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The ultimate goal of our research is to find a procedure that would enable us to generate, for a class of systems, nominal finite state machine models with guaranteed error bounds that are usable for robust controller synthesis [4]. Summary: We consider three notions of stability, input/output stability, external stability and incremental input/output stability, as they apply to deterministic finite state machine systems. We propose LP based algorithms for verifying stability of a system, or lack thereof, in the input/output and the external sense. We show that for a class of systems, incremental I/O stability and external stability are equivalent notions, stronger than the notion of I/O stability.

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New Tools for Hybrid Systems

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Abstract

We provide a final report on the research supported under AFOSR # FA9550-04-1-0052.

1 Robust Control Framework for Hybrid Systems

Our interest in deterministic finite state machines stems from our ongoing attempts to use these systems as approximations of dynamical systems having finite input and output alphabets [7]. The ultimate goal of our research is to find a procedure that would enable us to generate, for a class of systems, nominal finite state machine models with guaranteed error bounds that are usable for robust controller synthesis [4]. The issue of stability comes up in several ways in the context of this problem: For instance, Hankel model order reduction is applicable to stable LTI systems, is guaranteed to generate stable reduced order systems, and additionally provides a quantitative measure of the quality of the approximation in the form of induced gain bounds that are compatible with robustness analysis. Similarly for our problem, it makes sense to ask if there is some notion of stability that guarantees existence of good finite state machine approximations, and to ask whether the approximate models should preserve this notion of stability. It also makes sense to look for meaningful induced gain conditions to quantify the approximation error and to quantify the sensitivity of our model, and hence of the synthesized closed loop system, to perturbations due to unmodeled dynamics.

Various notions of stability have been explored for a range of discrete event and finite state systems. Stability in the sense of Lyapunov and asymptotic stability for logical discrete event systems were studied in [6]. The problem of output stabilization of discrete event systems was considered in [5], where stability was understood to mean that all state trajectories of the closed loop system pass through a given set E of interest infinitely often. Questions of stability of equilibria and attractiveness of invariant sets were addressed in [3]. Extensions of LaSalle's Invariance Theorem and Lyapunov's Indirect Method to hybrid automata were proposed. The focus our work is on input/output notions of stability that are more relevant to the approximation and control problems.

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Since our interest in deterministic finite state machines stems from problems of approximation and robust control, we would like to call a nominal system 'stable' if its output response is not highly sensitive to perturbations (for instance, due to unmodeled dynamics). Thus we explore three potentially useful notions of stability: The notion of input/output stability, which is widely used in classical LTI robust control. The notion of incremental input/output stability, which measures the sensitivity of the system to perturbations of the input, and which can be considered to be a viable alternative to input/output stability when the systems in question are nonlinear. The notion of external stability, which measures the sensitivity of the output of a system to changes in its initial conditions, and which was introduced in [7] and shown to be of relevance to the problem of approximating an analog state system with finite input and output alphabets by a deterministic finite state machine.

Summary: We consider three notions of stability, input/output stability, external stability and incremental input/output stability, as they apply to deterministic finite state machine systems. We propose LP based algorithms for verifying stability of a system, or lack thereof, in the input/output and the external sense. We show that for a class of systems, incremental I/O stability and external stability are equivalent notions, stronger than the notion of I/O stability.

1.1 A New Hybrid Control Architecture

A hybrid control structure is proposed which employs a finite state automaton (FSA) as well as a continuous-time feedback structure to be used for control of continuous-time plants. The motivation for exploring such a structure stems from real-world implementation issues that exist with current design methodologies for either strictly analog or strictly digital control. The main goal of this work is to investigate the utility of such a structure. First, we wish to determine whether the set of systems for which the given hybrid feedback structure can be designed is sufficiently broad, and we wish to find an efficient set of tools which can be used to analyze and design such feedback structures. Second, for the class of systems for which the structure is known to be stabilizing, we wish to compare its performance to already existing feedback structures. In particular, we wish to compare performance on measures such as system gain bounds ($\mathcal{L}_2$, $\mathcal{L}_\infty$, etc.), memory requirements, settling time, disturbance rejection, etc..

Much attention over the past year has been given to the specific case where the continuous-time plant is an unstable second order LTI system, and where the continuous-time portion of the feedback structure is a set of proportional gains. For this particular case, necessary and sufficient conditions for stabilizability of a given unstable plant have been discovered for a relaxation of the given problem in which the FSA has access to the full state of the plant. Moreover, the stabilizability of a given second order LTI plant can be characterized in terms of the classical control notion of root locus.

The goal of more recent work has been to investigate the stability and $\mathcal{L}_\infty$ gain-boundedness properties for second order LTI systems in which the finite state automaton has access to only a single output of the plant (rather than the full state vector). Standard Lyapunov/storage function methods for even simple examples have proved to be very difficult, so other methods of analysis have been under consideration. One such method which uses the fact that the system trajectories return to a particular region of the state space infinitely often has proved successful analysis-wise for a specific example. Investigation of this technique as a means of proving $\mathcal{L}_\infty$ gain boundedness for a general second order LTI plant has also been obtained. The results are documented in the conference papers [1, 2] which are under preparation for journal publication.

