**ABSTRACT**

Our research focuses on horizontal underwater acoustic communications. For high data rate applications, such as image transmission or multiuser networks at long ranges, coherent underwater acoustic communication is of great importance. In this direction, we consider multi-input multi-output (MIMO) transmit beamforming under the uniform elemental power constraint. This is a non-convex optimization problem, and the optimal transmit beamformer is usually difficult to construct. We will first find a beamforming solution in an ideal setup, and then consider practical finite-rate feedback methods required to implement the transmit beamforming. On the other hand, the highly time-varying and frequency-selective underwater acoustic channel consists of a great challenge for channel estimators necessary for coherent communications. Such difficulty is aggravated when multiple transducers and/or hydrophones are deployed. Therefore, we also investigate differential MIMO transmission and reception approaches tailored for doubly-selective MIMO channels. To enhance the system error performance, we notice that the three-dimensional space-time-frequency variation of the underwater channel can be exploited to provide three-dimensional diversity gain. Based on all these, we also develop a differential MIMO transceiver tailored for doubly-selective underwater channels.
LONG-TERM GOALS

Our proposed research will focus on the horizontal underwater acoustic communications (UAC). One important metric for evaluating UAC systems is the maximum achievable range-rate product. Building on co-located and/or distributed multiple transducers and hydrophones, we plan to develop rate-oriented and range-oriented multiple-input multiple-output (MIMO-)UAC systems, as well as additional range/rate enhancement via the deployment of distributed transceiver units which operate in a cooperative manner.

OBJECTIVES

Coherent underwater acoustic communication is of great importance for high data rate applications, such as, image transmission or multiuser networks at long ranges. We consider MIMO transmit beamforming under the uniform elemental power constraint. This is a non-convex optimization problem, and it is usually difficult to find the optimal transmit beamformer. We will first find a beamforming solution in an ideal setup, and then consider practical finite-rate feedback methods required to implement the transmit beamforming.

The time-varying and frequency-selective underwater acoustic channel consists of a great challenge for channel estimators necessary for coherent communications. Such difficulty is aggravated when multiple transducers and/or hydrophones are deployed. Therefore, we also investigate differential MIMO transmission and reception approaches tailored for doubly-selective MIMO channels. To enhance the system error performance, we notice that the three-dimensional space-time-frequency variation of the underwater channel can be exploited to provide three-dimensional diversity gain. Based on all these, our second objective is to develop a differential MIMO transceiver tailored for doubly-selective underwater channels.

APPROACH

1. MIMO Transmit Beamforming under Uniform Elemental Power Constraint

*The Cyclic Algorithm for MIMO Transmit Beamformer Design*

Consider an \((N_r,N_t)\) MIMO communication system in a quasi-static frequency-flat fading channel. At the transmitter, the symbol \(s \in \mathbb{C}\) is modulated by the beamformer \(w_t = \begin{bmatrix} w_{t,1} & w_{t,2} & \ldots & w_{t,N_t} \end{bmatrix}^T\). At the
receiver, after processing with the combining vector \( \mathbf{w} = [w_{r,1}, w_{r,2}, \ldots, w_{r,N_r}]^T \), the sampled baseband signal is given by \( y = \mathbf{w}^* \mathbf{H} s + \mathbf{n} \), where \( \mathbf{H} \in \mathbb{C}^{N_t \times N_r} \) is the channel matrix. Our problem at hand is

\[
\max_{\{\mathbf{w}, \mathbf{w}_r\}} |\mathbf{w}^* \mathbf{H} \mathbf{w}_r|^2, \quad \text{subject to } |w_{r,i}|^2 = \frac{1}{N_r}, \quad i = 1, 2, \ldots, N_t, \quad \|\mathbf{w}\| = 1. \tag{1}
\]

This is a non-convex optimization problem, which is usually difficult to solve, and no globally optimal solution is guaranteed. We solve it in the following way:

First, we consider the multi-input single output (MISO) case and propose a cyclic algorithm:

- **Step 0:** Set \( \mathbf{w}_r \) to an initial value (e.g., the left singular vector of \( \mathbf{H} \) for its largest singular value).
- **Step 1:** Obtain the \( \mathbf{w}_r \) that maximizes (1) for \( \mathbf{w}_r \) fixed at its most recent value. By taking \( \mathbf{w}^* \mathbf{H} \) as the “effective MISO channel,” the optimal solution is: \( \mathbf{w}_r = \frac{1}{\sqrt{N_r}} e^{j\mathbf{H}^* \mathbf{w}} \).
- **Step 2:** Determine the \( \mathbf{w}_r \) that maximizes (1) for \( \mathbf{w}_r \) fixed at its most recent value. The optimal \( \mathbf{w}_r \) is the MRC: \( \mathbf{w}_r = \frac{\mathbf{H} \mathbf{w}_r}{\|\mathbf{H} \mathbf{w}_r\|} \). Iterate Steps 1 and 2 until a given stop criterion is satisfied.

