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**REDUCED-ORDER MODELING OF THE RANDOM
RESPONSE OF CURVED BEAMS USING IMPLICIT
CONDENSATION (PREPRINT)**

Robert W. Gordon and Joseph J. Hollkamp

**Analytical Structural Mechanics Branch
Structures Division**

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Robert W. Gordon, Senior Aerospace Engr
Analytical Structural Mechanics Branch
Structures Division

//Signature//

Andrew G. Sparks, Ph.D., Chief
Analytical Structural Mechanics Branch
Structures Division

//Signature//

DAVID M. PRATT, Ph.D.
Technical Advisor
Structures Division

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Reduced-Order Modeling of the Random Response of Curved Beams using Implicit Condensation

Robert W. Gordon* and Joseph J. Hollkamp†

Air Force Research Laboratory, AFRL/VASM, Wright-Patterson AFB, Ohio 45433-7402

Accurate prediction of the response of aircraft skins to acoustic loading is important in the design of future air vehicles. Much work has been reported in recent years on prediction methods which reduce a finite element model to a reduced-order system of nonlinear modal equations. This body of work has shown good results for predicting the random response of flat structures. However, there have been few studies reported on reduced-order methods applied to structures with shallow curvature. Curvature complicates the analysis by introducing linear coupling of transverse and in-plane displacements. The implicit condensation and expansion (ICE) method, which eliminates the need for normal membrane vectors in the modal basis, has been shown to give accurate results for flat structures. This paper presents the results of a numerical study of the ICE method to predict the response of a thin, curved aluminum beam to random distributed loading. Power spectral densities of transverse and in-plane displacement from the ICE method agree very closely with those from direct integration of a full finite element model.

Nomenclature

$A_r(i,j,k)$	=	cubic stiffness term in the nonlinear stiffness function
$B_r(i,j)$	=	quadratic stiffness term in the nonlinear stiffness function
b	=	subscript indicating a quantity associated with bending (transverse) deformation
\bar{C}	=	damping matrix (generalized coordinates)
$f(t)$	=	applied force vector (physical coordinates)
$\mathbf{K}, \bar{\mathbf{K}}$	=	stiffness matrices (physical and generalized coordinates)
$\mathbf{K1}, \mathbf{K2}$	=	quadratic and cubic stiffness matrices (physical coordinates)
\mathbf{M}	=	mass matrix (physical coordinates)
m	=	subscript indicating a quantity associated with membrane (in-plane) deformation
\mathbf{p}, \mathbf{P}	=	response vector, matrix of response vectors (generalized coordinates)
\mathbf{T}	=	transformation matrix, the columns of which are the basis vectors
t	=	time
\mathbf{w}, \mathbf{W}	=	response vector, matrix of response vectors (physical coordinates)
ϕ_i, Φ	=	modal vector, modal matrix
θ	=	nonlinear internal force vector (generalized coordinates)
ω	=	natural frequency

I. Introduction

FUTURE air vehicle designs will require structures that can withstand extreme acoustic and thermal environments. Examples include reusable space access vehicles exposed to launch, hypersonic flight, and re-entry or stealthy aircraft with buried engines and ducted exhaust. Predicting the structural response to acoustic and thermal loads will be critical to achieving reliable vehicles. Today, accurate prediction of the large-amplitude nonlinear response of structures to random acoustic loading can only be performed by direct time integration of finite element models. Unfortunately, the computational cost of this approach makes it impractical for use as a design tool.

* Senior Aerospace Engineer, Structural Mechanics Branch, AFRL/VASM

† Senior Aerospace Engineer, Structural Mechanics Branch, AFRL/VASM

Modal models incorporating nonlinear terms were investigated by Nash¹ as a means of dramatically reducing the computational burden of numerical simulation for design and analysis. The approach uses a modal model including a few low frequency modes of the structure which are directly excited by the acoustic environment. Then, quadratic and cubic nonlinear terms in the modal coordinates are added. The coefficients of the nonlinear terms are only evaluated once rather than at every time step as required by direct integration of the full finite element model. Thus a very large nonlinear finite element model can be reduced to a low-order model with constant coefficients which can be integrated to produce response time histories in a matter of minutes on a personal computer.

The two primary challenges in developing accurate nonlinear modal models are determination of nonlinear stiffness coefficients and selection of modal basis vectors. Nonlinear stiffness coefficients can be computed using either direct or indirect approaches. In the direct approach, used by Nash¹ and Shi and Mei,² the full-order nonlinear stiffness matrices are evaluated with combinations of the modal vectors and then transformed to modal coordinates. This approach is effective but requires specialized finite element code. The indirect approaches use static nonlinear solutions of a full finite element model to determine stiffness coefficients. The static solutions can be performed in any commercial finite element code with nonlinear capability. The approach of Muravyov and Rizzi³ computes the nonlinear stiffness coefficients from static solutions to a set of enforced displacements comprised of combinations of the basis vectors. McEwan, et al.,⁴ developed a method which estimates the nonlinear coefficients from static solutions to a set of applied modal loads.

The selection of modal basis vectors is important in generating an accurate nonlinear modal model. The model must include all structural modes which are directly excited by the acoustic energy. These are typically low frequency normal modes with predominantly bending (transverse) deformation. In addition, the modal basis must be able to represent membrane (in-plane) displacements for the nonlinear modal model to be accurate. A fundamental characteristic of the large amplitude nonlinear response of flat beams and plates is the membrane displacement induced by finite bending displacement. One way to capture the membrane displacements is to explicitly include normal 'membrane' modes in the basis. Membrane modes are normal modes in which the deformation is primarily in the plane of the beam or plate. They tend to be much higher in frequency than bending modes and are not directly excited. Unfortunately, it can be difficult to identify membrane modes in a complex structure. Moreover, it is often not clear which or how many of them to include in the basis.

