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**THESIS**

**DESIGN AND IMPLEMENTATION OF A MOBILE PHONE  
LOCATOR USING SOFTWARE DEFINED RADIO**

by

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September 2007

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**DESIGN AND IMPLEMENTATION OF A MOBILE  
PHONE LOCATOR USING SOFTWARE DEFINED  
RADIO**

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Submitted in partial fulfillment of the  
requirements for the degree of

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# ABSTRACT

This thesis presents an approach for generating, detecting, and decoding a Global System for Mobile communications (GSM) signal using software defined radio and commodity computer hardware. Using software designed by the GNU free-software project as a base, standard GSM packets were transmitted and received over the air and their arrival times detected. A method is provided to use software analysis of multiple receivers to locate an emitter based on the information received by the software radio. Results and accuracy as well as limitations are shown based on initial testing. Complete implementation source code is provided in the appendices.

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# EXECUTIVE SUMMARY

Because of the rapid growth in the capability of general purpose commodity computers, many applications that traditionally are implemented only by special-purpose analog and digital circuitry are finding a new home in software. The advantages of software over hardware implementations can be many, including increased flexibility, accuracy, decreased cost and size, and improved performance. One particularly interesting application for communications engineers is software radio.

Software radio promises to be a leap forward in the capability and flexibility of traditional radio systems. Not only can a single software radio receive multiple types of signals on demand, but it can be configured to receive a wide range of frequencies, or even multiple frequencies at the same time. Additionally, a software radio is easily upgradeable, so the per-unit and maintenance costs of developing multiple software radio applications decrease as hardware is reused. Finally, the software in a software radio can easily be upgraded, keeping existing equipment up-to-date without having to upgrade the hardware.

But what is a software radio? Ideally, a software radio is nothing more than an analog-to-digital converter (ADC) attached to an antenna that digitizes the received signal which is then demodulated and processed entirely in software. Practically, the ideal software radio is not a very capable machine with present limitations in ADCs. Since most interesting radio signals have a much higher frequency than can be accurately digitized by today's ADCs, a real-world software radio will have some sort of radio-frequency (RF) front end that converts the frequency of interest to some intermediate frequency (IF).

One such device is a relatively inexpensive device called the Universal Software Radio Peripheral (USRP). The USRP consists of a motherboard which performs the analog-to-digital conversion of analog IF signals and interfaces with a personal computer and various daughter-boards which are responsible for converting various RF

frequencies to IF and sending that analog IF signal to the motherboard. This makes the USRP extremely flexible, as different daughter-boards can be used depending on the frequency of interest. A side benefit is that most of the important functionality can be implemented in a single motherboard, making the multiple daughter-boards inexpensive devices.

A signal that is particularly interesting is that used by mobile phones that comply with the Global Systems for Mobile communication (GSM) standard. Used globally, it is the most popular standard for mobile phones. It has a number of features that make receiving the signal challenging for a software radio, such as time-division multiplexing combined with slow frequency-hopping. However, the GSM standard is freely available and, as such, anyone with the ability and inclination to implement the standard in software can produce a conforming GSM software radio.

One of the primary problems in Information Warfare (IW) and signals intelligence is finding the location of the emitter of a signal. There are many methods to accomplish this task with various accuracy and complexity. One such method is to use multiple receivers in different locations and to measure the difference in time between when the signal was received at one receiver compared with another. This is called time-difference-of-arrival (TDOA) and has been successfully implemented in the past in hardware.

This thesis examines the feasibility of implementing a system for locating an emitter of a GSM signal using the TDOA method with software radio and the USRP. In order to accomplish this, the objectives of this research were as follows:

- Generate a typical packet of GSM traffic that is as close to mathematically perfect as possible.
- Transmit the GSM packet using software radio.
- Receive the GSM packet using software radio.
- Partially demodulate the GSM packet in software.
- Determine the time-of-arrival of the GSM packet.

- Perform a statistical analysis of the feasibility of performing geolocation with software radio.

These objectives were all completed with varying degrees of success due to limitations in software radio itself, the performance of general purpose computer hardware, and time.

The generation of the GSM signal in software was performed after careful analysis of the GSM specification, with special attention paid to the accuracy of the signal. Once the signal had been generated, it was sent repeatedly using the GNU Radio software and the USRP, so that the timing of the signal could be measured by the receiver.

A receiver was implemented in software that would take data from the USRP and demodulate it to the point where the arrival time could be measured. Partial demodulation of the digital data in the signal was sufficient for the arrival time measurement, so complete demodulation was not implemented. Subsequent to the demodulation, a software correlator was used to compare the received data with an ideal target sample in order to determine (within the accuracy of one digital sample) when the signal was received. This time data was checked against the known time of transmission and analyzed for accuracy.

The research found that implementing a TDOA software radio system is entirely feasible. However, significant portions of the implementation must make use of faster hardware like field-programmable gate arrays (FPGA) in order to perform the calculations in real-time. Precision of the location calculation using general purpose hardware and the USRP was on the range of 35-100 meters, with the accuracy being  $\pm 35$ -100 meters more than 90% of the time. Precision and accuracy were heavily dependent on the speed of the personal computer involved in the reception of the signal.

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## **DISCLAIMER**

The computer source code in the Appendices is supplied on an “as is” basis, with no warranties of any kind. The author bears no responsibility for any consequences of using these programs.

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## TABLE OF ACRONYMS

NPS	Naval Postgraduate School
GSM	Global System for Mobile communication
USRP	Universal Software Radio Peripheral
IW	Information Warfare
TDOA	Time-Difference-of-Arrival
FPGA	Field Programmable Gate Array
DSP	Digital Signal Processor
CPU	Central Processing Unit
PC	Personal Computer
JTRS	Joint Tactical Radio System
SCA	Software Communication Architecture
OSSIE	Open Source SCA Implementation::Embedded
ADC	Analog-to-Digital Converter
DAC	Digital-to-Analog Converter
DDC	Digital Down-Converter
GNU	Gnu's Not Unix
IF	Intermediate Frequency
RF	Radio Frequency
FM	Frequency Modulation
GMSK	Gaussian Minimum-Shift Keying
PGA	Programmable Gain Amplifier
CIC	Casaded Integrator-Comb
USN	Universal Serial Bus
NCO	Numerically Controlled Oscillator
SNR	Signal to Noise Ratio

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# I. INTRODUCTION

## A. BACKGROUND

Software defined radio, a relatively new field, promises to be a leap forward in the capability and flexibility of traditional radio systems. Instead of producing complex and inflexible analog hardware, a software radio tries to accomplish the same task in the digital domain using reconfigurable hardware such as field programmable gate arrays (FPGA), digital signal processors (DSP), or even general purpose processors such as those found in a personal computer. Although there are many advantages to processing radio signals in software, such as ease of manufacture and maintenance, the performance of software components, particularly the flexible general purpose processors, are just beginning to become powerful enough to handle the extraordinarily high data rates required for some applications. [1]

One particularly demanding application of a radio receiver is geolocation, or finding the location of the emitter of a signal based on the characteristics of the received signal. Performing geolocation with any reasonable degree of accuracy requires extremely sensitive and well-tuned equipment that can measure very small variations in receive time and frequency. [2]

But what is software defined radio, exactly? The term encompasses a lot of technologies and techniques, so an example of an ideal software defined radio will give the clearest understanding of what software radio is in general. An ideal software radio is an antenna attached directly to an analog-to-digital converter (ADC) that samples the signal coming in from the antenna. All subsequent demodulation, processing and decoding of the signal are done by software in the discrete digital domain. [1]

For various reasons, not the least of which is that the hardware required for an ideal software defined radio would be costly, practical software radios vary slightly from the ideal, whether by including special purpose hardware to perform demodulation or an analog radio-frequency (RF) front-end that converts the high radio fre-

quency signal to an intermediate frequency that is more easily sampled by an ADC. This allows the system to trade some flexibility for performance and practicality. [1]

Before this research was done, there was some doubt as to the ability of low-cost, consumer-grade hardware to perform geolocation using software defined radio and no hard data on how accurate it could be in practice. This research performs a theoretical analysis of how accurate such software should be, and experimental results show how well the theory corresponds to reality in practice.

## **B. OBJECTIVE AND APPROACH**

The objectives of this research were to:

- Generate a typical packet of GSM traffic that is as close to mathematically perfect as possible.
- Transmit the GSM packet using software radio.
- Receive the GSM packet using software radio.
- Partially demodulate the GSM packet in software.
- Determine the time of arrival of the GSM packet.
- Perform an analysis of the feasibility of performing geolocation with software radio.

To accomplish these objectives, it was necessary to calculate a discrete time signal prior to any transmission. Next, software and the USRP were used to send and receive the previously calculated and stored signal in real time and store the raw incoming data on the receiving computer. Finally, the raw data was analyzed by additional software to demodulate the signal and determine its time-of-arrival.

A number of different computer languages were used depending on the specific purpose of the software component. A Common Lisp environment called CLISP was chosen for the generation of the signal due to its native support of complex numbers,

high degree of mathematical precision, and ability to concisely implement mathematical algorithms. The GNU Radio free software project provided the libraries necessary to communicate directly with the USRP with very little overhead, and various scripts were written with the Perl computer language to perform the demodulation of a frequency modulated signal and handle translation of binary data into a readable textual representation that the gnuplot plotting program could then show graphically for further analysis.

## **C. RELATED WORK**

The GNU Radio project is an open-source software defined radio project that manages the design of the Universal Software Radio Peripheral and the low-level libraries used for its operation, as well as a relatively high-level software radio component system. Some of the GNU Radio software was used in this thesis.

The Open Source SCA Implementation Embedded (OSSIE) project is another open-source software defined radio project that focuses on compatibility with the Joint Tactical Radio System (JTRS) Software Communications Architecture (SCA). It provides a number of useful and educational tools for development and analysis of software radios.

Recently, many students at the Naval Postgraduate School (NPS) have written theses on either software defined radio or geolocation, including software radio designs for various standards such as Interim Standard 95B (IS-95B) and Institute of Electrical and Electronics Engineers (IEEE) 802.16 and 802.11a, software radio designs for general binary frequency shift keyed signals, and geolocation using the complex ambiguity function [3, 4, 5, 6, 7, 8].

## **D. THESIS ORGANIZATION**

The body of this thesis is divided into four chapters in order to indicate the level of technical detail. Chapter II focuses on general information about software

radio and GSM signals, and then discusses in specific theoretical detail how GSM and software radio work. Chapter III shows how the mathematical analysis can be put into practice by discussing the specific algorithms used to allow the software radio to transmit and receive a Gaussian minimum-shift-keyed (GMSK) signal. Chapter IV shows the results of that implementation, with graphs of the actual received signal and an analysis of the accuracy with which it was detected.

## II. ANALYSIS

### A. GLOBAL SYSTEM FOR MOBILE COMMUNICATION

The Global System for Mobile communication (GSM) is a worldwide standard for mobile telephone communication. Receiving a typical GSM signal presents many challenges due to the nature of the signal. In order to allow many simultaneous users, GSM uses a combination of time-division multiple access (TDMA), frequency-division multiple access (FDMA), and slow frequency-hopping. [9]

With TDMA, a channel is divided into time slots and a single recurring time slot is assigned to each user. Users transmit and receive in rapid succession, so the illusion is created that transmission and reception are occurring simultaneously. With FDMA, the total bandwidth available to a GSM provider is divided into many frequency bands so that multiple time-division multiplexed channels can be used simultaneously on different frequency bands. [9]

While this all seems simple enough, channels are not specified to use a given frequency on a permanent basis. Instead, GSM uses a form of slow frequency-hopping that means that the frequency for a user's channel will change for different time slots. In effect, this means a transmitting user will transmit on one frequency for the duration of a time slot, then change frequency according to a specified sequence and transmit during the next time slot. The same system is used for reception and is explained in greater detail below. [9]

#### 1. GSM Bursts

GSM uses a TDMA scheme for allowing multiple users access to a single frequency band. For a given frequency that is allocated in this manner, there are eight time slots that may be dedicated to individual users. These eight time slots compose a TDMA frame. [9: p. 215]

The basic unit of transmission in GSM is the burst. A burst is a transmission that occurs within a single time slot at a specific frequency [9]. The length of a

burst in bits is typically 147 (not including some number of guard bits that are not transmitted), however, there are shorter bursts of 87 bits that have a specific function [10]. The duration of the time slot in bits is 156.25, which allows for a short guard period to reduce interference with bursts starting in the next time slot [10].

There are many different types of GSM bursts, and each has a different information content depending on the purpose of the burst [10]. A normal burst is defined by the specification to carry voice and other data [10]. An access burst occurs whenever the mobile user must gain access to the network, such as when the phone is turned on, a call or text message is sent or received, or the user's location changes [9]. We are particularly interested in the access burst in this thesis due to the reasons for which it occurs and the specific physical properties of the burst.

*a. GSM Access Burst*

An access burst in GSM is 156.25 bits in length, including a guard period of 68.25 bits in which no transmission occurs [10]. The GSM standard defines the format of the access burst shown in Table 1 [10].

Table 1. Fields of a GSM access burst.

Bit Number	Length of Field (in bits)	Contents of field
0-7	8	extended tail bits
8-48	41	synchronization sequence
49-84	36	encrypted bits
85-87	3	tail bits
88-156	68.25	guard period

***b. Guard Period***

The guard period is the portion of each GSM burst which is provided to give the transmitter sufficient time to attenuate the transmitted signal. This period is provided to allow for the fact that the beginning or ending of a transmission cannot occur instantaneously, so the transient effect of the transmitter ramping up and down occurs during this time. [10]

***c. Tail Bits and Synchronization Sequence***

The tail bits are defined for each burst and are specific to that burst. In the case of an access burst, the first eight tail bits are as follows [10]:

Extended tail bits	0,0,1,1,1,0,1,0
--------------------	-----------------

The synchronization (or training) sequence is a specified sequence of bits that is used to recognize that a particular burst is received. In the case of the access burst, this is a 41 bit sequence starting at bit 8. The GSM specification gives three possible training sequences that can be used in an access burst, a primary and two alternates. These training sequences are shown in Table 2. [10] For the purposes of testing in this thesis, training sequence 0 is used exclusively. Since the bits are encoded using differential encoding, the last bit of the extended tail bits is needed to determine the first bit of the synchronization sequence.

## **2. GSM Frequency-Hopping**

In addition to TDMA, GSM spreads transmissions among multiple frequencies. In this way, many more than eight simultaneous users can be supported. [9]

Table 2. Access burst synchronization sequences.

Sequence Number	Sequence
0	0,1,0,0,1,0,1,1,0,1,1,1,1,1,1,1,0,0,1, 1,0,0,1,1,0,1,0,1,0,1,0,0,0,1,1,1,1,0,0,0
1	0,1,0,1,0,1,0,0,1,1,1,1,1,0,0,0,1,0,0,0, 0,1,1,0,0,0,1,0,1,1,1,1,0,0,1,0,0,1,1,0,1
2	1,1,1,0,1,1,1,1,0,0,1,0,0,1,1,1,0,1,0,1, 0,1,1,0,0,0,0,0,1,1,0,1,1,0,1,1,1,0,1,1,1

GSM employs a technique called slow frequency-hopping in order to increase security and reliability. Slow frequency-hopping is different from fast frequency-hopping in that one or more full channel symbols are transmitted during a single hop dwell. With GSM an entire burst is transmitted before the frequency is changed. [9: p. 219]

While the frequency-hopping in GSM is comparatively slow, it still presents a problem for reception in software radio, since either the center frequency must be changed rapidly in order to keep up with the frequency-hopping or the entire GSM bandwidth must be captured in order to pick out the channel of interest.

