DEPARTMENT OF SUPPLY

AUSTRALIAN DEFENCE SCIENTIFIC SERVICE

WEAPONS RESEARCH ESTABLISHMENT

TECHNICAL MEMORANDUM CPD 169

THEORETICAL ASSESSMENT OF "DEMON" PERFORMANCE

H.A. d'Assumpcao, M.E.

S U M M A R Y

An expression is obtained for the detection performance of a "DEMON" processor used to detect modulated signals imbedded in noise. It is shown that "DEMON" performance is degraded relative to the conventional power detector and that the "DEMON" threshold is more sharply defined than that of the power detector.

July 1970

POSTAL ADDRESS: The Director, Weapons Research Establishment,
Box 1424H, G.P.O., Adelaide, South Australia, 5001.

TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

Ships and submarines frequently radiate sounds underwater which exhibit distinct amplitude modulation. For example, the noise from a cavitating propeller is usually modulated at a frequency equal to the blade rate (i.e., the rate of rotation of the shaft multiplied by the number of blades of the propeller). This noise can be received by a passive sonar, and with training and in conjunction with other sounds, an operator can often, from the modulation, identify the type of vessel and estimate its speed.

The operator can be greatly assisted by rectifying the sonar signals to extract the modulation and spectrum analysing the result. This is termed "DEMON" processing. A block diagram of a DEMON processor is shown in figure 1. The writer is not aware of any published mathematical analysis of this processor. In this memorandum, an expression of its theoretical performance is derived.

It should be pointed out that the processor shown in figure 1 is not necessarily the optimum detector for such modulated signals. Tuteur(1) has made an attempt to determine the optimum processor and its performance, but the simplifying assumptions he makes are unrealistic, and as a consequence his results should be regarded as useful only in giving an upper bound to the performance which might be achieved. The present writer has not yet succeeded in determining the optimum processor under realistic assumptions.

2. ASSUMPTIONS

The following assumptions are made.

(i) The sound radiated from the vessel has the properties of zero-mean Gaussian noise which is amplitude modulated by a low-frequency sinusoidal wave. The resultant is henceforth referred to as the "signal".

(ii) The frequency of the modulating wave is known, but its phase is random.

(iii) The signal is contaminated by additive zero-mean Gaussian noise.

(iv) Both the signal and noise are wholly contained in a narrow bandwidth B (i.e., the centre frequency is much greater than B).

(v) The spectrum is computed for samples of data of duration T, where BT is an integer >> 1.

(vi) The average is taken of M spectra, where M >> 1.

(vii) Signal and noise are stationary, ergodic processes.

(viii) Signal and noise are independent.

Although it is unlikely that the modulating frequency would be known a priori, assumption (ii) above is nonetheless not unrealistic, because in practice one would compute the entire power spectrum over all the frequencies of interest.

3. NOTATION

Let the input waveform to the processor be described by

$$z(t) = \left\{1 + b\cos\left(\frac{2\pi Pt}{T} + \varphi\right)\right\} s(t) + n(t) , \tag{1}$$

where $s(t)$ and $n(t)$ are zero-mean Gaussian random waveforms,

P is an integer > 0,

$P \ll BT$ (assumption (i)),

$0 < b \leqslant 1$,

φ is random in $(0, 2\pi)$.

It is possible to express $s(t)$ and $n(t)$ in terms of low-frequency (complex) waveforms modulating a "carrier" frequency:

$$s(t) = \frac{1}{\sqrt{2}}\left\{\alpha(t)\exp(i2\pi f_o t) + \alpha^*(t)\exp(-i2\pi f_o t)\right\} \tag{2}$$

$$n(t) = \frac{1}{\sqrt{2}}\left\{\beta(t)\exp(i2\pi f_o t) + \beta^*(t)\exp(-i2\pi f_o t)\right\} \tag{3}$$

where $\alpha(t)$ and $\beta(t)$ are in general complex, and

$*$ denotes the complex conjugate.