The method of McEwan, et al.,⁴ referred to here as implicit condensation (IC), mitigates the problem of membrane basis selection by taking advantage of the inherent benefits of static load solutions to compute nonlinear coefficients as opposed to the use of static enforced displacement solutions as in Ref. 3. With static load solutions, load vectors which are linear combinations of only bending modes induce both bending and membrane displacements in the solutions—the membrane displacements occur naturally as a result of the geometric nonlinearity. The resulting modal equations include only bending generalized coordinates. The membrane effects are implicitly condensed into the bending mode equations. With enforced displacement solutions, membrane modes must be included, a priori, in the set of enforced displacements along with bending modes. The modal equations include both bending and membrane coordinates. The IC method does have a drawback, however. Physical membrane displacements cannot be recovered from the modal solution since the model has no membrane equations. Without membrane displacements, strains cannot be recovered from the finite element strain-displacement equations. Instead, a mapping approach is used to recover strains.

Hollkamp and Gordon⁵ proposed a method to recover physical membrane displacements directly from a bending-modes-only (IC) modal model. This approach, referred to as implicit condensation and expansion (ICE), synthesizes modal membrane displacements from bending displacements and then transforms them to the physical domain using a set of estimated membrane vectors. In Ref. 5, predicted membrane displacements for a clamped straight beam agreed very closely with results from direct time integration of a full finite element model.

The foregoing discussion applies to straight beams and flat plates in which bending and membrane displacements are coupled only through nonlinear stiffness terms. The situation is different for the more general case of curved beams or shells. Curvature causes linear coupling of bending and membrane displacements in addition to the nonlinear coupling. Normal modes are not strictly bending or membrane, but a combination of the two. The lower-frequency modes which respond directly to external loading tend to be bending-dominated. A second effect of curvature is the emergence of nonlinear softening behavior. For straight beams, the predominant effect of large displacements is increasing stiffness (hardening). For curved beams, however, stiffness can either increase or decrease (soften) depending on the specific geometry and loading. The hardening behavior is manifested through cubic nonlinear terms in the modal equations whereas softening occurs through quadratic nonlinear terms.

Recently, Przekop and Rizzi⁶ reported an application of the nonlinear modal approach from Ref. 3 to predict the random response of a beam with shallow curvature. They investigated the effect of several sets of transverse-dominated and in-plane dominated normal modes as basis vectors on the accuracy of the predicted response. Very

good agreement with direct integration results was achieved. The model which gave the best agreement used the eight lowest transverse-dominated modes of the beam plus eight additional higher-frequency, in-plane dominated modes. This work also pointed out that low-frequency, *anti-symmetric*, transverse-dominated modes—which are not directly excited by symmetric acoustic loading—can be excited parametrically in a curved structure.

The objective of the present work is to demonstrate the accuracy of the ICE method applied to a clamped beam with shallow curvature. Predicted power spectral densities (PSD's) are presented for two levels of distributed random loading on the beam. Excellent agreement is shown with results from direct integration of a full finite element model of the beam.

II. Reduced-Order Models

Finite element discretization of the acoustic response problem results in the nonlinear equations of motion

$$\mathbf{M}\ddot{\mathbf{w}} + [\mathbf{K} + \mathbf{K1}(\mathbf{w}) + \mathbf{K2}(\mathbf{w}, \mathbf{w})]\mathbf{w} = \mathbf{f}(t) \quad (1)$$

where \mathbf{M} is the mass matrix, \mathbf{K} is the linear stiffness matrix, \mathbf{w} is the displacement vector, and $\mathbf{f}(t)$ is a vector of external time-varying forces. Quadratic and cubic nonlinear stiffness matrices, $\mathbf{K1}$ and $\mathbf{K2}$, are linear and quadratic functions of the nodal displacements, respectively. The physical displacements are transformed into modal space by the relationship

$$\mathbf{w} = \Phi \mathbf{p} \quad (2)$$

where \mathbf{p} is a vector of modal amplitudes and

$$\Phi = [\phi_1 \phi_2 \dots \phi_n] \quad (3)$$

is the matrix of modal basis vectors or the transformation matrix. In a linear problem, the modal basis contains only a few low frequency modes which are directly excited by the external forces. This results in a much smaller number of modal displacements than physical displacements. Substituting Eq. (2) into Eq. (1), the reduced-order equations of motion become

$$\ddot{\mathbf{p}} + \bar{\mathbf{C}} \dot{\mathbf{p}} + \bar{\mathbf{K}}\mathbf{p} + \boldsymbol{\theta}(p_1, p_2, \dots, p_n) = \Phi^T \mathbf{f} \quad (4)$$

where

$$\bar{\mathbf{K}} = \text{diag}(\omega_1^2, \omega_2^2, \dots, \omega_n^2) \quad (5)$$

is the modal stiffness matrix, ω_i are the resonant frequencies of the modes included in the model, $\bar{\mathbf{C}}$ is a diagonal modal damping matrix, and $\boldsymbol{\theta}$ is a vector function containing the nonlinear terms. The equations are coupled only through the nonlinear function. The general form of the nonlinear function for the “rth” equation is

$$\theta_r = \sum_{i=1}^n \sum_{j=i}^n B_r(i,j) p_i p_j + \sum_{i=1}^n \sum_{j=i}^n \sum_{k=j}^n A_r(i,j,k) p_i p_j p_k \quad (6)$$

where the $B_r(i,j)$ and $A_r(i,j,k)$ terms express quadratic and cubic modal stiffness, respectively. The quadratic nonlinear terms are important only if the structure is curved or if membrane modes are included explicitly in the model.

The IC method estimates the nonlinear coefficients in Eq. (6) from a set of nonlinear static solutions obtained from a finite element code. The applied loads for the static solutions are linear combinations of the modal basis vectors. Only bending-dominated modes in the frequency range of the acoustic excitation are included in the basis. The method, as applied in Ref. 4, also neglects quadratic nonlinear terms since planar structures were considered. Quadratic terms are retained in the implementation reported here because the structure is curved.

