Analyzing the Performance of Multi-hop Underwater Acoustic Sensor Networks

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Abstract—Multi-hop underwater acoustic sensor networks constrain the performance of medium access control protocols. The efficiency of the well-known RTS-CTS scheme is degraded due to long propagation delays of such networks. Recently, interest in Aloha variants has surfaced; however, the performance of such protocols within the context of multi-hop networks is not well studied. In this paper, we identify the challenges of modeling contention-based medium access control protocols and present a model for analyzing Aloha variants for a simple string topology as a first step toward analyzing the performance of contention-based proposals in multi-hop underwater acoustic sensor networks. An application of the model suggests that Aloha variants are vary sensitive to traffic loads and network size.

Index Terms—Aloha Protocol, Medium Access Control, Underwater Acoustic Network

I. INTRODUCTION

UNDERWATER acoustic sensor networks (UASNs) are constrained by both link capacity and propagation delays. For such networks, traffic generally flows from the individual sensor nodes to a single gateway node that serves to interface the acoustic network with the external world. Each node is responsible for sending its own traffic to the gateway as well as forwarding all traffic from upstream nodes to the gateway. A medium access control (MAC) protocol for such networks must be tailored for the particular traffic pattern as well as the pertinent capacity constraints and propagation delays. In [1], it was found that the traditional RTS-CTS mechanism is inefficient in networks composed of more than just a few hops. Contention-based protocols that implement carrier sense mechanisms are also less effective for networks with extreme propagation delays unless large frames are used [1].

Contention-based protocols based on the simple Aloha protocol may be effective for such networks [2]; however, their performance in multi-hop environments subject to the specific traffic characteristics of an UASN is not as thoroughly understood. Before implementing such protocols in an operational network a more rigorous analysis of their performance expectation should be performed.

Theoretical analyses performed regarding MAC methods for underwater acoustic networks have so far focused on single hop topologies. In particular, an analysis of the performance of an Aloha variant was reported in [3]. The topology studied consisted of a single receiving node surrounded by multiple contending sources. No consideration was given to the impact of having to relay traffic across more than one hop. The unique characteristic of UASNs, as noted, is that traffic in these networks tends to have a particular flow pattern. As all sensor-generated traffic flows to the gateway (GW), the offered load within a particular single neighborhood is inversely related to the number of hops that neighborhood is from the gateway, increasing the vulnerability of traffic to congestion as the traffic approaches the gateway.

Other factors beyond the traffic characteristics of the network complicate the analysis of the performance of MAC protocols within the context of a multi-hop topology. These include the half-duplex nature of the communication, time-varying and space-varying signal propagation losses, which make it a challenge to model the transmission error rates and link connectivity, application constraints, such as reliable service, and complex topology implementations. While a comprehensive theory to address all of these factors is desirable, such complexity is beyond the scope of this paper. Rather, this paper establishes an initial step towards such a theory by defining the problem space and developing a model of the performance of Aloha within the context of a simple, multi-hop topology as depicted in Fig. 1. The model can be extended to more complex topologies, such as a tree composed of multiple string topologies.

The model provides a method for computing the expected network utilization and the probability of frame delivery to the gateway from an arbitrary sensor. The results offer insights useful in determining the appropriateness of an Aloha variant for such topologies. An application of the model indicates that Aloha variants may have applicability for simple UASNs with small loads.

II. PROBLEM SPACE

A. Target Protocols

Contention-based protocols decentralize medium access control so nodes access the medium without pre-coordination.
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with other nodes. Two general classes differentiate these protocols: Aloha-derivatives that do not consider the current state of the medium, and carrier-sense multiple access (CSMA) protocols that do consider it. For a CSMA protocol to work effectively a sensor must be able to determine with some degree of accuracy if its transmission will collide with transmissions by other nodes. The nature of the wireless medium makes such detections nontrivial. In the radio frequency (RF) domain this issue is addressed by a class of MAC protocols that avoids collisions by employing dynamic access reservation schemes, such as the well-known RTS-CTS exchange, that limit, but do not eliminate, contention.

A closed form analysis of CSMA for a single neighborhood is available in [4]. However, the analysis does not consider multi-hop topologies. An analysis is reported in [8] of the performance of the 802.11 CSMA implementation, with the RTS-CTS virtual carrier sense functionality, for specific multi-hop topologies, using Markov Chains. This analysis comments on the difficulty of modeling multi-hop topologies, as the contention neighborhoods are topology dependent. The analysis does not address the potential impact of differences in propagation delays between nodes, generally assumed to be negligible in RF networks. From [4] it is clear that propagation delays impact the performance of CSMA protocols, even over a single hop network. We believe that the relatively large propagation delays in UANs complicate the analysis of carrier sense based protocols for that environment, especially as these are compounded by the neighborhood interactions in multi-hop networks.

