REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
16-06-2008

2. REPORT TYPE
Technical Paper

3. DATES COVERED *(From - To)*

4. TITLE AND SUBTITLE

Modeling Electron Transport within the Framework of Hydrodynamic Description of Hall Thrusters (Preprint)

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
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5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER
33SP0853

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory (AFMC)
AFRL/RZSS
1 Ara Drive
Edwards AFB CA 93524-7013

8. PERFORMING ORGANIZATION REPORT NUMBER

AFRL-RZ-ED-TP-2008-237

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory (AFMC)
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Edwards AFB CA 93524-7048

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S NUMBER(S)
AFRL-RZ-ED-TP-2008-237

12. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution unlimited (PA #08240A).

13. SUPPLEMENTARY NOTES
For presentation at the 44th AIAA Joint Propulsion Conference, Hartford, CT, 20-23 July 2008.

14. ABSTRACT

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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
					Dr. Justin Koo
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**	SAR	8	**19b. TELEPHONE NUMBER** *(include area code)*
Unclassified	**Unclassified**	**Unclassified**			N/A

Submitted to JPC, 2008

Modeling electron transport within the framework of hydrodynamic description of Hall thrusters (PREPRINT)

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Abstract

In this paper we consider kinetic effects related to electron transport in the framework on hydrodynamic model of the plasma flow inside the Hall thruster channel. In particular, kinetics of the near wall conductivity (NWC) is analyzed and analytical expression is derived that takes into account the sheath effects. The NWC model is incorporated into the hydrodynamic model. In addition we consider an effect of SEE electron thermalization. It is found that current predicted by the analytical model can provide reasonable solution without any fitting parameters.

1. Introduction

A Hall thruster is one of the most advanced and efficient types of electrostatic propulsion for Earth-orbital applications.[1] The maximum thrust density of Hall thrusters is not limited by space charge effects because acceleration takes place in a quasi-neutral plasma. The Hall thruster discharge has an E×B configuration with applied radial magnetic and axial electric fields. Passing the electron current across a magnetic field leads to an electron closed drift or Hall drift. The unmagnetized ions are accelerated by the axial electric field producing the thrust. In Hall thrusters, some very basic plasma processes are closely related to practical characteristics such as the thruster efficiency and the thruster lifetime. The efficiency of the Hall thruster depends on the electron cross-field transport and power losses especially due to plasma-wall interaction. Moreover, the plasma-wall interaction can limit the thruster lifetime due to erosion of the channel walls.

In generally, two approaches for modeling plasma flows in Hall thrusters were undertaken in the past: particle simulation and hydrodynamic approach. A variation of the first approach is hybrid models in which ions and neutrals are treated as particles whereas electrons are treated as a fluid[2,3,4]. In this numerically expensive approach, however, very simplified boundary conditions are applied at the walls without considering the plasma-wall transition in details. In the second approach, the 1D hydrodynamic description for all species is employed[5,6,7,8]. However, due to restrictions of 1D analyses, the real boundary conditions at the wall were not considered. 2D hydrodynamic model of plasma flow in a Hall thruster suggests that plasma-wall interactions are rather complicated.[9,10,11,12,13] Any state-of-the-art Hall thruster model employs some anomalous cross-field electron mobility in order to reproduce experimental features. Generally there is no clear convincing evidence regarding which one of the possible anomalous transport mechanisms prevails in Hall thruster thus leaving this question unresolved. The electron conductivity is one of the long-standing problems related to Hall effect thrusters. This very important problem has various implications on fundamental issues of the Hall thrusters, such as current continuity, energy balance and ultimately on thruster efficiency. It was known for a long time that classical mechanism of electron collisions could not explain the electron transport experimentally observed in Hall thrusters. Until now, however, there is no consensus about which of the possible mechanisms of electron transport is most significant in Hall thrusters. One idea that was put forward early in Hall thruster history by Morozov relates the anomalous conductivity to near wall processes (so called near wall conductivity, NWC). This mechanism remains still questionable since current calculated by NWC is less than measured. On the other hand there is an idea about the role of the Bohm-type conductivity across magnetic field due to plasma turbulence. There is some limited indirect experimental evidence of Bohm conductivity, but it was obtained in low voltage regime of the Hall thruster. On the other hand most models of Hall thruster relies on either NWC (usually with a coefficient to correct for discharge current) or Bohm type. Thus the question about electron conductivity mechanism remains open from both experimental and theoretical points of view. Since all models rely on an assumption about electron transport mechanism and rely on some experimental data about integral discharge characteristics (such as discharge current) any modeling predictions are very limited to cases for which experimental data exist. It is obvious that development of a new thruster or predictions regarding existing thruster lifetime cannot be supported by state of the art models. Therefore there is an emerging need to develop high fidelity modeling capabilities.