The IC method must be modified to compute membrane displacements. The ICE method⁵ implements this procedure. The transformation matrix from the IC method is augmented as

$$\mathbf{T} = [\Phi_b \mathbf{T}_m] \quad (7)$$

where \mathbf{T}_m is the estimated membrane basis. The subscript, b, has been added to Φ to denote that it is comprised of bending modes. Note that the columns of \mathbf{T}_m are orthogonal to the columns of Φ_b . The vector of physical displacements can then be represented by

$$\mathbf{w} = \mathbf{w}_b + \mathbf{w}_m = \Phi_b \mathbf{p}_b + \mathbf{T}_m \mathbf{p}_m \quad (8)$$

where \mathbf{p}_b are the modal bending amplitudes, \mathbf{w}_b are the physical displacements spanned by the bending modal vectors, \mathbf{p}_m will be called generalized membrane amplitudes, and \mathbf{w}_m are the corresponding physical displacements. The vector, \mathbf{w}_m , contains displacements that are not spanned by the bending mode shapes. For convenience they are referred to as the membrane displacements.

The displacement vector in Eq. (8) is not partitioned, rather the displacement is separated into two additive vectors. Each physical displacement DOF has an entry in both vectors. Any general structure can be modeled. In the case of a planar structure, many of the entries will be zero in either \mathbf{w}_b or \mathbf{w}_m . In the case of a curved structure modeled in a Cartesian coordinate system, the vectors will be fully populated.

The expansion process by which \mathbf{T}_m , \mathbf{p}_m , and ultimately \mathbf{w}_m are derived is summarized here. A single static solution is represented by Eq. (8). Considering many static solutions, the equation becomes

$$\mathbf{W} = \Phi_b \mathbf{P}_b + \mathbf{T}_m \mathbf{P}_m \quad (9)$$

where the columns of \mathbf{W} , \mathbf{P}_b , and \mathbf{P}_m correspond to the individual solutions. The set of generalized membrane displacements, \mathbf{P}_m , can be synthesized from the modal bending displacements. A single column of \mathbf{P}_m is computed from

$$\mathbf{p}_m = [p_1^2 \ p_1 p_2 \ p_1 p_3 \ \dots \ p_1 p_n \ p_2^2 \ p_2 p_3 \ \dots \ p_2 p_n \ \dots \ p_{n-1}^2 \ p_{n-1} p_n \ p_n^2]^T \quad (10)$$

where the terms p_1 through p_n refer to the bending modal amplitudes in the corresponding single static solution. The set of generalized membrane amplitudes spans all the possible quadratic combinations of the bending modal amplitudes. The estimated membrane basis, \mathbf{T}_m , is then found by solving Eq. (9). Details of this derivation can be found in Ref. 5.

III. The Curved Beam and Computational Models

A. Beam Configuration

The curved beam configuration investigated in this study is taken from Ref. 6 and is shown in Fig. 1. The aluminum alloy beam had dimensions of 18.0 inches long (projected length) x 1.0 inch x 0.09 inches thick with an 81.25-inch radius of curvature. Both ends of the beam were clamped. The beam material elastic modulus was 10.6×10^6 psi, the shear modulus was 4.0×10^6 psi and the density was 2.588×10^{-4} lbf-sec²/in⁴.

B. Finite Element Model

A finite element model of the beam was constructed in ABAQUS⁷ using 80 B31 beam elements. This model was used to compute mode shapes and frequencies and perform the nonlinear applied loads solutions. Mode shapes and frequencies for the first eight modes of the beam are shown in Fig. 2.

Direct time integration of a second finite element model was performed using an in-house code written in Matlab⁸. This model also used 80 2-noded beam elements and had 237 degrees-of-freedom (DOF). A mass-proportional damping matrix was used to provide 2.0% critical damping at 258 Hz—the frequency of the first symmetric mode of the beam. A band-limited random excitation was applied as a uniform load on the model with a 0-1500 Hz frequency range. Two load levels were applied—0.2304 lb/in and 0.6517 lb/in—giving minimal and significant nonlinear response, respectively. These are the same load levels used in Ref. 6. Simulations were performed for 50 seconds at each load level with a time step of 1×10^{-5} second. It was important to generate relatively long time records so that statistical averaging of PSD's was adequate.

C. Reduced-Order Models

A reduced-order nonlinear model of the beam was computed using the IC method from a set of nonlinear static solutions. The lowest 8 normal modes of the beam were retained in the model. All of these modes had predominantly transverse displacements. The first seven of these were within the excitation bandwidth. Although the eighth mode was above the excitation bandwidth, it was included to be consistent with Ref. 6. No membrane-dominated modes were retained in the model since the implicit condensation approach does not require an explicit membrane basis. The reduced-order model was subjected to the distributed random loading used with the full model simulation. Fifty-second time records were computed using Newmark time integration with a time step of 2×10^{-5} second. The resulting modal displacements were then transformed to physical coordinates using the modal vectors. These physical displacements are the IC model results.

The integration results from the IC method were also used to obtain the ICE results. A set of 36 estimated membrane basis vectors was computed from the static nonlinear solutions using Eq. (9). The bending modal displacements obtained by the IC method were then expanded to produce the generalized membrane displacements using Eq. (10). The bending modal displacements and the generalized membrane displacements were transformed to physical coordinates using the augmented modal basis and Eq. (8).

IV. Discussion of Results

PSD's of displacement from the IC and ICE methods are compared to full finite element model simulation results in this section. Results are presented for four beam DOF: x- and y-direction displacements at the quarter point and x- and y-direction displacements at the center of the beam. Note that at the quarter point these displacements are approximate, but not identical, to the in-plane and transverse components, respectively.

