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Determining Training Demand for an Expanding Military Organisation

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ABSTRACT

This report discusses the disadvantages of a training plan whereby instructors don't return to the combat force after the expansion training period. We propose an efficient plan to calculate training demand for an expanding military force. The proposed plan takes care of the effect of instructors returning to the combat force, thus eliminating surplus instructors and reducing the number of trainees required. Two types of mathematical models are constructed for implementation of the proposed plan. Two application tools are created to facilitate the model solving. Examples to illustrate the benefits of applying the proposed plan are given.

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Determining Training Demand for an Expanding Military Organisation

Executive Summary

Many organisations experience difficulties in determining training demand during periods of expansion (or contraction) of their workforce, or when there is a period of re-training the workforce (such as when procedures or policy change, or when there is an upgrade or change in equipment employed by a workforce). Army is undergoing such a period. The requirement for an increase in the size of the military component of the Army as well as the introduction of new equipment and operating procedures for Hardened Networked Army has brought this problem to the fore for the Army. In 2002, Training Command – Army tasked DSTO to explore better ways of calculating training demand due to expansion of the combat force, in particular the demand for the Private Trainees at the Army Recruitment Training Centre.

The current approach to planning for an expanding military force is typically based on rules-of-thumb, intuition and experience. The intuitive approach has typically been based on an implicit assumption that soldiers who become instructors will not necessarily be returning to the combat force. The results in this report indicate that the current plan has the following disadvantages:

- Excessive reduction of personnel levels in the combat force
- Higher than usual training load in the expansion training year
- Surplus instructors after the expansion training year.

To address these disadvantages this report proposes a new approach to determining training demand to achieve and maintain the required expansion levels of a multi-rank military organisation. This approach uses the concept that instructors not required after the initial expansion training will return to the combat force. The after expansion training is to maintain the level of the expanded combat force. Two types of mathematical models, an analytic model and an optimisation model are constructed for implementation of the proposed approach. To help army officers in planning training demand for expansion, two computational tools have been developed to solve the models. With the approach proposed here, training planners should achieve expansion targets with an optimal training workforce.

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1. Introduction

In 2002, Training Command - Army (TC-A) tasked DSTO to find a better way of calculating training demand for the expansion of Australia's combat force, in particular the demand for the Private (PTE) Trainees at the Army Recruitment Training Centre (ARTC). The technical note by Wang, Vozzo and Galanis [1] presented two methods for deriving an analytic solution to calculate the training demand of an expanding military organisation. The analytic solution [1] provides the algorithm in implementing TC-A's training plan for expansion. The training plan requires graduates to fill vacancies in the combat force due to shifting staff to work as instructors. This training plan implicitly says that the instructors shifted from the combat force to TC-A won't return to the combat force since the vacancies will be filled by new graduates after the expansion training year. However, as pointed out in the previous report [1], TC-A's training plan is only one of many possible plans, and is not necessarily the optimal one.

TC-A's training plan does not explicitly consider the subsequent training required in order to replace those who will leave the expanded combat force.

The present work focuses on the investigation of alternative training plans. First, we point out the inefficiency of TC-A's training plan. Second, we propose an alternative, more efficient training plan in which some of the instructors will return to the combat force after the expansion training year. Finally, we offer two mathematical tools to implement our training plan, i.e., an analytical model based on the recursive method presented in the previous work, and an optimisation model that uses a mixed integer programming technique. We illustrate the impact of the training plans on the training workforce with simple examples.

For ease of communication in the discussion below, we name the TC-A training plan the 'pay-back-instructor' plan, meaning that TC-A will train to replenish all vacancies resulting from moving combat staff to become instructors, and the plan proposed in this report the 'instructor-returning' plan.

Throughout the report we assume that the parameter values such as graduation rates, instructor/trainee ratios, leaving rates are determined outside the models. Therefore, the models developed here cannot be used to determine what the appropriate instructor/trainee ratios or graduation rates should be.

2. 'Pay-back-instructor' plan

The analysis below aims to show that the 'pay-back-instructor' plan has the following disadvantages:

- Excessive reduction of personnel levels in the combat force
- Higher than necessary training load in the expansion training year
- Surplus instructors after the expansion training year.

The following case study exposes the disadvantages listed above.

Consider a one-rank combat force at steady state, i.e., the number of staff in the combat force is maintained at the specified level through so-called extant training. To add extra combat staff, D_0, to the current force, the 'pay-back-instructor' plan needs the number of trainees T_d to achieve the expansion goal and to replenish all vacancies resulting from moving combat staff to become instructors. This is obtained from ref [1]:

$$g_d T_d = D_0 + r_d T_d \ ,$$

where D_0 is the expansion demand, g_d and r_d are average graduation rate and instructor/ trainee ratio respectively. The term $r_d T_d$ is the number of vacancies resulting from moving combat staff to become instructors. From the above expression we obtain

$$T_d = \frac{D_0}{g_d - r_d} \ , \quad 0 < r_d < g_d \leq 1 . \tag{1}$$

Note that if $r_d \geq g_d$, the training system cannot achieve the expansion target since the number of graduates is at most equal to the number of vacancies resulting from moving combat staff to become instructors.

Let us consider an example where expansion demand is 100, graduation rate is 100% and instructor ratio is one half, i.e., $D_0 = 100$, $g_d = 1$ and $r_d = 1/2$. Equation (1) tells us that, under the 'pay-back-instructor' plan, TC-A needs to recruit $T_d = 100/(1-0.5) = 200$ trainees, and to shift $T_d \times r_d = 200 \times 1/2 = 100$ instructors from the combat force (and therefore 100 personnel shortfall in the combat force) to instruct the newly recruited 200 trainees in the expansion training year. After the expansion training year, TC-A will deliver $T_d \times g_d = 200 \times 1 = 200$ graduates to the combat force with 100 graduates to replenish (i.e., pay back) the 100 vacancies left by instructors and 100 graduates to expand the combat force. The results are summarised in the table below (negative numbers represent personnel shortfall relative to the steady state before expansion).

Table 1. Case study of 'pay-back-instructor' plan

Year	**Training Force**		**Combat Force**
	Trainee T_d	Instructor	Soldier
Year 1	200	100	-100
Year 2	x	100	200 (Graduate) Net increase=200-100=100

The question raised here is:

Does TC-A need the extra 100 instructors in the second training year?