**Finite-Rate Feedback for Transmit Beamforming Designs**

We consider herein vector quantization under uniform elemental power constraint (VQ-UEP) for finite-rate feedback. In this case, both the transmitter and the receiver maintain a codebook which is used to reconstruct the transmit beamformer. The generalized Lloyd algorithm is usually implemented for codebook construction. The design criterion is adopted to satisfy two conditions: the nearest neighborhood condition (NNC) and the centroid condition (CC). NNC is to find the optimal partition region for a fixed codeword, while CC updates the optimal codeword for a fixed partition region. The monotonically convergent property is guaranteed due to obtaining an optimal solution for each condition. We construct the codebook offline by maximizing the average receive SNR:

First, we generate a training set \( \{\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_N\} \) from a sufficiently large number of channel realizations. Next, starting from an initial codebook, we iteratively update the codebook according to the following two criteria until no further improvement is observed.

- **NNC:** for given \( \{\hat{\mathbf{w}}_i\}_{i=1}^N \), assign \( \mathbf{H}_n \) to the \( i \) th region \( S_i = \{\mathbf{H}_n : \|\mathbf{H}_n, \hat{\mathbf{w}}_i\| \geq \|\mathbf{H}_n, \hat{\mathbf{w}}_j\|, \forall j \neq i\} \), where \( S_i, i = 1, 2, \ldots, N \), is the partition set for the \( i \) th codeword \( \hat{\mathbf{w}}_i \).
- **CC:** for given partition \( S_i \), the updated \( \{\mathbf{w}_i\}_{i=1}^N \) satisfy \( \mathbf{w}_i = \arg \max_{\mathbf{w}_i} \mathbb{E}_a \left[ \|\mathbf{H}_n \mathbf{w}_i\|^2 | \mathbf{H}_n \in S_i \right] \), subject to \( |\mathbf{w}_{i,m}|^2 = \frac{1}{N_i}, \quad m = 1, \ldots, N_i, \quad \text{for } i = 1, 2, \ldots, N \). Let \( \mathbf{R}_i = \mathbb{E}_a \left[ \mathbf{H}_n^H \mathbf{H}_n | \mathbf{H}_n \in S_i \right] \) and \( \mathbf{R}_i^{1/2} \) be Hermitian square root of \( \mathbf{R}_i \).

For a given codebook \( \hat{\mathbf{W}} := \{\hat{\mathbf{w}}_1, \hat{\mathbf{w}}_2, \ldots, \hat{\mathbf{w}}_N\} \), the receiver first chooses the optimal beamformer as:

\( \hat{\mathbf{w}}_{\text{opt}} = \arg \max_{\mathbf{w}_i} \|\mathbf{H} \mathbf{w}_i\|^2 \). Then we feedback the index of \( \hat{\mathbf{w}}_{\text{opt}} \) to the transmitter with \( \log_2 N_v = B \) bits.

**Average Degradation of the Receive SNR**

We quantify below the effect of VQ-UEP with finite-bit feedback:

**Proposition 1** For i.i.d. MISO Raleigh fading channels, the average degradation of the receive SNR, for an \( N_r \)-antenna transmit beamforming system with an \( N_v = 2^B \)-size VQ-UEP codebook, can be
approximated as: \( D_c(B) \approx (N_t - 1) \cdot 2^B \left[ (2^{-B} + (1 - a)^{N_t-1})^{N_t} - (1 - a)^{N_t} \right] \sigma_h^2 - N_t(1 - a)\sigma_h^2. \)

where \( a = \frac{1}{N_t} + \frac{\pi N_t}{4(N_t-1)} \) and \( \sigma_h^2 \) is the covariance of each scalar channel.

The average degradation of the receive SNR can be proven to be monotonically decreasing with respect to non-negative real number \( B \). Given a degradation amount \( D_o \), this proposition provides a guideline to determine the necessary number of feedback bits.