Displacement PSD's for the 0.2304 lb/in input level are shown in Figs. 3-6. The y-direction PSD's are virtually identical to the full model simulation at the center and quarter point for IC and ICE models, as shown in Figs. 3 and 4. The x-direction PSD's, which are dominated by in-plane displacements, are shown in Figs. 5 and 6. The ICE model again shows nearly identical agreement with the full model. The IC model also shows very good agreement near the dominant peaks, but shows some small error near 150, 600, and 1100 Hz. This is probably due to the lack of membrane-dominated modes in the model basis. The bending-dominated modes in the model were an adequate basis for reproducing the major response peaks of the beam.

The predicted PSD's of the models at 0.6517 lb/in are compared in Figs. 7-10. Significantly nonlinear response occurred at this level. Again, the IC and ICE models show excellent agreement with the full model for the y-direction results at the center and quarter point of the beam (see Figs. 7 and 8). The ICE results also display excellent agreement with the full model simulation at both x-direction DOF as shown in Figs. 9 and 10. However, the IC model results show some error. The predicted x-displacement at the beam center, shown in Fig. 9, agrees very well at frequencies up to the dominant response peak at 250 Hz but begins to show significant deviation above this frequency. The lack of membrane-dominated vectors in the model basis is likely responsible. The predicted x-direction displacement at the beam quarter point, shown in Fig. 10, also shows error above 400 Hz, but the error is much less than it was at the beam center.

The IC method was able to reasonably predict x-displacements of the test beam because it was curved. The bending mode shapes of a curved beam contain both x-direction and y-direction DOF. If the beam was flat, the x-direction DOF in the bending mode shapes would be zero and the IC method would be unable to predict any x-displacements. For the curved beam, the IC prediction of the x-displacements was in error as the loading increased. The bending mode shapes were not a complete basis for these DOF. The ICE method, with its estimated membrane basis, did provide an adequate basis for the x-direction DOF. Furthermore, the ICE method worked directly on the IC results in a post-processing operation with little additional computational cost.

V. Conclusion

The nonlinear response of a curved beam with clamped ends subjected to random excitation was predicted using a reduced-order model computed by the implicit condensation and expansion (ICE) method. The first eight bending-dominated modes of the beam were retained in the model. No membrane-dominated modes were included. PSD's of displacement at four DOF on the beam were compared to results from direct time integration of a finite element model. The displacement PSD's were compared for two input force levels—one producing a small degree of nonlinear response and a second producing significant nonlinear response.

The primary advantage of the ICE method over other reduced-order methods is that membrane-dominated modes are not explicitly required in the construction of the reduced-order model. The model has equations for only the retained bending-dominant modes. The ICE method estimates a set of membrane basis vectors that are used in a

post processing expansion. Physical displacements predominated by nonlinear in-plane motion can be accurately recovered. This was demonstrated previously by the authors for straight beams.⁵

Displacement PSD's predicted by the ICE method showed excellent agreement with full model simulation results. In fact, the PSD's were virtually identical to those from the full model for both transverse-dominated and in-plane dominated DOF of the beam. Thus, the accuracy of the ICE method for beams with shallow curvature is verified. The predictions made without the estimated membrane basis vectors, from the IC method, predicted transverse-dominated displacements accurately but were significantly in error for in-plane dominated displacements.

Although the ICE method eliminates the need to identify and select membrane-dominated modes for the model, the question of which low-frequency, bending-dominated modes to include is not obvious. An effective approach for flat beams and plates is to include all modes which would be directly excited in a linear structure. This is obviously not the case in the present study, since the ICE model included four anti-symmetric modes of the beam and each responded significantly!

Finally, this study provides another step in the process of proving that reduced-order models can be an accurate and efficient substitute for a finite element model in predicting nonlinear response of structures to random excitation. However, a reduced-order model cannot capture the behavior of a real-world problem if the finite element model formulation does not include the underlying physics.

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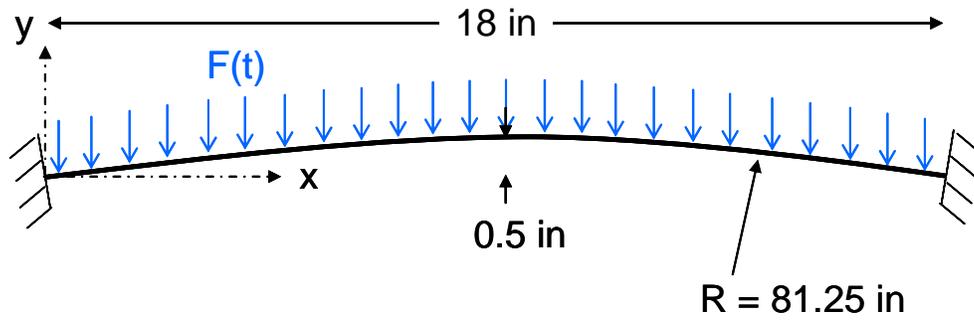


Figure 1. Geometry of the curved beam.

