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**RISK QUANTIFIED STRUCTURAL
DESIGN AND EVALUATION**



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TABLE OF CONTENTS

SECTION	PAGE
TABLE OF CONTENTS	iii
LIST OF FIGURES.....	iv
LIST OF TABLES	vi
FOREWORD.....	vii
1 INTRODUCTION.....	1
1.1 Background & Motivation	1
1.2 Objective & Approach	2
2 KEY PROBABILISTIC DESIGN DEFINITIONS	3
3 PROBABILISTIC DESIGN FRAMEWORK.....	6
3.1 Structural Design Technologies.....	6
3.2 Current Deterministic Design Framework.....	7
3.3 Probabilistic Risk-Based Design Framework.....	8
3.3.1 Probabilistic Assessment Using Weibull-Like Technique.....	11
3.4 Summary.....	11
4 PROBABILISTIC ASSESSMENT EXAMPLE.....	13
4.1 Overview of the UNIPASS Code	13
4.2 Aft-Deck Buckling Example.....	16
4.2.1 Conceptual Design Stage	16
4.2.2 Preliminary Design Stage	23
4.2.3 Detailed Design Stage.....	27
4.2.4 Final Design Stage	31
4.2.5 Aft-Deck Buckling Example Summary	31
5 CONCLUSIONS.....	32

LIST OF FIGURES

FIGURE		PAGE
1.	Comparison of basic structural design methods.....	6
2.	Basic framework for deterministic design.....	8
3.	Basic framework for probabilistic design	9
4.	Example of limit-state function dividing design space into safe and failure domains.....	13
5.	Example of cut sets created from a combination of limit-state functions.....	14
6.	Illustration showing typical first- and second-order failure surface approximations used for FORM and SORM probabilistic methods	15
7.	Examples of (a) uniform, (b) normal, and (c) lognormal distributions	16
8.	Basic components and assembly of representative aft-deck structure	16
9.	Representation of aft-deck structure for conceptual design stage.....	17
10.	Results from Excel Monte Carlo simulations of conceptual design using 1,000 trials	19
11.	Results from UNIPASS Monte Carlo simulations of conceptual design using 1,000 trials and 399,294 trials	19
12.	Results from UNIPASS Monte Carlo simulations of conceptual design using 1,000 trials with data subsequently processed in Excel	20
13.	Results from UNIPASS FORM/SORM simulations of conceptual design.....	21
14.	3D bar chart showing sensitivities of limit-state functions at various levels to the design variables from UNIPASS FORM simulation	22
15.	2D X-Y plot showing sensitivities of limit-state functions at various levels to the design variables from UNIPASS FORM simulation	22
16.	2D bar chart showing sensitivities of limit-state function at 95.62°F to the design variables from UNIPASS FORM simulation.....	23
17.	Simple clamped plate ABAQUS finite element model used for preliminary design stage.....	24
18.	Thermal buckling modes of preliminary design predicted by ABAQUS linear buckling analysis.....	24
19.	ABAQUS Monte Carlo simulation procedure	25
20.	Results from Python scripted Monte Carlo simulations of preliminary design using 500 trials.....	25
21.	Results from UNIPASS Monte Carlo simulations of preliminary design using 500 trials.....	26

LIST OF FIGURES (Concluded)

FIGURE	PAGE
22. Results from UNIPASS FORM/SORM simulations of preliminary design.....	26
23. Detailed ABAQUS finite element model including support structure used for detailed design stage.....	27
24. Thermal buckling modes of detailed design predicted by ABAQUS linear buckling analysis.....	28
25. Out-of-plane displacements measured on lower surface of exterior plate during experimental testing	29
26. Out-of-plane displacements predicted for upper surface of exterior plate for buckling mode at $\Delta T=422^{\circ}\text{F}$	29
27. Results from Python scripted Monte Carlo simulations of detailed design using 500 trials.....	30
28. Results from UNIPASS FORM simulation of detailed design using 200 trials.....	31

LIST OF TABLES

<u>TABLE</u>	<u>PAGE</u>
1. Nominal values of exterior plate used for aft-deck structure conceptual and preliminary design analyses	17
2. Distributions used for exterior plate for aft-deck structure conceptual and preliminary design analyses	18
3. Nominal values of exterior plate used for aft-deck structure detailed design analyses	28
4. Distributions used for exterior plate for aft-deck structure detailed design analyses	30

FOREWORD

This report documents investigation into the use of probabilistic methods for structural design and analysis of air vehicle structures. The work described was performed for the Air Force Research Laboratory Air Vehicles Directorate (AFRL/VA). The Air Force Program Manager for the effort was Dr. Eric Tuegel (AFRL/VASM). Dr. Steven E. Olson served as the University of Dayton Research Institute (UDRI) Principal Investigator. The work was performed within the Aerospace Mechanics Division of UDRI (Michael P. Bouchard, Division Head), in the Structures Group (Daniel R. Bowman, Group Leader). The UDRI DESP Program Manager is Mr. Michael L. Drake. The period of performance for this effort was from 11 February 2005 to 11 June 2006.

SECTION 1

INTRODUCTION

This report documents investigation into the use of probabilistic methods for air vehicle structural design and analysis. The following paragraphs provide details regarding the background and motivation for probabilistic design and analysis, and the specific objective and approach taken for this effort. Following sections list key definitions to be used throughout this report, outline the probabilistic design framework, and provide an example to demonstrate probabilistic design. Lastly, conclusions are made regarding the use of probabilistic methods for structural design and analysis.

1.1 Background & Motivation

Conventional air vehicle structural design utilizes factors of safety to account for uncertainty in the various parameters affecting structural performance, such as the geometric dimensions, material properties, or loading. The factors of safety provide a measure of protection to ensure that structural designs meet expectations. However, there are several disadvantages to the current design process. These disadvantages include:

1. Current aerospace design methods do not account for the random nature of most design parameters. Design parameters such as loads, geometry, material properties, and environment are assumed to be singly determined values (i.e. deterministic). This leaves the designer with no ability to assess reliability or, conversely, to identify risk. Historically, factors of safety have been sufficiently large to minimize the unknown risks in fielded structures.
2. New air vehicle concepts (e.g. reusable launch vehicles) depart dramatically from traditional operating environments and, as a result, utilization of historically based safety factors may not be appropriate. Development of new safety factors is economically unreasonable given the severe operating conditions and the limited production of such vehicles. Probabilistic methods are needed to quantify uncertainty and risk of these new air vehicle designs.
3. Lastly, current aerospace design methods are not affected by modifications to the manufacturing process or materials processing. For example, new manufacturing processes may increase the uncertainty in structural behavior or better material processing controls may reduce the uncertainty. Such changes in the vehicle fabrication will affect the vehicle reliability. However, these changes would not be explicitly captured in the conventional design process, but instead would remain lumped in the factor of safety specification. Probabilistic design can incorporate the effects of such changes.

For these reasons, probabilistic methods are being investigated for the next generation of aerospace vehicles.

1.2 Objective & Approach

The objective of this program is to demonstrate the benefits of a probabilistic design/risk assessment framework during all stages of aircraft structural design and evaluation. For discussion purposes, it is useful to define some general definitions relating directly to probabilistic structural design and analysis. Key definitions are given Section 2. In Section 3, a probabilistic design/risk assessment framework is outlined and the benefits of utilizing this framework are discussed. To demonstrate the framework, a benchmark problem of current interest to AFRL/VA is presented in Section 4. This problem is a design application relating to the thermal buckling response of a representative exhaust-washed aft deck structure. A series of deterministic and probabilistic analyses are performed for various levels of design refinement from conceptual design to preliminary design to detailed design. Lastly, conclusions made regarding the use of probabilistic methods for structural design and analysis are given in Section 5.

SECTION 2

KEY PROBABILISTIC DESIGN DEFINITIONS

Probabilistic structural design is a process to quantify uncertainty in structural performance by incorporating variations in the distributions of design parameters using statistical descriptions. As a result, it is recommended that the reader have a basic understanding of standard statistics terminology. For the purposes of this report, it will be useful to define some general definitions relating directly to probabilistic structural design and analysis. These definitions are given below.

Deterministic Design: The process of fashioning structural components based on nominal or mean values of the various design parameters. A single value quantifying system behavior is predicted using precise values for the design parameters.

Probabilistic Design: The process of fashioning structural components based on the distribution of various design parameters instead of nominal or mean values. The exact system behavior cannot be predicted, but the likelihood of certain behaviors over a range of values can be calculated.

Risk: The possibility of suffering damage or loss.

Risk Assessment: The identification and evaluation of the various possibilities of suffering damage or loss.

Probability of Failure: The likelihood that a component will be unable to perform its required function under specified conditions for a specified period of time. Probability of failure, P_f , is related to reliability, R , as: $P_f = 1 - R$

Reliability: The probability that a component will be able to perform its required function under specified conditions for a specified period of time. Reliability, R , is related to the probability of failure, P_f , as: $R = 1 - P_f$

System Reliability: The probability that a system will be able to perform its required function under specified conditions for a specified period of time. System reliability is a function of the reliabilities and relationships existing between various structural components.

Original Design Space: The space defined by the range of all possible values of a set of design random variables which can influence the reliability of a structure.

Standard Normal Space: The original design space transformed into a space defined by a set of independent normally distributed random variables having zero mean and unit variance.

Limit-State Function: A function that divides the design space into acceptable (safe) and unacceptable (failure) domains. The limit-state function, $g(x)$, is usually defined as: $g(x) = (\text{allowable function}) - (\text{response function})$ where the allowable function defines the acceptable level of response and the response function predicts the response based on a defined set of variables.

Cut Set: A collection of limit-state functions that define failure.

Component Problem: Models which contain a single cut set which contains only a single limit-state function.

Serial System Problems: Models which contain two or more cut sets, where each cut set contains only a single limit-state function. Serial systems are assumed to fail if any one of the limit-state functions fails.

Parallel System Problems: Models which contain a single cut set, where the cut set contains more than one limit-state function. Parallel systems are assumed to fail if all of the limit-state functions fail.

General System Problems: Models which are a combination of series and parallel systems.

