Fractal approaches to combat modelling

Defence Technology Agency

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See also ADM001929. Proceedings, Held in Sydney, Australia on July 8-10, 2003., The original document contains color images.

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Drivers

- Lack of models available. Simple “Lanchester” approaches the principal option
- Had a broad range of problems, including issues such as Recce and C2
- Agent-based models seemed like a good approach but not “Physics based”
Origins of our approach

- The physics of weather: more detail does not give better answers
- Simple fractal models seem to better reproduce statistics of weather than supercomputers
- Differential equations do not seem to be able to describe complex systems
- Self-organisation important
Hypothesis

- Assume combat is a self-organising system
- Further assume combat data can be characterised in terms of fractal dimensions
- Then, fractal dimension of combat data can be related to the attrition function
Blue attrition

Function of:
- Number of Red
- Time
- Kill probabilities
- Fractal dimension of distribution

\[
\frac{\Delta B}{\Delta t} = f(R, t, k_r, D)
\]
A convenient form is:

\[
\left\langle \frac{\Delta B}{\Delta t} \right\rangle \propto R k_r^{E(D)} \Delta t^{-F(D)}, \quad E + F = 1
\]

- Two parts: \( k \) and \( t \)
- Ensemble of runs with similar distribution of Red.
- Can choose by requiring a minimum casualty level.
- Reduces to the Lanchester equation.
Fractal pattern (MOUT)
The $k$ part:

- Different patterns for different behaviours, implies differing attrition rates.

![Graph showing different cases and their slopes: Case I slope = 1.33, Case II slope = 0.35, Case III slope = 0.16.](image)
The $t$ part:

- Implies that the attrition function itself should have a specific temporal structure.
- Should be intermittent and clustered
  i.e. when it rains it pours.
Historical data confirms our hypothesis!
Applications

Has implications for C2 and logistic loads etc.
\[ c = \left( \frac{P}{1.68} \right)^{-0.4} \]
## Casualty estimation

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Normalized estimate</th>
<th>Actual 1st Inf Div estimate (actual)</th>
<th>Actual 2nd Inf Div estimate (actual)</th>
<th>Actual 4th Inf Div estimate (actual)</th>
<th>Actual 2nd Arm Div estimate (actual)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>3.1</td>
<td>98 (92)</td>
<td>110 (88)</td>
<td>206 (186)</td>
<td>61 (64)</td>
</tr>
<tr>
<td>95%</td>
<td>4.1</td>
<td>130 (100)</td>
<td>145 (130)</td>
<td>271 (253)</td>
<td>81 (95)</td>
</tr>
<tr>
<td>99%</td>
<td>7.8</td>
<td>248 (159)</td>
<td>278 (319)</td>
<td>517 (470)</td>
<td>154 (160)</td>
</tr>
<tr>
<td>Mean</td>
<td>1.0</td>
<td>31.8</td>
<td>35.6</td>
<td>66.4</td>
<td>19.8</td>
</tr>
</tbody>
</table>
Understanding historical results

\[ C = \left( \frac{\text{Number of attackers}}{\text{Number of defenders}} \right)^{0.685} \]

- Thornton (UK)
- Osipov (Russia)
- Helmbold (US)

Inconsistent with Lanchester!
Fractal idea

- Check with agent-based models
- Behaviour (tactics) causes battles to evolve in similar (but not exactly the same) ways
- Could there be a fractal attractor at work?
Battles evolve into an attractor with the same fractal dimension for the same types of battles (related to ideas proposed by Jim Moffat)
Find values for $D$:
What this gives us

- Fractal nature of combat data tells us our models need to produce output consistent with fractals.
- Thus, fractals provide a method by which we can judge if the complexity is being characterised properly by our models.
- Can characterise sophisticated differences in forces by a single parameter!