1.2 Model Reduction of Hidden Markov Models

Hidden Markov Models have been used extensively in many fields of engineering and science in order to model large scale complex stochastic processes that evolve on concealed finite state spaces. The main tasks of statistical inference, forecasting, and decision making performed by means of mathematical abstractions of real world applications are to a large extend hindered by computational intractability issues associated with the large state space order of the underlying stochastic process. This situation is reflected by the commonly used term "curse of dimensionality".

The goal of this research is to alleviate that problem, by developing computational tools that perform model reduction of Hidden Markov Models. The main aspects of this work consist of defining appropriate measures of complexity, identifying relevant approximation metrics for control purposes, deriving reduced models by means of efficient algorithms and providing a priori error bounds that validate the low order dynamic systems. The computation of low complexity approximations of Hidden Markov Models also constitutes an integral part of a robust control framework for systems whose dynamics are governed by probabilistic laws, as well as nonlinear deterministic and hybrid systems.

Progress has been made by developing such tools for a related class of systems, namely discrete-time linear systems with white noise coefficients. This work has been summarized in [8]. In this work, we have shown how the Balanced-truncation algorithm, popular for LTI systems, can be extended to provide a reduced order model with *guaranteed* error bounds. The construction of the balancing transformation is obtained by solving LMIs. Current research is focused on mapping these techniques to the Hidden Markov Model case and is expected to be completed sucessfully in the following months.

1.3 Fundamental Limitations of Performance in the Presence of Finite Capacity Feedback

Motivated by applications, such as remote feedback, control in the presence of information constraints has received considerable attention. The exploration of such problems is exciting as they foster the interaction between the disciplines of Information Theory and Control. So far, research in this field has, primarily, directed its attention to stabilization.

The basic *framework* comprises a plant, a channel, an encoder and a decoder, which implicitly embeds a controller. Measurements of the plant's output must be encoded and sent through the channel. The information, received at the other end of the channel, is decoded and used to generate a control signal.

We have investigated the fundamental limitations of performance arising from finite capacity feedback, operating on arbitrary alphabets. Using Information theoretic principles, we have derived fundamental relations between key quantities. In particular, under stationarity assumptions, we show that the information rate at the channel, denoted as I, must satisfy:

$$I \geq \frac{1}{2\pi} \int_{-\pi}^{\pi} \max\{0, \log S\} dw + I_e$$

where S is the closed loop sensitivity and I_e is the information rate required for stabilization. As such, this inequality indicates that $I - I_e$ determines a fundamental limit on disturbance attenuation.

Using the same techniques, we have proven that Bode's integral formula holds in the presence of feedback over arbitrary alphabets. Both fundamental limitations must be always satisfied since they originate from distinct constraints: Bode's limitation results from causality, while the above limitation is associated with information rate constraints. These results are documented in [9, 10, 14].

1.3.1 Vehicle Routing: Dynamic Pick-up and Delivery Problem

Our research on the problem of control with communication constraints reported in [9, 10, 14] expanded to address problems concerning larger networks of systems. In here, we briefly describe our work on the control of networked mobile systems. Since many such formulations can result in intractable problems, a major objective of this project is to find abstracted models that capture some of the important tradeoffs in performance objectives and constraints which are also tractable in terms analysis and optimization. An example of such a simplified model of networked agents is the Dynamic Pickup and Delivery Problem (DPDP) described below as an example of vehicle routing problem, and is reported in [11, 12, 13].

Vehicle Dynamics: Assume there are n vehicles in a geographic area $\mathcal{A} \subset \mathbb{R}^2$, which is a convex, compact set with volume A. For simplicity of this exposition, we consider $\mathcal{A} = [0, \sqrt{A}]^2$. Each vehicle may move in any direction at any time with a velocity of magnitude $\leq v$. In the simplest case, v does not depend on n, however, if we want to consider congestion, we may allow the velocity to decrease as n increases.

Service Requests Messages (service requests) are generated according to a Poisson process with time intensity $\lambda(n)$. We assume that the rate of messages arriving to the system increases much faster than the number of vehicles in the system available to service these messages. Associated with each message j are source and destination locations denoted by $s(j) \in \mathcal{A}$ and $d(j) \in \mathcal{A}$ respectively. Source locations are independently and identically distributed (IID) in $\mathcal{A}$ according to the distribution $\phi_s : \mathcal{A} \to \mathbb{R}^+$. Similarly, destination locations are IID with distribution $\phi_d : \mathcal{A} \to \mathbb{R}^+$.

Objective The messages need to be picked up from their source locations and delivered to their destination locations by the vehicles. A message is picked up (delivered) when a vehicle spends a fixed on-site service time of $\bar{s}(n)$ at the source (delivery) location to pick up (deliver) the message.