### 3. GSM Channel Definition

A GSM burst is defined by both the time slot in which it occurs (eight time slots numbered 0-7) and the frequency at which it occurs. Since the frequency-hopping follows a specified pattern, a channel is defined by its time slot and frequency-hopping sequence. Figure 1 shows a pictorial representation of bursts occurring over time at different frequencies, where each colored box represents a burst.

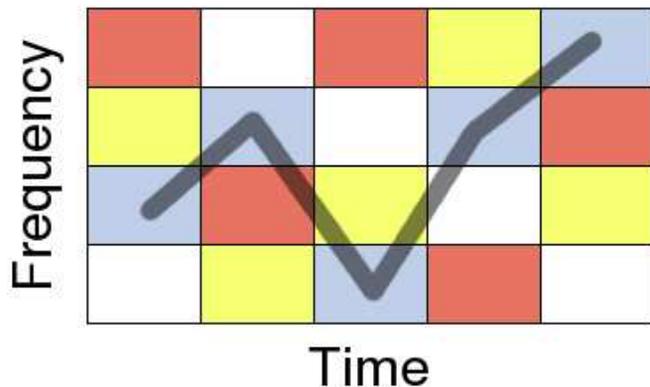


Figure 1. Illustration of a frequency-hopping GSM channel.

*a. GSM Control Channels*

The GSM specification distinguishes between traffic channels, which carry voice and other data, and control channels which are used to gain access to the traffic channels [10]. Also, the initial bursts that occur to gain access to control channels do not make use of frequency-hopping, simplifying the detection and decoding of control traffic [11]. For this reason, this research focuses on the bursts that are used to access a GSM network.

## B. GSM MODULATION

The following section describes the physical layer modulation in the GSM system. This does not include any discussion of higher level layer coding and interleaving, both of which exist in GSM, but are beyond the scope of this thesis.

The GSM system uses differential encoding of data bits and GMSK as its modulation scheme.

## 1. Differential Encoding

According to the GSM specification covering physical layer modulation, before data bits are modulated, they are encoded using differential encoding as follows [12]:

$$\hat{d}_i = d_i \oplus d_{i-1} \quad \text{where } \oplus \text{ denotes modulo 2 addition.} \quad (2.1)$$

The input to the differential encoder described in 2.1 is specified to consist of a series of zeros and ones. However, the output of the differential encoder is  $\pm 1$ , so the signal must be level-shifted as follows [12]:

$$\alpha_i = 1 - 2\hat{d}_i \quad (2.2)$$

The output of the differential encoder is  $\alpha_i$ , which is a differentially encoded polar signal.

## 2. GMSK Filtering

The binary data to be modulated using GMSK can be represented as a series of polar, differentially encoded impulses, as follows [12]:

$$\sum_{i=0}^{\infty} \alpha_i \delta(t - iT_b) \quad (2.3)$$

This set of impulses is convolved with a function  $g(t)$ , the impulse response of a Gaussian-shaped filter, to give the GMSK waveform [12]:

$$g(t) = h(t) * \text{rect} \left( \frac{t}{T_b} \right) \quad (2.4)$$

where the function  $\text{rect}(x)$  is the rectangular function with an area of one and a width of one bit duration [12] and is defined by

$$\text{rect} \left( \frac{t}{T_b} \right) = \frac{1}{T_b} \quad \text{for } |t| < \frac{T_b}{2} \quad (2.5)$$

$$\text{rect} \left( \frac{t}{T_b} \right) = 0 \quad \text{otherwise} \quad (2.6)$$

$T_b$  in Equation (2.5) is the bit duration, which is  $\frac{6}{1625 \times 10^3}$  seconds, or approximately 3.69 microseconds [12].  $h(t)$  is a Gaussian impulse response:

$$h(t) = \frac{\exp\left(\frac{-t^2}{2\delta^2 T_b^2}\right)}{\sqrt{2\pi}\delta T_b} \quad (2.7)$$

where  $\delta$  is defined to be inversely proportional to the bandwidth-time product [12]:

$$\delta = \frac{\sqrt{\ln(2)}}{2\pi BT_b} \quad \text{and} \quad BT_b = 0.3 \quad (2.8)$$

$BT_b$  (also called the bandwidth-time product) is the product of the the 3 dB bandwidth of the filter and the bit duration and is a constant (0.3 in the case of GSM). This affects the amount of inter-symbol interference in the GMSK signal. The 0.3 chosen for GSM is a compromise between error rate and spectral efficiency. [13: p. 319]

## C. THE UNIVERSAL SOFTWARE RADIO PERIPHERAL

The Universal Software Radio Peripheral (USRP) is a hardware device that makes reception and transmission of radio waves with consumer-grade, personal computer equipment possible. It consists of a motherboard containing various ADCs, digital-to-analog (DAC) converters, an FPGA, and slots for a number of daughter-boards, which determine the range of frequencies the board can send or receive. The USRP connects to the computer via a high-speed Universal Serial Bus (USB) port. [14]

### 1. Receive Capabilities

#### *a. Analog-to-digital Conversion*

The frequency band of the analog signals coming into the USRP motherboard is determined by the specific daughter-board that is used with the USRP. The daughter-board is responsible for providing the separated I and Q channels of the complex signal to the motherboard and possibly for converting the received real signal at RF to a complex signal at an intermediate frequency (IF). Some daughter-boards perform no conversion, so as to be useful with a variety of low frequency signals. The particular board used for this research, however, did perform conversion, so the analysis of its operation follows. [14]

The signal at the antenna of the daughter-board can be described as the signal  $r(t)$ , which has a carrier of frequency  $f_{RF}$  modulated by amplitude  $A(t)$  and phase  $\theta(t)$ :

$$r(t) = A(t) \cos(2\pi f_{RF} t + \theta(t)) \quad (2.9)$$

After the daughter-board performs frequency conversion and filtering to remove unwanted aliasing, the resulting complex signal analytic signal,  $S(t) = S_i(t) + jS_q(t)$ , can be described by

$$S_i(t) = A(t) \cos(2\pi f_{IF} + \theta(t)) \quad (2.10)$$

$$S_q(t) = A(t) \sin(2\pi f_{IF} + \theta(t)) \quad (2.11)$$

The preceding signals are then sampled by the ADCs after an optional amplification stage using a Programmable Gain Amplifier (PGA) on the USRP motherboard, which provides a gain of up to 20 dB. The USRP contains four 12-bit ADCs (each is dedicated to the I or Q samples of one of two possible channels) each running at  $64 \times 10^6$  samples/s, giving a theoretical upper frequency limit (for the intermediate frequency) of 32 MHz before artifacts due to aliasing are introduced. Note that two separate ADCs are required to convert the complex signal to a digital representation. The resulting sampled data is

$$S_i[n] = S_i(nT_s) \quad (2.12)$$

$$S_q[n] = S_q(nT_s) \quad (2.13)$$

where  $T_s$  is the sample interval,  $n = 0, 1, 2, \dots$  is discrete time, and  $S_i[n]$  and  $S_q[n]$  are the resulting in-phase and quadrature samples, respectively. [14]

**b. Conversion to Baseband**

After translation into the digital domain, the samples are processed by firmware on the FPGA. Each pair of incoming I and Q samples are treated as a single complex analytic sample  $S_i[n] + jS_q[n]$  by the multiplication algorithm in the FPGA which performs a complex multiplication resulting in a set of complex samples at baseband  $I[n] + jQ[n]$  and given by [14]

$$\begin{aligned} S[n] &= S_i[n] + jS_q[n] && \text{(Complex Analytic Signal)} \\ &= A(nT_s) [\cos(2\pi f_{IF}nT_s + \theta(nT_s)) + j \sin(2\pi f_{IF}(nT_s) + \theta(nT_s))] \\ &= A(nT_s)e^{j2\pi f_{IF}(nT_s) + \theta(nT_s)} \end{aligned} \tag{2.14}$$

$$\begin{aligned} I[n] + jQ[n] &= S[n]e^{-j2\pi f_{IF}(nT_s)} \\ &= A(nT_s)e^{\theta(nT_s)} && \text{(Complex Envelope)} \end{aligned} \tag{2.15}$$

$I[n]$  and  $Q[n]$  are the outputs of the first stage of the FPGA. Low-pass filtering is accomplished in the next stage in addition to decimation [14]. Decimation reduces the effective sample rate and, therefore, the signal bandwidth. Filtering is required because decimation itself will introduce aliasing if the decimation rate is large enough [15: p. 235].

**c. Aliasing**

Aliasing occurs because frequencies which differ by a multiple of the sampling rate are indistinguishable from each other after sampling [15: p. 11]. To illustrate this mathematically, the angular frequency  $\Omega_0$  and the angular digital frequency  $\omega_0$  are defined as:

$$\Omega_0 = 2\pi F_0 = \frac{2\pi}{T_0} \tag{2.16}$$

$$\omega_0 = \Omega_0 T_s = 2\pi \frac{F_0}{F_s} \text{radians} \tag{2.17}$$

where  $F_0$  is the frequency of interest of the continuous time signal and  $F_s$  is the sampling frequency. The digital frequency is shown to be dependent on the sampling frequency, which is an important distinction. Because decimation effectively lowers the sampling frequency  $F_s$ , the digital frequency that corresponds to a frequency  $F_0$  changes proportionally. [15]

One other important property of digital frequency is that any frequency  $\omega_1$  and another frequency  $\omega_2$  are indistinguishable from each other in the discrete domain provided that  $\omega_2 = \omega_1 + k2\pi$ , where  $k$  is any integer. This is why frequencies above the Nyquist rate are indistinguishable from frequencies below it and is the basis for understanding aliasing. Notice that the Nyquist frequency, which is defined as one-half the sample rate, corresponds to an angular digital frequency of  $\pi$ . Therefore, a discrete time signal's bandwidth must exist entirely between  $-\pi$  and  $\pi$  in order for aliasing not to occur. [15]

Figure 2 is an illustration of the effect of aliasing when the sampling frequency  $F_s$  is not large enough to separate the bandwidth of the target signal from its aliases, or repeated images, elsewhere in the frequency domain. The top graph is an example of a discrete time signal that does not exhibit any aliasing. The bottom graph is a representation of what happens if the same sampled signal is decimated such that  $\pi$  is less than  $\omega_1$ , the digital bandwidth of the original discrete time signal. This shows significant interference around the upper and lower portions of the baseband signal. Effectively, this requires that in addition to a decimation stage, a low pass filter must be used in order to prevent aliasing. The FPGA on the USRP includes a Cascaded Integrator-Comb (CIC) filter, which performs the low-pass filtering and decimation at a high speed. [14]

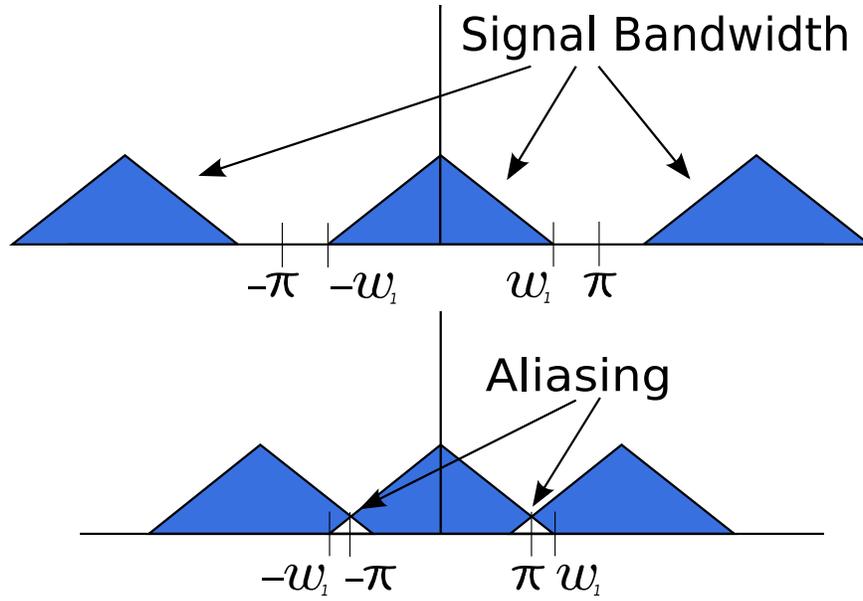


Figure 2. Illustration of aliasing due to down-sampling.

#### *d. Decimation*

The primary reason decimation is done is to reduce the data rate of the signal that the USRP transmits to the computer. Since the data rate of the USB connection is limited, not every sample can be sent without overflowing the USRP's buffer, so the decimation rate must be set appropriately to limit the signal data rate to a level that the USB connection can handle. [14]

All data is sent to the computer from the USRP in the form of 16-bit signed integers. The in-phase channel data is multiplexed with the Q channel data. In other words, the data stream always consists of an I channel integer followed by a Q channel integer. [14]

## **2. Transmit Capabilities**

The USRP's transmit capability is similar to its receive capability. For the most part, transmission is simply the reverse operation of reception, with a few minor differences.

The data to be transmitted is sent to the USRP over the USB connection. This data is then interpolated at a specified rate, which may be set programatically. Interpolation, like decimation, introduces aliasing into the signal, so the signal must be filtered afterward [15: p. 248]. Next, the signal is multiplied by a sinusoid at the intermediate frequency, converted to analog by the digital-to-analog converter (DAC) at a rate of  $128 \times 10^6$  samples/s, with the resulting signal sent to the daughter-board for transmission [14]. The  $128 \times 10^6$  sample/second DAC rate gives us a maximum intermediate frequency of 64MHz [14]. However, the maximum allowable intermediate frequency is limited to 44 MHz by the USRP software to make up-sampling and filtering practical [14, 16].

According to the USRP documentation, the required sample rate sent over the USB connection is

$$R_{USB} = \frac{F_{DAC}}{R_{interp}} \times channels \quad (2.18)$$

where  $F_{DAC}$  is the sample frequency of the DAC,  $R_{interp}$  is the interpolation rate, and *channels* is the number of simultaneous data signals being transmitted [16]. This sample rate is measured in terms of individual 16-bit integers being sent over the USB wire. This rate is important when generating the baseband signal, both in determining the rate at which to sample the ideal baseband signal and in providing the maximum rate across the USB connection to create a more accurate output signal.

### 3. Data Transfer Capabilities

The theoretical maximum transfer rate of the USB 2.0 connection that the USRP uses to transfer data is 480 Mbit/sec, but in practice the maximum is found to be about 32 MB/sec, or 256Mbit/sec [14]. Each complex sample consists of a 16-bit

signed integer for the I and the Q channels, totaling 32 bits per complex sample [14]. Therefore, the maximum sample rate coming in to (or going out of) the computer's USB port is

$$R_s = 256 \times 10^6 \frac{\text{bits}}{s} \times \frac{1 \text{ sample}}{32 \text{ bits}} = 8 \times 10^6 \frac{\text{samples}}{s} \quad (2.19)$$

This allows the calculation of the minimum sample interval

$$T_s = \frac{1}{8 \times 10^6} \frac{s}{\text{samples}} = 125 \text{ ns} \quad (2.20)$$

This minimum sample interval is used later to calculate the precision of performing geolocation using digital samples.