The Aloha-derivatives are relatively easier to analyze as they do not consider the state of the medium prior to transmission, mitigating much of the consideration for propagation delay impact. The state of the medium at the recipient location when the frame is being received is the crucial consideration. The relative propagation delays are only of concern when the frame must be received by multiple nodes for the same transmission, as this effectively increases the reception vulnerability period.

B. Complicating Factors

Several factors must be considered when analyzing the performance of a contention-based protocol for multi-hop UASNs.

Traffic Pattern and Traffic Flow: A sensor may generate traffic at random and independent times or periodically. Reporting of rare or extreme events uses the former sampling pattern, while trend monitoring uses the latter.

If samples are generated randomly, then the traffic arrival process may be modeled as a Poisson distribution. If the generation of samples is independent between sensors, then the composite of the samples will also be a Poisson distribution. Conversely, if the sample generation is periodic, then differences in traffic arrivals between nodes will be strictly dependent on the propagation delay between node pairs. One might consider the time at which a particular node begins its sampling process as a random variable. However, once the sampling process is started the frame submission interval will remain fixed for the duration of the node’s lifespan, unless modified by an external agent or the sampling algorithm. Thus, once a collision occurs for periodic traffic, all subsequent frames from the sources involved will continue to collide. These perpetual collisions eliminate the contribution of those nodes from the sensor network.

A further issue is the flow pattern of the traffic itself. In a UASN, all traffic flows to a single destination, the gateway, as described above. Thus, there is a concentration of traffic as it flows toward that gateway. This concentration increases the likelihood of collisions for either sampling strategy, but in the case of periodic sampling it increases the likelihood of some sensors experiencing perpetual frame losses.

This suggests that contention-based protocols may not be a good choice for UASNs employing periodic sampling. A scheme that emulates time division multiple access may be more appropriate, such as that proposed in [5].

Reliable Service Requirements: Many applications require delivery assurance. Such reliable service requires buffering and retransmitting frames until an acknowledgment is received. For a math analysis, buffering and retransmissions may invalidate the assumption of the Poisson distribution of traffic patterns at the nodes. Further, the potential for contention increases as the reception of acknowledgments introduces an additional vulnerability period.

Channel Model: Several characteristics of the water channel make it difficult to model the explicit performance. Beyond the propagation delay issue discussed above, one must consider the impact of half-duplex communications, time-variant and space-variant signal loss, and the possibility of unidirectional links.

Half-duplex communications favor the transmission of a particular node over the reception of a frame by that node. That is, if a node initiates a transmission while a reception is on-going or if a frame arrives while the node is currently transmitting then the frame reception always fails but the transmission does not, unless the frame collides with another at the downstream node. If frame arrivals overlap at a particular host then those frames are lost, from the perspective of that node. However, since one of those frames will always be from a downstream node given the string topology of Fig.1, then the topology will favor the downstream traffic, as it will only be lost if it collides with a frame at a node further downstream.

Up to this point we have only discussed frame loss due to collisions. A frame may be lost also because of transmission errors mainly induced by signal attenuations in the transmission channel. The time-varying and location-varying nature of signal attenuations in the UASN environment results in large and complex changes in the signal loss pattern over time and space [2], which makes it a formidable challenge to develop an accurate statistical model of transmission error for these networks.

The variance of signal loss over time and space may devolve some of the links, assumed to be bi-directional, to unidirectional. Thus, the network may become partitioned, with the upstream segment being isolated from the gateway. In a more general topology, this may not necessarily partition the network but require separate paths for acknowledgments to be returned if reliable service be implemented.
C. Performance Metrics

Throughput and delay are common metrics for evaluating network performance. The throughput must consider the typical traffic flow. Since all traffic in the UASN flows to the gateway, only the traffic reaching the gateway reflects the network throughput. This is different than ad hoc networks, where traffic may indeed flow between any arbitrary node pair or more typically between 1-hop neighbors. It also raises concern over scalability issues, as discussed in [6]. One method of assessing the throughput is to determine the utilization of the network. From this one can derive the effective throughput.

The nature of the traffic content is a determining factor in the relative importance of latency. If the use of the sample data is time-sensitive then minimizing the delay of a particular sample becomes important. If, however, the use of the data is not time-critical then the success of the delivery of the sample may be more important than the latency of that sample.