We shall decompose $\alpha(t)$ and $\beta(t)$ into their respective Fourier components:

$$\alpha(t) \simeq \sum_{n=1}^{BT} x(n)\exp(i2\pi\, nt/T) \tag{4}$$

$$\beta(t) \simeq \sum_{n=1}^{BT} y(n)\exp(i2\pi\, nt/T) , \tag{5}$$

$$\begin{aligned} \text{where } x(n) &= \frac{1}{T}\int_0^T \alpha(t)\exp(-i2\pi\, nt/T)\,dt, \\ y(n) &= \frac{1}{T}\int_0^T \beta(t)\exp(-i2\pi\, nt/T)\,dt. \end{aligned} \tag{6}$$

For BT >> 1,

$$\langle x(n)\ x^*(m)\rangle \simeq S(n)\ \delta_{nm}\ ,$$
$$\langle y(m)\ y^*(m)\rangle \simeq N(n)\ \delta_{nm}\ , \tag{7}$$

where S(n) is the unmodulated signal power component at the frequency corresponding to the index n,
and δ_{nm} is the Kronecker delta,

$$\begin{aligned} &= 0\ ,\ n \neq m, \\ &= 1\ ,\ n = m. \end{aligned} \tag{8}$$

N(n) is the noise power component at the frequency corresponding to the index n,

$\langle\quad\rangle$ denotes an ensemble average.

By virtue of assumption (viii),

$$\langle x(n)\ y^*(m)\rangle = 0, \text{ for all n, m,} \tag{9}$$

and from assumption (iv)

$$x(n) = y(n) = 0 \text{ for } n < 1 \text{ and } n > BT \tag{10}$$

In what follows, we shall make extensive use of the following theorem for the moment of complex gaussian processes(2) :

$$\langle z(m_1)\ z(m_2)\ldots z(m_r)\ z^*(n_1)\ z^*(n_2)\ldots z^*(n_s)\rangle = 0 \text{ if } r \neq s,$$
$$= \sum_{\pi} \langle z(m_{\pi(1)})\ z^*(n_1)\rangle\ \langle z(m_{\pi(2)})\ z^*(n_2)\rangle \ldots \langle z(m_{\pi(s)}) z^*(n_s)\rangle \quad \text{if } r = s \tag{11}$$

where $\pi(\ )$ denotes a permutation.

Specifically, if r = s = 2,

$$\langle z(m_1)\ z(m_2)\ z^*(n_1)\ z^*(n_2)\rangle = \langle z(m_1)\ z^*(n_1)\rangle\langle z(m_2)\ z^*(n_2)\rangle + \langle z(m_1)\ z^*(n_2)\rangle\langle z(m_2)\ z^*(n_1)\rangle \tag{12}$$

4. MATHEMATICAL DEVELOPMENT

After squaring and low-pass filtering z(t), only terms remaining are those not containing f_o, and we have, from equations (1) to (3):

$$\left\{1 + b\cos\left(\frac{2\pi Pt}{T} + \varphi\right)\right\}^2 |\alpha(t)|^2 + |\beta(t)|^2$$

$$+ \left\{1 + b\cos\left(\frac{2\pi Pt}{T} + \varphi\right)\right\}\left\{\alpha(t)\beta^*(t) + \alpha^*(t)\beta(t)\right\}$$

After substituting equations (4) and (5) into this expression, and a little manipulation, we have:

$$\left\{1 + \frac{b^2}{2} + b\exp\left(\frac{i2\pi Pt}{T} + i\varphi\right) + b\exp\left(-\frac{i2\pi Pt}{T} - i\varphi\right)\right.$$

$$\left. + \frac{b^2}{4}\exp\left(\frac{i4\pi Pt}{T} + i2\varphi\right) + \frac{b^2}{4}\exp\left(-\frac{i4\pi Pt}{T} - i2\varphi\right)\right\} .$$

$$\cdot \sum_{n,m} x(n)\, x^*(m) \exp\left\{i2\pi(n-m)\, t/T\right\}$$

$$+ \sum_{n,m} y(n)\, y^*(m) \exp\left\{i2\pi(n-m)t/T\right\}$$

$$+ \left\{1 + \frac{b}{2}\exp\left(\frac{i2\pi Pt}{T} + i\varphi\right) + \frac{b}{2}\exp\left(-\frac{i2\pi Pt}{T} - i\varphi\right)\right\} .$$

$$\cdot \sum_{n,m}\left[x(n)\, y^*(m)\exp\left\{i2\pi(n-m)t/T\right\} + x^*(n)y(m)\exp\left\{-i2\pi(n-m)t/T\right\}\right] \tag{13}$$