## D. SIGNAL DETECTION

### 1. Correlator

Determining that the signal of interest has been received is a matter of correlating the incoming signal with the known training sequence that has been modulated with GMSK. The sampling frequency associated with this known sequence should match the sampling frequency of the data coming in over the USB link.

Let the signal of interest be sampled signal  $X$  and the incoming sampled signal be signal  $Y$ . From the USRP,  $X$  will be a multiplexed complex signal with 16-bit I and Q samples, respectively. In hardware, we normally perform separate convolutions on the I and Q channels and sum the result [17: p. 232]. Theoretically this is not necessary in software, however, since we can construct an identical multiplexed complex signal and correlate against that. Unfortunately, test results show that other factors prevent this straightforward approach from working easily and will be discussed in Chapter IV.

Correlation consists of performing a non-time-reversed convolution between  $X$  and  $Y$ . In the discrete domain, we may define the correlation at a certain time  $t$  as follows, assuming the input  $X$  is the same length as the target signal  $Y$

$$C = \sum_{i=0}^{n-1} X_i Y_i \quad (2.21)$$

where  $n$  is the number of samples contained in the signal of interest. If the signals  $X$  and  $Y$  match exactly, the magnitude of  $C$  is large and a spike occurs in the graph of  $C$  over time.

However, since we are receiving the signal continuously,  $Y$  does not simply consist of only  $n$  samples but may be arbitrarily long. Because of this, we must keep shifting the samples in  $Y$  by one, such that  $Y_1$  becomes the new  $Y_0$ ,  $Y_2$  becomes the new  $Y_1$ , and so forth. The output of the correlator at each iteration is checked to see if its value is above a threshold, at which point we can be reasonably certain that the target signal has been received. Additionally, the maximum value of the correlator output that is above the threshold allows us to determine the exact time that the signal was received to the nearest sampling interval. [17]

## **E. GEOLOCATION**

Location of an emitter in this system is done by measuring the time difference of arrival between identical listening posts. This requires at least two posts to calculate the distance from either post and at least three to perform an accurate triangulation. [2]

### **1. Requirements**

In the case of only two listening posts on either side of an emitter, the emitter can be located on a hyperbolic curve between the two posts. The hyperbolic curve is a representation of all points where the signed difference of the distance between either post (focus) and the emitter is a constant. This constant is equal to the distance traveled by light between the time the signal is first detected (at one post) and the time it is detected at the second post.[2: p. 174]

Using only two posts, an accurate measurement of distance can be made, but the emitter can still only be said to be located anywhere on a hyperbolic curve that is infinitely long.

By using another listening post, two more such curves can be found. Ideally, the intersection of these curves occurs at the exact location of the emitter. Accuracy and precision can be increased by using more receivers. Since any single curve is dependent on the time difference between exactly two receivers, the number of unique curves  $N$  that can be plotted based on the number of listening posts  $L$  is

$$N = \frac{L(L-1)}{2} \quad (2.22)$$

## 2. Precision

Because the USB 2.0 link is the bottleneck for data transfer into the computer and the precision of the geolocation directly depends on the sample interval, the maximum precision can be calculated. From Equation (2.20), the minimum sample interval is 125 nanoseconds. In that time, an RF signal will travel a distance

$$D_s = T_s c = (125 \times 10^{-9})s \times (3 \times 10^8) \frac{m}{s} = 37.5 \text{ m} \quad (2.23)$$

## 3. Limitations and Workarounds

Although four listening posts provide six curves and improved accuracy, calculation of the points on each curve and the intersection of those points, particularly in the presence of measurement error, is computationally intensive. Additionally, even the slightest measurement error means that the hyperbolic curves have no simultaneous solution or point where every curve intersects. Also, the nature of hyperbolic equations imparts a floating-point inaccuracy to computer calculations dependent on which part of the curve the emitter is actually located. Finally, calculation of a hyperbolic curve involves trigonometric functions which are not practical for heavy real-time usage, even with modern hardware. Therefore, another approach is needed.

### *a. Graphical Method*

The most obvious method is to use graphical analysis to show the likely position of the emitter. To illustrate this method, we assume there exist four listening posts located in a square configuration on a Cartesian coordinate system at points

(0,0), (0,1000), (1000,0), and (1000,1000). This creates a  $1000 \times 1000$  grid between the posts. This grid can be superimposed on any two dimensional surface with grid coordinates translated to map coordinates, latitudes and longitudes, etc.

The emitter can be found by calculating the likelihood of it being located at any coordinate on the grid based on the time differences of arrival between the listening posts. Since the output of the algorithm is a two-dimensional matrix of probabilities, the output is easily represented as a computer graphic. This method is used to simulate finding the location of the emitter in Chapter IV.

***b. Lookup Table Method***

The complexity and run-time of the location calculation can be reduced even further. Ideally, the result of the calculation is a coordinate on the computed grid based on a series of time-differences-of-arrival. Note that each point P on the grid ideally maps to a unique set of time-differences between three listening posts:

$$P_{ij} \text{ maps to } [\tau_{01}, \tau_{02}, \tau_{12}] \tag{2.24}$$

where  $\tau_{ab}$  is the time-difference-of-arrival between two listening posts  $a$  and  $b$ . Therefore, the calculation of the ideal time differences of each point on the grid can be done prior to the deployment of the system. Then the identification of the most likely location of an emitter can be reduced to a table lookup/interpolation of the time differences, which could happen in real time. The only limitation is the storage space required to hold such a table and the fact that movement of the receivers requires a table update. As such, the lookup table approach is not practical for a system in which the receivers are mobile.

## **F. CONCLUSION**

We have examined the mathematical theory behind generation, transmission, reception, and detection of the time of arrival of a GSM access burst in this chapter. Also discussed were the theoretical limitations of the accuracy and precision of a software radio. What remains is the translation of that theory into actual algorithms that can be used in software. The translation is challenging because continuous time signals are generated in discrete time for transmission. The following chapter focuses on overcoming those challenges.

### III. IMPLEMENTATION

#### A. GMSK SIGNAL GENERATION

##### 1. GMSK Filter Implementation

The GMSK definition in Chapter II must be used to generate a GMSK signal. However, the GMSK signal is defined in the continuous time domain and must be converted to the discrete domain before it can be used. There are two rates to be concerned with when performing this conversion: the GMSK bit rate,  $R_b = 1/T_b$ ; and the sampling rate that is used to convert the continuous GMSK signal to the discrete domain.

Equation (2.1) through Equation (2.8) suggest that a series of impulses representing the data bits should be convolved with the defined impulse response function, which itself contains a convolution:

$$x(t) = \sum_{i=0}^{\infty} \alpha_i \delta(t - iT_b) * \left[ h(t) * \text{rect} \left( \frac{t}{T_b} \right) \right] \quad (3.1)$$

where  $*$  implies convolution. The result  $x(t)$  is the signal which subsequently frequency modulates the signal carrier. Because convolution is an associative and commutative operation [18], we can rewrite the process as

$$x(t) = \left[ \sum_{i=0}^{\infty} \alpha_i \delta(t - iT_b) * \text{rect} \left( \frac{t}{T_b} \right) \right] * h(t) \quad (3.2)$$

$$= b(t) * h(t) \quad (3.3)$$

Instead of performing two convolutions on the continuous data and then sampling that, we can simply perform a discrete convolution on a discrete function  $h[k]$  and a discrete series of rectangular pulses  $b[k]$ . The functions  $h[k]$  and  $b[k]$  are equal to the sampled continuous  $h(t)$  and  $b(t)$ , respectively, and are given by

$$h[k] = h(nT_s) \quad (3.4)$$

$$b[k] = b(nT_s) \quad (3.5)$$

where  $T_s$  is equal to some sample time. The rectangular pulses in  $b[k]$  represent the data-bearing impulses already convolved with the discrete  $\text{rect}[x]$  function and, therefore, look like

$$(1, 1, 1, 1, 1, -1, -1, -1, -1, -1, \dots) \quad \text{if } \alpha_i = [1, -1, \dots] \quad (3.6)$$

The above is equivalent to sampling the input data  $b(t)$  at some rate  $R_s = n/T_b$ , where  $n = 5$ , and the time difference between the successive bits 1 and  $-1$  is  $T_b$ . In other words, each sample represents  $T_b/n$  seconds. The result is achieved simply by repeating the input data bits  $n$  times, which saves significant computation time and is less complex to implement than performing multiple convolutions.

## 2. Determining the Sample Rate

Before generating a discrete-time GMSK signal, the appropriate sample rate must be determined. The required sample rate for transmission is determined solely by the configuration of the USRP, which expects samples at a rate dependent on the number of channels used, the frequency of the DAC, and the interpolation rate.

The DAC frequency on the USRP is constant at a rate of  $128 \times 10^6$  complex samples/s. The GMSK generation function generates pairs of 16-bit integers where each pair represents one complex sample. The only variable is the interpolation rate. This is a design decision. A lower interpolation rate results in a more accurate signal, while a higher one requires less bandwidth from the USB link. Since the USRP in our case is dedicated to transmitting a signal, we can use all of the USB bandwidth if necessary.

Equation (2.19) shows that the maximum USB bandwidth is  $8 \times 10^6$  complex samples/s. Substituting this into Equation (2.18), along with the constants already mentioned, we get an interpolation rate of 32, which is the lowest acceptable interpolation rate that does not saturate the USB connection.

Since signal accuracy is important for this experiment, the minimum interpolation rate of 32 is used. Thus, the complex sample rate can be calculated, (keeping in mind that a complex sample consists of two 16-bit samples for the I and Q channels):

$$R_{s_{tx}} = \frac{128 \times 10^6}{32} = 4 \times 10^6 \frac{\text{samples}}{s} \quad (3.7)$$

$$T_{s_{tx}} = \frac{1}{R_{s_{tx}}} = 2.5 \times 10^{-7} s \quad (3.8)$$

$R_{s_{tx}}$  is the minimum required sample rate for our discrete generated signal. Note that any integer multiple  $nR_{s_{tx}}$  works equally well – we simply have to select every  $n^{\text{th}}$  sample to send to the USRP. However, there is another rate to consider, and that is the GMSK bit rate of  $R_b = 1625 \times 10^3/6$  (bits/second) [12]. We have seen that it is very convenient if the sample rate used is some integer multiple of the GMSK bit rate. So we now have two different rates that we would like to have as factors of the overall sample rate, and we need to find integers  $n_0$  and  $n_1$  such that:

$$n_0 R_{s_{tx}} = n_1 R_b \quad (3.9)$$

$$\frac{n_0}{n_1} = \frac{R_b}{R_{s_{tx}}} = \frac{1625 \times 10^3}{6(4 \times 10^6)} = \frac{13}{96} \quad (3.10)$$

The overall sample rate  $R_s$  that we would like to use is  $13(4 \times 10^6)$  or  $192(1625 \times 10^3/6)$ , both of which are equal to  $52 \times 10^6$ . This ensures that no interpolation has to be done when calculating samples for either the GMSK bit rate or the USB sample rate.

### 3. Baseband Signal Generation

To understand how the baseband signal is generated, a review of the source code is helpful. The Common Lisp code generates a complex baseband frequency modulated signal, one sample at a time. It keeps track of the phase angle of the last sample using the `phase-angle` variable. The `reset` function is called to set the phase angle to zero. The signal is generated by calling the `modfm` function repeatedly, with the interval between the samples and the instantaneous frequency as arguments.

The `modfm` function calculates the new phase angle based on the old phase angle and the phase change that is calculated based on the new frequency and the sample interval. A complex number is returned that represents the complex amplitude of the current sample. The preceding is accomplished by the code:

```
;;; fm signal
(let ((phase-angle 0))

  (defun reset ()
    (setf phase-angle 0))

  (defun modfm (sample-duration freq)
    (let ((phase-inc (* 360 sample-duration freq))) ;;in degrees
      (setf phase-angle (+ phase-angle phase-inc))
      (exp (* j (radian phase-angle))))))
```

The preceding code implements the mathematical function

$$\text{modfm}(\theta_p, T_{stx}, F_\Delta) = e^{j(\theta_p + (T_{stx} \times F_\Delta))} \quad (3.11)$$

where  $\theta_p$  is the phase angle calculated from the previous sample,  $T_{stx}$  is the sample duration, and  $F_\Delta$  is the instantaneous frequency to be encoded. The result is a complex vector which can be separated into I and Q samples for transmission.

This function, in combination with another function that performs a linear translation of voltage to frequency, is called a numerically controlled oscillator (NCO). It takes a normalized input voltage and produces a complex baseband FM signal as its output. One of the most desirable features of an NCO is that it generates a signal that is as close to mathematically perfect as possible and the output does not vary with temperature or other factors that affect an analog voltage controlled oscillator. This is why it is practical to generate the GMSK signal with a simple algorithm instead of modeling the more complex analog circuits in software [1].

Figure 3 shows an example of the output of this NCO algorithm where the instantaneous frequency starts at  $-10$  Hz and slowly increases to  $10$  Hz.

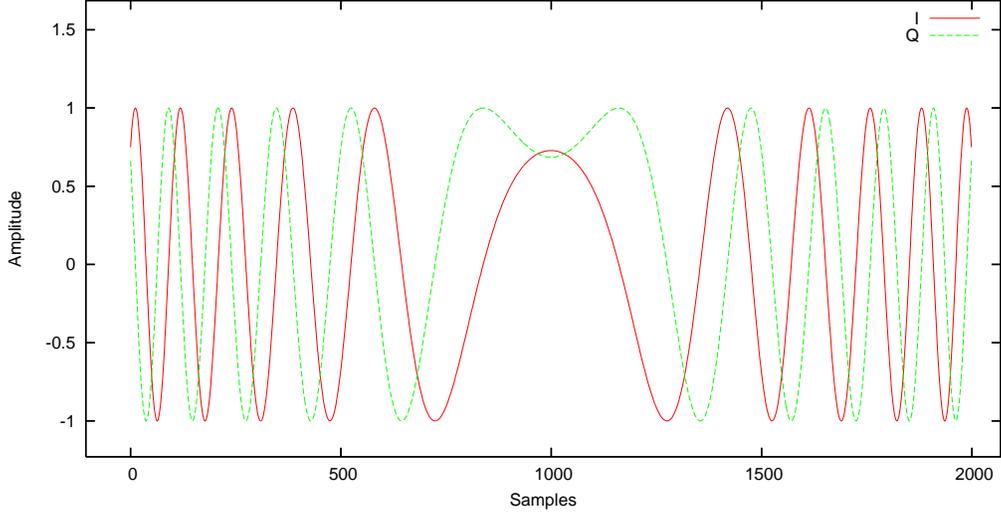


Figure 3. Complex FM baseband signal from  $-10$  to  $10$  Hz.

#### 4. Application of a GMSK Filter

Once we can frequency modulate a baseband signal, we must calculate the modulating signal, which is the GMSK filtered bit stream. The bit stream used was the first synchronization sequence specified by the GSM specification [10: p. 21]. The result of generating the bit-stream such that it is equivalent to a GSM bit-stream at the previously calculated sample rate  $R_s$  from Equation (3.9) is shown in Figure 4.