Probability Analysis: Determination of the probability of failure for a given model, along with the associated sensitivities to the design variables and the most likely conditions for a specified limit-state function level.

Inverse Probability Analysis: Determination of the limit-state function level and most likely conditions to produce a given probability of failure, along with the associated sensitivities.

Probability Density Function (PDF): A statistical expression that shows how the collection of potential responses is distributed. A probability density function must be non-negative everywhere and the integral of the function over its entire range must be unity.

Cumulative Distribution Function (CDF): A statistical expression that describes the probability that a potential response is less than or equal to a specified value. Mathematically, the cumulative distribution function is the integral of a continuous probability density function.

PDF/CDF Analysis: Determination of the PDF/CDF curves over a specified range of the limit-state function and with a minimum or fixed number of points in each curve.

Most-Probable-Point (MPP): The point in the failure domain which is closest to the origin in standard normal space and is the most likely failure point. The majority of failure probability is associated with the probability density of the failure domain around this point.

SECTION 3

PROBABILISTIC DESIGN FRAMEWORK

In the aerospace community, it has become increasingly important to assess risk, identify parameters that drive risk, and minimize the risk given other program constraints. Probabilistic structural design, unlike traditional methods, provides a means to quantify the inherent risk of a design and to quantify the sensitivities of design variables to that risk. The following sections give a brief introduction to structural design technologies, review the current deterministic design process, and present the framework for probabilistic risk-based design.

3.1 Structural Design Technologies

Structural design can be classified into three basic methods: deterministic methods, statistical methods, and probabilistic methods. Current aerospace structural design utilizes deterministic methods. The design parameters are assumed known and structural performance is evaluated using physics-based process models with the design parameters as inputs. Any scatter in the design parameters, and resulting uncertainty in the performance, is accounted for using historically-based safety factors. Statistical methods, on the other hand, utilize statistics from fielded systems to evaluate the uncertainty in structural performance. However, these techniques do not utilize any physics-based process models and, to be accurate, require significant amounts of performance data which may be flawed, incomplete, or difficult to obtain. Probabilistic methods combine key aspects of the deterministic and statistical methods. Scatter in the design parameters is incorporated and the physics-based process models are used to investigate uncertainty in performance from which reliability estimates can be made. Figure 1 (taken from www.predictionprobe.com) shows a comparison of the three design methods.

Predictive Technology Comparison Chart			
METHODOLOGY APPROACH	DETERMINISTIC	STATISTICAL	PROBABILISTIC
Utilizes physics based behavioral model	✓	✗	✓
Considers inherent uncertainties, modeling uncertainties, lack of data, human error, measurement error	✗	✗	✓
Compensates for unknowns using:	SF	SM	SM
Utilizes past performance data to improve accuracy	✓	✓	✓
Does not require event's past performance data to develop behavioral model	✓	✗	✓
Quantifies safety measures	U	U	✓
Quantifies prediction accuracy	U	U	✓

✓ TRUE ✗ FALSE U UNKNOWN SF SAFETY FACTORS SM STATISTICAL METHODS

Figure 1. Comparison of basic structural design methods

Aerospace structural design is typically performed in various stages including conceptual design, preliminary design, detailed design, and final design. At each step in the design process, deterministic or probabilistic analyses can be performed. During the conceptual design stage, the feasibility of potential design configurations is investigated using simple “back-of-the-envelope” calculations as a quick assessment for a particular structural design. Once a design concept has been shown to be feasible, the preliminary design stage examines additional aspects of the design using basic analyses and calculations. The detailed design stage, the next step in the design process, incorporates further design elements and may include sophisticated analyses and calculations. Lastly, a final design is created from which the structure is fabricated. For aerospace components, the final design is often validated through experimental testing. The current deterministic design framework and the probabilistic design framework, which can be applied at any stage in the design process, are discussed in further detail in the following paragraphs.

3.2 Current Deterministic Design Framework

Current aerospace structural design utilizes deterministic analyses and calculations at the various stages of the design process. The basic framework for deterministic design is shown in Figure 2. The design variables, such as the geometric dimensions, material properties, or loading, are assumed to be singly determined values (i.e. deterministic). Typically mean or nominal values are utilized. The response of the structure is assessed using physics-based process models with the design parameters as inputs. Based on the predicted response and specified allowable response values, which also are assumed to be deterministic, a factor of safety calculation is performed. The calculated factor of safety is compared with factors of safety which historically have been sufficiently large to minimize the unknown risks in fielded structures. From this comparison, a decision is made as to whether a given design is safe. However, no indication is given as to how reliable a design may be.

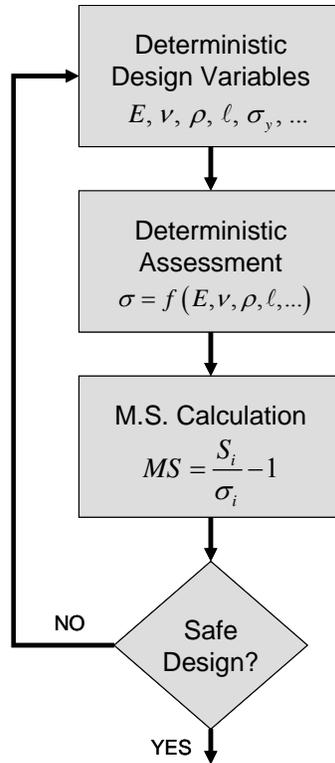


Figure 2. Basic framework for deterministic design

As an example, consider an aerospace structure under a specified design load. A deterministic analysis is performed to determine the stress at each point in the structure. These stress values are compared against lower limit strength values determined experimentally as recorded in MIL-HDBK-5 for metals or MIL-HDBK-17 for composites. At each point in the structure, the margin of safety – equal to the strength/stress ratio minus one – is computed. If the margin of safety is greater than zero at each point in the structure, the design is considered safe. If not, the structural design is modified until this criterion is met. This conventional deterministic design process ensures that the minimum strength is greater than the maximum stress in the part. This approach leads to safe designs, but since the effects of variability are not considered, the reliability cannot be determined. It is possible to estimate the reliability using statistical methods provided a significant quantity of fabricated hardware is available for testing. However, such techniques will be unreasonable for future air vehicle concepts, such as reusable launch vehicles, due to the severe operating conditions these components will likely experience and the limited production of such vehicles.

3.3 Probabilistic Risk-Based Design Framework

Probabilistic analysis serves as a means to determine how the variability in loading, geometry, materials, and environment affect the design reliability and the contribution of each design parameter to the overall risk. The objective of probabilistic design is not to establish a particular margin of safety, but rather to achieve a specified level of reliability. Figure 3 shows

the basic framework for probabilistic design. In a probabilistic design environment, all design input parameters are considered to be statistically varying. As in the deterministic design framework, the response of the structure is assessed using physics-based process models with the design parameters as inputs. However, since the design parameters are specified as distributions rather than specific values, a distribution in response values is obtained. In addition, the results of a probabilistic structural analysis may include a number of failure modes and potential failure locations. The allowable response values also are specified as distributions and the probability of failure can be computed statistically as the joint probability that the predicted response values exceed the specified allowable values. From the probability of failure calculations, the reliability of the design can be quantified. Sensitivity information also may be used to identify key design parameters which contribute to the risk and the best options for risk reduction.

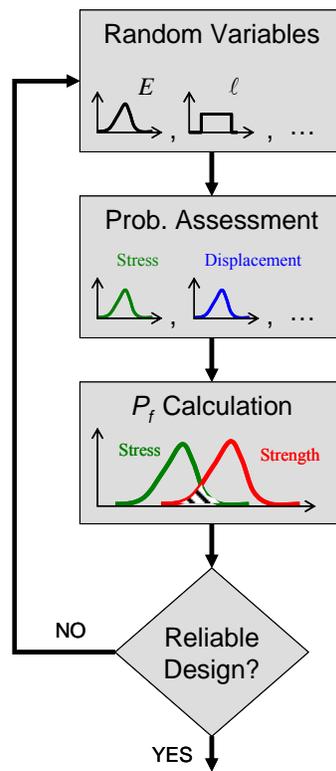


Figure 3. Basic framework for probabilistic design

There are several major issues which have hindered the use of probabilistic design practices. These issues include the computational and time resources required for probabilistic assessments, the definition of target reliabilities to ensure a structure meets its design criteria, and the definition of appropriate ways to perform probabilistic assessments. These issues are discussed in further detail in the following paragraphs. Satisfactory resolution of these issues is required prior to the widespread adoption of probabilistic technologies.

Recent technological advances have alleviated many of the issues relating to resources. The physics-based process models used for structural design, such as finite element analysis, may require significant computational resources and solution time for even a single analysis.

Probabilistic assessment typically requires a number of analyses. As a result, mathematical techniques have been developed to provide intelligent means of selecting a subset of solutions to perform. A number of commercial probabilistic analysis software packages, which incorporate these mathematical techniques and can interface with external physics-based analysis codes, have been created. Two of the more popular packages are UNIPASS (**UN**ified **P**robabilistic **A**ssessment **S**oftware **S**ystem) from Prediction Probe Incorporated in Irvine, California, and NESSUS (**N**umerical **E**valuation of **S**tochastic **S**tructures **U**nder **S**tress) from the Southwest Research Institute in San Antonio, Texas. For this program, the Air Force has chosen to utilize the UNIPASS code; however, other codes could potentially be utilized to perform the probabilistic structural design and risk assessment techniques discussed in this report. An overview of the UNIPASS code is given in the following section and its use is demonstrated.

Another major obstacle relates to the definition of target reliabilities to ensure a structure meets its design criteria. For air vehicles, overall allowable failure rates (e.g. 1 failure per 10^7 flight hours) or associated reliability levels can be specified. However, these reliabilities are for the overall vehicle and not any specific structural component. For design purposes, it would be beneficial to define the required reliability of individual components. For discrete systems, where the structural response can be quantified using a few parameters, it is possible to define specific structural reliabilities based on the overall reliability. Techniques such as fault tree analysis can be utilized to define which particular events (e.g. failure of a specific structure) or combination of events result in system failure. Once such combinations are assigned, the reliabilities of particular events can be calculated to assess the overall system reliability. These techniques work well for discrete systems, but difficulties are encountered when trying to apply such techniques to continuous systems. As discussed below for probabilistic assessment of continuous systems, it may be necessary to employ techniques where the responses through the entire volume of a structure are of interest.