2. Orthogonal Space-Time Block-Differential Modulation over Underwater Acoustic Channels

Our approach hinges upon the delay-Doppler spread channel characterization and a block-by-block transmission and receiver processing. Without loss of generality, let us consider a SISO doubly-selective UAC channel with impulse response \( h(t; \tau) \) and frequency response \( H(f; \tau) \) which is also known as the delay-Doppler spread function [1]. The I/O relationship of such a channel can then be obtained as \( y(t) = \int_{-f_{max}}^{f_{max}} \int_0^{\tau_{max}} x(t - \tau) H(f; \tau) e^{i2\pi ft} d\tau df \), where \( \tau_{max} \) is the maximum delay spread and \( f_{max} \) is the maximum Doppler spread. Consider a block-by-block transmission with \( N \) symbols per block and symbol duration \( T \). It follows that the time- and frequency-domain resolutions of the corresponding discrete-time equivalent system are \( T \) and \( (NT_s)^{-1} \), respectively. Hence, the discrete-time equivalence of the above I/O relationship is:

\[
y(n) = \sum_{k=0}^{K} x(n - k) \sum_{l=0}^{L} h_q(l) e^{i\omega(q-K)n}, \quad q \in [0, L].
\]

(3)

Then the system I/O relationship becomes \( y(n) = \sum_{l=0}^{L} h(n; l) x(n - l) + z(n) \), or \( y = Hx + z \).

For a frequency-selective but time-invariant channel, \( H \) is Toeplitz, which can be diagonalized by FFT and IFFT operations together with cyclic prefix (CP) to enable differential encoding. In the time-varying case, however, the elements on (sub)diagonals are different. Fortunately, based on the BEM model, one could diagonalize the channel matrix via two FFT-like operations, one deals with the multipath-frequency conversion as in conventional OFDM, the other the time-Doppler conversion \( (n - q \text{ in (3)}) \). This diagonalization will then enable differential coding. Special considerations will be added to enable Doppler-multipath diversity. On top of these, we also employ the differential orthogonal space-time block code (OSTBC) [3], [4] to harvest the space diversity.

WORK COMPLETED

1. MIMO Transmit Beamforming under Uniform Elementary Power Constraint

First, we consider the bit-error-rate (BER) performance of our beamformer with perfect CSI at the transmitter. For comparison purposes, we also implement the conventional transmit beamforming design without the uniform elemental power constraint (Con TxBm) and the conventional design with peak power clipping (TxBm with Clipping), in addition to our own (UEP TxBm). Fig. 1 shows the BER comparison for (4,2) and (8,8) MIMO systems. Con TxBm achieves the best performance since it is not under the uniform elemental power constraint. Under the uniform elemental power constraint, UEP TxBm schemes outperform TxBm with Clipping. We consider next the effect of vector quantization. We use m-bit VQ-UEP with \( N_v = 2^m \), \( m=2,4,6,8 \) to quantize the transmit beamformer for a (4, 2) MIMO system. Fig. 2 shows that VQ-UEP outperforms TxBm with Clipping when \( B = 4, 6, \ldots \)
8, although the latter is with the perfect feedback. By using the 6-bit VQ-UEP, the (4,2) MIMO system only suffers from slight performance loss compared to UEP TxBm with perfect feedback. When more bits are used, we can further close the small gap. In Fig. 3, we carry out Monte-Carlo simulations for a (4,1) system and plot the numerically simulated degradation result. We can see that the closed-form expression in Proposition 1 gives a very accurate approximation.

2. Orthogonal Space-Time Block-Differential Modulation over Underwater Acoustic Channels

Eq. (3) implies that the \( \sum_{l=1}^{L+1} n \) time-varying channel parameters \( h(n; l) \) can be captured by the \( (Q + 1)(L + 1) \) time-invariant (within one block) channel parameters \( h_q(l) \). In practice, however, we could only observe \( h(n; l) \). The first problem at hand is how to form \( h_q(l) \) from the observed \( h(n; l) \) s.

**Lemma 1** With \( W_b = \left[ \omega_0, \omega_{b+1}, ..., \omega_{b+QB} \right]^T \) and \( N = B \times (Q + 1) \), we have: \( W_b^H W_b = (Q + 1) I_{Q+1} \); i.e., \( g(l) = \frac{1}{Q+1} W_b^H [h(b; l), h(b + B; l), ..., h(b + QB; l)]^T \) where \( g(l) = [h_0(l), ..., h_Q(Q)]^T \).

Notice that \( W_b \) is reminiscent of the FFT matrix. This lays the foundation for our 2-dimensional FFT-based differential approach, which is detailed in the following steps.

**Step 1: CP Insertion and Removal**

At the transmitter, every frame is partitioned into \( P \) subblocks each containing \( (Q + 1)M \) symbols. Every \( M \) symbol segment is augmented by a cyclic prefix (CP) of length \( L \), which is removed at the receiver. The transmitted symbol frame structure is shown in Fig. 4. As a result of this step, the channel matrix \( H \) becomes block-diagonal (see Fig. 5).