2.1 Training demand after expansion

Since the combat force has achieved its expansion target at the end of the expansion training year, the extra training demand x (relative to the steady state before expansion), for TC-A, is to maintain the net increase in the combat force at the level of 100. Therefore, extra training demand is given by

$$x = l_d * D_0 / g_d \ , \qquad (2)$$

where l_d is the soldier leaving rate in the combat force[1] (due to discharge, lateral transfer, promotion). Therefore, for the specific case studied in Table 1, setting $l_d = 0.1$, the extra number of trainees needed to sustain the expanded combat force is $x = 0.1 \times 100 / 1 = 10$. Consequently, TC-A only needs $x \times r_d = 10 \times 1/2 = 5$ new instructors to sustain the new steady state from the second training year onwards. The example here shows that, *under the 'pay-back-instructor' plan*, the training force will have 95 surplus instructors after the completion of expansion training.

The key point is that *not* all the vacancies in the combat force left by soldiers to work as instructors are *genuine*[2], because these shifted instructors can return to the combat force once they are not needed in the training force. A cost-effective training plan, we believe, should consider the effect of surplus instructors returning to the combat force at the end of the expansion training year. In the next section, we provide an analytical model for implementation of the proposed 'instructor-returning' plan and use examples to demonstrate its advantages.

3. 'Instructor-Returning' Plan

The analysis in the last section has shown that under the 'pay-back-instructor' plan, the training force will end up with excess instructors because it is assumed that instructors do not return to the combat force. In this section we derive the training demand and the number of instructors required using the idea that excess instructors after expansion training will return to the combat force. The on-going training requirement following the expansion year is to maintain the level of the expanded combat force.

3.1 Analytical Model for One Rank Expansion

The analysis below is a special case of the multi-rank expansion derived in the next section. For completeness, we explain the steps of the derivation for single rank here. In this plan the number of graduates $T_d g_d$ should be enough to fill the expansion target positions D_0 plus the number of extra instructor positions I^d needed after expansion training. Mathematically our plan is described by $g_d T_d = D_0 + I^d$.

[1] We neglect the effect of instructors leaving for simplicity in the discussion of one-rank expansion.

[2] By genuine, we mean the vacancies to be filled by training.

The number of instructors I^d is determined from the number of trainees E_d after the expansion to maintain the expanded combat force and the instructor ratio r_d. That is, $I^d = r_d E_d$. Therefore we now have $g_d T_d = D_0 + r_d E_d$.

The number of trainees E_d is determined from the number of graduates required to maintain the expanded force. That is, $g_d E_d = l_d D_0$. Applying these substitutions into $g_d T_d = D_0 + I^d$ yields

$$g_d T_d = D_0 + \frac{r_d l_d D_0}{g_d} = (1 + \frac{r_d l_d}{g_d}) D_0 .$$

Dividing both sides by g_d yields the equation for the instructor-returning plan for single rank system

$$T_d = (1 + l_d * r_d / g_d) D_0 / g_d . \qquad (3)$$

We now analyse the advantage of our 'instructor-returning' plan by this solution and leave the details in derivation for a multi-rank expansion for a later section.

For the case investigated in section 2: $D_0 = 100$, $g_d = 1$, $r_d = 0.5$ and $l_d = 0.1$, equation (3) reads $T_d = (1 + 0.1 * 0.5/1) \times 100/1 = 105$. Under the 'instructor returning' plan, in the expansion training year, TC-A should recruit T_d=105 trainees and shift $T_d \times r_d \cong 53$ instructors[3] from the combat force (and therefore 53 combat force shortfall). After the expansion training year, TC-A will deliver $T_d \times g_d = 105 \times 1 = 105$ graduates, and return 48 instructors to the combat force. The combat force has 5 graduates and 48 returning instructors to fill the 53 vacancies left by instructors and 100 graduates for expansion. The extra 5 instructors staying at TC-A for the new steady state training, will train an extra 10 trainees every year from the 2nd year onwards to maintain the net increase in combat force at the level of 100.

Thus, the 'instructor-returning' plan eliminates the disadvantages of the 'pay-back-instructor' plan as follows:

- Less reduction of combat force in expansion training year (53 versus 100)
- Less training load in the expansion training year (105 versus 200)
- No surplus instructors after expansion training (0 versus 95).

For comparison, we summarise the training requirements under two training plans for the case study in Table 2.

[3] Non-integer numbers have been rounded up to next integers for practical application.

Table 2.Training demand from 'instructor-returning' and 'pay-back-instructor' plans

		Pay-back Plan	Returning Plan	Difference	Percentage Difference (%)
Expansion Year	Trainees	200	105	95	47.5
	Instructors	100	53	47	47
Steady State	Trainees	10	10	0	0
	Instructors	100	5	95	95

The results in the table show that 'instructor-returning' plan can provide significant improvement in the determination of training demand and instructor numbers for a single rank system. We further examine the impact of two training plans on the human resource planning for a multi-rank expansion example in a later section.

3.2 Analytical Model for Multi-Rank Expansion

Now we consider expansion of a four-rank system described by the following narrative:
The Army wants to **expand its combat force** in four ranks of ***a*, *b*, *c*** and ***d***, with the rank ***d*** as the highest, by A_0, B_0, C_0 and D_0. The problem is to determine the numbers of trainees and instructors needed to achieve the expansion goal in one training year. We denote the number of trainees required by T_a, T_b, T_c and T_d for each of these four ranks.

We summarise the definitions for all input parameters needed in Table 3.

Table 3: Parameters for expansion of a four-rank military workforce

Rank (λ)	***a***	***b***	***c***	***d***
Expansion (Δ_λ)	A_0	B_0	C_0	D_0
Officer Leaving rate (l_λ)	l_a	l_b	l_c	l_d
Graduation Rate (g_λ)	g_a	g_b	g_c	g_d
Instructor/Trainee Ratio (r_t^i)	r_a^i $(i \ge a)$	r_b^i $(i \ge b)$	r_c^i $(i \ge c)$	r_d^i $(i \ge d)$

In Table 3, we defined $r_\lambda^i (0 \le r_\lambda^i < 1)$ as the ratio of i-rank instructor for λ-rank trainee. We also assumed that the minimum instructor rank is one rank above the trainee rank. The leaving rate l_λ applies to λ-rank officers no matter whether they serve in the combat force or the training force.

The analytic solution to calculate T_λ $(\lambda = a, b, c, d)$, under the 'instructor-returning' plan, is obtainable by the recursive method[1].