After filtering using the Gaussian filter, the result is the modulating wave shown in Figure 5.

This modulating wave is decimated by a factor of  $n_0 = 13$  so that the resulting sample rate matches the target sample rate of the USRP USB connection,  $2 \times 10^6$ . It is then used to modulate a complex baseband signal with the same sampling rate. Thus, when the baseband signal is interpolated by the USRP at an interpolation rate of 32, the samples match exactly the USRP's DAC rate of  $128 \times 10^6$  samples/s.

At this point, the complex data shown in Figure 6 is ready to be sent to the USRP. The computer program used to send the data is listed in Appendix D.



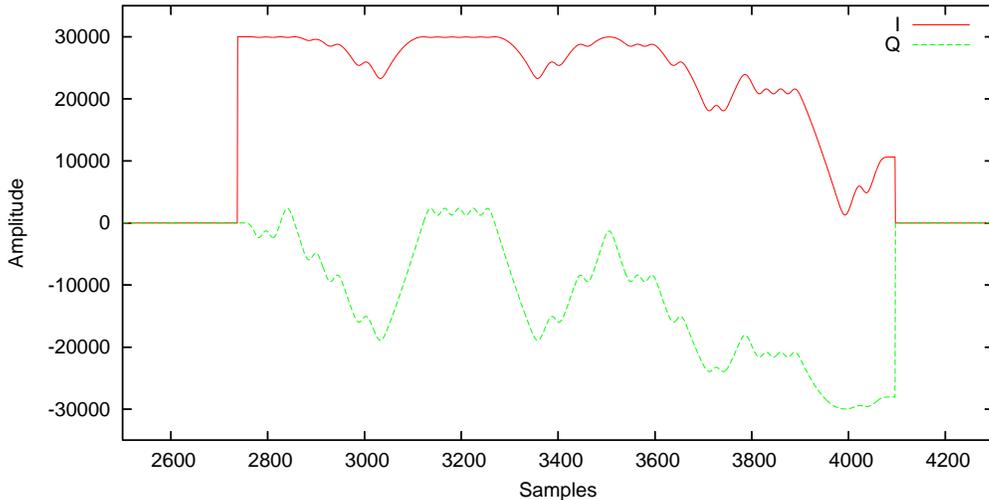


Figure 6. Modulated access burst, I and Q channels.

## B. GMSK DECODING

Since a GMSK signal is essentially a frequency modulated signal, demodulation of GMSK using FM demodulation techniques is straightforward and produces good results [13: p. 320].

### 1. FM Demodulation

The only data coming in from the USRP are the I and Q samples. Because we can set the decimation rate and know the sample rate of the ADC, we also know the time between samples. The instantaneous frequency of the incoming signal must be found and converted to the amplitude of the modulating signal in order to perform an FM demodulation.

The instantaneous frequency cannot be found exactly for any particular sample, but the change in phase between two successive samples can. This change in phase divided by the time between samples gives a close approximation of the frequency of the signal between those two samples [13]. Figure 7 illustrates how this is done. It shows two FM samples on the complex plane, with the angle  $\theta$  between them being the difference in phase.

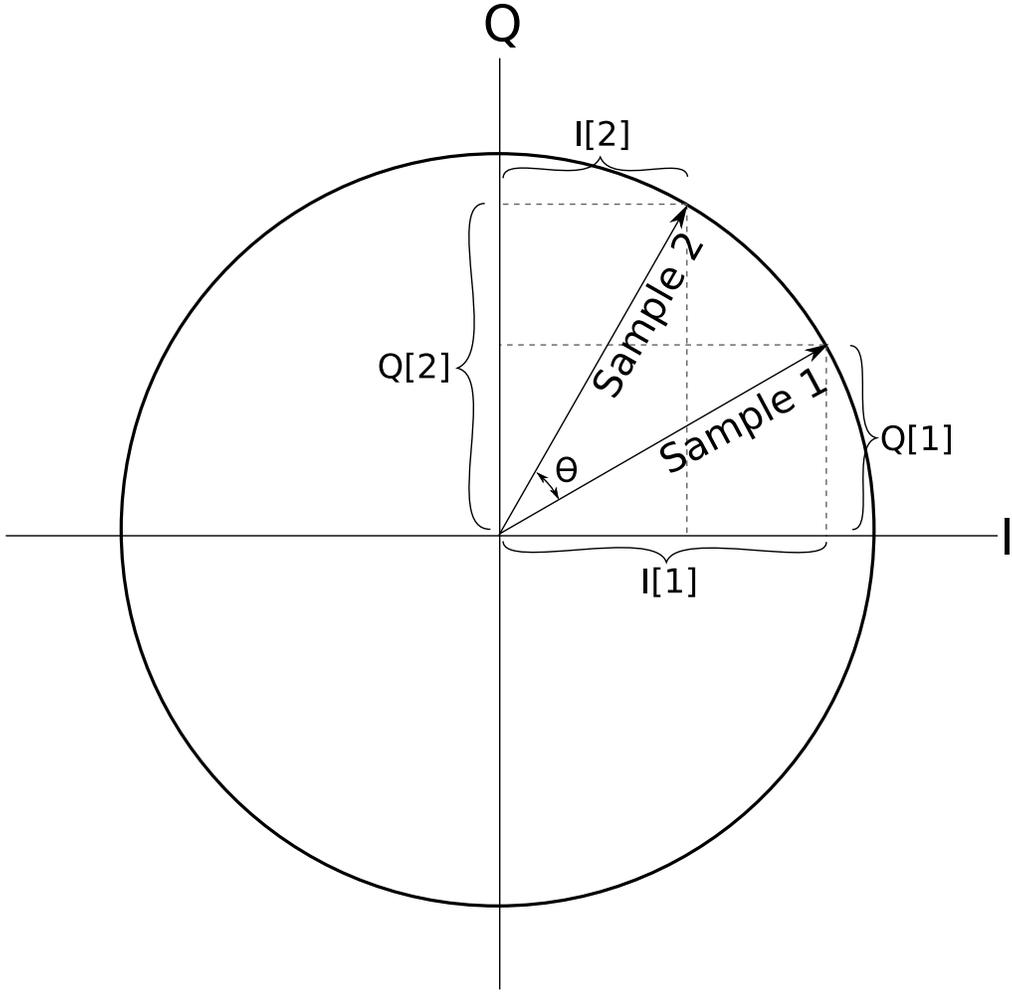


Figure 7. Two successive FM samples.

The instantaneous frequency can be calculated using two sequential samples without using the computationally expensive (and possibly inaccurate) arctan function. The instantaneous frequency is

$$F_i = \frac{d\theta}{dt} \quad (3.12)$$

If we let

$$k(t) = \frac{q(t)}{i(t)} \quad (3.13)$$

and

$$\theta(t) = \arctan(k(t)) \quad (3.14)$$

then taking the derivative of Equation (3.13), we get

$$\frac{dk}{dt} = \frac{i \frac{dq}{dt} - q \frac{di}{dt}}{i^2} \quad (3.15)$$

Computing the derivative of Equation (3.14), we get

$$\begin{aligned} \frac{d\theta}{dt} &= \frac{1}{1 + (k)^2} \frac{dk}{dt} \\ &= \frac{1}{1 + \left(\frac{q}{i}\right)^2} \frac{i \frac{dq}{dt} - q \frac{di}{dt}}{i^2} \\ &= \frac{i \frac{dq}{dt} - q \frac{di}{dt}}{i^2 + q^2} \end{aligned} \quad (3.16)$$

which is equal to the instantaneous frequency. In the discrete domain,  $di/dt$  and  $dq/dt$  can be calculated by dividing  $(q[x] - q[x - 1])$  and  $(i[x] - i[x - 1])$  each by the sample duration. The results here are consistent with the result of the discussion found in [19].

## 2. Determining Sample Rate

Ideally, we want to be able to use the lowest possible decimation rate when receiving to get the most accurate results possible. According to Equation (2.19), the maximum sample rate the USB connection can handle is  $8 \times 10^6$  samples/s. This corresponds to a decimation rate of eight, since the ADCs in the USRP operate at a rate of  $64 \times 10^6$  samples/s, as shown by

$$R_{USB} = \frac{R_{ADC}}{R_{decim}} \quad (3.17)$$

where

$$R_{decim} = \frac{64 \times 10^6}{8 \times 10^6} = 8 \quad (3.18)$$

where  $R_{USB}$  is the data rate limit of the USB connection, and  $R_{ADC}$  is the rate of the USRP's ADC.

However, initial testing showed that over longer data collection runs, the rate at which the data could be transferred to the hard disk from memory became a limiting factor. Since a single buffer overrun means that samples are lost and the time of arrival calculation is incorrect, the decimation rate must be set high enough to ensure all data can be copied to the hard disk for the entire collection without any overruns. Testing indicated that a decimation rate of 16 produced a rare overrun, and a decimation rate of 32 was high enough so that no overruns occurred. For the purposes of the following illustration of the demodulation process, however, a decimation rate of 16 was used as it was the best compromise between performance and precision.

A decimation rate of 16 gives the incoming sample rate for the received signal:

$$R_{s_{Rx}} = \frac{64 \times 10^6}{R_{decim}} = \frac{64 \times 10^6}{16} = 4 \times 10^6 \frac{samples}{s} \quad (3.19)$$

### 3. Generating Target Sequence

In order to detect the GSM training sequence, we need to generate that sequence at the  $R_{s_{Rx}}$  sample rate. This was done exactly the same way as for the transmission signal by generating a baseband FM signal as in section (A.4). The difference here is that the further steps of modulating a carrier signal were not taken, since our received signal is frequency demodulated prior to correlation. The results of this calculation are, therefore, the same as those of the intermediate step in the generation of the transmission signal, shown in Figure 5.

### 4. Correlation

In order to perform the correlation, the incoming signal samples must be multiplied by the target signal samples. An array of the target samples is created in order to perform the multiplication and summation as in Equation (2.21). The target sequence is simply the training sequence portion of the access burst shown in Figure 5. These target samples are placed in the array in non-reversed time order such that

the oldest samples (in time) occupy the lowest array address (leftmost) as in Figure 8. This way, the oldest incoming samples match with the oldest target samples and the newest with the newest. Note that this method requires filling an input memory buffer from low to high addresses in order to perform the correlation. For some specific hardware, it might be faster to reverse the target sequence in time.

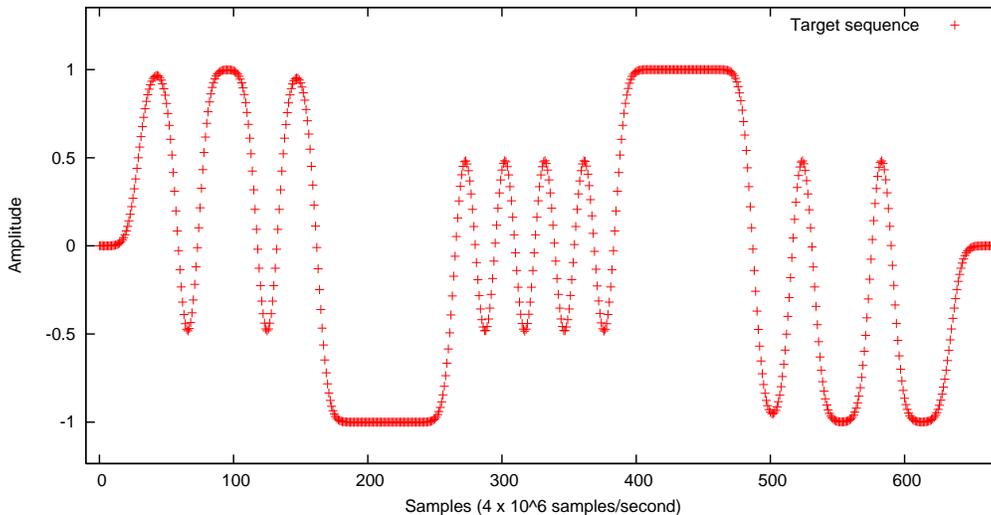


Figure 8. Target GSM synchronization sequence.

## 5. Storage Requirements

The volume of data coming in from the USRP is quite large, so a very large amount of permanent storage is required to capture data for analysis. Recall from Equation (3.18) that data comes in at  $8 \times 10^6$  samples/s when using a decimation rate of eight. Since each complex sample consists of four bytes of data (two bytes each for I and Q), we can calculate the minimum amount of storage required for a certain amount of recording time from

$$\begin{aligned} \text{Disk space} &= \left(4 \frac{\text{bytes}}{\text{sample}}\right) \left(8 \times 10^6 \frac{\text{samples}}{\text{s}}\right) \left(60 \frac{\text{s}}{\text{min}}\right) \\ &= 1.92 \times 10^9 \frac{\text{bytes}}{\text{min}} \end{aligned} \quad (3.20)$$

Equation (3.20) shows that with a decimation rate of eight, a minute of recording takes up nearly two gigabytes of hard disk space. With the typical size of commodity hard drives today being 80-100 gigabytes, it would take under an hour to completely fill the disk. Obviously, doubling the decimation rate would halve this requirement, at the expense of geolocation precision. However, subsequent processing of the data might increase the amount of storage needed. This is the case with the GNU Radio tools, which store each complex sample as a pair of four byte floating point values for a total of eight bytes per complex sample.

The storage requirements make it necessary to either do real-time or near-real time processing of the data or to offload the data regularly to make room for new collection.

## C. CONCLUSION

This chapter discussed all of the implementation details that are necessary for constructing a software radio based on the mathematical theory discussed in Chapter II. What follows are the results of putting this implementation to the test, using two computers each connected to a separate USRP, one to perform the signal generation and transmission, and the other to receive the signal and perform the partial demodulation and detection of the time-of-arrival. The following chapter also includes an analysis of the actual accuracy and precision of the system as a whole.

## IV. EXPERIMENTATION

Now that the theory and implementation has been discussed, a set of real-world results helps to understand how well the implementation works in practice and how feasible a software radio locator actually is using commodity hardware. The experiment conducted uses two laptop computers, two USRPs with daughter-boards that allow transmission and reception at 900 GHz, and some customized software discussed in Chapter III.

### A. SET UP

For the experiment, two computers were used, each attached to its own USRP. They were configured according to the parameters listed in Tables 3 and 4. The two USRPs were set on a desk next to each other, with the antenna of each about one foot apart.

Table 3. Transmitting computer settings.

DAC Rate	$128 \times 10^6 \text{ samples/s}$
Interpolation Rate	32
Sample Rate	$4 \times 10^6 \text{ samples/s}$
File Size	16384 bytes = 4096 Complex Samples (16 I bits followed by 16 Q bits)

Table 4. Receiving computer settings.

ADC Rate	$64 \times 10^6 \text{ samples/s}$
Interpolation Rate	16
Sample Rate	$4 \times 10^6 \text{ samples/s}$
File Size	Variable, depending on amount of time receiving.

## B. TRANSMISSION

The transmitted signal samples for the GSM access burst were calculated in accordance with the procedure outlined in previous chapters and stored in a 16384 byte file. This file size allows for 4096 complex samples. Since the burst itself only takes 665 samples, the file was padded with samples of zero amplitude so that transmission essentially pauses for a 3431-sample time period when not sending the access burst. A small program was written with the GNU Radio software to send the contents of the file repeatedly until the operator hit the Enter key on the keyboard. In effect, the same access burst signal was sent repeatedly, every 4096 samples.