**Step 2: Interleaving and De-Interleaving**

Lemma 1 implies that recovery of \( h_q(l) \) using \( W_b \) requires equally-spaced \( h(n, l) \)s. Hence, we perform interleaving and de-interleaving at the transmitter and receiver, respectively: \( x := \Theta^T \tilde{x} \) and \( y := \Theta y \), where \( \Theta / \Theta^T \) takes every \( PM \) columns/rows. As a result, the I/O relationship becomes \( \tilde{y} = \tilde{H} \tilde{x} + \tilde{z} \), where \( \tilde{H} := \Theta H \Theta^T \) consists of block-quasi-circulant matrices on its diagonal as depicted in Fig. 6. Because of the diagonal structure of \( \tilde{H} \), the I/O relationship can be represented by shorter streams: \( \tilde{y}(p) = \tilde{H}(p) \tilde{x}(p) + \tilde{z}(p), \quad p \in [0, P - 1] \).

**Step 3: Time-Doppler FFT**

Now we have an equivalent channel matrix \( \tilde{H}(p) \) which consists of \( \text{diag}(h(n; l)) \). Lemma 1 indicates that, in order to obtain \( g(l) \), we need to multiply \( W_b^H \) with \( h(n, l) \); that is, to multiply

\[
\Omega(p) := \begin{bmatrix}
W_b^H & 0_{(Q+1)\times(Q+1)} & \cdots & 0_{(Q+1)\times(Q+1)} \\
0_{(Q+1)\times(Q+1)} & W_b^H & \cdots & 0_{(Q+1)\times(Q+1)} \\
\vdots & \vdots & \ddots & \vdots \\
0_{(Q+1)\times(Q+1)} & 0_{(Q+1)\times(Q+1)} & \cdots & W_b^H \\
\end{bmatrix}
\]

with \( \tilde{y}(p) \) at the receiver. In the meantime, \( \tilde{x}(p) = U(p) \otimes 1_{Q+1} \) is also needed at the transmitter, where \( U(p) \) is the \( M \times 1 \) data vector. Then, the I/O relationship becomes \( \hat{y}(p) = (Q + 1)G U(p) + \tilde{z}(p) \), where \( G \) is the \( M(Q + 1) \times M \) equivalent channel matrix (see Fig. 7), which is independent of \( p \).

**Step 4: Multipath-Frequency FFT**

It is well known that IFFT at the transmitter and FFT at the receiver can diagonalize a circulant matrix. The transmitted vector can be formed as \( U(p) = F_M^H u(p) \), where \( F_M \) is the \( M \)-point FFT matrix. And the received vector \( v(p) := \tilde{F}_M \tilde{y}(p) \), with \( \tilde{F}_M := [I_{Q+1} \otimes f_1, ..., I_{Q+1} \otimes f_M] \). Therefore \( v(p) = \tilde{D} u(p) + \eta(p) \) is now the I/O relationship with \( \tilde{D} \) being a block-diagonal matrix containing the frequency responses of channel vectors \( g(q) = [h_q(0), ..., h_q(L)]^T \), as depicted in Fig. 7.
Step 5: MIMO Differential Encoding

It is now straightforward to do differential encoding if the system is single-input single-output (SISO). When multiple transducers are deployed, we extended the differential OSTBC [3], [4] for time-invariant flat-fading channels to doubly-selective channels. Without loss of generality, we considered the case consisting of $N_t = 2$ transducers and $N_r = 1$ hydrophones. We group 2 consecutive subblocks into one and index it with $P_b$. The differential encoding goes as: $\begin{bmatrix} \hat{\mathbf{u}}_1(p_b), \hat{\mathbf{u}}_2(p_b) \end{bmatrix} = [D_{d1}(p_b), D_{d2}(p_b)] \begin{bmatrix} \mathbf{u}_1(p_b - 1) - \mathbf{u}_2(p_b - 1)^* \\ \mathbf{u}_2(p_b - 1) - \mathbf{u}_1(p_b - 1)^* \end{bmatrix}^*$, where $D_{d1}(p_b)$ and $D_{d2}(p_b)$ are diagonal data matrices, $\mathbf{u}_1(p_b)$ and $\mathbf{u}_2(p_b)$ are from the $P_b$-th block. These are then mapped to different transducers (subscript) as: $\hat{\mathbf{u}}_1(2p_b) = \mathbf{u}_1(p_b), \mathbf{u}_1(2p_b + 1) = -\mathbf{u}_2(p_b)^*$ and $\hat{\mathbf{u}}_2(2p_b) = \mathbf{u}_2(p_b), \mathbf{u}_1(2p_b + 1) = \mathbf{u}_1(p_b)^*$. With $\mathbf{v}(2p_b)$s denoting the received blocks, the ML decoder is

$$D_{c1}(p_b) = \arg\min_{\mathbf{D}_{d1}} \left\| \frac{\mathbf{v}(2p_b)\mathbf{v}(2p_b - 1)^* + \mathbf{v}(2p_b + 1)\mathbf{v}(2(p_b - 1) + 1)}{\mathbf{v}(2p_b - 1)^2 + \mathbf{v}(2(p_b - 1) + 1)^2} - \mathbf{D}_{d1}(p_b) \right\|.$$