Control Policies

A control policy, π, is a set of decision making rules that decides the pickup and delivery schedule of arriving messages, based on a set of constraints on the information available to the vehicle. In this paper, we consider policies $\pi = (\pi_A, \pi_S)$ that can be decomposed into two components, assignment and service. An assignment policy, π_A, describes how a centralized controller assigns arriving messages to vehicles on a real-time basis. A service policy, π_S, describes how each vehicle performs the pickup and delivery of its assigned messages. We assume that neither the vehicles nor the centralized assignment controller have any knowledge of individual messages before they arrive although the overall message arrival process and source and destination distributions are known.

Information Structure We further restrict the set of assignment policies according to the information available to the controller in making message assignments. For example, we can consider two types of information structures. The first is the *Source only information.* In such a case, when a message arrives, its source location is known to the centralized controller, which is consequently communicated to the appropriate vehicle. Vehicles, however, do not know the destination of messages until they pick them up. The second is the *Source-destination information.* In this case, when a message arrives, both its source and also its destination location are known to the centralized controller and both are consequently communicated to the appropriate vehicle.

Performance Metrics: One metric that has received much attention in queuing theory is the *average delay.* The delay of message j, denoted $W(j)$, is defined to be the elapsed time between the message's arrival to the system and its delivery to its destination location. This includes any time the message waits to be picked up, the onsite service time for pickup, travel time on the vehicle before arriving at the delivery location, and finally onsite service time for delivery. The quantity W is defined to be

$$W = \limsup_{j \to \infty} \mathbb{E}[W(j)]. \tag{1}$$

W is the steady state expected value of the delay of a message in the system. We say that the system is stable if $W < \infty$. If π is a stable policy, $W(\pi)$ is defined to be the limiting expected delay associated with that policy. Note that $W(\pi) = W(\pi_A, \pi_S)$ is a function of both the assignment policy and the service policy.

Problem Definition: Given the problem defined above, we seek stable policies that minimize that average delay per message. Equivalently, we must solve the following optimization problem:

$$\min_{(\pi_A, \pi_S)} W(\pi) \qquad \pi \text{ is a stabilizing policy}$$

This problem minimizes the weighted sum of average delays over all of the vehicles according to a joint constraint on the policies used by the individual vehicles. If the minimum is finite, each of the $W_i(\pi)$ must be finite as well and the system is stable.

We call this problem the Dynamic Pickup-Delivery Problem (DPDP). It is important to note that $OPT1$ is still a difficult optimization problem. However, we will discuss how the complexity of this problem can be substantially reduced by considering the behavior of this problem for large n.

Below we summarize the results for the Source-only information structure.

Theorem 1

(a) For any policy in Π_{SO} under the Source Only information structure, the average delay per message scales as

$$W_{SO} \geq \gamma^2 \left(\frac{\lambda(n)A}{v^2(1-\rho)^2 n} \right) - \frac{n(1-2\rho)}{2\lambda(n)} \tag{2}$$

with constant $\gamma = 2/3\sqrt{2\pi}$. For $\lambda(n) = \Omega(n)$ this bound scales as $W_{SO} = \Omega(\lambda(n)A/v^2(1-\rho)^2 n)$.

(b) Further, there exists a policy using Source Only information, for which the average delay scales as $O(\lambda(n)A/v^2(1-\rho)^2 n)$ for all $\lambda(n)$. Therefore the lower bound scaling is achievable.

Similarly, we summarize the results for the Source-Destination information structure.

Theorem 2

(a) For any policy in Π_{SD} under the Source-Destination information structure, the average delay per message with $\lambda(n) = \Omega(n)$ scales as

$$W_{SD} \geq \frac{\gamma^2}{4} \frac{\lambda(n)A}{v^2(1-\rho)^2 n^{3/2}} \tag{3}$$

with constant $\gamma = 2/3\sqrt{2\pi}$.

(b) Further, there exists a policy using Source and Destination information for which the delay scales as $O(\lambda(n)A/v^2(1-\rho)^2 n^{3/2})$ for $\lambda(n) = \Omega(n^{3/2})$. Therefore the lower bound scaling is achievable in heavy traffic.

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1. Munther Dahleh, Professor.
2. Alex Megretski, professor.
3. Nuno Martins. Post doctoral fellow.
4. Keith Santarelli, student.
5. Danielle Tarraf, student.
6. Georgios Kotsalis, student.
7. Holly Waisanen, student.

3 Awards

- Prof. Dahleh received the George Axelby best paper award on a paper co-authored with Prof. Bamieh and Prof. Paganini in 2003.
- Prof. Dahleh received the ACC 2006 Hugo Schuck Award for theory, for the paper entitled, "Fundamental Limitations of Performance in the Presence of Finite Capacity feedback, co-authored with Nuno Martins.

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