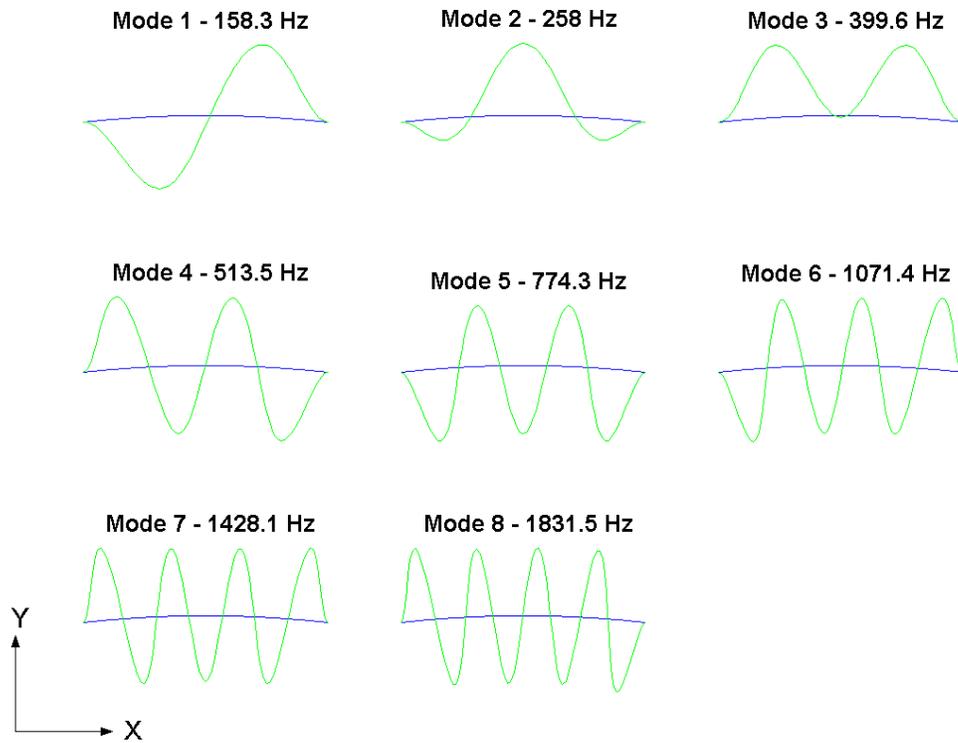


Figure 2. Mode shapes of the curved beam from the finite element model.

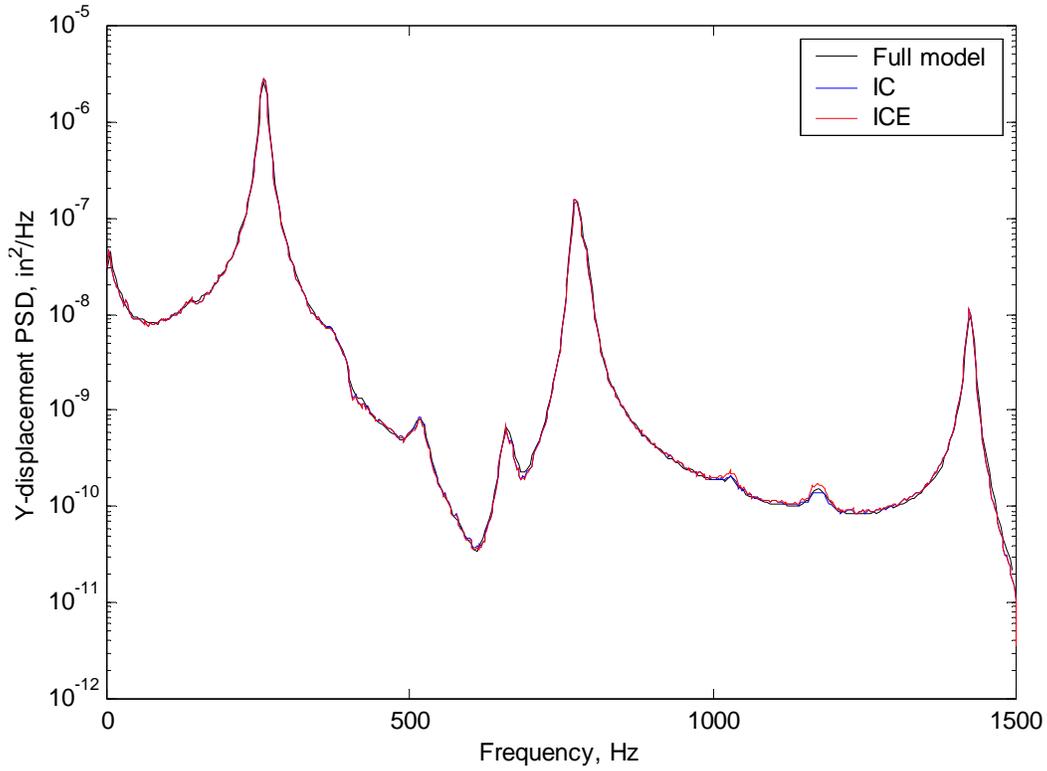


Figure 3. Y-displacement PSD at the beam center, 0.2304 lb/in.