The last major obstacle relates to the techniques used to perform probabilistic assessment of aerospace structures. Some of these techniques will be presented in Section 4. The techniques generally function well for discrete systems where the structural response can be quantified using a few parameters. For example, in the thermal buckling example presented in the following section only the temperature change required to initiate buckling is of interest. Many structural problems, however, address continuous systems. As an example, consider loads in a wing structure which are distributed among spars, ribs, and skins. Deterministic analyses typically focus on only the worst location (such as the maximum stress location in the wing example) in a particular structure, since the remainder of the structure should be at least as “safe” as this worst location. However, the entire structure, and not just the worst location, contributes to the reliability. As a result, probabilistic assessment of continuous systems must consider the contributions from the entire structure. Currently, appropriate techniques to perform probabilistic assessment of continuous systems have not been identified. One of the more promising techniques considered under this program is to use techniques similar to Weibull analysis, where the responses through the entire volume of a structure are of interest. The following paragraphs provide a brief description of this technique and its potential application to aerospace structural design.

3.3.1 Probabilistic Assessment Using Weibull-Like Technique

In brittle materials, the strength can vary widely based on the random distribution of minor flaws in the material. As such, conventional design practices for brittle materials based on factor of safety estimates are not effective because there is not a well-defined strength value. To design brittle components, the inherent variability of the material strength must be considered. This led Weibull and others to develop a risk-based design technique that evaluates design concepts in terms of reliability and/or failure probability. Such an approach might be viable for probabilistic assessment of aerospace structures.

The basic approach in Weibull analysis is to estimate the reliability of an entire structure by combining the reliabilities of each volume element of the structure. The resulting expression for the structural component reliability, R , is:

$$R = e^{-\sum \left(\frac{\sigma_i}{S} \right)^m \left(\frac{v_i}{V} \right)} \quad (1)$$

where m and S are Weibull parameters to account for the material strength variation, σ_i is the maximum stresses in the element of volume v_i , and V is a reference volume.

Given the results of a single finite element analysis of a structure under a particular load, it becomes straightforward to estimate the reliability of the structure by summing the reliability contributions of each element in the structure. This calculation provides a simple means to compute the reliability of a continuous structural system and, as such, is a good candidate for a reliability-based design framework.

The Weibull reliability estimate accounts for the material strength variation, but does not account for stress variation due to geometric, loading, or other material variations. Probabilistic assessment techniques, like those discussed in the Section 4, are required to account for these additional uncertainties and their impacts on the structural reliability.

3.4 Summary

In the aerospace community, it has become increasingly important to assess risk, identify parameters that drive risk, and minimize the risk given other program constraints. The current aerospace structural design process utilizes deterministic analyses and calculations at the various stages of the design process. A “safe” design is created, but no indication is given as to how reliable a design may be. Probabilistic analysis serves as a means to determine how the variability in loading, geometry, materials, and environment affect the design reliability and the contribution of each design parameters to the overall risk. However, there are several major issues which have hindered the use of probabilistic design practices. These issues include the computational and time resources required for probabilistic assessments, the definition of target reliabilities to ensure a structure meets its design criteria, and the definition of appropriate ways to perform probabilistic assessments. Satisfactory resolution of these issues is required prior to the widespread adoption of probabilistic technologies.

In the following section, an overview of the UNIPASS code is given along with an example of probabilistic assessment. As discussed above, only the temperature change required to initiate buckling is of interest for the example problem. Therefore the structure can be considered a discrete system where the buckling temperature change is the sole output response.

Since the representative exhaust-washed aft deck structure has been fabricated to study the potential for buckling, no target reliabilities or minimum temperature change criteria has been specified. As a result, reliability calculations and a determination as to whether the structure meets design criteria have not been performed. However, the probabilistic assessment capability described in the following section is a critical element of an overall risk-based design process.

SECTION 4

PROBABILISTIC ASSESSMENT EXAMPLE

To demonstrate probabilistic assessment, and the advantages to utilizing such a process, it is useful to consider an example problem. A benchmark problem of current interest to AFRL/VA has been selected. This problem is a design application relating to the thermal buckling response of a representative exhaust-washed aft deck structure. The UNIPASS code has been utilized to perform probabilistic assessments at various stages of the design process. The following paragraphs provide a brief overview of the UNIPASS code and present the aft-deck thermal buckling example.

4.1 Overview of the UNIPASS Code

UNIPASS is a commercial software package which can be utilized to perform probabilistic analysis. The UNIPASS software can model uncertainties, compute probabilities, identify most likely outcomes, analyze risk, identify key drivers, and perform sensitivity analysis. Capabilities of the UNIPASS software are discussed briefly below.

In probabilistic analysis, the failure probability of a system is typically expressed in terms of limit-state functions. These limit-state functions are expressed in terms of the random variables of the problem and divide the design space into safe and failure domains as shown in Figure 4. Cut sets are created from the limit-state functions to define when failure will occur. For example, Figure 5 shows the safe and failure domains for a problem where failure occurs if either the g_1 or g_2 limit-state functions fails (a serial system problem). Problem types are categorized according to the number of cut sets and the number of limit-state functions in each cut set. UNIPASS has the capability to perform four different problem types: component problems, serial system problems, parallel system problems, and general system problems. Definitions for these various problem types have been given in Section 2.

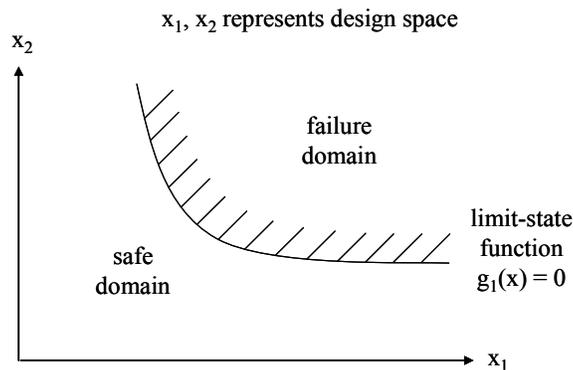


Figure 4. Example of limit-state function dividing design space into safe and failure domains

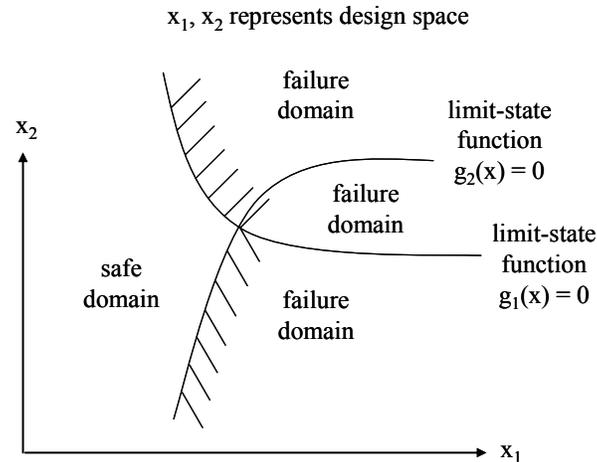


Figure 5. Example of cut sets created from a combination of limit-state functions

In addition to the four different problem types, UNIPASS can perform three different types of analyses: probability analyses, inverse probability analyses, and PDF/CDF analyses. Definitions of these various analyses types also have been given in Section 2. It should be noted that, within UNIPASS, inverse probability analyses and PDF/CDF analyses can only be performed for component problems. For most analyses, sensitivity information is available which indicates the degree to which each random variable and its distribution parameters contribute to the uncertainty in the output quantities.

UNIPASS employs six different probabilistic methods in solving all three types of analyses for both component and system problems. These six methods include:

- First Order Reliability Methods (FORM) – FORM methods replace the limit-state surface with a first-order polynomial approximation of the limit-state function at the most probable point (MPP) on the failure boundary as shown in Figure 6;
- Second-Order Reliability Methods (SORM) – SORM methods are similar to FORM methods except the limit-state surface is replaced with a second-order polynomial approximation of the limit-state function at the MPP as also shown in Figure 6;
- Simulation Methods (SM) – for simulation methods, deterministic analyses are performed for a series of sample points (i.e. a set of random variables) which are randomly sampled from their probability distributions;
- Importance Sampling Methods (ISM) – importance sampling methods are a subclass of simulation methods where the sample points are skewed towards the MPP to improve accuracy or reduce the number of simulations required;
- Response Surface Methods (RSM) – response surface methods calculate the true limit-state function at several points in the design space and use these results to approximate the true limit-state surface with a second order polynomial; and,

- Mean Value-Based Methods (MVBM) – mean value-based methods utilize first-order Taylor series expansion of the limit-state functions around the mean values of the random variables by computing the first and second moments (i.e. mean and standard deviation) of the limit-state function.

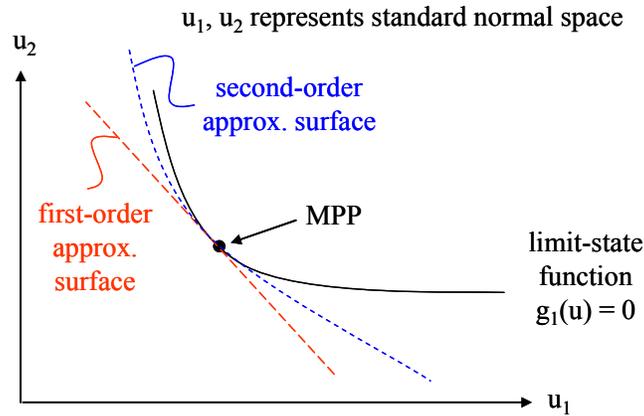


Figure 6. Illustration showing typical first- and second-order failure surface approximations used for FORM and SORM probabilistic methods

For many of the solution methods shown above, the efficiency and accuracy is related to identifying the MPP. This point is important because the majority of failure probability is contributed from the probability density of the failure domain around the MPP. Various algorithms have been investigated for MPP identification, since different problems require different optimization procedures to avoid erroneous solutions or lack of convergence. UNIPASS currently offers 11 different algorithms, in both the original design space and standard normal space, for MPP identification. Further details on these algorithms can be found in the UNIPASS documentation.