## C. RECEPTION

For reception, the GNU Radio software was used to read the multiplexed I and Q samples from the USRP and immediately write them to a file. No further processing was done to the samples at this stage.

After the reception was complete, demodulation was performed on the raw I and Q samples by a separate program, listed in Appendix G. It should be noted that the demodulation was not done in real time. The GNU Radio software does have the capability to demodulate FM signals in real time. This was tested and does indeed work. However, as discussed below, additional flexibility in the FM demodulation was needed, so a separate program was used instead.

### 1. Initial Results

The first FM demodulation, shown in Figure 9, shows a significant amount of power when no signal is being transmitted. In fact, the noise is significantly more powerful than the signal after being FM demodulated. This is to be expected, since the white noise has large random variations in instantaneous frequency, but this problem had to be solved.

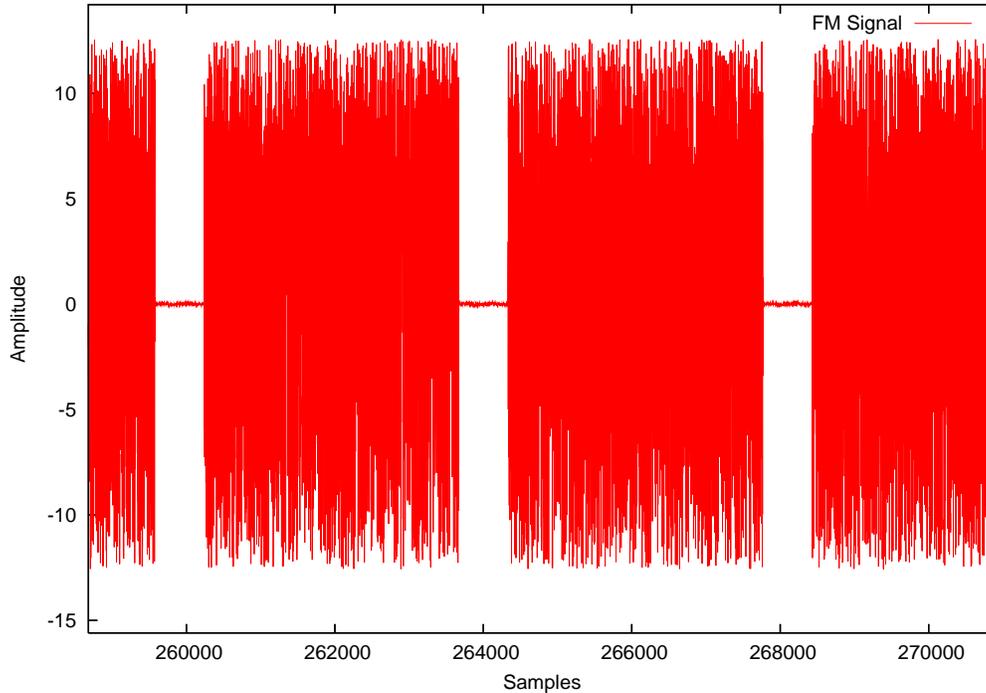


Figure 9. Demodulated GSM signal, no power measuring.

Since the carrier frequency is only present when the GSM signal is being transmitted, a standard FM demodulator is not sufficient for an accurate demodulation and detection of the signal. What is needed is an additional component of the FM demodulator that measures the power of the I and Q channels for each sample to detect the presence of a signal, in which case the FM demodulator is turned on. Otherwise, no demodulation occurs and the white noise passes through unchanged.

After implementing the power detection component, the results show a significantly better signal-to-noise ratio (SNR) after demodulation. However, large transients occur at the edge of the access burst bits, caused by finding the instantaneous frequency of the first signal sample with respect to the noise sample immediately preceding it. For this reason, a limiter was added to the FM demodulator to cut off these large spikes in amplitude. Figure 10 shows the result, with the bursts in red being repeated regularly. Notice how the I and Q channels vary from burst to burst. This is due to a slight frequency mismatch between the USRP's center frequency

and the frequency of the carrier. Although this does not affect the FM demodulated signal very much, the fact that a non-zero frequency component has been introduced into the I and Q channels means that correlating against the raw I and Q data is not possible.

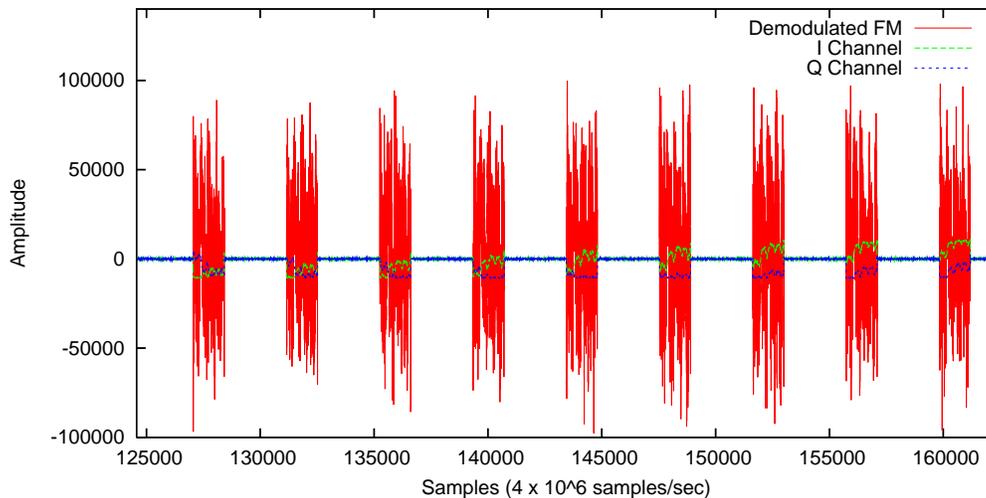


Figure 10. Demodulated GMSK signal, repeated bursts.

Figure 11 shows a magnified view of a single burst with the I and Q channels superimposed on it. This gives a much better idea of what the signal actually looks like upon reception.

## 2. Signal-to-Noise Ratio

The signal power and noise power of the incoming I and Q channels were measured while performing the demodulation and the SNR was found to be 29.3 dB. This was found by calculating the average signal-plus-noise power ( $P_{SN_{avg}}$ ) and the average noise power ( $P_{N_{avg}}$ ) during the demodulation and using

$$SNR = 10 \log_{10} \left( \frac{P_{SN_{avg}} - P_{N_{avg}}}{P_{N_{avg}}} \right) \quad (4.1)$$

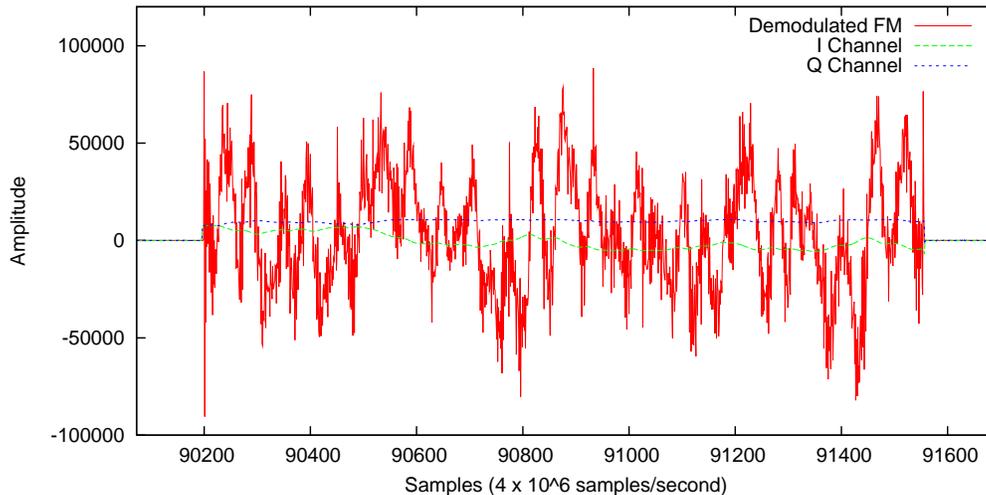


Figure 11. Demodulated GMSK signal, single burst.

## D. CORRELATION

Correlation was done after reception using the C++ program listed in Appendix H. This program takes the previously calculated portion of the access burst training sequence (not the entire access burst) shown in Figure 8 and the incoming signal, which consisted of multiple repeated access bursts transmitted every 1.024 milliseconds, or every 4096 samples, at the rate of  $4 \times 10^6$  samples/s.

### 1. Algorithm

Since correlation shows a sharp peak when the target matches the incoming signal, the algorithm stored the index of the sample of each local maximum above a certain threshold and compared those indexes to calculate the number of samples between each maximum. The threshold was used because, otherwise, all local maxima would be seen as peak values, and we only wanted the peaks that actually represented a signal match.

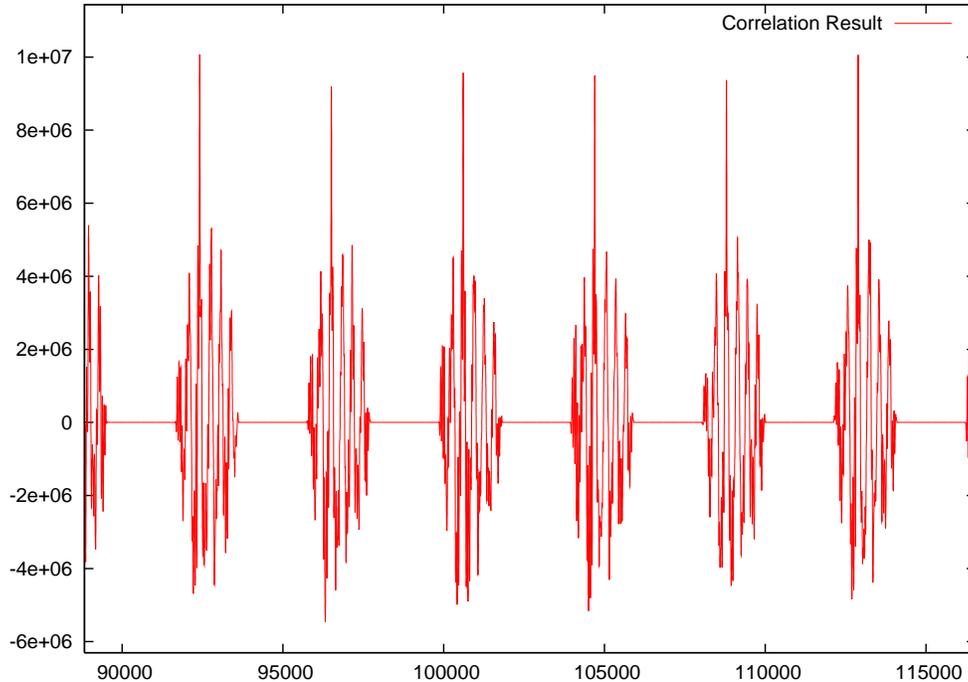


Figure 12. Result of correlation with target sequence.

This correlation algorithm, although fairly simple to implement and easy to understand, has an inherent inaccuracy that can be expected to cause an error of a single sample. This happens when the maximum value of the correlation is nearly centered between two samples, so that the two peak values above the threshold have nearly equal magnitude. In other words, if the sampling rate were twice as high, neither of the samples at the lower sampling rate would be the maximum, but the sample in between would be. Figure 13 illustrates this error.

The inherent inaccuracy of the algorithm, coupled with a small error from noise, only accounts for occasional errors of  $\pm 1$  sample. Testing showed that the maximum error was much greater, so it was apparent that other factors such as channel noise play a much larger role in the error than the algorithm, so no effort was made to increase the algorithm's accuracy.

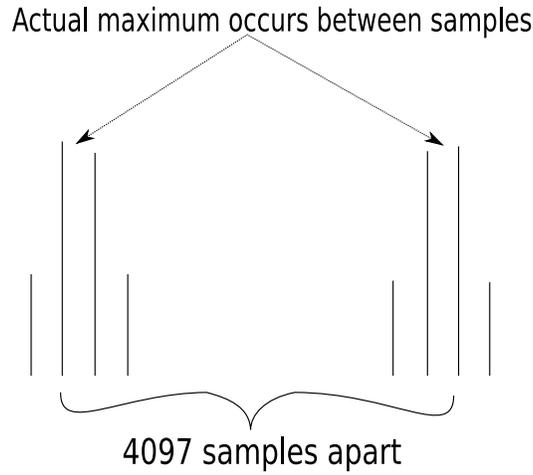


Figure 13. Algorithmic jitter in correlator.

## 2. Results

A sample of the results for a 0.1 s reception is shown in Table 5. Complete results for the same 0.1 s reception are shown in Appendix A. Statistics based on result data from a much longer six second reception (which is too large to reproduce in this thesis) are shown in Table 6 and Figure 14.

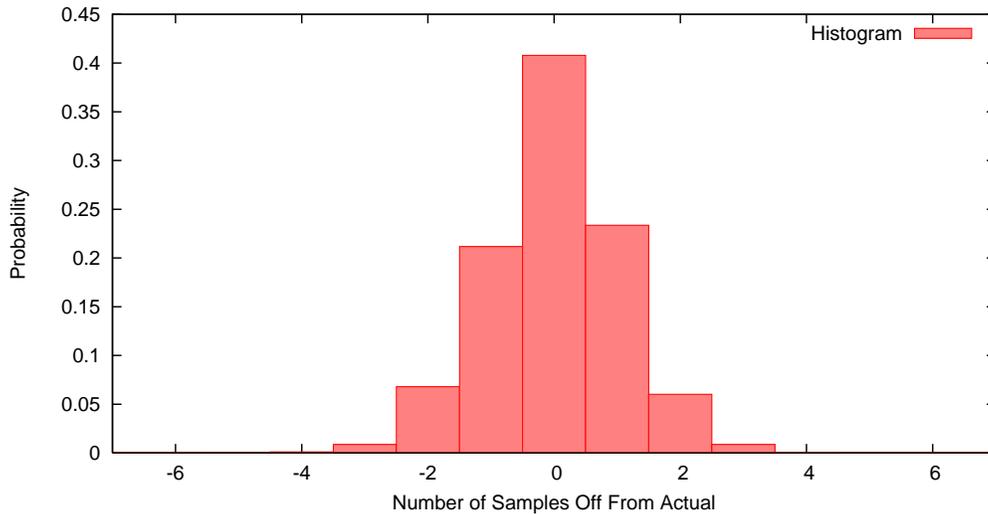


Figure 14. Histogram of sample errors based on 5101 data points.

Table 5. Correlation output data.

Sample	Number of Samples Between Maxima	Mean Number of Samples Between Maxima	Error
8396	4098	4098	-2
12492	4096	4097	0
16587	4095	4096.33	1
20683	4096	4096.25	0
24779	4096	4096.2	0
28875	4096	4096.17	0
32972	4097	4096.29	-1
37069	4097	4096.38	-1
41163	4094	4096.11	2
45258	4095	4096	1
...	...	...	...
241867	4096	4096.02	0
245964	4097	4096.03	-1
250059	4095	4096.02	1
254156	4097	4096.03	-1
258252	4096	4096.03	0

The results are encouraging in that the mean number of samples is extremely close to the actual value. Also, the mean error is much lower than one, so given enough sample data, this algorithm is very likely to produce an accurate result. As shown in the graph of the experimental histogram in Figure 14, the vast majority of estimates of the time-of-arrival are within one or two samples of the actual value.