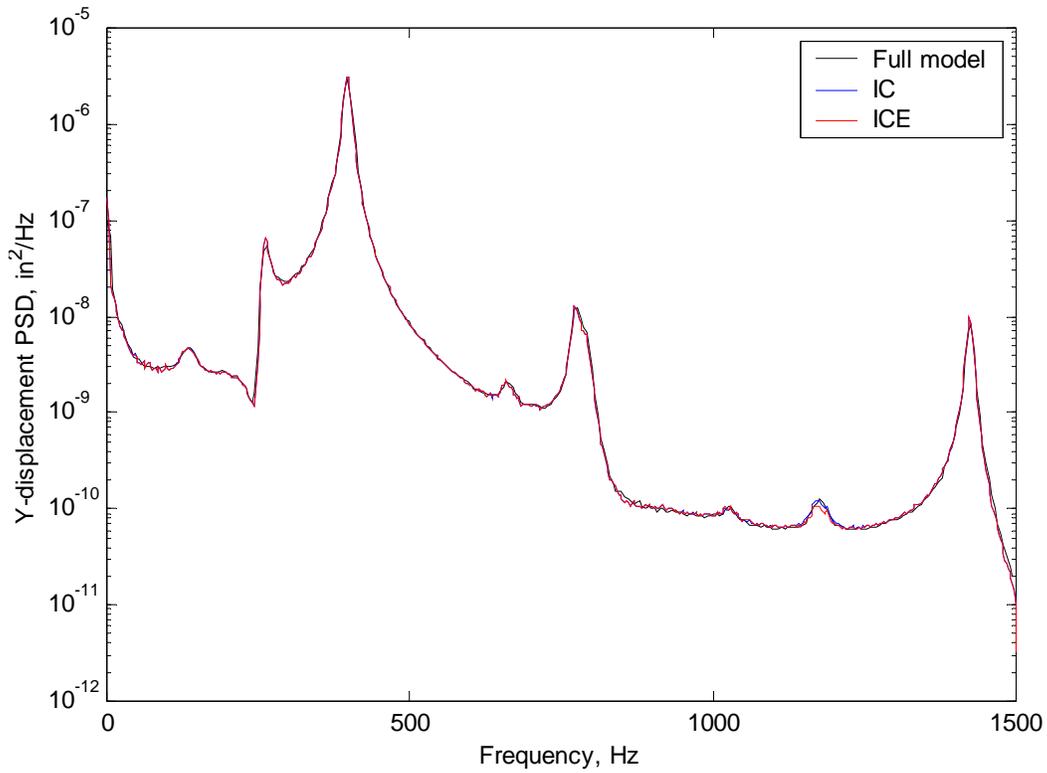


Figure 4. Y-displacement PSD at the beam quarter point, 0.2304 lb/in.

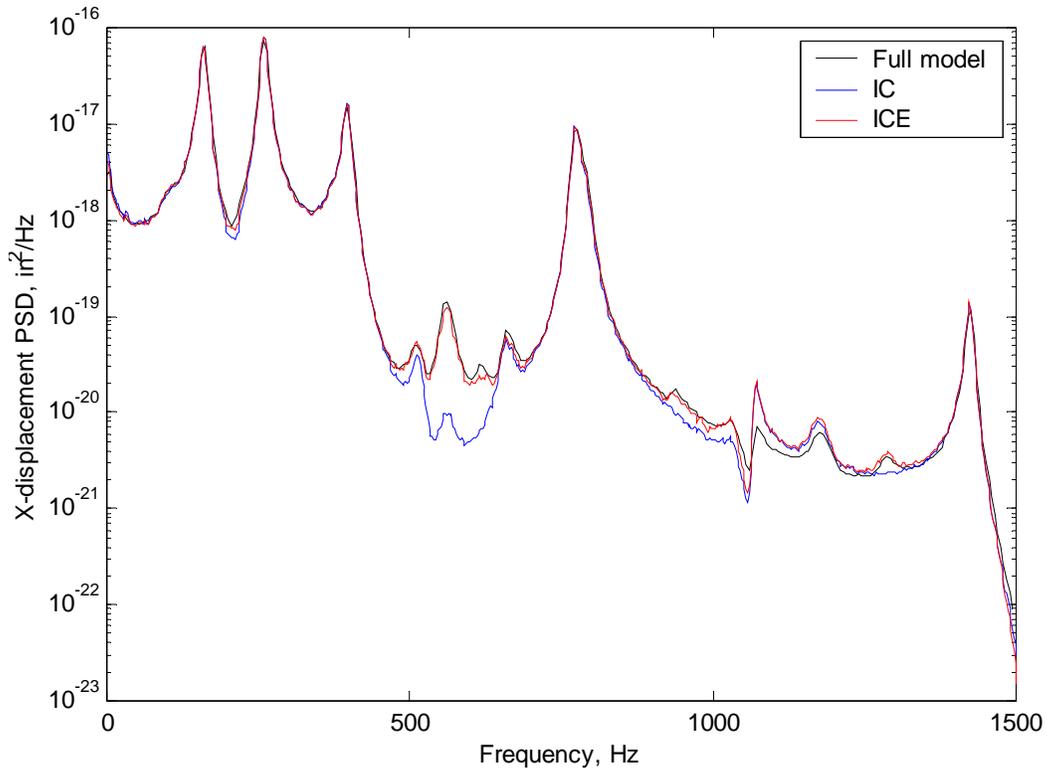


Figure 5. X-displacement PSD at the beam center, 0.2304 lb/in.

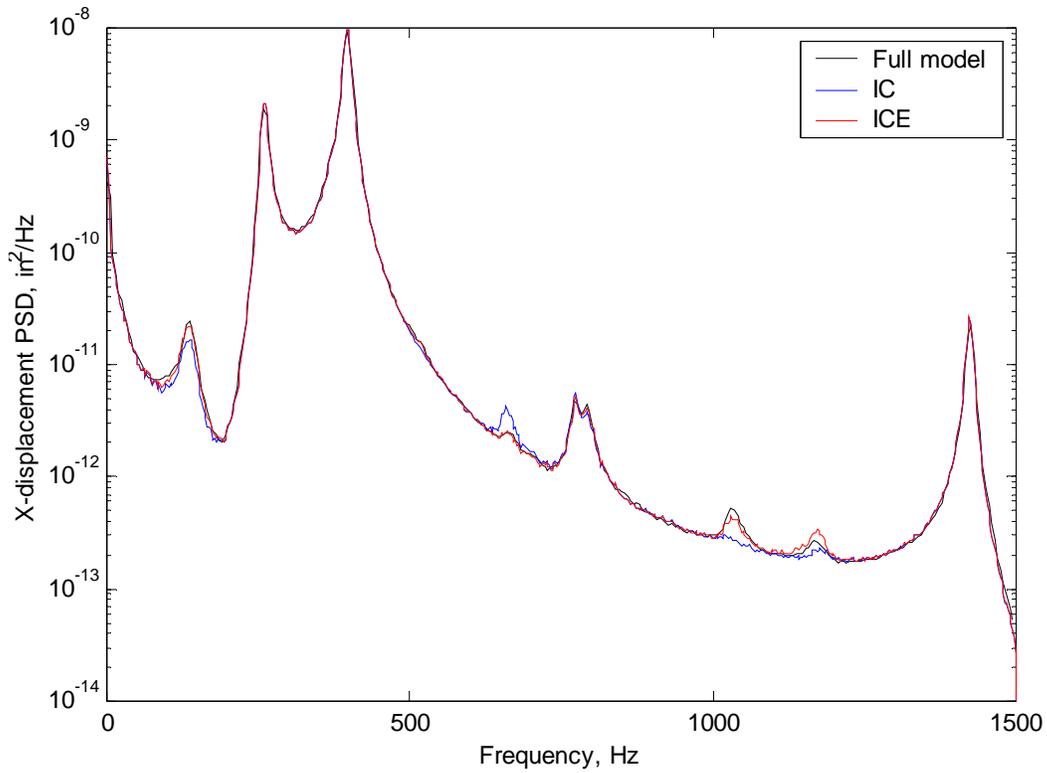


Figure 6. X-displacement PSD at the beam quarter point, 0.2304 lb/in.