Additional features of UNIPASS include 37 different probability distributions (e.g. uniform, normal, lognormal, etc.) which can be used to define four different classes of random variables. Examples of a few of the more common distributions are shown in Figure 7. The distributions include the major types appropriate for aerospace structural design such as lognormal distributions for material properties, or uniform or truncated normal distributions for geometric dimensions. The four different classes of random variables include independent variables and dependent variables, as well as variables with distributions defined by parameters which are either independent or dependent variables.

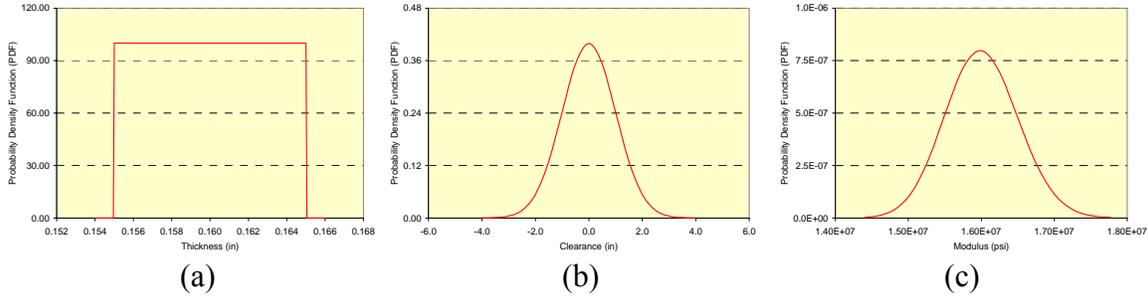


Figure 7. Examples of (a) uniform, (b) normal, and (c) lognormal distributions

UNIPASS also can interface with external codes, such as finite element analysis software, to perform the probabilistic analyses. UNIPASS includes a NASTRAN interface and a text-based generic interface, as well as tools to create a customized interface. These interfaces allow UNIPASS to automatically prepare the required input for external codes and to extract the desired output information. For the studies presented in this report, the text-based generic interface has been used to link UNIPASS with the ABAQUS finite element code.

4.2 Aft-Deck Buckling Example

The benchmark problem selected to demonstrate probabilistic assessment is a design application relating to the thermal buckling response of a representative exhaust-washed aft deck structure. The basic components and assembly of the design are shown in Figure 8. The objective is to design the structure to prevent buckling under anticipated thermal loading. A series of deterministic and probabilistic analyses have been performed for various levels of design refinement from conceptual design to preliminary design to detailed design. Efforts at each of the design stages are discussed below.

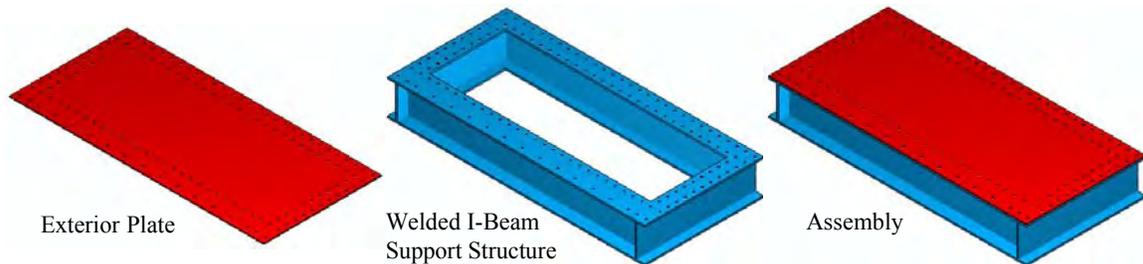


Figure 8. Basic components and assembly of representative aft-deck structure

4.2.1 Conceptual Design Stage

During the conceptual design stage, the initial step in the design process, simple calculations and/or analyses are performed to validate the design concept. For this problem, the aft-deck support structure can be considered rigid and buckling of an edge-loaded plate can be

investigated. This is shown schematically in Figure 9. Estimates of the critical buckling stresses, σ'_x and σ'_y , can be found using equations for a flat plate with all edges clamped.¹

$$\sigma'_x + \frac{a^2}{b^2} \sigma'_y = 1.1 \frac{Et^2 a^2}{1-\nu^2} \left(\frac{3}{a^4} + \frac{3}{b^4} + \frac{2}{a^2 b^2} \right) \quad (2)$$

where a and b are plate dimensions corresponding to the distances between the inner rows of fasteners on the exterior plate, t is the thickness of the plate, and E and ν are the material modulus and Poisson's ratio, respectively. The stress resulting due to a uniform temperature change can be found as:

$$\sigma = \frac{E\alpha\Delta T}{1-\nu} \quad (3)$$

where α is the coefficient of thermal expansion (CTE) and ΔT is the change in temperature. Nominal values of the parameters used for deterministic analyses are listed in Table 1. Assuming that the critical buckling stresses are equal (i.e. $\sigma = \sigma'_x = \sigma'_y$) the temperature change required to buckle the clamped plate is 86.1°F.

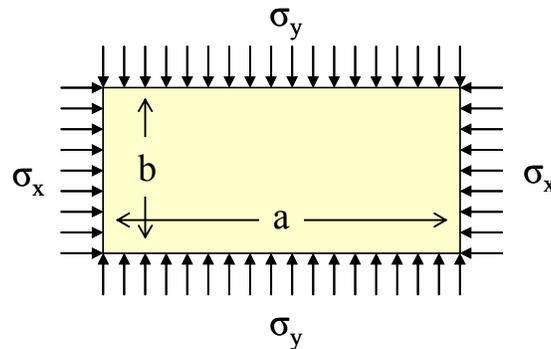


Figure 9. Representation of aft-deck structure for conceptual design stage

Table 1. Nominal values of exterior plate used for aft-deck structure conceptual and preliminary design analyses

Parameter	Nominal Value
Plate Length, a	32.5 in
Plate Width, b	12.0 in
Plate Thickness, t	0.160 in
Elastic Modulus, E	16.0×10^6 psi
Poisson's Ratio, ν	0.30
CTE, α	5.0×10^{-6} in/in/°F

¹ Roark, R. and W. Young, Roark's Formulas for Stress and Strain, 6th edition, New York, NY: McGraw-Hill, 1989.

Based on the result from this deterministic analysis, one would assume that buckling would not occur at temperature changes below 86.1°F. However, due to the uncertainty in the various design parameters, there is some likelihood that buckling will occur at lower temperature changes. Similarly, there is some likelihood that buckling will not occur until larger temperature changes. Probabilistic analysis provides a technique to investigate the distribution of temperature changes that would cause buckling. For probabilistic analysis, it is necessary to define the distributions in the design parameters. Table 2 lists the distributions used for the probabilistic conceptual design studies. A coefficient of variation of 3% has been assumed for the lognormal-distributed material properties and reasonable tolerances have been assumed for the uniform-distributed geometric parameters.

Table 2. Distributions used for exterior plate for aft-deck structure conceptual and preliminary design analyses

Parameter	Distribution	Mean Value	Standard Deviation or Tolerance (\pm)
Plate Length, a	Uniform	32.5 in	± 0.100 in
Plate Width, b	Uniform	12.0 in	± 0.100 in
Plate Thickness, t	Uniform	0.160 in	± 0.005 in
Elastic Modulus, E	Lognormal	16.0×10^6 psi	0.48×10^6 psi
Poisson's Ratio, ν	Lognormal	0.30	0.0099
CTE, α	Lognormal	5.0×10^{-6} in/in/°F	0.15×10^{-6} in/in/°F

To determine the distribution of temperature changes that would cause buckling, initially Monte Carlo simulations have been performed. The Monte Carlo technique is a simulation method where a large number of predictions are made using variables randomly sampled from their distributions. The actual distribution in buckling temperature change is assumed to be represented by the distribution in the predicted results. This assumption improves as larger numbers of simulations are performed. Monte Carlo simulations have been performed using both Microsoft Excel and UNIPASS. Predicted distributions in buckling temperature from the Excel and UNIPASS simulations are shown in Figures 10 and 11, respectively. For the Excel simulations, 1,000 trials have been performed. The cumulative distribution function (CDF) has been calculated from the results of the individual trials and the probability density function (PDF) has been estimated from the CDF. The mean temperature change is 86.2°F and the standard deviation is 4.2°F. For the UNIPASS simulations, two different sets of Monte Carlo simulations have been performed. For the first set, 1,000 trials have been performed. As with the Excel simulations, the mean temperature change is 86.2°F and the standard deviation is 4.2°F. For the second set, 399,294 simulations have been performed such that the coefficient of variation in the probability of failure curve is less than 0.05. For this set, the mean temperature change is 86.3°F and the standard deviation is 4.2°F. As seen in the figure, the UNIPASS results from the first set of 1,000 trials appear quite noisy. The result appears noisy since only 1,000 trials have been performed and UNIPASS has tried to group the results into a relatively large number of bins (buckling temperature ranges). The raw data has subsequently been reprocessed, using the same method used for the Excel simulations, with a smaller number of bins. The CDF has been calculated from the results of the individual trials and the PDF has been estimated from

the CDF. Figure 12 shows the predicted distributions in buckling temperature based on the reprocessed data.