Thus, three metrics are apparent with respect to sensor network performance: the utilization, the frame latency, and the delivery probability of frames from particular sensor nodes. Each of these is dependent upon the success rate of frame reception at each hop, which we address in the next section.

III. A Specific Analysis

As a first step in the analysis of the suitability of Aloha variants for UASNs, we derive a model of their performance over a simple, multi-hop, string topology, as depicted in Fig.1. We assume the transmission range of each node is only sufficient to reach its 1-hop neighbors and the interference range is less than the distance to any 2-hop neighbor. Each node immediately forwards any frame it receives from its upstream neighbor. This analysis does not consider a reliable service model, rather it assumes a frame is lost if it collides with any other frame. It does not consider the potential capture effect, where a single frame may be recovered from a collision if its signal strength is sufficiently larger than that of the other frames involved in the collision.

A. Problem Formulation

Traffic Pattern: Each sensor node is assumed to randomly generate a frame containing sensor data at an average rate of \( \lambda \) frames per second. The generation of samples is independent between sensors, both locally, should a node have more than one sensor, and between sensor nodes. It is assumed the frame generation for each sensor follows a Poisson distribution. We further assume a constant frame size and uniform transmission rate for all sensors, resulting in a constant frame transmission time, denoted by \( T \). Therefore, the offered load (original frames) of each sensor node is \( \lambda T \).

Performance Metrics: In this analysis we focus on the utilization of the network. We are also interested in the probability of frame delivery from each sensor node to the gateway.

Since the effective throughput of the network is the traffic received by the gateway from the final node in the string, the throughput must be analyzed with respect to the achievable utilization of the link between the last node, \( O_n \), and the gateway. The utilization of the network, denoted by \( U(n) \), is the same as the utilization of the final link. This utilization is dependent upon the successful reception of frames from \( O_n \) by the gateway.

The successful reception of a frame from \( O_i \) at node \( O_{i+1} \) depends on the state of \( O_{i+1} \): whether it is idle, currently overhearing the transmission of a frame by its downstream neighbor \( O_{i+2} \), or currently sending a frame itself. These constraints are independent. The success probability of \( O_i \)’s transmission, \( P_i \), is the success probability of its frame’s reception by \( O_{i+1} \). More formally stated, this is:

\[
P_i = \Pr \{ \text{successful reception at } O_{i+1} | \text{ frame transmitted by } O_i \} .
\]

The problem, then, is to derive each \( P_i \) and relate that to the traffic load at the gateway. Once they are obtained, the likelihood that a particular frame from \( O_i \) reaches the gateway is simply \( \prod_{j=i}^{n} P_j \). This is precisely the probability that the frame is successfully received, in turn, by every downstream node. The end-to-end delay of a particular frame can then be assessed by considering the probability that the frame succeeds in traversing the network and the cumulative transmission and propagation delays along the path to the gateway.

B. Derivation of \( P_i \) and \( U(n) \)

Since a node does not consider whether or not a reception is already on-going at one of its 1-hop neighbors before it transmits, the vulnerability period during the reception of one frame is twice the frame transmission time, i.e., \( 2T \). To determine the reception success probability at \( O_{i+1} \), we must identify all possible contention sources (we do not consider frame loss due to physical phenomena other than transmission range). As we assume the interference range is less than the distance between any 2-hop neighbor-pairs, only traffic generated by 1-hop neighbors of the recipient of interest must be considered, as shown in Fig.2. Thus, we must determine the likelihood that any node in the contending node set, \( C_i = \{ O_2, O_{i+1}, O_{i+2} \} \), will inject traffic such that it arrives at the reception point at any time during the reception of the frame of interest.

Given that each node generates independent, identically distributed frames with an inter-arrival rate of \( \lambda \) and that the generation of a sample at one node is independent of the generation of a frame at any other node, the aggregate traffic at a node \( O_i \) can be modeled with a Poisson distribution. We denote the aggregate traffic rate for \( O_i \) as \( \lambda_j \). The probability that no
traffic is generated by $O_j$, during a frame’s reception vulnerability period is:

$$e^{-2T}((\lambda_j)_{2T})^{0} = e^{-2T}(\lambda_j). \quad (3.1)$$

Given each node originates frames at the same rate, we have:

$$\lambda_1 = \lambda, \quad \lambda_2 = \lambda(1 + P_1), \quad \lambda_3 = \lambda(1 + P_1P_2 + P_2), \ldots,$$

$$\lambda_i = \lambda \left(1 + \sum_{k=1}^{i} \prod_{h=1}^{k} P_h \right), \quad \ldots, \quad \lambda_n = \lambda \left(1 + \sum_{k=1}^{n-1} \prod_{h=1}^{k} P_h \right),$$

$$\lambda_i = \lambda \left(1 + \sum_{k=1}^{i} \prod_{h=1}^{k} P_h \right),$$

respectively. In general, we have:

$$\lambda_i = \lambda \left(1 + \sum_{k=1}^{i} \prod_{h=1}^{k} P_h \right), i = 1, \ldots, n. \quad (3.2)$$