The fundamental equation in the derivation is:

$$N_\lambda = \Delta_\lambda + N_{\lambda+1} + I^\lambda, \qquad (\lambda = a,b,c,d) \tag{4}$$

where $N_\lambda = g_\lambda T_\lambda$ is the number of graduates, Δ_λ is the expansion goal defined in the second row of Table 3 and I^λ is the number of λ-rank instructor positions needed after expansion training.

We note that the second term, $N_{\lambda+1}$, on the right hand side of equation (4) addresses the 'chain-training' demand [2], or 'suck-up' effect as referred to by TC-A [1], which is due to the hierarchical nature of military organisations as exposed by a causal-loop analysis of system dynamics[2][4]. We note again that the existence of the third term I^λ on the right hand side of the above equation is due to the closedness of military organisations, as also shown by the analysis using the causal-loop tool of system dynamics [2].

For the four rank expansion specified by the input parameters in Table 3, we have the following equations to solve:

$$g_d T_d = D_0 + I^d = D_0 + \sum_{\lambda=a}^{d} r_\lambda^d E_\lambda \tag{5}$$

$$g_c T_c = C_0 + g_d T_d + I^c = C_0 + g_d T_d + \sum_{\lambda=a}^{c} r_\lambda^c E_\lambda \tag{6}$$

$$g_b T_b = B_0 + g_c T_c + I^b = B_0 + g_c T_c + \sum_{\lambda=a}^{b} r_\lambda^b E_\lambda \tag{7}$$

$$g_a T_a = A_0 + g_b T_b + I^a = A_0 + g_b T_b + r_a^a E_a \tag{8}$$

where the numbers of trainees E_λ ($\lambda = a,b,c,d$), required after the expansion training year to sustain the expanded force, are determined by the following equations:

$$g_d E_d = l_d (D_0 + I^d) = l_d (D_0 + \sum_{\lambda=a}^{d} r_\lambda^d E_\lambda) \tag{9}$$

$$g_c E_c = l_c (C_0 + I^c) + g_d E_d = l_c (C_0 + \sum_{\lambda=a}^{c} r_\lambda^c E_\lambda) + g_d E_d \tag{10}$$

$$g_b E_b = l_b (B_0 + I^b) + g_c E_c = l_b (B_0 + \sum_{\lambda=a}^{b} r_\lambda^b E_\lambda) + g_c E_c \tag{11}$$

$$g_a E_a = l_a (A_0 + I^a) + g_b E_b = l_a (A_0 + r_a^a E_a) + g_b E_b \tag{12}$$

Equations (9) – (12) are solved first for four unknown E_λ ($\lambda = a,b,c,d$), while the training demand required to achieve the expansion goal is found by solving the equations (5) – (8) recursively, starting from the d-rank.

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[4]'Chain-training' or 'suck-up' effect is the propagation of training demand from higher ranks to lower ranks.

Notice that higher-than- d -rank instructor related quantities do not appear in equations (5) – (12). Higher rank instructors are external resources, while lower ranked instructors (from a to d) must be produced within the four-rank training system.

For simplicity, we now assume that all instructors are from one rank above the trainees[5], i.e., $r_\lambda^i = 0$ ($i \neq \lambda$). The solutions of equations (9) – (12) are

$$E_\lambda = \frac{l_\lambda \Delta_\lambda + g_{\lambda+1} E_{\lambda+1}}{g_\lambda - r_\lambda l_\lambda}, \qquad (\lambda = a, b, c, d) \qquad (13)$$

with $E_{d+1} = 0$, $g_\lambda > r_\lambda l_\lambda$ and $r_\lambda \equiv r_\lambda^\lambda$ denoting the instructor/trainee ratio for this special case.

Notice that $I^\lambda = r_\lambda E_\lambda$, and the solution of equations (5) – (8) reads

$$T_\lambda = \frac{\Delta_\lambda + g_{\lambda+1} T_{\lambda+1} + I^\lambda}{g_\lambda}, \qquad (\lambda = a, b, c, d) \qquad (14)$$

with $T_{d+1} = 0$.

3.3 Implementation

The analytic solution (equations (13) and (14)) has been implemented in a Microsoft Excel spreadsheet.

When the Excel file is opened and the macros are enabled, the explanation sheet shown in Figure 1 pops up.

When the 'Run the Application' button in the explanation sheet is clicked, a dialog window for input parameters appears (Figure 2). After completing the table, click 'OK' to perform the calculation. The results of the calculation will be displayed together with the values of input parameters in the 'Report' sheet (Figure 3). To view charts of the results, click, 'View 1st Year Demand Chart' or 'View 2nd Year Demand Chart' buttons on the 'Report' sheet. To start a new calculation, click 'View Explanation Sheet' to go back to the beginning.

In the input dialog box displayed in Figure 2, the lowest rank is in the first column, with ranks increasing to the right. If there is no expansion required for a particular rank, set the expansion value to zero but fill in the rest of the information.

[5] This is, on average, an acceptable assumption according to TC-A.