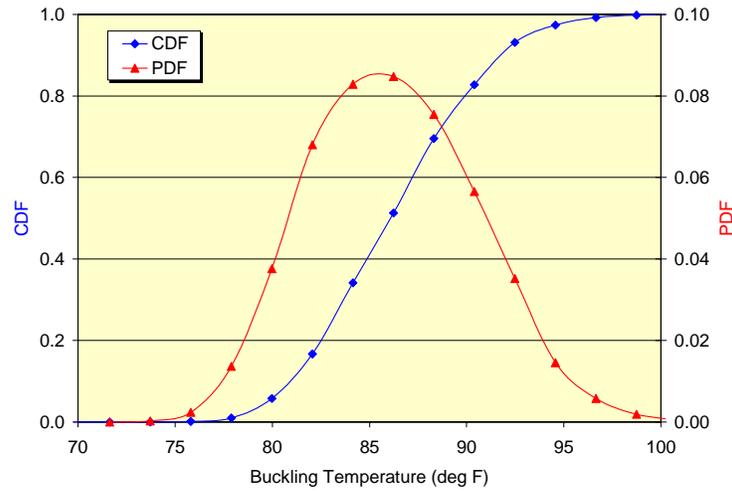


Figure 10. Results from Excel Monte Carlo simulations of conceptual design using 1,000 trials

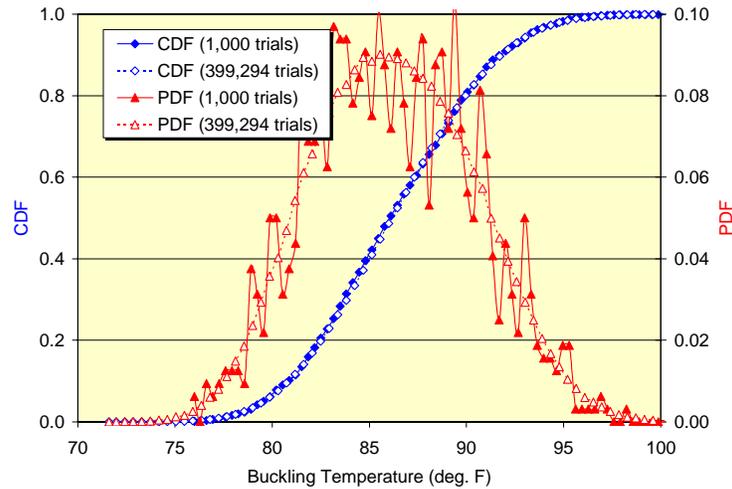


Figure 11. Results from UNIPASS Monte Carlo simulations of conceptual design using 1,000 trials and 399,294 trials

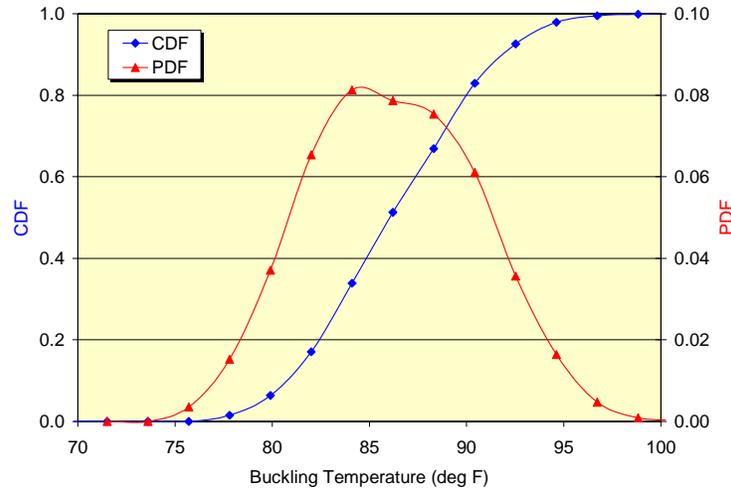


Figure 12. Results from UNIPASS Monte Carlo simulations of conceptual design using 1,000 trials with data subsequently processed in Excel

Monte Carlo simulations generally are not very efficient, particularly for problems with a large number of random variables. As the number of random variables increases, the number of Monte Carlo simulations required for an accurate representation of the response distribution also increases. For this relatively simple demonstration problem, Monte Carlo simulations have been performed to demonstrate the simulation method technique, as well as to provide a baseline for which to compare other probabilistic solution techniques.

In addition to the Monte Carlo simulations, FORM and SORM simulations have been performed using UNIPASS. Figure 13 shows the results from these analyses. The FORM and SORM simulations utilize 1,063 and 3,181 limit-state function evaluations, respectively. As seen in the figure, the second-order polynomial approximation used for the SORM simulation produces a more accurate distribution than the first-order approximation used for the FORM simulation at the expense of additional limit-state function evaluations. For this simple problem (consisting of only the basic buckling equation) the number of limit-state function evaluations required for accurate PDF/CDF predictions is relatively high compared to the number required for accurate Monte Carlo solutions. However, for more complex problems, the FORM and SORM solutions generally provide reasonable estimates of the PDF/CDF curves with much fewer limit-state functions than simulation method techniques.

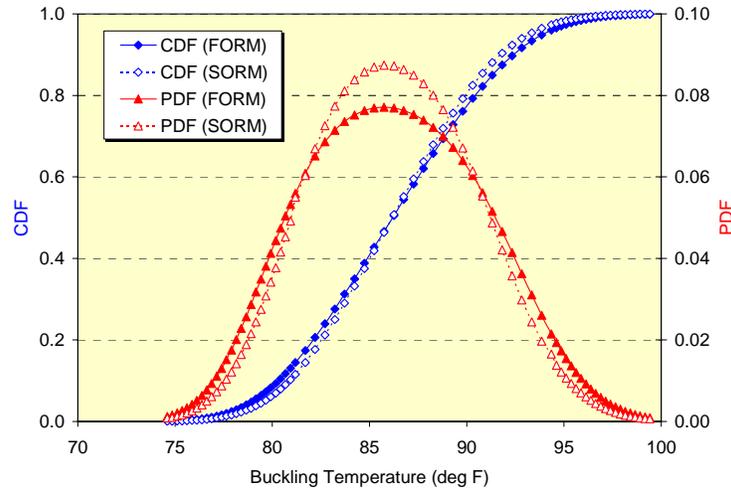


Figure 13. Results from UNIPASS FORM/SORM simulations of conceptual design

Results from the UNIPASS simulations also typically include sensitivity information, which may be extremely useful for design purposes, particularly at early stages in the design process. The sensitivity information highlights those random design variables which most significantly affect a specific design response. For example, Figures 14 through 16 show sensitivity results from the UNIPASS FORM simulation of the conceptual design. Figure 14 is a 3D bar chart showing sensitivities of the limit-state function at various levels to the design variables. The limit-state function for this example problem is the buckling temperature change. For each design variable, the change in the failure probability is shown when the mean value of one random variable is varied by one standard deviation and the rest of the design variables remain unchanged. This results in a dimensionless sensitivity measurement which can be used to compare the relative importance of the random variables regarding their mean values. Similar results, in a different format, are shown in a 2D X-Y plot in Figure 15. It is also possible to investigate the sensitivities at a particular value of the limit-state function. Figure 16 shows the design variable sensitivities at a buckling temperature change of 95.62°F. The sensitivity information highlights those design parameters which most significantly affect the buckling temperature change. As shown in the figures, the plate thickness and coefficient of thermal expansion have a fairly significant effect on the buckling temperature change, the plate width and Poisson's ratio have a much less significant effect, and the plate length has a relatively insignificant effect. This information is extremely useful when design changes are required.

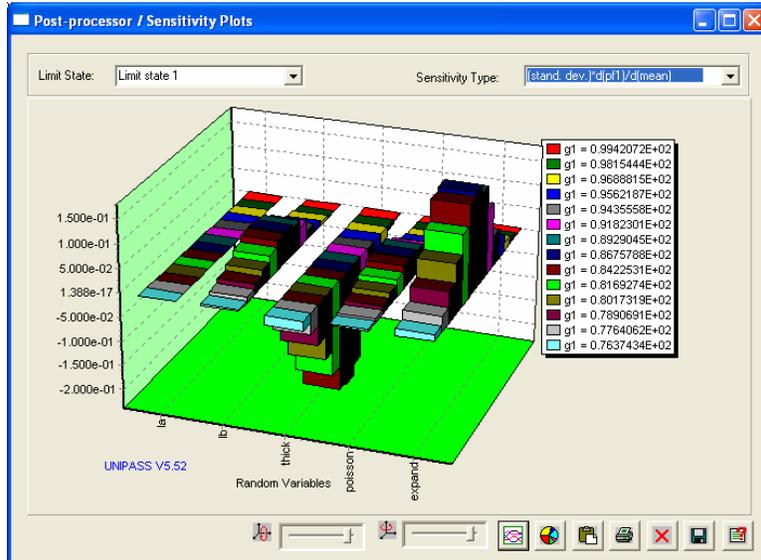


Figure 14. 3D bar chart showing sensitivities of limit-state functions at various levels to the design variables from UNIPASS FORM simulation

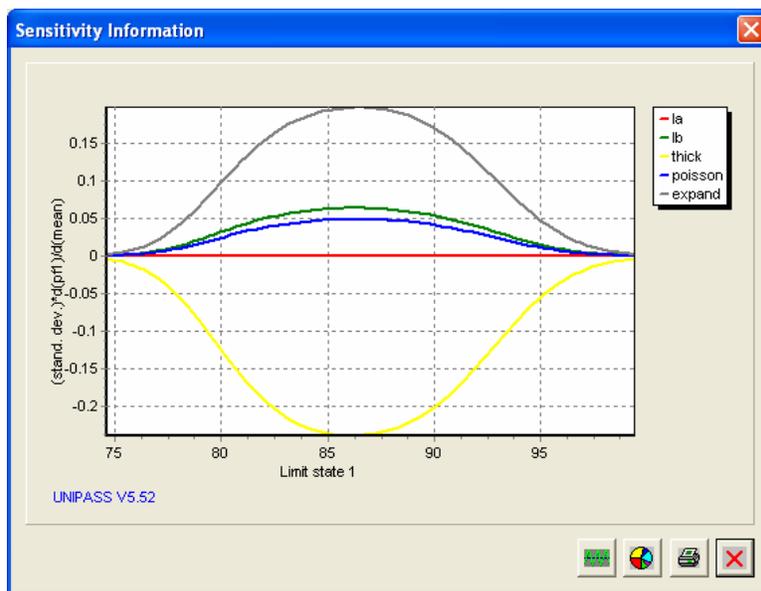


Figure 15. 2D X-Y plot showing sensitivities of limit-state functions at various levels to the design variables from UNIPASS FORM simulation