2. Modeling of the electron transport

A 2D macroscopic (hydrodynamic) model of the channel was developed[9]. A 2D plasma flow domain is considered that has lateral boundaries near the dielectric wall. We will start with a brief description of the plasma flow model. We consider a model region with magnetic and electric field. The plasma is assumed quasi-neutral. A plasma is considered with “magnetized” electrons and “unmagnetized” ions (i.e., $\rho_e \ll l \ll \rho_i$,) where ρ_e and ρ_i are the Larmor radii for the electrons and ions, respectively, and l is a characteristic length of the system. In this model we will assume that the magnetic field has only a radial component. The numerical analysis is similar to that developed previously.[5,14] Boundary conditions are described in details elsewhere.[5] Plasma quasi-neutrality is assumed and therefore the plasma presheath-sheath interface is considered to be the lateral boundary for the plasma flow region. The electron temperature is calculated along the centerline as a balance between Joule heating, ionization and wall losses. The dielectric wall effect is taken into account by introducing an effective coefficient of secondary electron emission (SEE). Partial electron thermalization is taken into account parametrically and it was found that it has very strong effect on global discharge characteristic. In particular due to partial electron thermalization, effective SEE coefficient decreases[15],[16].

3. Near wall conductivity

Recently new formulation for NWC effect was proposed[17]. In this paper we incorporate the NWC formulation in the electron transport model. In the NWC formulation the current density (z component) can be calculated as follows:

$$j_{ez} = \int_{-\infty}^{\infty}\int_{\alpha_y}^{\infty}\int_{-\infty}^{\infty} f(v) V_z dV_x dV_z dV_y \quad (1)$$

In this case, one can arrive at the following expression for the electron current density:

$$j_{ew} = 2n_0 \frac{E}{B}\left(\frac{m}{2\pi k T_w}\right)^{1/2} \exp\left(\frac{e\Delta\varphi_z}{kT_w}\right)\exp\left(\frac{e\Delta\varphi_y}{kT_w}\right)\int_{\sqrt{\frac{2e\Delta\varphi_y}{m}}}^{\infty}\exp\left(-\frac{mV_y^2}{2\pi k T_w}\right)\sin\left(\omega\frac{y}{V_y}\right)dV_y \quad (2)$$

In this case, a new function can be introduced which is the integral in Eq. 2

$$Q(s) = \int_{\sqrt{\frac{e\Delta\varphi_y}{kT_w}}}^{\infty}\exp(-\theta^2)\sin\left(\frac{s}{\theta}\right)d\theta \quad (3)$$

where *s* is the non-dimensional distance from the wall (*y* direction) and the function *Q(s)* determines the current distribution as a function of that distance.
The current density due to NWC can be expressed as follows:

$$j_{ew} = \frac{2}{\sqrt{\pi}} n_0 \frac{E}{B}\exp\left(\frac{e\Delta\varphi_z}{kT_w}\right)\exp\left(\frac{e\Delta\varphi_y}{kT_w}\right)\times Q(s) \quad (4)$$

One can see that the function *Q(s)* provides the dependence of the current density on the distance from the wall. It is interesting to note 2 effects incorporated in Eq.4, namely potential drop across the sheath, $\Delta\varphi_y$ and the effective potential drop along the wall, $\Delta\varphi_z$. Both effects have strong effect on NWC current as it was shown previously[17].

For instance, the NWC current is shown in Fig.1 with sheath voltage as a parameter. It can be seen that the sheath voltage leads to decrease of the current as well as current concentration near the wall.

4. Effect of partial electron thermalization

Recently significant attention was paid into the problem of interaction of the SEE electrons with plasma electrons, i.e. electron thermalization[15,16]. In particular this effect leads to anisotropy in electron distribution function and creates significant beam-like electron population that interacts with channel wall. Ultimately this effect alters energy balance in the plasma bulk as well as particle flux to the wall. In this paper we incorporate this effect into the model by considering its effect on the electron energy, current as wall as potential drop across the sheath.

From the current balance at the dielectric wall one can find that:

$$\Gamma_i = \Gamma_e(1 - s + \alpha s) \quad (5)$$

Where s is SEE coefficient, α is the electron thermalization coefficient that reduces effective SEE.