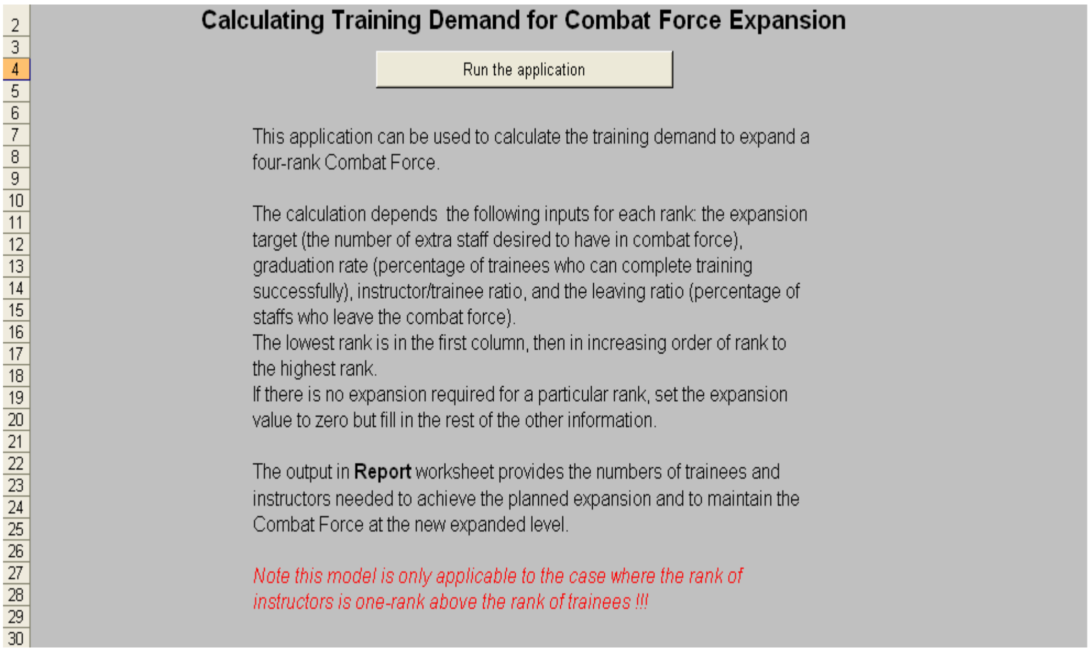


Figure 1: The explanation sheet

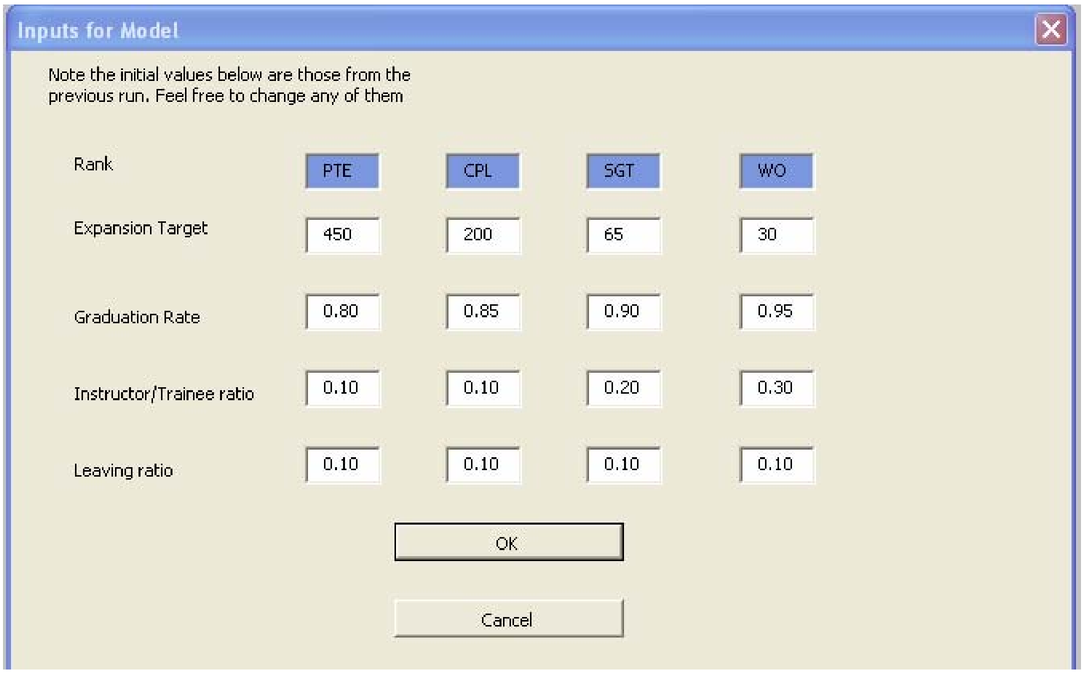


Figure 2: Input dialog window

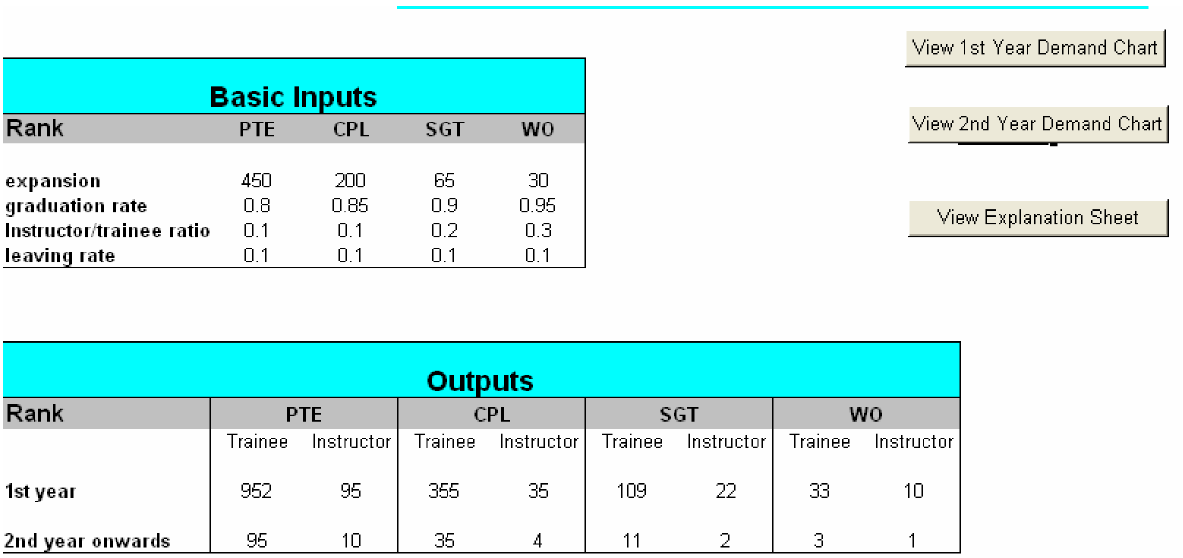
Now we illustrate the 'back end' of the application tool for a four rank expansion example.

3.4 Example

Assume an Army unit requires an expansion of four ranks as follows:

	Rank			
	PTE	CPL	SGT	WO
Expansion target	450	200	65	30
Graduation rate	0.8	0.85	0.9	0.95
Instructor/trainee ratio	0.1	0.1	0.2	0.3
Separation rate	0.1	0.1	0.1	0.1

Figure 2 shows the input dialog window. Figure 3 shows the report sheet from the application.



Basic Inputs				
Rank	PTE	CPL	SGT	WO
expansion	450	200	65	30
graduation rate	0.8	0.85	0.9	0.95
Instructor/trainee ratio	0.1	0.1	0.2	0.3
leaving rate	0.1	0.1	0.1	0.1

Outputs								
Rank	PTE		CPL		SGT		WO	
	Trainee	Instructor	Trainee	Instructor	Trainee	Instructor	Trainee	Instructor
1st year	952	95	355	35	109	22	33	10
2nd year onwards	95	10	35	4	11	2	3	1

Figure 3: The numerical result displayed in Report sheet

The results in Figure 3 are displayed in Figures 4 and 5.

