_T$ vs. $Q_L$
Curvature in each of the panels shows the response for constant Quality.
3-D Surface: $T$ vs. $Q_L$
3-D Surface: $Q_T$ vs. $Q_L$
Response Analysis

- Desired – Combat Outcome (Total Kills) increases as performance moves from degraded to enhanced
- Actual – matches desired outcome
- Results – model is sensitive to the 3 factors in the direction hypothesized
## Significance Across Components

<table>
<thead>
<tr>
<th>Factor</th>
<th>DF</th>
<th>IF</th>
<th>A2G</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
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<td>Model</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$T$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$Q_T$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$Q_L$</td>
<td>*</td>
<td>*</td>
<td></td>
<td>*</td>
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<tr>
<td>$T^2$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<tr>
<td>$Q_T^2$</td>
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<td>*</td>
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<tr>
<td>$Q_L^2$</td>
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<td>$TQ_T$</td>
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<tr>
<td>$Q_TQ_L$</td>
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<td></td>
<td>*</td>
<td>*</td>
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<tr>
<td>Factor</td>
<td>DF</td>
<td>IF</td>
<td>A2G</td>
<td>TOTAL</td>
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<td>------</td>
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<tr>
<td>$\beta_0$</td>
<td>286.13</td>
<td>141.07</td>
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<tr>
<td>$T$</td>
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<td>234.66</td>
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<tr>
<td>$Q_T$</td>
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<tr>
<td>$T^2$</td>
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<td>-48.30</td>
<td>121.70</td>
<td>90.89</td>
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<tr>
<td>$Q_T^2$</td>
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<td>47.57</td>
<td>65.50</td>
<td>147.43</td>
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<tr>
<td>$Q_L^2$</td>
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<td>-20.69</td>
<td>-68.10</td>
<td>-117.77</td>
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<tr>
<td>$TQ_T$</td>
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<td>7.21</td>
<td>-30.36</td>
<td>-14.85</td>
</tr>
<tr>
<td>$TQ_L$</td>
<td>9.42</td>
<td>-1.72</td>
<td>-32.05</td>
<td>-24.34</td>
</tr>
<tr>
<td>$Q_TQ_L$</td>
<td>-13.43</td>
<td>-4.90</td>
<td>-31.58</td>
<td>-49.90</td>
</tr>
</tbody>
</table>
Battlefield Interactions

Maximum Artillery Range
Battlefield Interactions

Maximum Artillery Range

Maximum Direct-Fire Weapon Range
Battlefield Interactions

Maximum Artillery Range

Maximum Direct-Fire Weapon Range
Battlefield Interactions

Maximum Artillery Range

Maximum Direct-Fire Weapon Range
Conclusions

• The model is:
  – Sensitive to the C3I parameters of Timeliness, Quantity, and Quality of Information

• The Face-Centered CCD is:
  – Statistically Powerful
  – Robust
  – Capable of providing significant insights
Wrap-Up

• Topics Covered:
  – History from early days to Code of Best Practices
  – Types of experiments and why we do them
  – Strategies for experimentation
  – Basic comparison techniques
  – Analysis of Variance (ANOVA)
  – Complex variations of ANOVA
  – Importance of checking the diagnostic statistics
  – Practical military modeling example

Thank you for attending today.