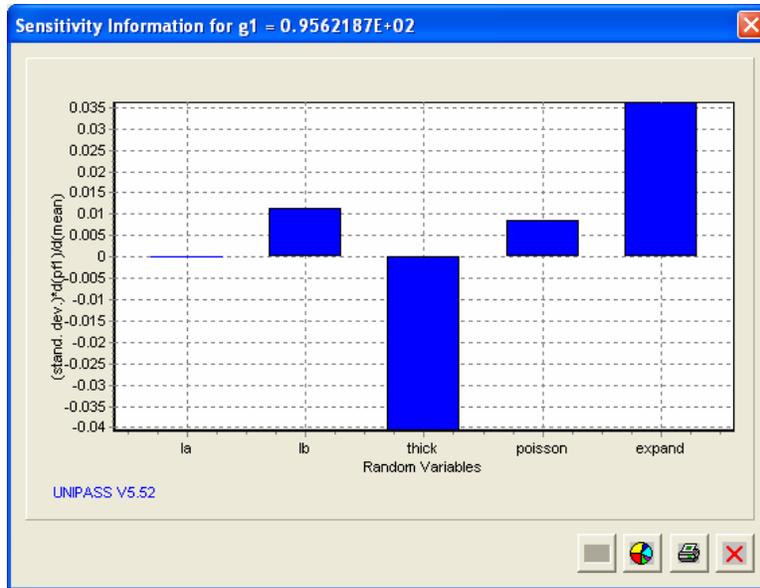


Figure 16. 2D bar chart showing sensitivities of limit-state function at 95.62°F to the design variables from UNIPASS FORM simulation

4.2.2 Preliminary Design Stage

Once a design concept has been validated, a preliminary design is created and additional analyses or calculations performed to evaluate its performance. The analyses or calculations performed during the preliminary design stage are more detailed than those from the conceptual stage, but still include various simplifying assumptions. For the aft-deck example, the preliminary design stage utilizes a simple clamped plate ABAQUS finite element model, as shown in Figure 17. The plate is modeled using continuum shell elements (SC8R elements in ABAQUS) and the temperature change is assumed to remain uniform throughout the plate. The plate dimensions correspond to the distances between the inner rows of fasteners on the exterior plate.

Deterministic analyses have been performed using the nominal values shown in Table 1. As shown in Figure 18, ABAQUS linear thermal buckling analysis yields two closely spaced buckling modes at predicted temperature changes of 85.6°F and 85.9°F. This result compares well with the 86.1°F temperature change predicted by the equations used in the conceptual design stage. Based on this deterministic result, one would assume that buckling would not occur at temperature changes below 85.6°F. However, due to the uncertainty in the various design parameters, there is some likelihood that buckling will occur at lower temperature changes. Similarly, there is some likelihood that buckling will not occur until larger temperature changes. Probabilistic analysis can be used to investigate the distribution in temperature changes that cause buckling.

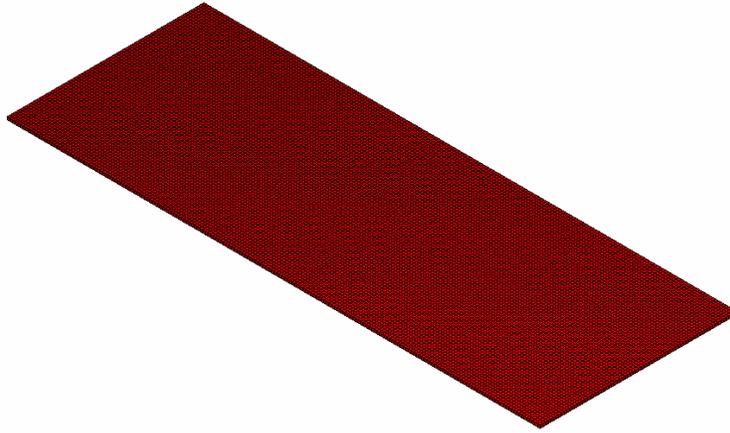


Figure 17. Simple clamped plate ABAQUS finite element model used for preliminary design stage

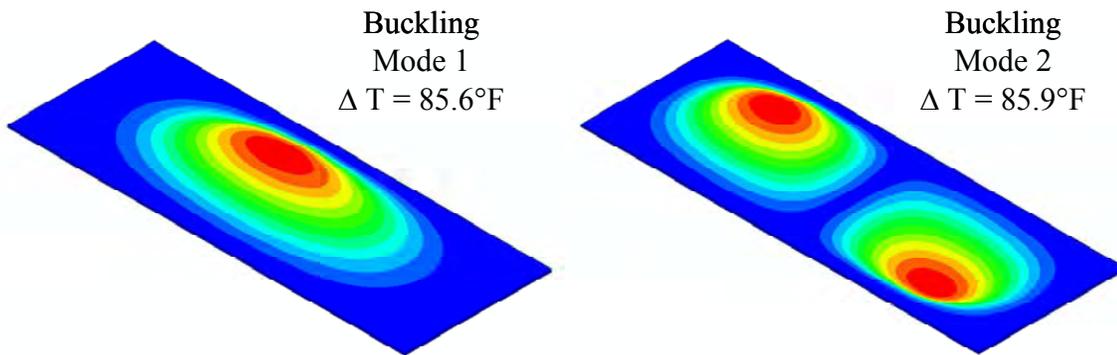


Figure 18. Thermal buckling modes of preliminary design predicted by ABAQUS linear buckling analysis

As in the conceptual design stage, initially Monte Carlo simulations have been performed to investigate the distribution in buckling temperature changes. As noted above, Monte Carlo simulations generally are not very efficient, particularly for problems with a large number of random variables. However, Monte Carlo simulations have been performed to demonstrate the simulation method techniques, as well as to provide a baseline for which to compare other probabilistic solution techniques. For the conceptual stage, such simulations could easily be performed using the derived equation. The preliminary design stage, however, utilizes an ABAQUS finite element model that must be updated based on the variables randomly sampled from their distributions. Distributions for the random variables have been shown in Table 2. Python scripts have been written to randomly sample these distributions, automatically generate finite element models using these variables, and analyze and process the finite element results. This process is repeated to create multiple predictions of the buckling temperature change. Figure 19 shows a graphical representation of the ABAQUS Monte Carlo simulation procedure. This procedure has been performed both within and outside of the UNIPASS framework. Figure 20 shows the predicted distribution in buckling temperature change from the Monte Carlo

simulations performed outside of UNIPASS. These simulations include 500 trials and require approximately 330 CPU minutes of processor time on a 3.2 GHz Pentium 4 single-processor system. As with the earlier results, the CDF has been calculated from the results of the individual trials and the PDF has been estimated from the CDF. The mean temperature change is 85.9°F and the standard deviation is 4.3°F. Similar results from the simulations performed with UNIPASS are shown in Figure 21. These simulations also include 500 trials and require approximately 330 CPU minutes of processor time. The raw data from the individual predictions has been processed outside of UNIPASS using the same methods used for the data shown in Figure 20. For the UNIPASS simulations, the mean temperature change is 86.1°F and the standard deviation is 4.2°F.

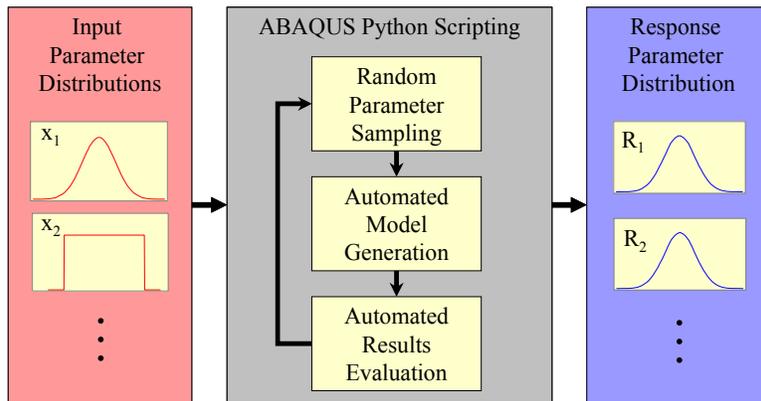


Figure 19. ABAQUS Monte Carlo simulation procedure

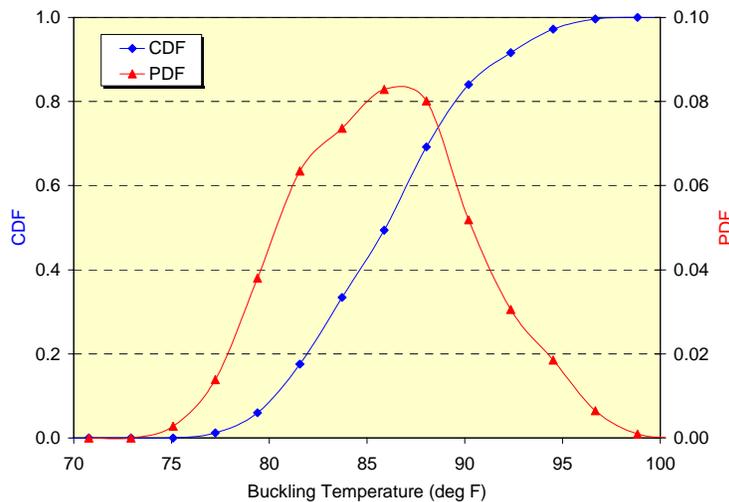


Figure 20. Results from Python scripted Monte Carlo simulations of preliminary design using 500 trials

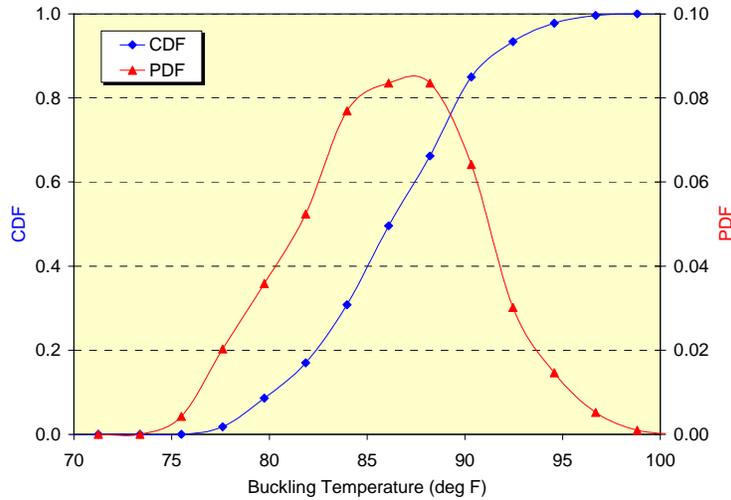


Figure 21. Results from UNIPASS Monte Carlo simulations of preliminary design using 500 trials

In addition to the Monte Carlo simulations, FORM and SORM simulations have been performed using UNIPASS. Figure 22 shows the results from these analyses. The FORM and SORM simulations have been performed simultaneously, with the SORM simulations utilizing 434 limit-state function evaluations and requiring approximately 280 CPU minutes of processor time. For these simulations, the convergence tolerances in UNIPASS have been loosened and only 14 points have been created on the PDF/CDF curves. It is anticipated that, with tighter tolerances and more points, smoother PDF/CDF curves would be predicted.