Consider now multiplying this by $\cos(2\pi Pt/T)$ and integrating over $(0,T)$. Noting that

$$\frac{1}{T}\int_0^T \exp\left\{i2\pi(n-m)\, t/T\right\} dt = \delta_{nm} , \tag{14}$$

we have

$$
\begin{aligned}
&\tfrac{1}{2}\left(1+\frac{b^2}{2}\right)\sum\left\{x(n)\ x^*(n+P)+x(n)\ x^*(n-P)\right\}\\
&+\frac{be^{i\varphi}}{2}\sum\left\{x(n)\ x^*(n+2P)+x(n)\ x^*(n)\right\}\\
&+\frac{be^{-i\varphi}}{2}\sum\left\{x(n)\ x^*(n)+x(n)\ x^*(n-2P)\right\}\\
&+\frac{b^2}{8}\,e^{i2\varphi}\sum\left\{x(n)\ x^*(n+3P)+x(n)\ x^*(n+P)\right\}\\
&+\frac{b^2}{8}\,e^{-i2\varphi}\sum\left\{x(n)\ x^*(n-P)+x(n)\ x^*(n-3P)\right\}\\
&+\tfrac{1}{2}\sum\left\{y(n)\ y^*(n+P)+y(n)\ y^*(n-P)\right\}\\
&+\tfrac{1}{2}\sum\left\{x(n)\ y^*(n+P)+x(n)\ y^*(n-P)\right\}\\
&+\frac{b}{4}\,e^{i\varphi}\sum\left\{x(n)\ y^*(n+2P)+x(n)\ y^*(n)\right\}\\
&+\frac{b}{4}\,e^{-i\varphi}\sum\left\{x(n)\ y^*(n)+x(n)\ y^*(n-2P)\right\}\\
&+\tfrac{1}{2}\sum\left\{x^*(n)\ y(n-P)+x^*(n)\ y(n+P)\right\}\\
&+\frac{b}{4}\,e^{i\varphi}\sum\left\{x^*(n)\ y(n-2P)+x^*(n)\ y(n)\right\}\\
&+\frac{b}{4}\,e^{-i\varphi}\sum\left\{x^*(n)\ y(n)+x^*(n)\ y(n+2P)\right\}
\end{aligned}
\qquad (15)
$$

A similar expression (except for a change of sign inside each curly bracket, and the presence of i outside each term) obtains after multiplying by $\sin(2\pi Pt/T)$ and integrating over $(0,T)$.

These two outputs (i.e., after cosine and sine multiplications and integrating) are squared and summed. The algebraic manipulations are simplified if we note that the two outputs take the forms:

$$\text{cos:} \quad (p + q) + (r + s) + \ldots$$

$$\text{sin:} \quad (ip - iq) + (ir - is) + \ldots,$$

and that after squaring and summing the only terms remaining are the cross products:

$$4\,pq + 4\,ps + 4\,qr + 4\,rs + \ldots$$

Making use of equations (7), (9) and (12), we find that the expression for the mean after squaring and summing eventually reduces to:

$$\left(1 + \frac{b^2}{2}\right)^2 \sum S(n)S(n+P) + \frac{b^4}{16}\sum S(n)S(n+3P) + \frac{b^4}{16}\sum S(n)S(n-P)$$

$$+ b^2 \sum S(n)S(n+P) + b^2 \sum S^2(n) + b^2 \left\{ \sum S(n) \right\}^2 + \sum N(n)N(n+P)$$

$$+ \sum S(n)N(n+P) + \sum S(n)N(n-P) + \frac{b^2}{4}\sum S(n)N(n+2P) + \frac{b^2}{4}\sum S(n)N(n-2P)$$