The probability of a successful transmission by $O_1$ and corresponding reception at $O_2$ are dependent upon the contending node set, $C_i$. This probability is:

$$P_1 = e^{-2(\lambda_1 + \lambda_2 + \lambda_3)T}. \quad (3.3)$$

Equation (3.3) reflects that a reception of a frame from $O_1$ is successful only if no other frame arrives at $O_2$ during one vulnerability period, either from $O_1$ or $O_2$, or by $O_3$ initiating a transmission thereby blocking the frame reception. In general, the probability of a successful reception of a frame from $O_i$ is dependent upon $C_i$ and is:

$$P_i = e^{-2(\lambda_1 + \lambda_2 + \lambda_3)T}, \quad i = 2, \ldots, n-2, \quad (3.4)$$

$$P_{n-1} = e^{-2(\lambda_1 + \lambda_2)T}; \quad P_n = e^{-2(\lambda_2)T}. \quad (3.5)$$

Combining equations (3.2), (3.4), and (3.5), we can obtain $n$ nonlinear equations with respect to $n$ variables: $\lambda_1, \lambda_2, \ldots, \lambda_n$. However, we are not able to derive closed-form solutions from these equations. In the next section, we will present a numerical method for calculating the aggregate load at each node.

Let $U_i$ denote the utilization of the link from $O_i$ to $O_{i+1}$. Then, $U_i = \lambda_i \cdot P_i \cdot T$. The utilization of the network is simply:

$$U(n) = U_n = \lambda_n \cdot P_n \cdot T \quad (3.6)$$

The load increases with each successive hop toward the base station until no more traffic can be supported, whereupon the generation of a new frame causes at least one other frame to be lost due to collision at one or more of the members of the respective contending node set. Thus, the probability of successful reception decreases with each hop, with the exception of the last two nodes in the string, as their contending node sets are smaller. The ratio of the reception success of two successive nodes is:

$$\frac{P_i}{P_{i-1}} = e^{-2(\lambda_1 + \lambda_2 + \lambda_3)T} = e^{-2(\lambda_1 + \lambda_3)T} < 1; \quad i = 2, \ldots, n-2. \quad (3.7)$$

Thus, the sustainable load for each host decreases exponentially with the length of the network. Specific values of $\lambda_i$ must be used to calculate, iteratively, the resulting values of $P_1, P_2, \ldots, P_n$ for a given sampling rate of $\lambda$, due to the interdependence of the values.

The effective throughput of the sensor network, or the good network throughput, denoted by $S(n)$, can be expressed as follows:

$$S(n) = \lambda_n \cdot P_n \cdot L \cdot \alpha., \quad (3.8)$$

where $L$ is the average data frame size in bits and $\alpha$ the average fraction of data bits in each data frame received by the gateway.

### IV. Performance Evaluation

Given a traffic load and the number of sensors in the string, we can obtain the aggregate traffic load of each node by solving $n$ non-linear equations given in Equation 3.2. This can
be achieved by solving the following minimization problem,

$$\min_{\lambda_1, \lambda_2, \ldots, \lambda_n} \sum_{i=1}^{n} \left[ F_i(\Lambda) - \lambda_i \right]$$

(4.1)

where

$$\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n),$$

and

$$F_i(\Lambda) = \lambda_i \left( 1 + \sum_{k=i}^{n} \left( \prod_{j=k+1}^{n} P_j \right) \right),$$

where $$P_i$$ is a function of $$\Lambda$$ as given by equations (3.4) and (3.5).

We found that the Nelder-Mead simplex method has been quite effective to solve this minimization problem [7]. It is straightforward to calculate $$P_i$$ and $$U(n)$$ once each $$\lambda_i$$ is obtained. In the following we present results for various values of the network size $$n$$ and the per-sensor load $$\lambda \cdot T$$.

Without loss of generality, we set $$T$$ to 1. We let $$\lambda = 0.002, 0.01, 0.1, \text{ or } 0.5$$ to vary the per-sensor load.