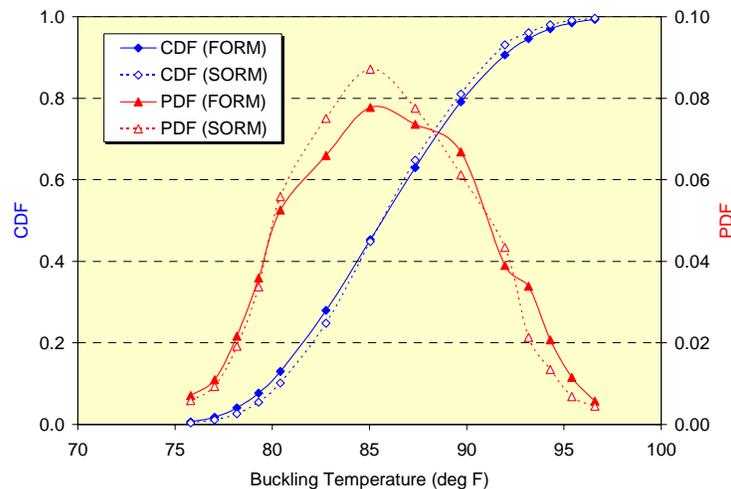


Figure 22. Results from UNIPASS FORM/SORM simulations of preliminary design

4.2.3 Detailed Design Stage

Studies during the preliminary design stage utilize a simplified model consisting of only the exterior plate of the representative exhaust-washed aft deck structure. For the detailed design stage, the entire assembly, including both the exterior plate as well as the support structure, is modeled. The detailed model is shown in Figure 23. The entire model uses continuum shell elements (SC8R elements in ABAQUS) and a uniform temperature change is imposed only on the exterior plate. The support structure is assumed to maintain a constant temperature. Contact conditions are imposed between the exterior plate and the support structure, and displacements at nodes around the perimeter of corresponding bolt holes on each part have been tied. Boundary conditions on the inboard flange of the support structure have been imposed to simulate the conditions during experimental testing of the part.

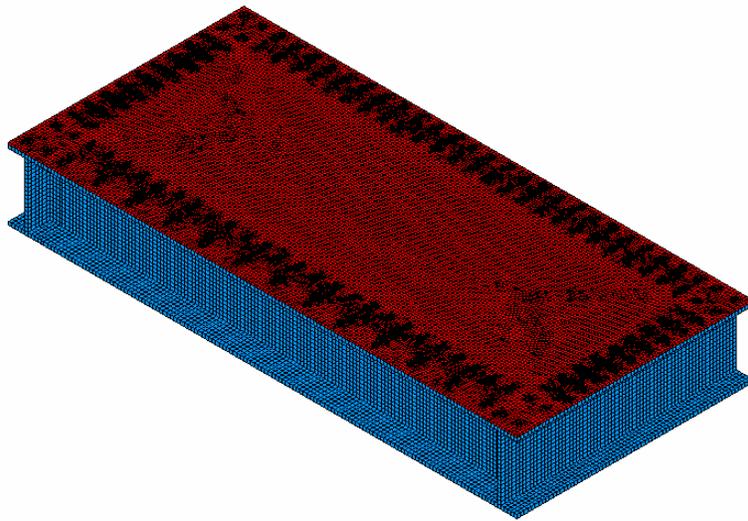


Figure 23. Detailed ABAQUS finite element model including support structure used for detailed design stage

Deterministic analyses have been performed using the nominal values shown in Table 3 for the exterior plate. The support structure is given nominal steel properties, with an elastic modulus of 30.0×10^6 psi and a Poisson's ratio of 0.33, and the length and width of the support structure are adjusted to match the corresponding dimensions of the exterior plate. As shown in Figure 24, ABAQUS linear thermal buckling analysis yields two closely spaced buckling modes at predicted temperature changes of 422°F and 435°F. This result highlights the influence of boundary conditions on the buckling behavior, as the conceptual and preliminary design stage results predicted buckling at a temperature change of approximately 86°F. It should be noted, however, that this discrepancy between the preliminary and detailed results would occur regardless of whether deterministic or probabilistic analyses are performed. The discrepancy emphasizes the effects that assumptions can have on the design performance. Based on the deterministic result, one would assume that buckling would not occur at temperature changes below 422°F. However, due to the uncertainty in the various design parameters, there is some

likelihood that buckling will occur at lower temperature changes. Similarly, there is some likelihood that buckling will not occur until larger temperature changes. Probabilistic analysis can be used to investigate the distribution in temperature changes that cause buckling.

Table 3. Nominal values of exterior plate used for aft-deck structure detailed design analyses

Parameter	Nominal Value
Overall Plate Length	36.3 in
Overall Plate Width	16.8 in
Plate Thickness	0.160 in
Elastic Modulus	16.0×10^6 psi
Poisson's Ratio	0.30
CTE	5.0×10^{-6} in/in/°F

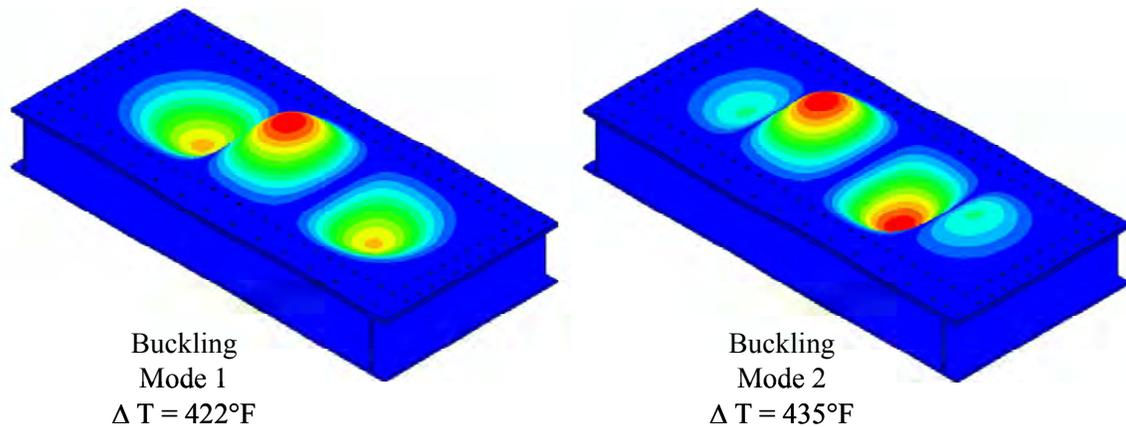


Figure 24. Thermal buckling modes of detailed design predicted by ABAQUS linear buckling analysis

Limited experimental testing has been performed using a test specimen fabricated based on the nominal dimensions and material properties shown in Table 3. For this testing, the upper surface of the exterior plate has been heated using an array of quartz lamps. The support structure is attached to additional fixturing. Out-of-plane displacement measurements have been made on the inboard surface of the exterior plate using a non-contacting optical system. Figure 25 shows out-of-plane displacements measured on the lower surface of the exterior plate during the experimental testing. Since radiant heating is used for the plate, thermal gradients exist throughout the assembly and, as a result, the precise temperature at which this buckling occurs is unknown. At least qualitatively, results from the linear buckling analyses compare well with the measured displacements. Figure 26 shows out-of-plane displacements predicted on the upper surface of the exterior plate for the buckling mode at $\Delta T = 422^\circ\text{F}$. As seen from the figures, the measured and predicted buckled shapes compare well, although the analysis predicts that the plate would buckle in the opposite direction than that measured.