$$+ \frac{b^2}{2}\sum S(n)N(n) \qquad (16)$$

In expression (16), the sums are taken over the range (1,BT); it should be noted that from the initial assumption (iv),

$$S(n) = N(n) = 0 \text{ for all } n < 1 \text{ and } n > BT.$$

We now make the additional assumption that except at the band limits, both the signal and noise spectra are locally slowly varying, i.e.,

$$S(n + 2P) \simeq S(n + P) \simeq S(n),$$

$$\text{and} \quad N(n + 2P) \simeq N(n + P) \simeq N(n).$$

Expression (16) for the mean output is then approximated by

$$b^2 \left\{ \sum S(n) \right\}^2 + \left(1 + 3b^2 + \frac{3}{8}b^4\right) \sum \left\{ S(n) \right\}^2$$

$$+ \sum \left\{ N(n) \right\}^2 + (2 + b^2) \sum \left\{ S(n)N(n) \right\} \qquad (17)$$

We are also interested in the variance of the output when signal is absent. In the absence of signal, $S(n) = 0$, and the mean output is simply $\sum \left\{ N(n) \right\}^2$. The output without signal is

$$\sum_{n,m} y(n)\, y^*(n + P)\, y(m)\, y^*(m - P)$$

The mean square output is

$$\sum_{k,\ell,m,n} \left\langle y(k)\, y^*(k+P)\, y(\ell)\, y^*(\ell - P)\, y(m)\, y^*(m+P)\, y(n)\, y^*(n-P) \right\rangle \qquad (18)$$

Using equations (7) and (11), we find, after considerable algebraic manipulation, that the mean square output for noise only is

$$2 \left\{ \sum N(n) \right\}^2 + 6 \sum \left\{ N(n) \right\}^4 \qquad (19)$$

The variance of the output is therefore

$$\text{mean square output - (mean output)}^2 = \left[\sum \left\{ N(n) \right\}^2 \right]^2 + 6 \sum \left\{ N(n) \right\}^4 \qquad (20)$$

The signal-to-noise power ratio at the output of the processor may be defined to be:

$$d \triangleq \frac{(\text{change in mean with signal present})^2}{\text{variance without signal}}$$

As we add M independent spectra, it can readily be seen that the signal-to-noise output power ratio will be

$$d = \frac{M \left[b^2 \left\{ \sum S(n) \right\}^2 + \left(1 + 3b^2 + \frac{3}{8} b^4\right) \sum \left\{ S(n) \right\}^2 + (2 + b^2) \sum \left\{ S(n) N(n) \right\} \right]^2}{\left[\sum \left\{ N(n) \right\}^2 \right]^2 + 6 \sum \left\{ N(n) \right\}^4} \qquad (21)$$

For the special case in which both (unmodulated) signal and noise have flat spectra, let

$$S(n) = S, \; N(n) = N \text{ for all } n$$

Defining

$$K \triangleq BT,$$

$$\frac{S}{N} \triangleq \mu,$$

we have

$$d = \frac{KM \left\{ b^2 K \mu^2 + \left(1 + 3b^2 + \frac{3}{8} b^4\right) \mu^2 + (2 + b^2) \mu \right\}^2}{K + 6} \tag{22}$$

In most cases of practical interest, K >> 1, μ << 1. Hence

$$d \simeq M\mu^2 \left\{ b^2 K\mu + (2 + b^2) \right\}^2 \tag{23}$$

If M is sufficiently large, the output will, by the central limit theorem, be approximately gaussian, and could therefore be defined by second-order statistics. Further, we could then use curves for Receiver Operating Characteristics (as derived, for example, in references 3 and 4) directly; these give a plot of the probability of detection versus the probability of false alarm for various values of d^2 (see figure 2), which in our case is simply the output signal-to-noise ratio.

It should be noted that in equations (21) to (23) μ is not the input signal-to-noise ratio, because S is the power spectral density of the unmodulated signal. The modulation will increase the actual input signal-to-noise power ratio to a value of $\left(1 + \frac{b^2}{2}\right) \mu$.