Fig. 3 shows the aggregate traffic load of each node ($$\lambda_i$$, $$i=1,...,n$$) for different values of the string size, $$n$$. When the load is small ($$\lambda = 0.002, 0.01, \text{ or } 0.1$$), $$\lambda_i$$ increases when $$i$$ increases, regardless of $$n$$. This observation matches our intuition as each node has to forward the frames received from the previous node to the next node. This tendency becomes weak when the load increases. Evidently, when $$\lambda = 0.5$$, $$\lambda_i$$ becomes almost a constant regardless of $$i$$. This is because as more collisions occur due to higher traffic loads each node gradually reaches saturation status.

Then, we expect $$P_i$$ decreases with $$i$$ due to the increase of the traffic at each node. Fig. 4 shows that no matter what string size is chosen, $$P_i$$ deceases except at the last two nodes due to their smaller contending node sets. When the load exceeds $$\lambda = 0.5$$ each node has reached its saturation status and $$P_i$$ becomes flat.

Fig. 5 shows that the network utilization increases with $$n$$ when the per-sensor load is very small. We expect more frames arrive at the gateway as the string size increases as long as the nodes have not reached their saturation status. We also expect that when the string size becomes large, the nodes close to the gateway will become saturated. Then the utilization will no longer increase. This can be observed when $$\lambda = 0.01$$. When $$\lambda = 0.1$$ or 0.5, all nodes are saturated, or almost saturated, and the utilization is flat regardless of the string size.

Fig. 6 shows the network utilization versus the load for a given string size of eight. Initially, as the per-sensor load
increases, but before the nodes reach their saturation status, more frames arrive at the last node to be passed to the gateway. However, with the load surpassing a certain threshold, too many collisions occur, reducing the total number of frames arriving at the gateway. This is exactly what Fig. 6 shows. The utilization first increases with the load. It reaches the maximum when the load is about 0.5. After that, it starts decreasing because of the many collisions. This is precisely the performance characteristic of Aloha applied to single-hop networks.

V. RELATED WORK
The design and performance of MAC protocols for wireless networks have undergone significant study. In particular, the performance of the IEEE 802.11 MAC has been well studied, for both the basic access mechanism (CSMA) and the four-way handshake used to provide virtual carrier sense of hidden nodes. The results of these studies can benefit the design and analysis of protocols for UANs and UASNs in particular.

Gupta provides a model of the four-way handshake performance in single hop, ring, and mesh topologies using Markov Chains [8]. The traffic pattern it considers is not like that expected in a UASN, in that the load is not cumulative with each hop. However, it provides a useful means for considering the impact of back-off and retransmission issues. Bisnik provides an analysis of the end-to-end delay for collision avoidance schemes in multi-hop ad hoc wireless networks [9]. The results underscore the importance of the traffic pattern and average hop-count with respect to the network throughput and delays. The model considers the relationship between the locality of traffic and the maximum achievable throughput. For a UASN, as considered in this paper, all traffic is destined for the gateway, thus locality is low, for which [9] suggests the throughput will be low as compared to networks where traffic tends to be localized.

The upper bound on the performance of a land sensor network was presented in [5], and the bound is achievable with a perfect scheduling algorithm. The result is applicable to UASNs as the traffic pattern considered was the same as that of a UASN. An analysis of contention-based protocols specifically for UAN networks was conducted by [3]; however, it considered only hosts within the one-hop neighborhood of the gateway so that the effort of traffic forwarding was not pertinent. This paper specifically considers the performance of Aloha variants in multi-hop UASNs.

MAC Design efforts for UANs cover the spectrum of techniques employed by wireless networks, to include channel allocation [1,8], carrier-sense techniques using duty cycles [11], adaptive reservation-based protocols [12], and topology-aware protocols [13]. Each of these designs seems to offer certain advantages for specific UAN issues. However, their expected behaviors have not been substantiated by theoretical analyses in the context of multi-hop routing as considered by this paper.

VI. CONCLUSIONS
This paper reports the initial results of our research toward developing an analytic model to address the performance of contention-based protocols within the context of UASNs. It identified issues that complicate the analysis of such protocols. It then presented a model for analyzing simple Aloha variants without reliable service guarantees. While the model makes several simplifying assumptions, several key conclusions can be drawn from the application of the model to a string topology. Notably, since such networks are subject to saturation as the number of nodes or the per-sensor load increase, care must be taken in their implementation to insure the sensor data is able to reach the gateway. Except for very small loads, saturation occurs in less than five hops and within three hops for the optimal load. Once the network is saturated, frames from upstream nodes have a very small probability of reaching the gateway.

The limiting factor in the performance of Aloha variants is collisions. Avoiding collisions is the goal of refinements to this protocol class. Further analysis and refinement of this model are necessary to address the impact of these refinements within the context of UASNs.

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