Table 6. Correlation statistics for six second reception.

Number of Maxima Found, Rejecting Outliers	5101
Mean Number of Samples between Maxima	4095.995
Mean Error	$5.293 \times 10^{-3}$
Error Variance ( $\sigma_{err}^2$ )	1.14739
Error Standard Deviation ( $\sigma_{err}$ )	1.07212

Additionally, the error in this experiment is actually not as bad as the data shows. An error in arrival time of one or two samples not only causes an error in the measurement of time between the arriving burst and the previous burst, but also between the the arriving burst and the burst immediately following it. Essentially, this means that any single measurement error will show up as two measurement errors in the data. There is no way to fix this, however, since we cannot be certain about the accuracy of any single measurement. It is enough to note that the measured error in the data is actually an upper bound.

### **3. Software Radio Limitations**

A number of limitations of software radio became apparent during the test runs. Some of these limitations are inherent to software radio, while others are merely a matter of not having a powerful enough hardware configuration to accomplish certain tasks. The latter type of limitations would vanish with more expensive hardware or with hardware that has a similar cost to the hardware used here but that is designed with a focus on performance rather than flexibility.

#### *a. Hard Drive Throughput*

The most glaring limitation was the speed of the computer's storage mechanism compared to the incoming data rate of the USRP. It became apparent that the hard drive write speed was a limiting factor in the time it took to process the information coming in from the USRP. Short sub-second test runs resulted in no overflow of data, but runs of longer than a second resulted in the USRP's buffer overflowing whenever data was being written to the hard drive. This means that chunks of data of indeterminate size were lost whenever the operating system could not write data to the hard drive fast enough. Obviously, if one were dependent on timing the incoming signal based on the number of samples received (which we were), a buffer overflow is catastrophic. This is why it was necessary to reject the outliers when doing the statistical analysis of the accuracy of the correlation.

### *b. Processing Speed*

The next limitation, which the test data does not show, is that the processing speed of a general-purpose CPU is not fast enough to perform the FM demodulation and correlation in real time. Although the correlator.cpp program used to perform the correlation was not highly optimized for this problem, its run time still gives us a good idea of the scale of the problem. For a 0.1 second reception, the correlation took over two seconds to perform, a factor of 20. Only a cluster of the fastest general purpose CPUs can keep up with the incoming data rate in this case. Reduction of the decimation rate from 16 to eight or lower in order to get a more accurate result compounds this problem.

Until general-purpose CPU speeds become much faster, the real-time FM demodulation and correlation portion of the software radio is much more effectively performed by a special-purpose processor, high performance DSP, or FPGA.

### *c. USB Data Rate*

The USB data rate, although theoretically able to provide up to  $8 \times 10^6$  samples/s, worked much better with a decimation rate of 16, which corresponds to an incoming rate of  $4 \times 10^6$  complex samples/second. A decimation rate of eight was found to result in too many buffer overflows to be usable for estimating the time-of-arrival. Even if the system were capable of handling a decimation rate of eight, not having to do decimation at all would mean a much greater potential to accurately determine arrival time. A software radio receiver that used a higher data rate connection, like Gigabit Ethernet or PCI, would be capable of providing a higher data rate. Again, this would likely result in hardware that is more expensive and less flexible than the USRP, so this limitation should be seen as a trade-off.

## **E. LOCATING THE EMITTER**

Unfortunately, the precision achieved using the USRP did not allow testing using multiple USRPs as receivers to see how accurately they could locate an emitter.

Using a decimation rate that would allow for no buffer overflows, the emitter would have been out of range of the receiver before a difference in time-of-arrival could be detected. Instead, a simulation was conducted using the experimentally determined precision as a guide.

Suppose that three receivers were stationed at the corners of a  $1000 \times 1000$  space grid. The spaces can represent any distance, so meters were chosen since they give a good range to measure the accuracy graphically. Figure 15 shows these three receivers and an emitter located at point (200, 300) on the grid, with an assumed precision of 1 m. Notice how the three calculated curves for the time-difference-of-arrival intersect at a single point. This represents a very accurate and precise measurement that would be possible if the sampling rate of the USRP were four times higher ( $256 \times 10^6$  samples/s) and no decimation were used. The precision can be verified using Equation (2.23) where  $T_s = 1/256 \times 10^6$  samples/s. This is not currently possible with the USRP.

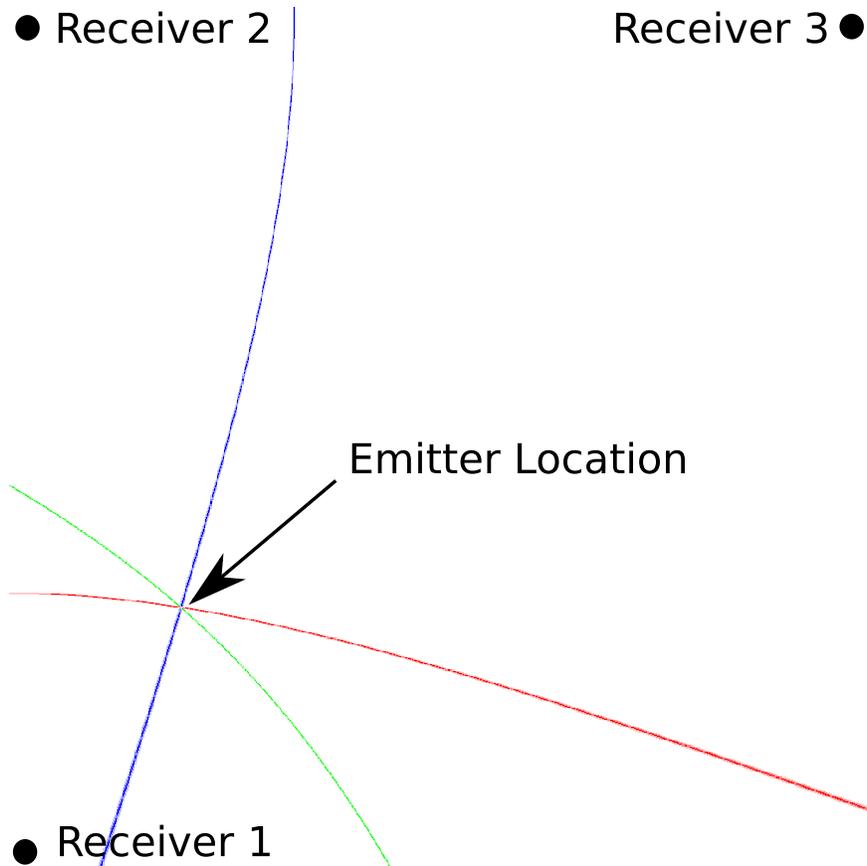


Figure 15. Emitter location with one-meter precision.

Figure 16 shows a much more realistic simulation with a 37 meter precision, the highest possible precision for the USRP using a decimation rate of eight, as shown in Section (E.2) of Chapter II. The darker colored bars indicate the region where the emitter could be located if no errors in the amount of samples were made, which occurs with a probability of 0.41 according to experimental data. The slightly lighter color indicates an error of one sample (0.44 probability), and the lightest color indicates an error of two samples (0.13). In effect, the darkest color band represents

the inherent 37 meter imprecision of the system based on Equation (2.23) and each successively lighter color band around it represents another 37 meter (or single sample) imprecision. Based on the experimental histogram shown in Figure 14, the emitter would be located within the intersection of the colored curves more than 98% of the time.

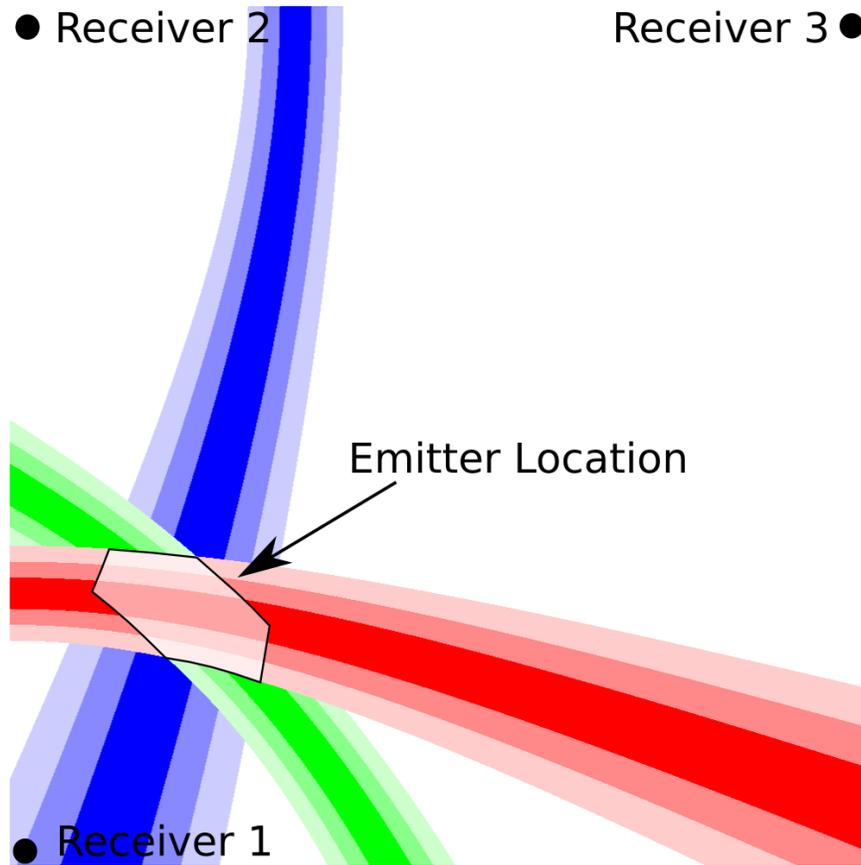


Figure 16. Emitter location with 37-meter precision.

## F. CONCLUSION

At this point, the implementation of the software radio has been tested and has shown to be reasonably precise, such that it could be used in applications where pinpoint accuracy is not required. This chapter has shown that minor modifications to the hardware specifications could result in a much greater accuracy and precision

using the same algorithms. The following chapter discusses the conclusions that can be taken from this research and the possible directions future research could take.

## V. CONCLUSION

### A. CONCLUSIONS

This research has shown that performing geolocation of a GSM signal using software defined radio is possible, and even accurate, with some caveats. The USRP is a very low-cost device that was designed for flexibility rather than performance but has the potential to perform a geolocation with an 80 meter radius in over 98 percent of the test cases. This is certainly good enough for applications where pinpoint accuracy is not required. The accuracy and precision can be much greater with minor changes in hardware.

The most important limitation is the slow speed of general-purpose hardware that necessitates performing the analysis of received data in non-real time, which creates other limitations having to do with lack of fast storage space, processing power, etc. This points to the need to push more functionality out from software and onto the hardware, which would reduce the flexibility. However, by using reconfigurable, programmable hardware like FGPAs, most of the benefits of software radio would be retained while still gaining the performance required for real-time operation.

### B. RECOMMENDATIONS

Time constraints precluded taking the next step in improving this system, which would be to alter the code on the USRP's FPGA to perform the correlation of the target signal with the received data in hardware instead of in software. That way, the USRP could simply send the number of samples and the correlation data instead of sending the raw samples to the computer for processing. Additionally, since the FPGA performs the decimation of the signal by default, this could be changed such that no decimation occurred prior to the correlation. This would allow for an accuracy of about 5 meters using only the USRP.

The idea of performing correlation of the I and Q signals without performing the FM demodulation could work if either an RF front-end were developed that could exactly match the carrier frequency and phase or if the phase and frequency error between the carrier and the receiver were to be corrected in software. Correction of the phase and frequency of the received I and Q signals in software may turn out to be more computationally expensive than simply doing the FM demodulation. However, time constraints prevent exploration of this more fully.

Finally, the USRP is not the only hardware platform that can be used to implement a software radio. The algorithms and software developed in this thesis could be used nearly unchanged with more powerful hardware to get much better results. Specifically, an RF front-end and antenna developed specifically for the reception of GSM signals would likely perform much better for the reception of that signal than the USRP daughter-board designed with a range of signals in mind.

## APPENDIX A. CORRELATION DATA

Sample	Number of Samples Between Maxima	Mean Number of Samples Between Maxima	Error
8396	4098	4098	-2
12492	4096	4097	0
16587	4095	4096.33	1
20683	4096	4096.25	0
24779	4096	4096.2	0
28875	4096	4096.17	0
32972	4097	4096.29	-1
37069	4097	4096.38	-1
41163	4094	4096.11	2
45258	4095	4096	1
49356	4098	4096.18	-2
53451	4095	4096.08	1
57548	4097	4096.15	-1
61644	4096	4096.14	0
65740	4096	4096.13	0
69835	4095	4096.06	1
73931	4096	4096.06	0
78027	4096	4096.06	0
82123	4096	4096.05	0
86219	4096	4096.05	0
90314	4095	4096	1
94410	4096	4096	0
98506	4096	4096	0
102607	4101	4096.21	-5

106697	4090	4095.96	6
110795	4098	4096.04	-2
114891	4096	4096.04	0
118987	4096	4096.04	0
123084	4097	4096.07	-1
127179	4095	4096.03	1
131275	4096	4096.03	0
135372	4097	4096.06	-1
139467	4095	4096.03	1
143563	4096	4096.03	0
147660	4097	4096.06	-1
151755	4095	4096.03	1
155849	4094	4095.97	2
159947	4098	4096.03	-2
164042	4095	4096	1
168139	4097	4096.02	-1
172235	4096	4096.02	0
176331	4096	4096.02	0
180427	4096	4096.02	0
184523	4096	4096.02	0
188620	4097	4096.04	-1
192715	4095	4096.02	1
196811	4096	4096.02	0
200907	4096	4096.02	0
205003	4096	4096.02	0
209097	4094	4095.98	2
213196	4099	4096.04	-3
217291	4095	4096.02	1

221387	4096	4096.02	0
225482	4095	4096	1
229579	4097	4096.02	-1
233675	4096	4096.02	0
237771	4096	4096.02	0
241867	4096	4096.02	0
245964	4097	4096.03	-1
250059	4095	4096.02	1
254156	4097	4096.03	-1
258252	4096	4096.03	0

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## APPENDIX B. UNDERSTANDING COMMON LISP EXAMPLES

Many of the code examples in this thesis are written in Common Lisp. Common Lisp has several advantages for non-real-time software radio applications, including native support of complex numbers, support for rational numbers that don't introduce floating point error, and a very concise syntax that makes algorithms easy to understand, once you understand how Lisp works. This very short introduction to Common Lisp is intended to provide a good enough background so that the examples included in this thesis are clear.

### 1. S-EXPRESSIONS

Lisp programs are composed of called s-expressions, which take the following form:

```
(function argument0 argument1 ...)
```

An s-expression consists of a pair of parenthesis, a function, and any required arguments to that function following the function name. A classic example may remind you of a reverse Polish notation calculator. Here is a list of expressions followed by the result of evaluating them:

```
(+ 20 22) => 42  
(* 2 3 4) => 24  
(- 9 8)   => 1
```

S-expressions may be nested, so that the result of one expression can be used as the argument to another:

```
(+ 20 (* 11 2)) => 42
```

For the most part, Lisp programs consist of a series of nested s-expressions that are evaluated sequentially. The result of a Lisp program (or function) is the value returned by the final s-expression.