It is also noteworthy that the results obtained here are considerably more pessimistic than those obtained by Tuteur(1), who assumes knowledge of the phase of the modulating signal. Essentially, his output signal-to-noise ratio varies with μ^2, whereas the result derived here, which assumed random phase, varies with μ^4 for large K.

5. COMPARISON WITH SIMPLE ENERGY DETECTOR

By way of comparison, consider the simple energy detector shown in figure 3. The output signal-to-noise ratio, defined as before to be

$$d \triangleq \frac{(\text{change in mean with signal present})^2}{\text{variance without signal}}$$

where?

may be shown, after the usual algebra, to be

$$d = \left(1 + \frac{b^2}{2}\right)^2 \mu^2 \, BTM \tag{24}$$

Comparing equations (23) and (24), we find that for the Demon processor, with small signals, the output signal-to-noise power ratio varies as μ^4, whereas with the simple power detector it varies as μ^2.

Hence it is to be expected that in general detection using a simple power detector would take place before Demon classification could be effected.

6. EXAMPLE

By way of a simple example, consider the case of noise and (unmodulated) signal spectra which are flat. It is required to calculate the value of μ which will give a probability of detection of 90%, and a probability of false alarm of 0·01%. Other given parameters are:

$B = 10^3\,\text{Hz}$,

$T = 10\,\text{s}$,

$M_1 = 100$,

$b = 1\cdot0$

From figure 2, it is seen that d = 25. Hence

$$100\ \mu^2\ \{10^4\ \mu + 3\}^2 = 25$$

i.e.,

$$\mu \simeq 7 \times 10^{-3}\ ,$$

and the actual signal-to-noise input ratio is

$$\left(1 + \frac{b^2}{2}\right)\mu \simeq 10^{-2}$$

or about -20 dB.

Under the same circumstances, the ordinary power detector of broad-band signals would have a detection threshold of -25 dB.

7. CONCLUSIONS

An expression has been obtained for the output signal-to-noise ratio of the Demon processor. It is found that in general the detection (using conventional power detection) would occur before Demon classification could be effected.

For small signals, the signal-to-noise power ratio out of a Demon processor falls as the fourth power of the input signal-to-noise ratio, whereas that of the power detector falls as its square. This means that the threshold of the Demon processor can be expected to appear more sharply defined than that of the power detector.

As would be expected, the performance of the Demon processor falls rapidly as the depth of modulation of the signal is reduced.

REFERENCES

No.	Author	Title
1	Tuteur, F.B.	"Detection of Wide-Band Signals Modulated by a Low-Frequency Sinusoid". Appendix A-4 from "Processing of Data from Sonar Systems", by R.A. MacDonald et al., Yale University, Connecticut, September 1963, AD-420575.
2	Reed, I.S.	"On a Moment Theorem for Complex Gaussian Processes". Trans. IRE on Information Theory, Vol. IT-8, 194, April 1962.
3	Peterson, W.W., Birdsall, T.G. and Fox, W.C.	"The Theory of Signal Detectability". Trans. IRE on Information Theory, Vol.IT-4, 171, 1954.
4	Urick, R.J.	"Principles of Underwater Sound for Engineers". McGraw-Hill, 1967, p309.