We note that, while this application only codes the case where instructors are from one rank above trainees, the implementation of the solution of equations (5) – (12) for a general instructor rank structure is not difficult. It only requires setting the instructor/trainee ratios to appropriate values.

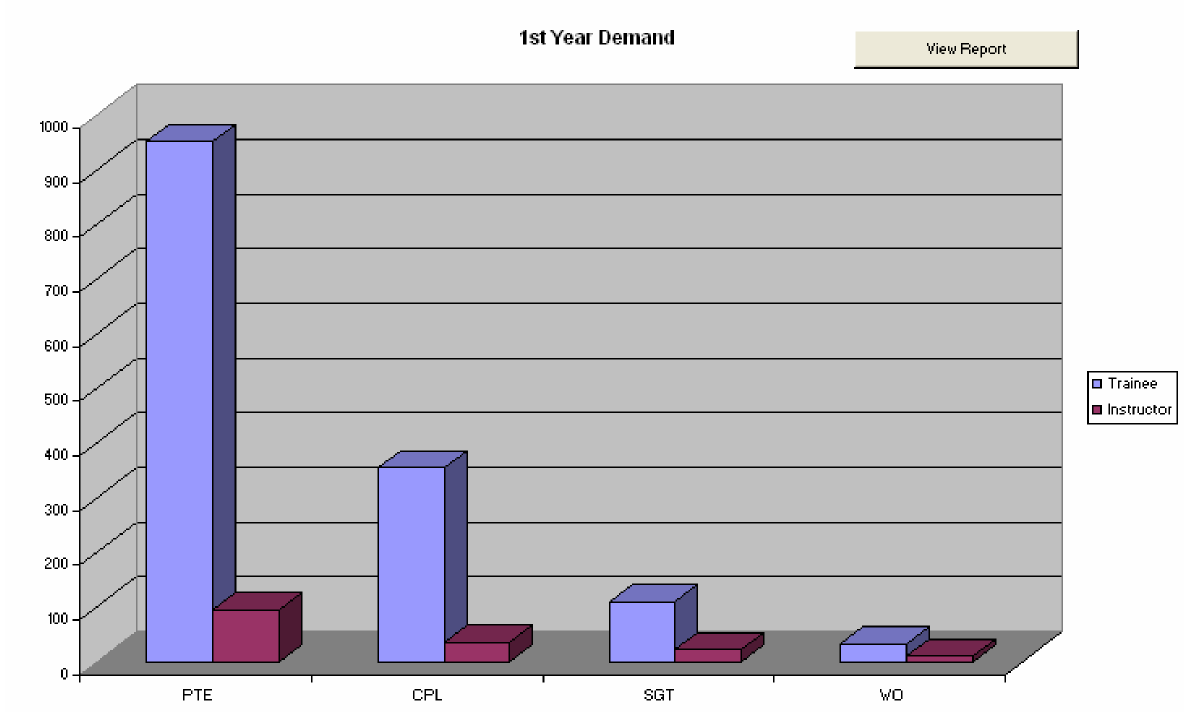


Figure 4: Expansion training requirement

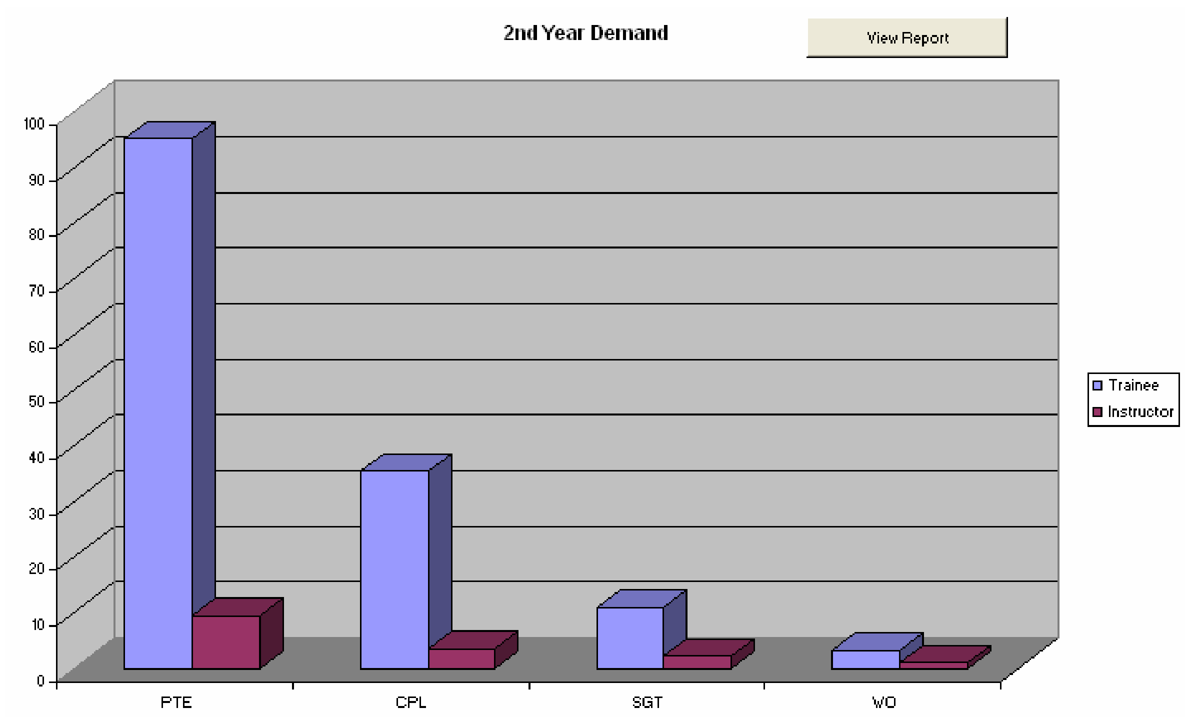


Figure 5: Sustainment training requirement

Although the application tool assumes that the time to achieve the expansion is one training year, the extension to multiple training year expansion could be carried out by applying the solution for the one-year expansion repeatedly. For example, suppose the expansion is planned to increase Δ^t_λ ($\lambda = a, b, c, d$) numbers of λ-rank combat staff in t-th year ($t = 1,2,...m$) with m denoting the number of years to achieve the expansion. Then, the number of λ –rank trainees, required in the t-th expansion year, is given by:

$$T^t_\lambda = T_\lambda(\Delta^t_\lambda) + (1-\delta_{1t})\sum_{t'=1}^{t-1} E(\Delta^{t'}_\lambda) \quad (t = 1,2,...,m), \tag{15}$$

and the number of trainees, required to sustain the expanded force after m expansion training years is:

$$E^{m+1}_\lambda = \sum_{t=1}^{m} E_\lambda(\Delta^t_\lambda), \tag{16}$$

where δ_{1t} is the Kronecker delta function[6], $T_\lambda(\Delta^t_\lambda)$ and $E_\lambda(\Delta^t_\lambda)$ are obtained by inserting the expansion target Δ^t_λ into the solution in Section 3.2. The implementation in Excel/VBA application tool is straightforward.