Lisp has quite a few built-in functions, but wouldn't be very useful unless you could define your own. This is done using what is called a special form in Lisp, specifically the special form `defun`. Suppose we want a function that takes a single argument and adds 3 to it:

```
(defun add3 (x)
  (+ x 3))
```

The first part of the `defun` form is the word `defun`, which indicates that we're defining a function. The second item is the name of the function. The third part is a list of arguments, enclosed in parentheses. The reason `defun` is called a special form is because the argument list is not evaluated, as an s-expression would be. It's treated differently. The last part of the `defun` form is called the body, and consists of a series of s-expressions, the last of which produces the result of the function.

The argument list contains a symbol used to represent the value that will be passed into `add3` when it is called. The final (and only) s-expression in the `add3` `defun` is `(+ x 3)`, so the `add3` function will return this value as its result:

```
(add3 4) => 7
```

Just as in most other programming languages, in Lisp it may become necessary to define local variables, which are variables which only have meaning within the scope of the function. This is done with another special form, called `let`:

```
(defun add-stuff (x)
  (let ((y 4)
        (z 5))
    (+ x 3)))
```

Let works just like the mathematical term, in that it assigns a value to a symbol (or multiple symbols.) That assignment exists everywhere inside the parentheses that began the let form, so this is valid syntax:

```
(let ((a 0))
  (defun get-a ()
    a)
  (defun set-a (x)
    (setf a x)))
```

The preceding code defined two functions that had access to the `a` symbol, `get-a` and `set-a`. Executing `(set-a 4)` followed by `(get-a)` would return the value 4.

Since everything in Common Lisp is an s-expression, the preceding covered just about all that is necessary to know about the syntax of Common Lisp to understand the code included in this thesis.

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## APPENDIX C. GMSK.LISP

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;
;; File: gmsk.lisp
;; Author: Ian Larsen
;; Description: Functions for generating and
;;              modulating a digital gmsk signal.
;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;; Define j as a complex number
(defparameter j #C(0 1))

;; Converts degrees to radians
(defun radian (x)
  (/ (* x pi) 180))

;; FM Modulator
;; Calculates a complex number based on the
;; phase angle of the last sample, the sample interval,
;; and the sampling frequency.
;; The reset function sets the phase angle to zero.
(let ((phase-angle 0))

  (defun reset ()
    (setf phase-angle 0))

  (defun modfm (sample-duration freq)
    (let ((phase-inc (* 360 sample-duration freq))) ;;in degrees
      (setf phase-angle (+ phase-angle phase-inc))
      (exp (* j (radian phase-angle))))))

;; This function generates the Gaussian shaped impulse response
;; specified by the GSM spec. To avoid division by zero errors,
;; we generate the positive side and mirror it to the negative.
;; This is also more efficient.
(defun impulse-response (Tb tm-seq)
```

```

(let* ((delta (/ (sqrt (log 2)) (* 2 pi 0.3)))
      (response (map 'array
                    (lambda (tm)
                      (/
                        (exp (/
                          (* -1 (expt tm 2))
                          (* 2 (expt delta 2) (expt Tb 2))))
                        (* (sqrt (* 2 pi)) delta Tb)))
                    tm-seq)))
      (concatenate 'array
                   (reverse response)
                   response)))

;; Takes a stream of bits and outputs a differentially
;; encoded stream of the same length. The optional last
;; parameter allows us to specify what the state of the
;; prior encoded bit should be. It defaults to 1, in
;; compliance with the GSM spec, which states the differential
;; encoder's state at the start of encoding should be as if
;; a series of 1's had just been encoded.
(defun diff-encoder (bits &optional (last 1))
  (let ((p last))
    (map 'array (lambda (x)
                  (let ((d (logxor x p)))
                    (setf p x)
                    (- 1 (* 2 d))))
          bits)))

;; Takes a list of bits and returns the same bitstream,
;; with the bits repeating n times.
;; This is how we generate rectangular symbols from impulses.
(defun make-bitstream (bits n)
  (let ((result #()))
    (map nil (lambda (x)
              (setf result
                    (concatenate
                     'array
                     result
                     (make-array n :initial-element x))))
          bits))
  (diff-encoder bits))

```

```

result))

;; Make a sequence of numbers, starting with startnum,
;; and incrementing by increment. Defaults to 1 as the
;; increment. Similar to Matlab's linspace function.
(defun make-seq (startnum length &optional (increment 1))
  (let ((a (make-array length :initial-element startnum)))
    (map 'array (let ((acc (- increment)))
                  (lambda (y)
                    (setf acc (+ acc increment))
                    (+ y acc)))
          a)))

;; Generate the gmsk Gaussian filter based on values
;; in the GSM spec.
(defun gsm-gmsk (sample-interval)
  (let* ((Tb 6/1625000) ;; bit duration
         (BTb 0.3) ;; bandwidth time product
         (B (/ BTb Tb))
         (filter-length
          (* 2 (/ Tb sample-interval)))) ;;should be an integer
    (format t "Filter length: ~a~%" filter-length)
    (impulse-response
     Tb (make-seq 0 filter-length sample-interval))))

;; This will take an array of numbers, find the
;; absolute maximum, and divide all numbers by that.
;; This is a linear scaling normalization.
(defun normalize (l)
  (let ((max (abs (reduce (lambda (a b)
                          (if (> (abs a) (abs b))
                              (abs a)
                              (abs b)))
                        l))))
    (if (> max 0)
        (map 'array
             (lambda (x)
               (/ x max))
             l)
        l)))

```

```

1)))

;; Perform convolution of stream and base.
;; base should be longer than stream.
(defun convolve (stream base)
  (let* ((len (length stream))
         (iter (+ (length stream) (length base) -1))
         (result (make-array iter :initial-element 0)))
    (dotimes (count iter nil)
      (setf (aref result count)
            (reduce #'+
                    (map 'array
                        (lambda (r s)
                          (* r s))
                        (if (> len count)
                            (subseq stream (- len count 1))
                            stream)
                        (if (> len count)
                            base
                            (subseq base (- count len -1)))))))
      result))

;; Return an array of every "rate" samples from array x.
(defun decimate (x rate)
  (let ((result ()))
    (loop for i from 0 to (1- (length x)) do
      (if (eq (mod i rate) 0)
          (setf result
                (concatenate 'array result (list (aref x i))))))
    result))

;;; This sets everything in motion (main procedure)
;;; results are stored in the four files, filt.dat, gauss.dat,
;;; bitstream.dat, and gmsk.asc
;;; gmsk.asc is the final signal that we'll be sending, the rest
;;; make nice graphs so you can see what's happening at each stage.
(let* ((filtout (open "filt.dat" :direction :output
                     :if-exists :supersede :if-does-not-exist :create))

```

```

(gaussout (open "gauss.dat" :direction :output
           :if-exists :supersede :if-does-not-exist :create))
(bitsout (open "bitstream.dat" :direction :output
           :if-exists :supersede :if-does-not-exist :create))
(gmskout (open "gmsk.asc" :direction :output
           :if-exists :supersede :if-does-not-exist :create))
(buffer-size 4096) ;;number of samples in gmsk.asc
(Rs 52000000)      ;;Fast sample rate that we decimate later
(Ts (/ 1 Rs))     ;;Sample duration
(n0 13)           ;;Decimation rate to get 4x10^6 samples/sec
(n1 192)          ;;Number of times we repeat each bit
(delta 1625000/24) ;;1/4 270.833333 as per GSM spec (Rappaport pg 566)
(mag 30000)       ;;amplitude

(reset) ;;Sets phase angle to zero

(let* ((filter (gsm-gmsk Ts)) ;;make gaussian filter
      (bits (make-bitstream ;; create the access burst bits
        '(0 0 1 1 1 0 1 0 ;;extended tail bits
          0 1 0 0 1 ;;training sequence: 41 bits
          0 1 1 0 1
          1 1 1 1 1
          1 1 0 0 1
          1 0 0 1 1
          0 1 0 1 0
          1 0 0 0 1
          1 1 1 0 0 0

          0 1 0 1 1 0;;encrypted bits: 36 bits
          0 1 0 1 1 0
          1 0 1 1 0 0
          0 0 1 0 0 1
          1 0 0 1 0 1
          0 1 0 1 1 1

          0 0 0 ;;tail bits
        ) n1))
      ;; now filter the access burst
      (filtered (normalize (convolve filter bits)))
      ;; decimate to get proper sample rate
      (modwave (decimate filtered n0))
      ;; create padding for 4096 samples

```

```

(zerobuffer (make-array
             (* (ceiling (/
                    (- buffer-size
                       (length modwave))
                    2))
              2)
             :initial-element 0)))

;; Write gaussian filter to file
(map nil (lambda (x)
          (format gaussout "~A~%" x))
      (normalize filter))

;; Write raw bits to file
(map nil (lambda (x)
          (format bitsout "~A~%" x))
      bits)

;; Write filtered bits to file
(map nil (lambda (x)
          (format filtout "~A~%" x))
      modwave)

;; Write GMSK signal with zero padding
(map nil (lambda (x)
          (format gmskout "~A~%" x))
      zerobuffer)
(map nil (lambda (x)
          (let ((complex (modfm Ts (* delta x))))
              (format gmskout "~f~%~f~%"
                        (* mag (realpart complex))
                        (* mag (imagpart complex))))
          modwave)
      (map nil (lambda (x)
                (format gmskout "~A~%" x))
            zerobuffer)

;; Close files, and we're done.
(close filtout)
(close gaussout)
(close bitsout)
(close gmskout))

```

## APPENDIX D. TRANSMIT.PY

```
#!/usr/bin/python

#####
# File: transmit.py          #
# Author: Ian Larsen        #
# Description: Uses the gnuradio python #
# module to tune the USRP and transmit #
# the raw GMSK I/Q data to it.      #
#                                  #
#####

from gnuradio import gr
from gnuradio import usrp

def build_graph ():

    fg = gr.flow_graph ()

    #set the USRP as our destination
    dest = usrp.sink_c(0, 32) #32 is the interpolation rate
    print "Interp rate: " + str(dest.interp_rate())
    print "DAC Freq:      " + str(dest.dac_freq())
    print "N Channels:    " + str(dest.set_nchannels(1))

    #Tune the USRP and daughterboard to the proper frequency
    subdev = usrp.selected_subdev(dest, (0, 0))
    print dest.pga_min()
    print dest.pga_max()
    dest.set_pga(0, -0.0) #20 dB of gain
    dest.set_pga(1, -0.0) #20 dB of gain
    print dest.pga(0)
    print dest.pga(1)
    freq = dest.tune(0, subdev, 900e6)
    if freq:
        print "Frequency: 900e6"

    #set up the gmsk data as the source
    src = gr.file_source(gr.sizeof_gr_complex, "gmsk.bin", True)
```

```
#connect the source to the destination
fg.connect (src, dest)

return fg

#main loop
if __name__ == '__main__':
    fg = build_graph ()
    fg.start ()
    raw_input ('Press Enter to quit: ')
    fg.stop ()
```

## APPENDIX E. RECEIVE.PY

```
#!/usr/bin/python

#####
# File: receive.py          #
# Author: Ian Larsen       #
# Description: Uses the gnuradio python #
# module to tune the USRP and receive #
# the raw GMSK I/Q data from it,      #
# demodulate it using an FM demodulator, #
# and store the result in file "fmraw.dat" #
#                                     #
#####

from gnuradio import gr
from gnuradio import usrp
import time

def build_graph ():

    #set decimation rate and center frequency
    decim = 16
    center_freq = 900e6

    #build flow graph
    # this is simple, it comes in from the USRP,
    # into the demodulator, and out to a file.
    fg = gr.flow_graph ()
    u = usrp.source_c(0, decim)

    #select the flex900 daughterboard
    subdev = usrp.selected_subdev(u, (0,0))
    g = subdev.gain_range()

    #set the gain and tune
    subdev.set_gain((g[0] + g[1]) * 0.5)
    freq = u.tune(0, subdev, center_freq)
```

```
#define the components
sink0 = gr.file_sink(gr.sizeof_float, "fmraw.dat")
demod = gr.quadrature_demod_cf(4)

#connect the components
fg.connect (u, demod)
fg.connect (demod, sink0)
#return completed flow graph
return fg

# Main - only receive for 0.2 seconds, otherwise
# we'll get a huge amount of data
if __name__ == '__main__':
    fg = build_graph ()
    fg.start ()
    time.sleep(0.2)
    fg.stop ()
```

## APPENDIX F. RECEIVE2.PY

```
#!/usr/bin/python

#####
# File: receive2.py          #
# Author: Ian Larsen        #
# Description: Uses the gnuradio python #
# module to tune the USRP and receive #
# the raw GMSK I/Q data from it,      #
# and store the result in file "fmraw.dat" #
#                                     #
#####

from gnuradio import gr
from gnuradio import usrp
import time

def build_graph ():

    # set decimation rate and center frequency
    decim = 16
    center_freq = 900e6

    #build flow graph
    fg = gr.flow_graph ()
    u = usrp.source_c(0, decim)

    #get access to flex900 daughterboard
    subdev = usrp.selected_subdev(u, (0,0))

    #set gain and tune
    g = subdev.gain_range()
    subdev.set_gain((g[0] + g[1]) * 0.5)
    freq = u.tune(0, subdev, center_freq)

    #generate component - just a file sink here.
    sink0 = gr.file_sink(gr.sizeof_gr_complex, "fmraw.dat")
```

```
#connect output of USRP straight to file
fg.connect (u, sink0)

#return the flow graph
return fg

# Main - only receive for 0.1 second, otherwise
# we'll get a huge amount of data
if __name__ == '__main__':
    fg = build_graph ()
    fg.start ()
    time.sleep(0.1)
    fg.stop ()
```

## APPENDIX G. FM\_DEMOD.PL

```
#!/usr/bin/perl

#####
# File: fm_demod.pl          #
# Author: Ian Larsen        #
# Description: Reads 8-byte binary #
# complex IQ samples stored by gnuradio #
# and demodulates them as an FM signal. #
# Signal power is measured in order to #
# decide when an FM signal is being #
# received, as opposed to just noise. #
# A limiter is employed to eliminate #
# transients that occur when the FM #
# demodulator is turned on and off. #
#                               #
#####

use strict;

#open input and output files
open(INFILE, shift) or die "Couldn't open input file.\n";
my $outname = shift or die "Couldn't open output file.\n";
open(FMOUT, ">$outname.fm") or die "Couldn't open output file.\n";
open(FMBIN, ">$outname.bin") or die "Couldn't open output file.\n";
open(IOUT, ">$outname.i") or die "Couldn't open output file.\n";
open(QOUT, ">$outname.q") or die "Couldn't open output file.\n";
#set decimation rate, 16 is default.
my $decim = shift || 16;

# Find sample duration based on decimation rate
my $dt = 1 / (64000000 / $decim);
print "Sample interval: $dt\n";

my $oldI = 0;
my $oldQ = 0;
my $power = 0;
my $thresholdPower = 1e13; #130dB
my $signalNoisePower = 0;
my $signalTime = 0;
```