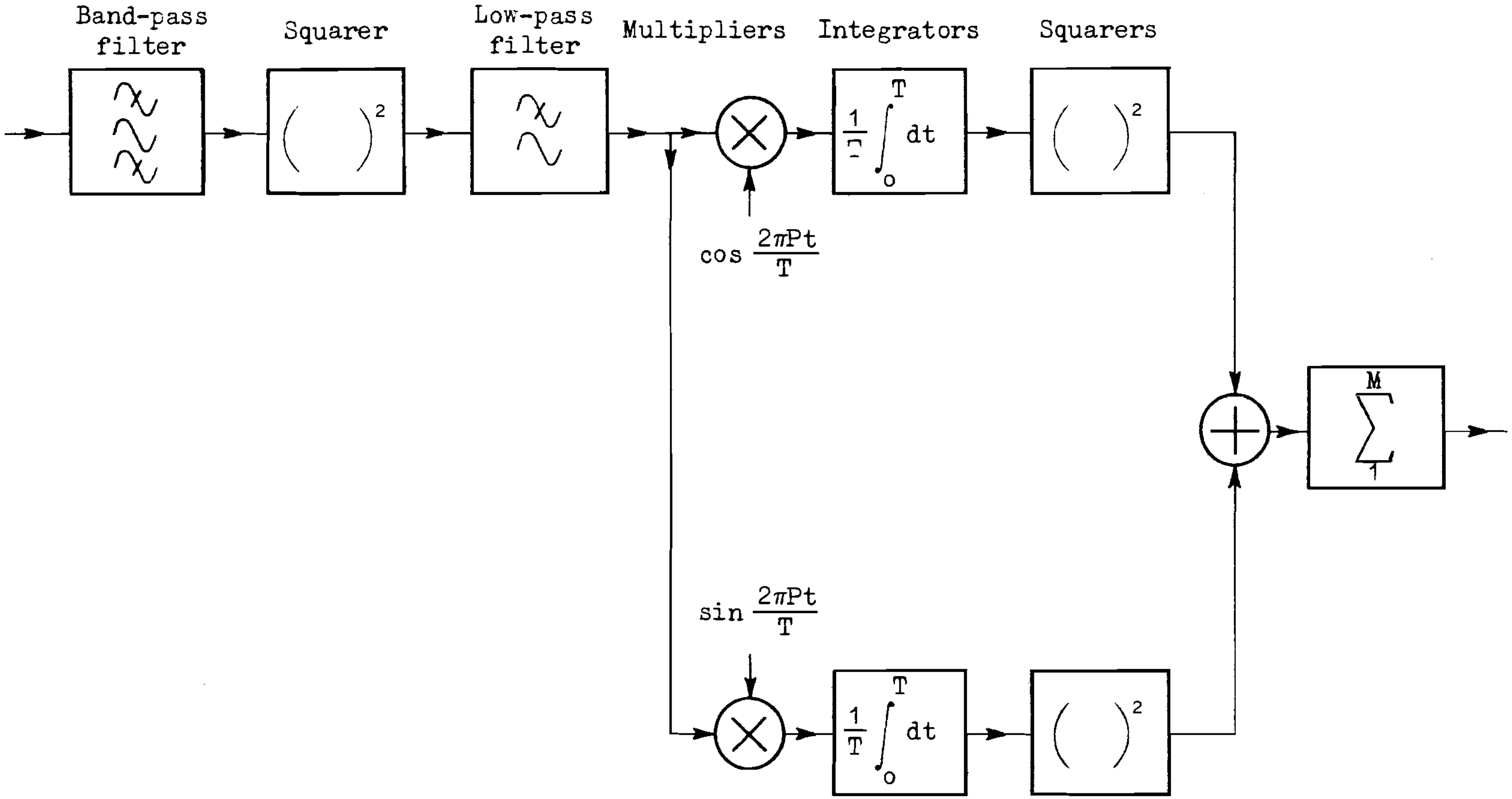

Figure 1. Block diagram of DEMON processor

UNCLASSIFIED

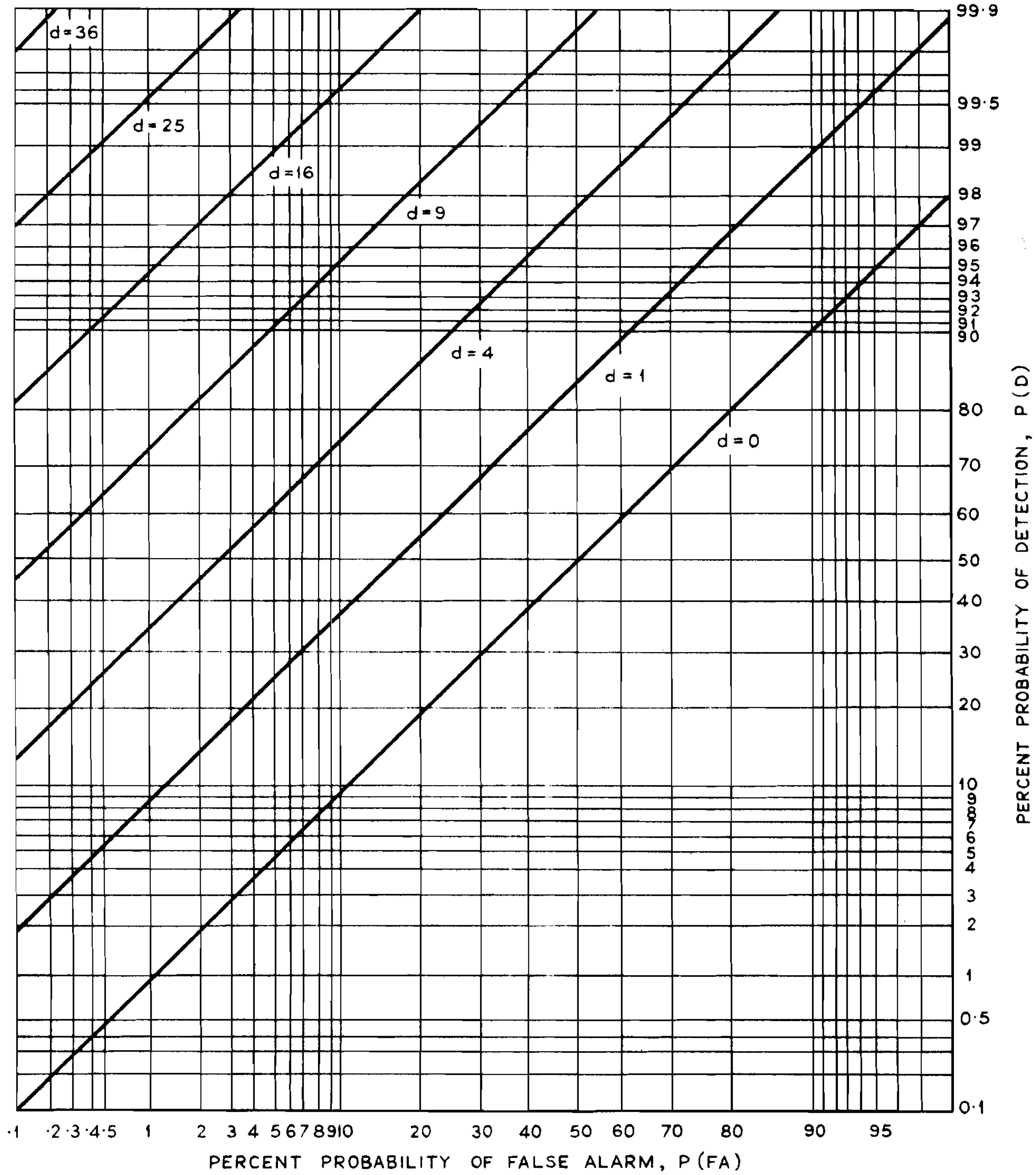

Figure 2. Receiver operating characteristics

UNCLASSIFIED

DOCUMENT-CONTROL DATA

1. Security Classification: (a) Complete document: ~~Confidential~~ UNCLASSIFIED (b) Title in isolation: ~~Confidential~~ UNCLASSIFIED (c) Summary in isolation: ~~Confidential~~ UNCLASSIFIED	2. Establishment, Document, Type, Number, Wing Tech. Memo. CPD 169 3. Document Date July 1970

4. Title and Sub-title

THEORETICAL ASSESSMENT OF "DEMON" PERFORMANCE

5. Personal author(s) (Show affiliations of author(s) if different to issuing establishment) H.A. d'Assumpcao

6. Corporate Author Weapons Research Establishment

7. Summary

An expression is obtained for the detection performance of a "DEMON" processor used to detect modulated signals imbedded in noise. It is shown that "DEMON" performance is degraded relative to the conventional power detector and that the "DEMON" threshold of detection is more sharply defined than that of the power detector.

8. Descriptors	9. Cosati Codes 10. Task Reference Number RD69 50/1
11. Library distribution (Libraries of Australian Defence Group to which copies will be sent)	12. Sponsoring Agency Reference Navy