5. Numerical simulations

Effect of electron thermalization on the peak electron temperature is shown in Fig. 2. One can see that complete thermalization leads to peak electron temperature saturation, while in the case of a partial thermalization peak electron temperature increases linearly with discharge voltage. Current voltage characteristics of the discharge are shown in Fig. 3. It can be seen that electron partial thermalization does not have noticeable effect on the discharge current. Potential and electron temperature distribution along the channel is shown in Fig.4. One can see that electron temperature significantly increases due to effect of partial thermalization. Analytical expression for the NWC was incorporated into hydrodynamic model and calculated self-consistently electron current without any fitting parameters. These results are shown in Fig.5. NWC electron current is normalized by the average electron current that corresponds to experiment. One can see that this model provides the current level close to that measured experimentally.

The 2D structure of the NWC current was investigated by applying the hydrodynamic model to the P5 thruster based on simplification that uses the radial component of the magnetic field. Electric field and sheath potential were calculated self-consistently. The NWC current was calculated using the local E/B ratio, with $Q(s)$ evaluated by averaging $Qi(s_L)$ and $Q_U(s_L)$, the contributions from the inner and outer wall, respectively. In this work, we assumed that the potential drop in the direction tangential to the wall is negligible, yielding $\Delta\phi_z = 0$. Fig.6 shows the results for $\Delta\phi_w/T_w$=0, 0.5, 0.7 and 1.0, respectively. As expected, for the case of no sheath drop, the j_{ew} term is limited to the near-wall region. Addition of sheath potential drop results in formation of strong oscillations in the current distribution.

6. Summary

In summary, in this paper we attempted to include kinetic effects related to electron transport in the framework on hydrodynamic model of the plasma flow. Kinetics of the near wall conductivity (NWC) is analyzed and analytical expression is derived that takes into account the sheath effects. It is interesting to note that NWC current predicted by the analytical model can provide reasonable solution without any fitting parameters.

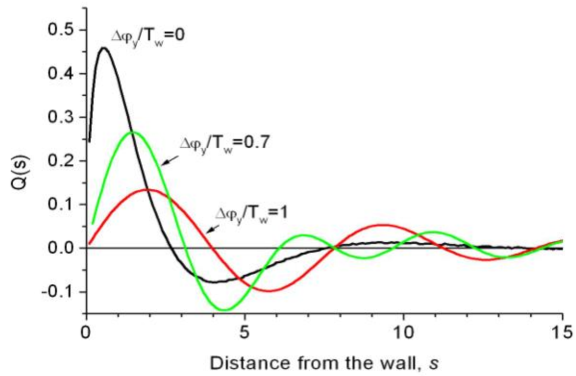


Fig.1. Normalized NWC current distribution with sheath potential drop as a parameter.

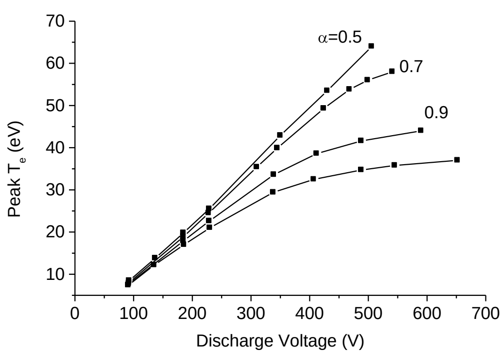


Fig. 2. Effect of the partial SEE electron thermalization on electron temperature. α=1 corresponds to complete thermalization.

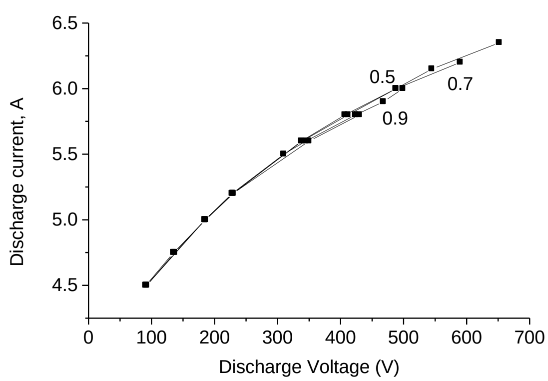


Fig.3. Effect of the partial SEE electron thermalization on discharge current.

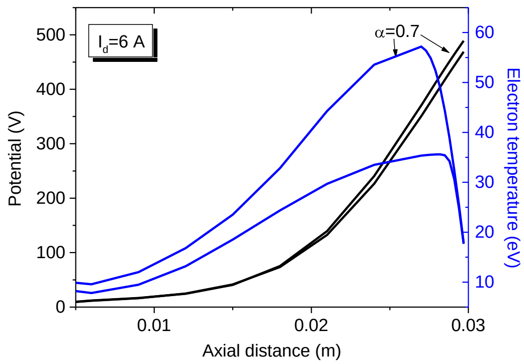


Fig.4. Effect of the partial SEE electron thermalization on electron temperature and potential distribution along the channel.