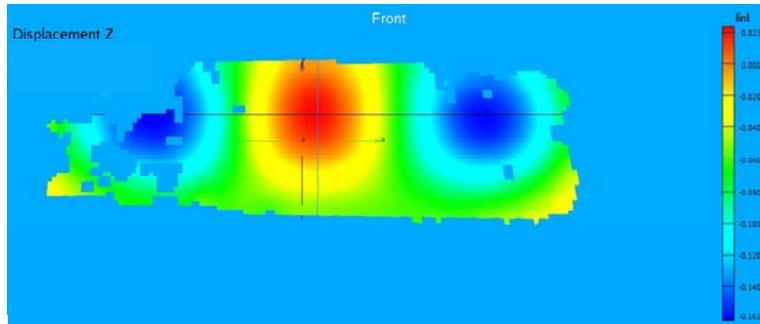


Figure 25. Out-of-plane displacements measured on lower surface of the exterior plate during experimental testing

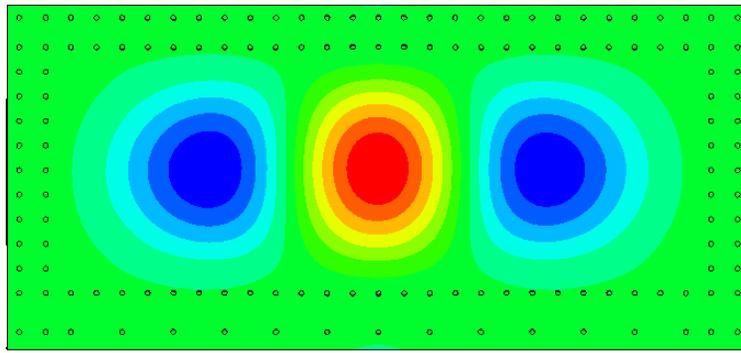


Figure 26. Out-of-plane displacements predicted for upper surface of the exterior plate for buckling mode at $\Delta T = 422^{\circ}\text{F}$

Utilizing the detailed design model, Monte Carlo simulations have been performed to investigate the distribution in buckling temperature changes. As previously mentioned, Monte Carlo simulations generally are not very efficient, particularly for problems with a large number of random variables. However, Monte Carlo simulations have been performed to demonstrate the simulation method techniques, as well as to provide a baseline for which to compare other probabilistic solution techniques. Similar to the preliminary design stage, the detailed design stage utilizes an ABAQUS finite element model which must be updated based on the variables randomly samples from their distributions. Distributions for the random variables are listed in Table 4. Python scripts have been written to randomly sample these distributions, automatically generate finite element models using these variables, and analyze and process the finite element results. This process is repeated to create multiple predictions of the buckling temperature change, and has been performed both within and outside of the UNIPASS framework. Figure 27 shows the predicted distribution in buckling temperature change from the Monte Carlo simulations performed outside of UNIPASS. These simulations include 500 trials and require approximately 10,000 CPU minutes (approximately 1 week) of processor time on a 3.2 GHz Pentium 4 single-processor system. As with the earlier results, the CDF has been calculated from the results of the individual trials and the PDF has been estimated from the CDF. The mean temperature change is 413.8°F and the standard deviation is 23.0°F .

Table 4. Distributions used for exterior plate for aft-deck structure detailed design analyses

Parameter	Distribution	Mean Value	Standard Deviation or Tolerance (\pm)
Overall Plate Length	Uniform	36.3 in	± 0.100 in
Overall Plate Width	Uniform	16.8 in	± 0.100 in
Plate Thickness	Uniform	0.160 in	± 0.005 in
Elastic Modulus	Lognormal	16.0×10^6 psi	0.48×10^6 psi
Poisson's Ratio	Lognormal	0.30	0.0099
CTE	Lognormal	5.0×10^{-6} in/in/ $^{\circ}$ F	0.15×10^{-6} in/in/ $^{\circ}$ F

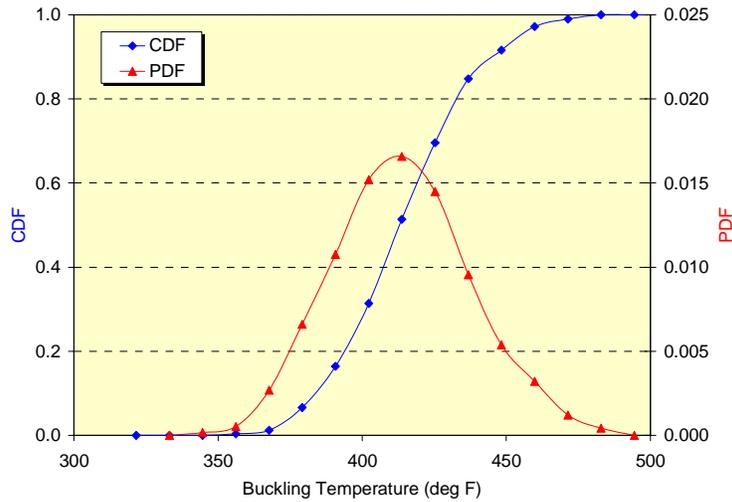


Figure 27. Results from Python scripted Monte Carlo simulations of detailed design using 500 trials

It is anticipated that UNIPASS Monte Carlo simulations would yield results similar to those from the Python scripted simulations. Therefore, due to the large computational resources required, UNIPASS Monte Carlo simulations have not been performed. However, a UNIPASS FORM solution has been performed, with results shown in Figure 28. For this solution, the convergence criteria have been loosened from the default values and the maximum number of iterations has been set to 200 trials. Approximately 4,000 CPU minutes (approximately 2.75 days) of processor time are required. Results from the UNIPASS FORM solution differ fairly significantly from the scripted Monte Carlo results. It is believed that the lack of significant digits in the ABAQUS buckling solution may be a major contribution to this discrepancy. For the FORM solution, UNIPASS predicts the MPP by calculating sensitivities of the buckling temperature to changes in the design parameters. Without sufficient significant digits, relatively small changes in the design parameters may not yield corresponding changes in the buckling temperature. The UNIPASS FORM result highlights the need to have a good understanding of the design parameters as well as the process models used for either probabilistic or deterministic evaluations.

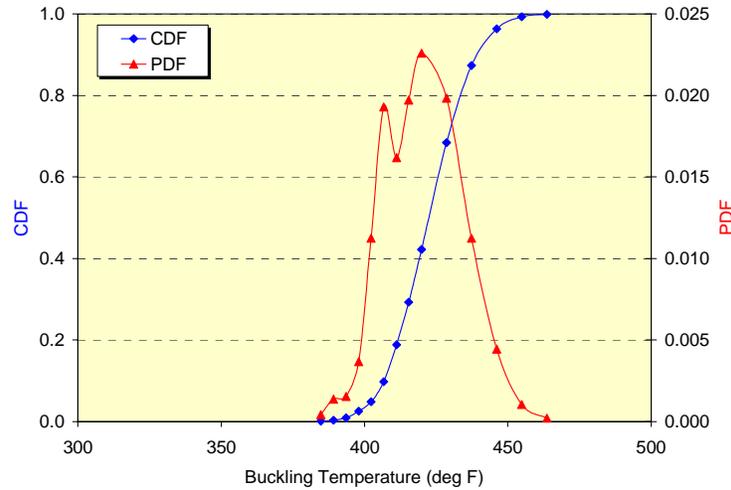


Figure 28. Results from UNIPASS FORM simulation of detailed design using 200 trials

4.2.4 Final Design Stage

For the aft-deck buckling example, final design studies have not been performed. However, such studies would likely utilize models similar to those used for the detailed design studies with refinements made in several areas. One area where refinement would likely prove beneficial is in the fastener modeling to incorporate the variations in fastener stiffness or preload. This refinement may involve modeling the actual fastener geometry or utilizing subscale models to determine the precise fastener characteristics. Additional areas of refinement might include investigating the effects of fastener alignment, thermal profiles, and heating rate. The previous studies have been based on the assumption that the fasteners are all collinear whereas actual parts would have some variation in fastener-to-fastener alignment. The previous studies also assumed a uniform temperature change in the exterior plate while an actual part would involve thermal gradients based on the thermal loads (e.g., radiant heating of the exterior plate) and losses (e.g., thermal conduction from the exterior plate into the support structure or convection to surrounding environment) in various regions.

4.2.5 Aft-Deck Buckling Example Summary

In the preceding analyses, it has been shown that probabilistic techniques can be applied to address uncertainty at every stage of the design process. The sensitivity information that is generated is very useful in driving toward more robust designs. The probabilistic approach provides a mechanism by which to quantify risk during the design process, but comes at some cost. Specifically, the number of analyses which need to be performed typically increases significantly. Additionally, as the design progresses to the final design, identification and validation of design parameters can become extremely difficult. As such, the probabilistic assessment techniques are best suited for use early in the design process.

SECTION 5 CONCLUSIONS

Current aerospace structural design utilizes deterministic methods. The design parameters are assumed known and the structural performance is evaluated using physics-based process models with the design parameters as inputs. However, there are several disadvantages to these current methods. First, the current methods do not account for the random nature of most design parameters. Historically, factors of safety have been sufficiently large to minimize the unknown risk in fielded structures. Second, new air vehicle concepts, such as reusable launch vehicles, depart dramatically from traditional operating environments. As a result, utilization of historically based safety factors may not be appropriate. Lastly, current aerospace design methods are not affected by modifications to the manufacturing process or materials processing. Such changes will affect reliability, but are not captured by the current methods.

Probabilistic design methods can overcome these disadvantages. Probabilistic analysis serves as a means to determine how the variability in loading, geometry, materials, and environment affect the design reliability and the contribution of each design parameter to the overall risk. The objective of probabilistic design is not to establish a particular margin of safety, but rather to achieve a specified level of reliability. There are several major issues which have hindered the use of probabilistic design practices. These issues include the computational and time resources required for probabilistic assessments, the definition of reliability design criteria, and the definition of appropriate ways to perform probabilistic assessments. Satisfactory resolution of these issues is required prior to the widespread use of probabilistic technologies.

Recent technological advances have alleviated many of the issues relating to resources. Probabilistic design typically requires a number of physics-based analyses, such as finite element analyses. Commercial probabilistic analysis software packages, such as UNIPASS, have been created which incorporate mathematical techniques to provide an intelligent means of selecting a subset of solutions to perform. Issues relating to the definition of target reliabilities and probabilistic assessments remain. Many structural problems address continuous systems, where the entire structure, and not just the worst location, contributes to the reliability. Techniques must be developed to enable the definition of target reliabilities for continuous systems, as well as to perform probabilistic assessments of such systems. A brief description of one possible technique, similar to Weibull analyses performed for brittle materials, has been given.

To demonstrate probabilistic assessment, and the advantages to utilizing such a process, an example problem has been presented. This problem is a design application relating to the thermal buckling response of a representative exhaust-washed aft-deck structure. The UNIPASS code has been utilized to perform probabilistic assessments at various stages of the design process. It has been shown that probabilistic techniques can be applied to address uncertainty at every stage in the design process. The sensitivity information that is generated is very useful in driving toward more robust designs. The probabilistic approach provides a mechanism by which to quantify risk during the design process, but comes at some cost. Specifically, the number of analyses which need to be performed typically increases significantly. Additionally, as the design progresses to the final design, identification and validation of design parameters can become extremely difficult. As such, the probabilistic assessment techniques are best suited for use early in the design process.