Simulations and comparisons:

Putting everything together, the transceiver structure is shown in Fig. 8. Using this structure, we performed simulations with three types of channels: time-invariant (TI), time-varying (TV) and experiment-extracted (EXP) (see Fig. 10). Fig. 9 depicts the diversity differences among the BER curves. Comparing the two curves using QPSK and $k = 3$ coding over TI channels, we see that the coding in [5] provides diversity advantage. Doppler diversity can be observed by comparing the TI and TV cases. From the curve generated using the SISO scheme in [6] and the one using our scheme over TV channels, the spatial diversity advantage is evident. Fig. 9 also shows the performance degradation encountered by plain OFDM. With experiment-extracted channels, the BER curve exhibits an error floor due to the noise in the estimated channels. Recall that the BEM has a requirement on $f_{max}$ and the noise spikes introduce high frequency components. Therefore, this heavy tail is expected to disappear in real experiments.

RESULTS

Our MIMO transmit beamformer accounts for the uniform elemental power constraint, which is critical for the power efficiency and lifetime of high-rate UAC transmitters. While this problem is a difficult non-convex optimization problem, we managed to develop a high-performance low-complexity cyclic algorithm. We also considered practical finite-rate feedback and studied the average SNR degradation. Although the channel assumptions we employed still differ from realistic underwater acoustic channels, our methods can be readily extended to the underwater acoustic applications.

Doubly-selective MIMO UAC channels are ample in diversity but are difficult to estimate. We are among the first to explore full-diversity differential approaches in such channels. Our simulation results using statistically modeled and experiment-extracted channels are promising. In addition to field tests, we will also continue working on improving the data rate of our diversity differential transceivers.

IMPACT/APPLICATIONS

The natural bandwidth limitations of coherent underwater acoustic channel suggest a technical breakthrough. MIMO signal processing is a promising bandwidth efficient method to high data rate and high quality services. Taking both coherent and differential approaches, our promising results are expected to favorably impact high-rate long-range MIMO-UAC designs.
Figure 1: Performance comparison of various transmit beamformer designs with perfect CSI at the transmitter: (a) the (4, 2) case, and (b) the (8, 8) case. [Graph: Our design (red) is much better than existing approach (blue), and is close to the design without uniform power constraint (black).]
Figure 2: Performance of vector quantization under uniform elemental power constraint with various bits feedback, for a (4,2) system. Note that 8-bit VQ-UEP and UEP TxBm with perfect feedback curves almost coincide with each other.

[graph: With 8-bit for channel feedback, little performance degradation observed.]

Figure 3: Average degradation of the receive SNR for a (4,1) MISO system.

[graph: Our theoretical prediction of feedback SNR loss agrees closely with simulations.]
Figure 4. Transmitted symbol frame structure
[graph: A transmitted frame with \( P = 2, \ Q = 2 \)]

Figure 5. Sub-transmitting matrix structure
[graph: quasi-circulant structure of each sub-channel matrix, \( M \times M \)]

Figure 6. I/O after (de-)interleaving
[graph: quasi-circulant structure of each sub-channel matrix, \( M(Q + 1) \times M(Q + 1) \)]
Figure 7. Multipath-Frequency FFT
[graph: equivalent I/O, block-circular, circular, block-diagonal]

Figure 8. Overall transceiver structure
[graph: transmitter: differential encoding, serial to parallel, LCF encoding, IFFT, interleaving, insert CP;
receiver: remove CP, de-interleaving, FFT, differential decoding]
Figure 9. Bit error ratio vs. signal to noise ratio
[graph: our scheme has the best performance, achieving 3-dimensional diversity, traditional OFDM degrades over time-varying channels, our scheme also perform well over real channels]

Figure 10. One snapshot of the underwater acoustic channel extracted from the experiment
[graph: Amplitude vs. delay index, the channel delay may as long as 40 taps]
(Courtesy Dr. Daniel Kilfoyle)
REFERENCES


PUBLICATIONS


PATENTS