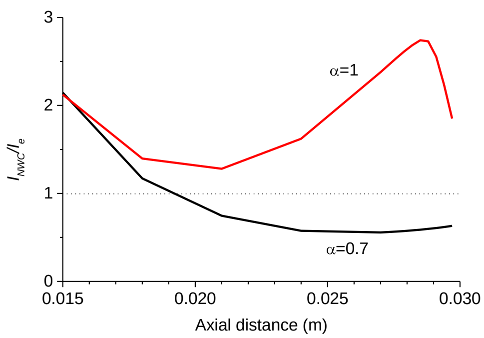
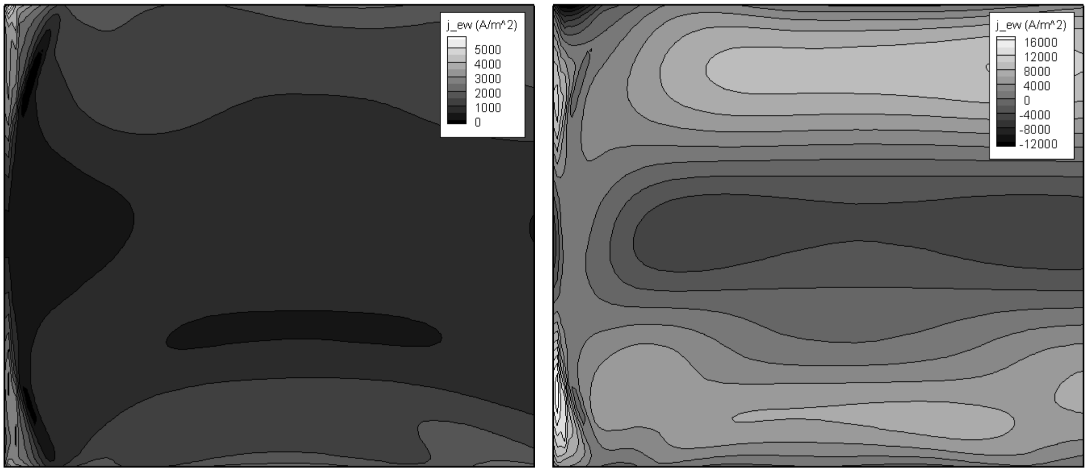


Fig. 5. Effect of the partial SEE electron thermalization on electron current.



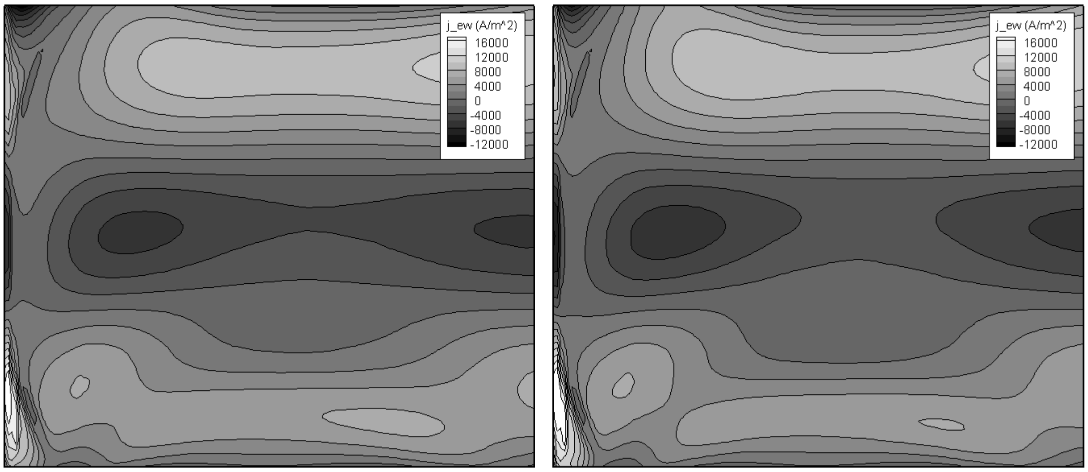


Fig. 6. Electron current computed using the near wall conductivity model for $\Delta\phi_y/T_w$=0, 0.5, 0.7 and 1.0, respectively.

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