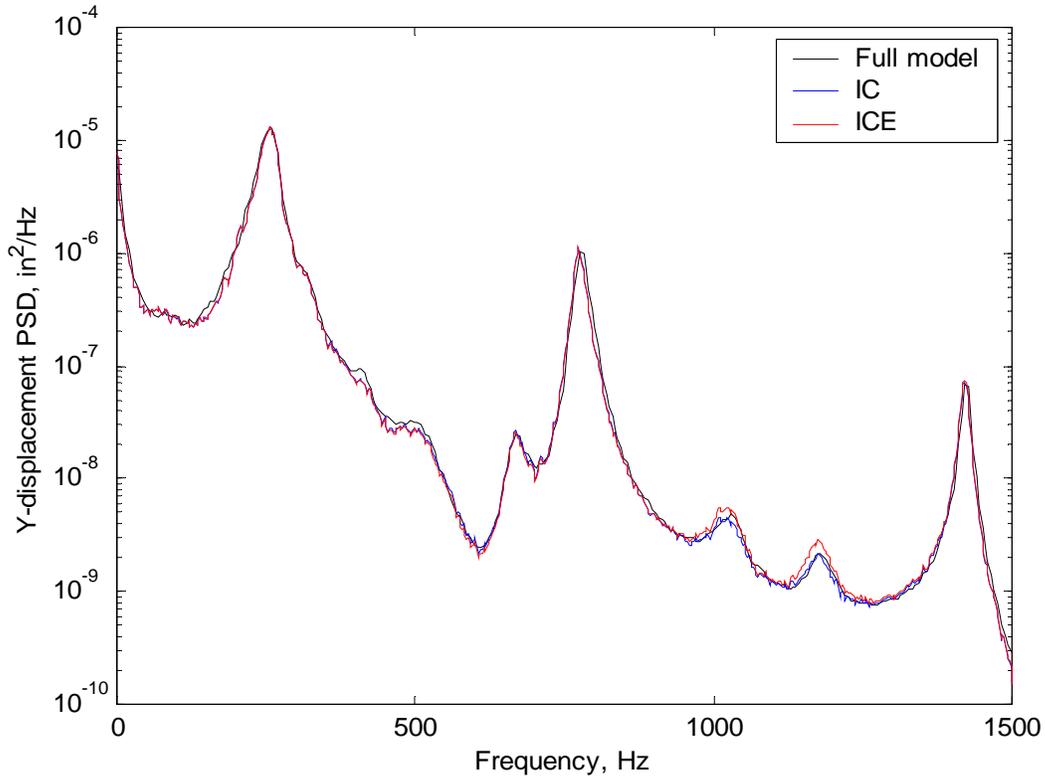


Figure 7. Y-displacement PSD at the beam center, 0.6517 lb/in

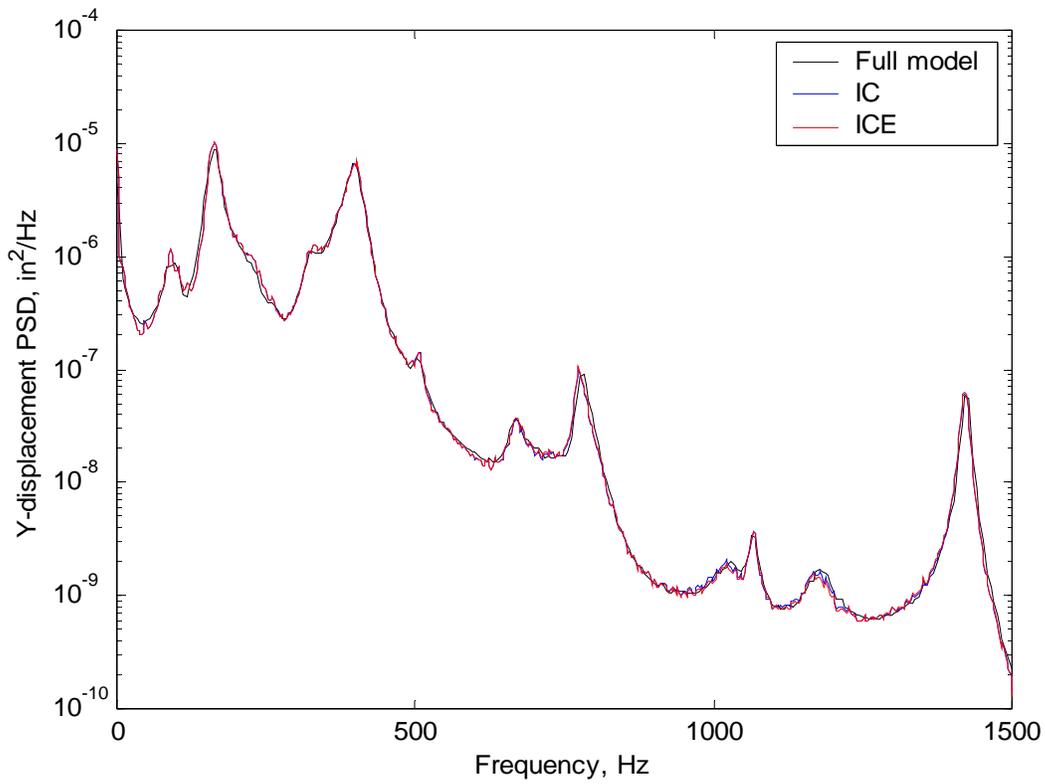


Figure 8. Y-displacement PSD at the beam quarter point, 0.6517 lb/in