In section 4 we present the second mathematical model, an optimisation model and its implementation of the 'instructor-returning' plan.

4. Mathematical Programming Model

The expansion planning is framed as an optimisation problem which requires minimum cost in terms of workforce (instructors and trainees) to achieve a given expansion target. The mathematical programming techniques used in developing the model here can be found in [3]. The optimisation tool is implemented via ILOG OPL Studio software [4], and can deal with arbitrary instructor rank structure and multi-period expansion training requirement. Note that the model can be implemented in any other optimiser with Integer programming functionality. The model is described below.

4.1 Optimisation Model for 'Instructor-Returning' Plan

We define I^t_λ as the number of instructors required to train soldiers to be promoted to rank λ on graduation during training period *t*. T^t_λ is the number of soldiers to be trained during period *t* for promotion to rank λ on graduation.

[6] The Kronecker delta is defined as: $\delta_{ij} = 1$ (for $i = j$; =0 for $i \neq j$)

Instructor Trainee Relationship:

$$r_\lambda T_\lambda^t \le I_{\lambda,}^t \forall(\lambda \in Rank, t \in period)$$

Note that we have an inequality, not equality, to speed up the optimisation process and to reduce the possibility of infeasible solutions as instructor variables are integer.

Graduate Trainee relationship:

$$N_\lambda^t \le g_\lambda T_\lambda^t, \forall(\lambda \in Rank, t \in period)$$

The number of graduates is no more than the number of trainees times the graduation rate.

Net increases for each rank

$$\delta_\lambda^t \ge N_\lambda^t - N_{\lambda+1}^t, \forall(\lambda \in Rank, t \in period)$$

This constraint ensures that the net increase in each rank is at least the difference between soldiers being promoted to the rank and those being promoted from the rank.

Rank levels after each training period

$$S_\lambda^t \le (1 - l_\lambda) \times S_\lambda^{t-1} + \delta_\lambda^t, \forall(\lambda \in Rank, t \in period)$$

This constraint updates the number of soldiers in each rank at the end of each training period. The updating of the expansion training rank levels is done by discounting the previous number by the separation rate and then adding the net graduation numbers (that is the difference between those graduates promoted to the rank and those promoted from the rank). We use an inequality here rather than an equation because the end level S_λ^t is an integer variable. This will speed the algorithm and reduce the possibility of infeasibility since it is harder to satisfy an equation than an inequality. Note that S_λ^t variables' values will be pushed to the higher end of the inequality because of the need to meet expansion target requirements represented by the expansion target constraints below.

Expansion Targets:

Expansion targets must be met at the end of each expansion training period.

$$S_\lambda^t \ge \sum_{t'=1}^{t} \Delta_\lambda^{t'}, \forall(\lambda \in Rank, t \in period)$$

The last expansion training period should produce enough graduates to meet the expansion targets and the number of instructors required for steady-state training.

$$S_\lambda^{NP-1} \ge \sum_{t'=1}^{NP-1} \Delta_\lambda^{t'} + \sum_j \gamma_{j\lambda} x_{j\lambda}, \forall \lambda \in Rank$$

$x_{j\lambda}$ is the number of rank λ instructors required for rank j trainees and $\gamma_{j\lambda} = 1$ if rank λ qualify as instructors for rank j trainees, zero otherwise.

During steady-state training there should be enough graduates to maintain the new levels of the expanded combat force.

$$N_{\lambda}^{NP} \geq l_{\lambda} \sum_{t'=1}^{NP-1} \Delta_{\lambda}^{t'} + \sum_{j} \gamma_{j\lambda} x_{j\lambda} + N_{\lambda+1}^{NP}, \forall \lambda \in Rank.$$

The number of instructors for rank λ trainees is at most the sum of qualified instructors from all ranks that are assigned to instruct rank λ trainees.

$$I_{\lambda}^{NP} \leq \sum_{j \geq \lambda} \gamma_{j\lambda} x_{j\lambda}.$$

Starting level values

$$S_{\lambda}^{0} = initial_{\lambda}, \forall \lambda \in Rank$$

This constraint assigns the end level before expansion to initial values.

We have chosen the minimisation of the number of personnel (instructors and trainees) in the training system as the driver of the optimisation process.

Minimise $\sum_{\lambda \in ranks, t \in periods} (I_{\lambda}^{t} + T_{\lambda}^{t})$ subject to the above constraints. The resource costs and limits can be included if that is deemed to be important in the determination of training demand.

This concludes the description of model constituents, now we briefly explain the software implementation of the model.

4.2 Implementation

The mixed integer programming model has been coded in ILOG OPL Studio software, see Appendix A.

At the moment input parameter data can be read from a text data file. The output can be obtained directly from OPL Interface Development Environment (IDE) or written to a formatted text file.

The output includes:

- The number of instructors required for each rank during each training period.
- The number of trainees required for each rank during each training period.
- The expected number of graduates in each rank at the end of each training period.
- The expected number of soldiers (addition to the extant force) in each rank.

As an illustration, the ILOG application tool is applied to the following multi–training-period expansion example.

4.3 Example

We re-state the example in section 3.4. An Army unit requires an expansion of its four ranks as given in Table 4.

Table 4: Input parameters and the expansion goal

	Rank			
	PTE	CPL	SGT	WO
Expansion target	450	200	65	30
Graduation rate	0.8	0.85	0.9	0.95
Instructor ratio	0.1	0.1	0.2	0.3
Separation rate	0.1	0.1	0.1	0.1

However, the training planner determines that it will take three training periods to achieve the required expansion and the unit has provided the following targets for the expansion training.

Table 5: Multi-Period Expansion targets

Training Period	**Rank**			
	PTE	CPL	SGT	WO
1	100	50	20	10
2	300	150	50	20
3	450	200	65	30

The expansion targets in Table 5 are relative to the steady state before the 1st expansion training period. This means that the expansion target is, for the private rank, 100 net increase in the combat force at the end of the 1st expansion training period, 300 net increase at the completion of the 2nd expansion training period (i.e., 200 net increase relative to the previous training period), and 450 net increase at the end of the final expansion training period (i.e., a further 150 net increase relative to the 2nd expansion training period).