```

my $noisePower = 0;
my $noiseTime = 0;
my $data;

# Read an IQ sample, one at a time
while (read(INFILE, $data, 8))
{
    # translate binary float data into
    # a form we can use
    my ($i, $q) = unpack("ff", $data);

    # write IQ samples to files
    print IOUT "$i\n";
    print QOUT "$q\n";

    # Avoid division by zero
    if ($i == 0)
    {
        $i = 0.0001;
    }
    if ($q == 0)
    {
        $q = 0.0001;
    }

    my $fm = 0;

    # Check signal power.
    # If it's over the threshold, we demodulate
    # by differentiating the phase to get
    # instantaneous frequency.
    # Otherwise, we output a zero.
    $power = ($i * $i + $q * $q) / $dt;
    if ($power > $thresholdPower)
    {
        $fm = ((($i * ($q - $oldQ) / $dt) -
                ($q * ($i - $oldI) / $dt)) /
                (($i * $i) + ($q * $q)));
        if ($fm > 100000 || $fm < -100000)
        {
            $fm = 0;
        }
    }
}

```

```

        # Keep track of total signal power
        # for SNR measurement
        $signalNoisePower += $power;
        $signalTime += $dt;
    }
    else
    {
        # Keep track of total noise power
        # for SNR measurement
        $noisePower += $power;
        $noiseTime += $dt;
    }

    # Write output to files
    print FMOUT "$fm\n";
    print FMBIN pack("f", $fm);

    $oldI = $i;
    $oldQ = $q;
}

#print results
my $avgSNP = ($signalNoisePower / $signalTime);
my $avgNP = ($noisePower / $noiseTime);
print "Average signal plus noise power: $avgSNP \n";
print "Average noise power: $avgNP \n";
print "Calculated average signal power: " . ( $avgSNP - $avgNP ) . "\n";
print "SNR: " . (( $avgSNP - $avgNP ) / $avgNP) . "\n";

close INFILE;
close FMOUT;
close FMBIN;
close IOOUT;
close QOUT;

```

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## APPENDIX H. CORRELATOR.CPP

```
// File: correlator.cpp
// Author: Ian Larsen
// Description: Perform correlation of an incoming
// signal and determine the time of arrival
// by finding the maximum correlation value.
// Show the absolute and average number of
// samples between maxima.
// Calculate average error for known sample distance.
```

```
using namespace std;
```

```
#include <iostream>
#include <fstream>
#include <math.h>
#include <stdint.h>
#include <unistd.h>
#include <sys/mman.h>
#include <sys/types.h>
#include <sys/stat.h>
#include <fcntl.h>
```

```
#define TARGETSIZE 665
#define THRESHOLD 8e6
```

```
//Perform cumulative sum of multiplication
//between two arrays
float
multAdd (float *data, float *target, int size)
{
    float result = 0;

    for (int i = 0; i < size; i++)
    {
        result += data[i] * target[i];
    }
}
```

```

    }

    return result;
}

int
main(int argc, char *argv[])
{
    ifstream target;
    ofstream output;
    ofstream output1;
    float targetArray[TARGETSIZE]; //length of target data in samples
    float *data = NULL;
    int fd = open("incoming.dat", O_RDONLY);

    if (fd < 0)
    {
        cerr << "Couldn't open incoming data file" << endl;
        return 1;
    }

    //open ideal sampled signal file
    target.open("target.dat");
    if (!target)
    {
        cerr << "Couldn't open target file" << endl;
        return 1;
    }

    //load target data into array
    for (int i = 0; i < TARGETSIZE; i++)
    {
        target >> targetArray[i];
    }
    target.close();

    //map first 1 megabyte of target file into memory
    data = (float *) mmap(0x0, 0x100000, PROT_READ, MAP_SHARED, fd, 0);

    cout << "Data mapped to: " << data << endl;
}

```

```

//open output files
output.open("correlated.dat");
output1.open("statistics.dat");
if (!output || !output1)
{
    cerr << "Couldn't open output file" << endl;
    return 1;
}

//Do correlation for entire input stream
int numtimes = (0x100000 / (sizeof(float))) - TARGETSIZE;
float max = -1;
int maxOccuredAt, lastMaxAt = 0;
int maxes = 0;
int maxError = 0;
double total = 0;
double totalError = 0;
double totalVariance = 0;

for (int i = 2000; i < numtimes; i++)
{
    float cumSum = multAdd(&data[i], targetArray, TARGETSIZE);
    //Only check for maximum beyond a certain threshold,
    //otherwise we'd find all local maxima, which we don't want.
    if (cumSum > THRESHOLD)
    {
        if (cumSum > max)
        {
            max = cumSum;
            maxOccuredAt = i;
        }
    }

    //After we've found a max, we report it
    //only when we've dipped below the threshold.
    //This is how we make sure the maximum is the
    //maximum for the entire set of points above
    //the threshold.
    else if (cumSum < THRESHOLD && max > 0)
    {
        if (lastMaxAt > 0)
        {

```

```

        int difference = maxOccuredAt - lastMaxAt;
        int error = 4096 - difference;

        if (abs(error) > abs(maxError))
    {
        maxError = error;
    }

        maxes++;
        totalError += error;
        totalVariance += error ^ 2;
        total += maxOccuredAt - lastMaxAt;

        cout << "Max at: " << maxOccuredAt
            << ", " << maxOccuredAt - lastMaxAt
            << " samples between maxima. Average: "
            << total / maxes
            << " Error: " << error
            << endl;

        //LaTeX tabular output
        output1 << maxOccuredAt << " & "
            << maxOccuredAt - lastMaxAt << " & "
            << total / maxes << " & "
            << error << "\\\\" << endl;
    }
    lastMaxAt = maxOccuredAt;
    max = -1;
}

//save correlated data point to file
output << cumSum << endl;
}

//Report statistics
cout << "Total Samples: " << total << endl;
cout << "Number of maxima: " << maxes << endl;
cout << "Average: " << total / ((double) maxes) << endl;
cout << "Maximum Error: " << maxError << endl;
cout << "Mean Error: " << totalError / ((double) maxes) << endl;
cout << "Variance: " << totalVariance / ((double) maxes) << endl;
cout << "Std Dev: " << sqrt(totalVariance / ((double) maxes)) << endl;

```

```
output.close();  
output1.close();  
return 0;  
}
```

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# APPENDIX I. STATISTICS.PL

```
#!/usr/bin/perl

#####
#                                     #
# File: statistics.pl                 #
# Author: Ian Larsen                  #
# Description: Calculates statistics of #
# received correlation data.          #
# The results can be graphed in a    #
# histogram.                          #
#                                     #
#####

my $totalError = 0;

my %bins = ();

for ($i = -10; $i <= 10; $i++)
{
    $bins{$i} = 0;
}

my $count = 0;
my $outliers = 0;

open(INFILE, "statistics.dat");
while($val = <INFILE>)
{
    chomp($val);

    #reject outliers
    if ($val > -90 && $val < 90)
    {
        $totalError += $val;
        $bins{$val}++;
        $count++;
    }
    else
```

```

    {
        $outliers++;
    }
}
close(INFILE);

print "Rejected $outliers outliers.\n";
print "Data Points: $count\n";
open(OUTFILE, ">histogram.dat");
for ($i = -8; $i <= 8; $i++)
{
    print OUTFILE "$i " . ($bins{$i} / $count) . "\n";
    print "$i: " . $bins{$i} . " " . ($bins{$i} / $count) . "\n";
}
close(OUTFILE);

my $mean = $totalError / $count;

print "Mean: $mean\n";
#calculate variance

$count = 0;
open(INFILE, "statistics.dat");
while($val = <INFILE>)
{
    chomp($val);

    #reject outliers
    if ($val > -90 && $val < 90)
    {
        $totalVariance += ($val - $mean) ** 2;
        $count++;
    }
}
close(INFILE);

print "Variance: " . $totalVariance / $count . "\n";
print "Std Dev: " . sqrt($totalVariance / $count) . "\n";

```

## APPENDIX J. DISTANCE.LISP

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;; File: distance.lisp                                     ;;
;; Author: Ian Larsen                                    ;;
;; Description: Define a grid with                       ;;
;; listening posts.                                     ;;
;; Then calculate delays for                             ;;
;; each point on the grid.                              ;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;; Calculate distance between two points
(defun distance-between (a b)
  (let ((x (- (car b) (car a)))
        (y (- (cadr b) (cadr a))))
    (values
     (sqrt (+ (expt x 2) (expt y 2)))
     x
     y)))

;; Functions for colorizing the grid
;; and saving the output to a file
(let ((grid (make-array (* 4 1000 1000)
                       :initial-element #xff
                       :element-type '(unsigned-byte 8))))

  (defun clear-grid ()
    (setf grid (make-array (* 4 1000 1000)
                          :initial-element #xff
                          :element-type '(unsigned-byte 8))))

  (defun color-point (x y &optional (color #x333333ff))
    (if (or (< x 0) (< y 0) (> x 999) (> y 999))
        ()
        (progn
         (setf (aref grid
                    (* 4 (+ x (* 1000 (- 999 y)))))
               (logand (ash color -24) #xff))
```

```

      (setf (aref grid
              (+ 1 (* 4 (+ x (* 1000 (- 999 y))))))
            (logand (ash color -16) #xff))
      (setf (aref grid
              (+ 2 (* 4 (+ x (* 1000 (- 999 y))))))
            (logand (ash color -8) #xff))
      (setf (aref grid
              (+ 3 (* 4 (+ x (* 1000 (- 999 y))))))
            (logand color #xff))))

(defun save-rgb ()
  (let ((file (open "output.rgba" :direction :output
                   :element-type '(unsigned-byte 8)
                   :if-does-not-exist :create
                   :if-exists :supersede)))
    (write-sequence grid file)
    (close file)))

;; calculate the delay in grid spaces that
;; each listening post would see for a certain
;; coordinate
(defun calc-delays (point &rest lps)
  (let* ((delays
          (loop for lp in lps collect
                (distance-between point lp))))
    delays))

;; Go through entire grid and color each point
;; according to how close it is to actual
;; delay values. Precision can be varied.
(defun test-grid (&optional (precision 1))
  (let* ((emitter '(200 300))
         (lp0 '(0 0))
         (lp1 '(0 999))
         (lp2 '(999 999))
         (d0 (distance-between emitter lp0))
         (d1 (distance-between emitter lp1))
         (d2 (distance-between emitter lp2))
         (lp01diff (floor (- d1 d0)))
         (lp02diff (floor (- d2 d0))))
    ))

```

```

(lp12diff (floor (- d2 d1))))
(format t "~A ~A ~A%" (- d1 d0) (- d2 d0) (- d2 d1))
(clear-grid)
(loop for x from 0 to 999 do
  (loop for y from 0 to 999 do
    (color-point
     x
     y
     (let* ((l (calc-delays
                (list x y) lp0 lp1 lp2))
            (time01 (floor
                    (- (second l) (first l))))
            (time02 (floor
                    (- (third l) (first l))))
            (time12 (floor
                    (- (third l) (second l))))))
      (cond ((< (abs (- time01 lp01diff))
                precision)
             #xff0000ff)
            ((< (abs (- time01 lp01diff))
                 (* 2 precision))
             #xff8888ff)
            ((< (abs (- time01 lp01diff))
                 (* 3 precision))
             #xffccccff)

            ((< (abs (- time02 lp02diff))
                 precision)
             #x00ff00ff)
            ((< (abs (- time02 lp02diff))
                 (* 2 precision))
             #x88ff88ff)
            ((< (abs (- time02 lp02diff))
                 (* 3 precision))
             #xccffccff)

            ((< (abs (- time12 lp12diff))
                 precision)
             #x0000ffff)
            ((< (abs (- time12 lp12diff))
                 (* 2 precision))
             #x8888ffff)
            (t))))))

```

```
((< (abs (- time12 lp12diff))
      (* 3 precision))
 #xcccffff)

(t
 #xffffffff))))))

(save-rgb))

(test-grid 37)
```

## LIST OF REFERENCES

- [1] Jeffrey H. Reed. *Software Radio: A Modern Approach to Radio Engineering*. Prentice Hall, 2002.
- [2] David Adamy. *EW101: A First Course in Electronic Warfare*. Artech House, 2001.
- [3] Upendra Ramdat. Software Communications Architecture (SCA) compliant software radio design for Interim Standard 95B (IS-95B) transceiver. Master's thesis, Naval Postgraduate School, 2007.
- [4] Kian Wai Low. Software Communications Architecture (SCA) compliant software defined radio design for IEEE 802.16 wirelessman-OFDM™ transceiver. Master's thesis, Naval Postgraduate School, 2006.
- [5] Wai Kiat Chris Leong. Software defined radio design for an IEEE 802.11A transceiver using open source Software Communications Architecture (SCA) implementation::embedded (OSSIE). Master's thesis, Naval Postgraduate School, 2006.
- [6] Glenn D. Hartwell. Improved geo-spatial resolution using a modified approach to the complex ambiguity function (CAF). Master's thesis, Naval Postgraduate School, 2005.
- [7] Juan Sanfuentes. Software Defined Radio Design for Synchronization of 802.11a Receiver. Master's thesis, Naval Postgraduate School, 2007.
- [8] Juan P. Svenningsen. Modeling, simulation and implementation of a non-coherent binary-frequency-shift-keying (BFSK) receiver-transmitter into a field programmable gate array (FPGA). Master's thesis, Naval Postgraduate School, 2005.
- [9] Michel Mouly and Marie-Bernadette Pautet. *The GSM System for Mobile Communications*. Cell & Sys, 1992.
- [10] 3<sup>rd</sup> Generation Partnership Project; Technical Specification Group GSM/EDGE. *3GPP TS 45.002: Radio Access Network; Multiplexing and multiple access on the radio path (Release 7)*, 2007.
- [11] Gunnar Heine. *GSM Networks: Protocols, Terminology, and Implementation*. Artech House, 1998.
- [12] 3<sup>rd</sup> Generation Partnership Project; Technical Specification Group GSM/EDGE. *3GPP TS 45.004: Radio Access Network; Modulation (Release 6)*, 2005.

- [13] Theodore S. Rappaport. *Wireless Communications: Principles and Practice 2<sup>nd</sup> Edition*. Prentice Hall, 2002.
- [14] Dawei Shen. *Tutorial 4: The USRP Board*. <http://www.nd.edu/~jnl/sdr/docs/tutorials/4.html>. (Accessed September 2007).
- [15] Roberto Christi. *Modern Digital Signal Processing*. Brooks/Cole, 2004.
- [16] *GNU Radio 3.0 Documentation: usrp\_standard\_tx Class Reference*. [http://gnuradio.org/doc/doxygen/classusrp\\_\\_standard\\_\\_tx.html](http://gnuradio.org/doc/doxygen/classusrp__standard__tx.html). (Accessed September 2007).
- [17] John G. Proakis. *Digital Communications*. McGraw Hill, 2001.
- [18] Alan V. Oppenheim and Alan S. Willsky. *Signals and Systems*. Prentice Hall, 1983.
- [19] Vladimir Vassilevsky. *FM Demodulation*. <http://www.dsprelated.com/showmessage/67382/1.php>. (Accessed September 2007).

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