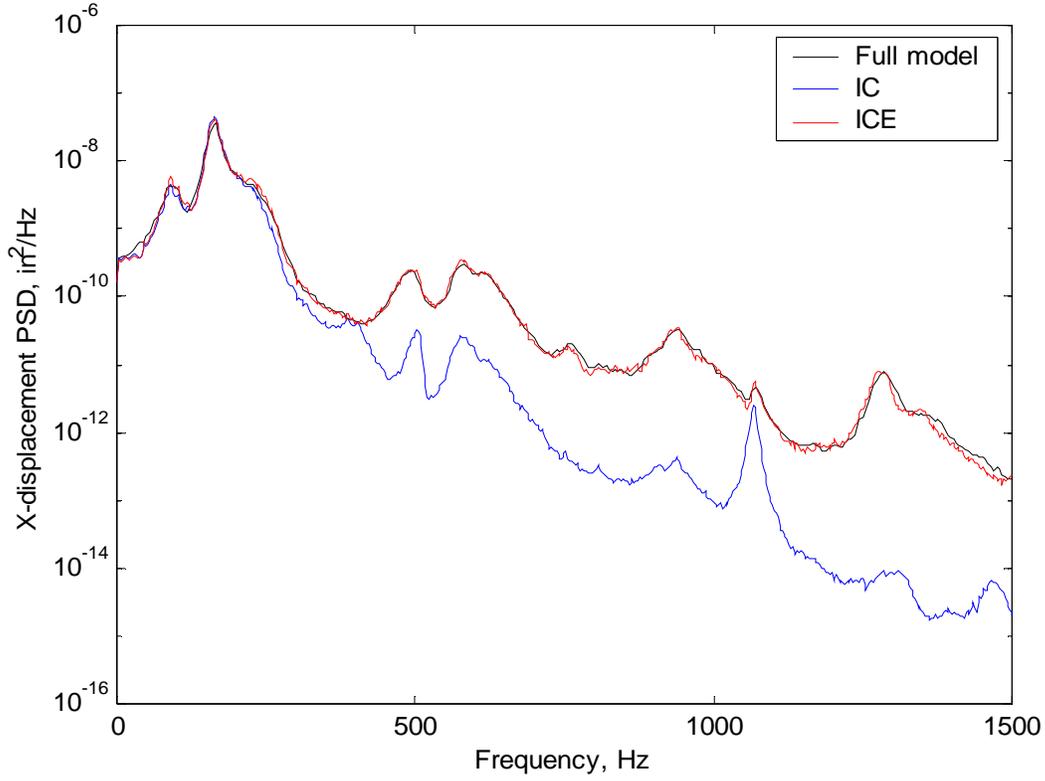


Figure 9. X-displacement PSD at the beam center, 0.6517 lb/in

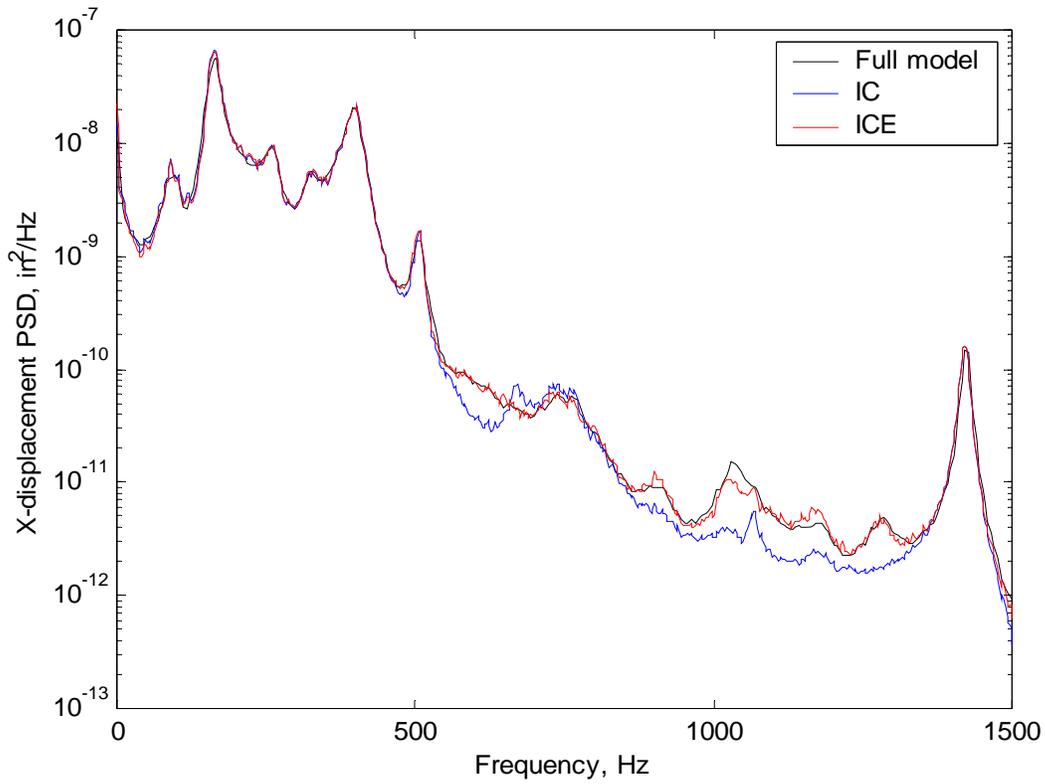


Figure 10. X-displacement PSD at the beam quarter point, 0.6517 lb/in