This information is written into a data file that OPL Studio will read from as shown in the sample input below:

```
nbperiods = 4;
nbranks = 5;
graduationrate = [0.8,0.85,0.9,0.95,1];
instructorratio = [0.1,0.1,0.2,0.3,1];
Expansiontarget = [[0,100,300,450,450][0,50,150,200,200][0,20,50,65,65];
[0,10,20,30,30][0,0,0,0,0]];
Separationrate = [0.1,0.1,0.1,0.1,0];
initial = [0,0,0,0,0];
indicator = [[0,1,0,0,0][0,0,1,0,0][0,0,0,1,0][0,0,0,1,0][0,0,0,0,1]];
```

The results from the ILOG Mixed Integer Optimiser are shown in Tables 6 to 9.

Table 6 gives the numbers of trainees required to achieve the expansion targets. The numbers in the ‘Steady State’ column are the required trainees to sustain the expanded force.

Table 6: The numbers of trainees required to achieve the expansion targets

Rank	Training Period			
	1	2	3	Steady State
PTE	225	448	370	94
CPL	95	175	137	36
SGT	34	48	46	12
WO	11	12	18	4

The results in Table 7 are obtained by multiplying the Table 6 results with the corresponding instructor/trainee ratios defined in Table 4.

Table 7: The numbers of instructors needed

Rank	Training Period			
	1	2	3	Steady State
PTE	23	45	37	10
CPL	10	18	14	4
SGT	7	10	10	3
WO	4	4	6	2

Table 8 presents the numbers of graduates after training, which are obtained by multiplying the Table 6 results with their corresponding graduation rates as specified in Table 4.

Table 8: Numbers of graduates deliverable to Combat Force

Rank	Training Period			
	1	2	3	Steady State
PTE	180	358	296	75
CPL	80	148	96	30
SGT	30	43	37	10
WO	10	11	12	3

Finally, Table 9 displays the net increase of number of staff, relative the steady state before expansion, in combat force.

Table 9: Net increase of number of staff in Combat Force

Rank	Training Period			
	1	2	3	Steady State
PTE	100	210	180	45
CPL	50	106	75	20
SGT	20	32	24	7
WO	10	11	17	3

In section 5, using the two mathematical models presented and their implementation tools, we analyse the impact of the two training plans on the workforce planning for a four-rank expansion problem.

5. Impact of training plans on workforce planning

The 'pay-back-instructor' plan assumes that instructors do not return to the combat force after expansion training period is completed. This will generate more training capability than is required and will result in surplus instructors. In the proposed 'instructor-returning' plan, there is no need to train others to replenish all the vacancies left by instructors during expansion training. Some personnel who are transferred from the combat force to be instructors will return to the combat force after completing their tour of duty at Training Command. This premise means that there will be less training capability. To make this point clear, we analyse the results from both the 'pay-back-instructor' plan and the 'instructor-returning' plan for the example of four-rank expansion in one training period as specified in section 3.4.

Table 10: Comparison of 'Pay-Back' and 'Returning' Instructor plans

	Trainees Trained			
Rank	**'Pay-Back' Plan**	**Returning Plan**	**Difference**	**% Difference**
PTE	1194	952	242	25%
CPL	454	355	99	28%
SGT	156	109	47	43%
WO	47	33	14	42%
Total	*1851*	*1449*	*402*	*28%*
	Instructors Needed			
Rank	**'Pay-Back' Plan**	**Returning Plan**	**Difference**	**% Difference**
PTE	120	96	24	25%
CPL	46	36	10	28%
SGT	32	22	10	45%
WO	15	10	5	50%
Total	*213*	*164*	*49*	*30%*

The Table 10 results are displayed in figures 6 and 7.

For this example, which requires a total of 745 personnel expansion over all four ranks, the analysis of the training demand for the 'Pay-Back' plan and the 'Returning' instructor plan shows that the former over estimates trainee demand by 402, and has 49 surplus instructors, which means total over-demand by 451 that is 28% more than the Returning Instructor Plan.

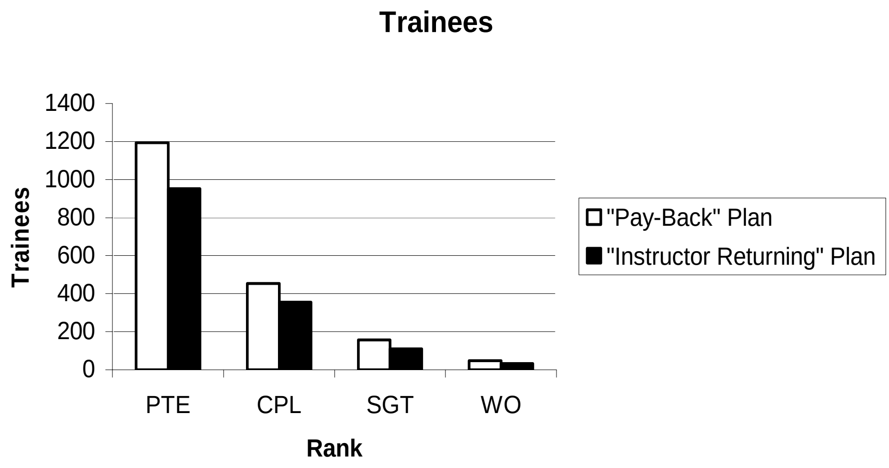


Figure 6: Number of trainees required for 'Pay-Back' and 'Returning' Instructor Plans

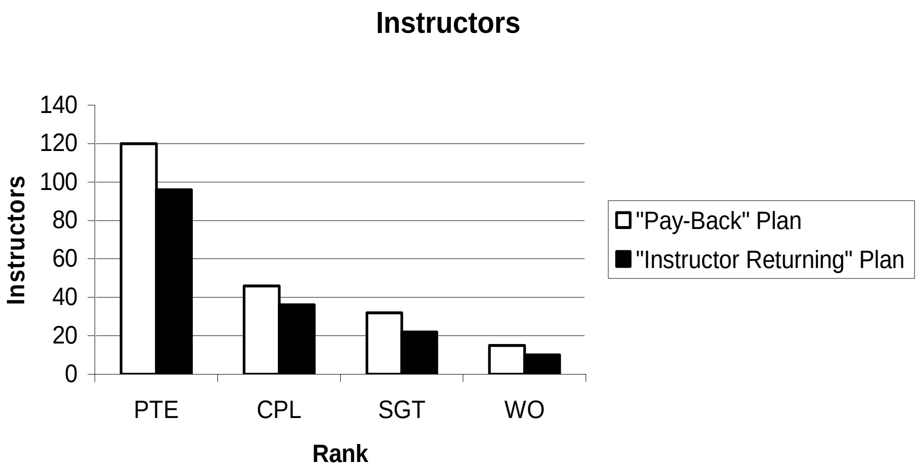


Figure 7: Number of instructors required for 'Pay-back' and 'Returning' Instructor Plans

6. Summary

This report has pointed out the drawbacks of using the 'pay-back-instructor' plan in determining training demand for an expanding military organisation. It is found that the plan has several disadvantages such as over-training load, excessive reduction of personnel in the combat force during the expansion training year and surplus instructors after the expansion training is over. An alternative training plan, which includes instructors returning to the combat force when their training service is no longer required, is proposed. The proposed 'instructor-returning plan' eliminates the disadvantages mentioned above.

For the proposed 'instructor-returning' plan, two mathematical models, an analytic model and an optimisation model based on mixed integer programming technique, are constructed to implement the plan. For the constructed models, two application tools, an Excel/VBA tool based on the analytical model, and an ILOG application tool based on the optimisation model, are created to facilitate training demand determination.

With the example presented above, we have demonstrated the advantages of 'instructor-returning' plan. The example shows that a significant reduction in training demand, consequently a reduction of cost in workforce, will result when the proposed plan is used instead of the pay-back plan.

Finally, it is worth pointing out that the proposed training plan and the mathematical models will help planners in the calculation of optimal workforce requirement to achieve their expansion goal under the conditions of guaranteed resources[7] in trainees and instructors plus steady graduation and leaving rates. The models are not supposed to be applicable in the investigation of dynamic response of the training system to unexpected 'shock', such as noticeable fluctuations in the numbers of leaving officers and failed trainees, insufficient new recruits or promotion-qualified officers. A further study to answer this 'What would happen if …' type of questions and to test control policies in designing a robust training system is presented in a companion report[5].

7. References

1. Wang, J., A. Vozzo, and G. Galanis, *Calculating the Training Demand in an Expanding Military Organisation: an Analytical Solution*. 2004, DSTO-TN-0608.
2. Wang, J., *A Review of Operations Research Applications in Workforce Planning and Potential Modelling of Military Training*. 2004, DSTO-TR-1688.
3. Williams, H.P., *Model Building in Mathematical Programming*. 1985: John Wiley & Sons.
4. ILOG, *ILOG OPL Studio 5.0*. 2006.
5. Wang, J., *A System Dynamics Simulation Model for a Four-Rank Military Workforce*. 2007, DSTO-TR-2037.

[7] We have only considered the human resource part of the whole training system.

Appendix A: ILOG OPL Studio Code of Mathematical programming model

```
/*********************************************
 * OPL 4.0 Model
 * Author: EgudoR
 * Creation Date: Mon Nov 14 15:04:39 2005
 *********************************************/
int   nbperiods = ...;
int   nbranks = ...;
range rank = 1..nbranks;
range nlastrank = 1.. nbranks-1;
range period = 0..nbperiods;
float gradationrate[rank] = ...;
float instructorratio[rank] = ...;
float Expansiontarget[rank][period] = ...;
float Seperationrate[rank] = ...;
int indicator[rank][rank] = ...;
int   initial[rank] = ...;
dvar int+   instructors[rank][period];
dvar int+ instructrain[rank][rank];
dvar int+  Trainees[rank][period];
dvar int+ graduates[rank][period];
dvar int+ Net_Period_increase[rank][period];
dvar int+ End_level[rank][period];
constraint ct1;
constraint ct2;
constraint ct3;
constraint ct4;
constraint ct5;
constraint ct6;
constraint expansion_training;
constraint Steady_state_training;

minimize
  sum(j in 1..nbranks, t in 1.. nbperiods) (instructors[j][t]+Trainees[j,t]);
  // sum(j in 1..nbranks, t in 1.. nbperiods) graduates[j][t];
subject to {
 ct1=  forall(j in rank, t in period)
    instructorratio[j]*Trainees[j,t] <= instructors[j][t];
  ct2= forall(j in rank, t in period)
    graduates[j,t] <= gradationrate[j]*Trainees[j,t];
   ct3=  forall(j in nlastrank, t in period)
    Net_Period_increase[j][t]>= graduates[j,t]-graduates[j+1,t];
```

```
ct4 =    forall(j in nlastrank)
     End_level[j,0]==initial[j];
  ct5 =   forall(j in 1..nbranks-1, t in 1..nbperiods)
      End_level[j,t]<= (1-Seperationrate[j])*End_level[j,t-1]+(graduates[j,t]-graduates[j+1,t]);
  // This constraint updates the number of soldiers in each rank at the end of each period
  // by discounting the previous number and adding the net graduates
  // (that is difference between those graduates promoted to the rank and those promoted
  // from the rank)
    expansion_training =  forall(j in rank, t in period)
      End_level[j,t]>= Expansiontarget[j,t];
   // This constraint ensures that the expansion targets of each rank are met at the end of
   // each training period.
  ct6  =  forall(j  in  rank)            End_level[j,nbperiods-1]>=Expansiontarget[j,nbperiods-1]
+sum(Iinrank)indicator[i,j]*instructrain[i,j];
  Steady_state_training = forall(j in nlastrank)
       graduates[j,nbperiods]         >=         Seperationrate[j]*Expansiontarget[j,nbperiods-1]+
graduates[j+1,nbperiods];
       forall(j in rank)
      instructors[j,nbperiods] <= sum(i in rank)indicator[j,i]*instructrain[j,i];
   // This contraint ensures that during steady state enough trainees graduate to maintain
   // the new expanded levels of the combat force.

}
```

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19. ABSTRACT

This report discusses the disadvantages of a training plan whereby instructors don't return to the combat force after the expansion training period. We propose an efficient plan to calculate training demand for an expanding military force. The proposed plan takes care of the effect of instructors returning to the combat force, thus eliminating surplus instructors and reducing the number of trainees required. Two types of mathematical models are constructed for implementation of the proposed plan. Two application tools are created to facilitate the model solving. Examples to illustrate the benefits of applying the proposed plan are given.