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Student Paper

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An Approximation Technique for Belief Revision in Timed Influence Nets*

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Abstract

The paper presents an approach for belief updating in Timed Influence Nets. Influence Nets provide graphical representation of causal or influencing relationships in complex situations. They are used to model and evaluate courses of actions in certain domains and to compare the performance of actions based on the desired outcome. In Timed Influence Nets, the impact or effect of these actions on target variables is not instantaneous. This is modeled by adding communication and processing delays in the model. The paper provides a technique for updating the beliefs of variables in the model over time once new evidence is received about some of the variables in the model. The objective is to assess the behavior of the variables of interest as a function of both the timing of actions and the receipt of evidence on indicators, thus providing aid to decision makers in the revision of the planned courses of actions.

1 Introduction

Probabilistic Belief Networks have gained popularity in last two decades to model uncertainty [Charniak, 1991], [Jensen, 2001], [Neapolitan, 2003], and [Pearl, 1987]. Commonly referred to as Bayesian Networks (BN), these belief networks use a graph-theoretic representation to explicitly show the dependencies among variables in a particular domain. Formally BNs are Directed Acyclic Graphs (DAG) in which nodes represent random variables while an edge connecting two nodes (typically) represents causal relationship (though it is not required that the connection be causal) between the two variables. The relationship between a node and its parents is defined by a Conditional Probability Table (CPT) for all combination of parents' states. The joint distribution over the random variables present in the network can be expressed as

$$P(x_1, \dots, x_n) = \prod_i P(x_i | pa(x_i))$$

where $pa(x_i)$ represents a configuration of the set of parents of variable x_i .

These networks have been primarily designed to simplify the intractable task of joint probability distribution elicitation. They have been usually applied without considering an explicit representation of time. In the past few years, efforts have been taken to integrate the

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notion of time and uncertainty [Figueroa and Sucar, 1999], [Galan and Diez, 2002], [Hanks et al., 1995], [Kjaerulff, 1992], and [Santos and Young, 1999]. The popular approach of modeling processes having temporal dependencies is to discretize the time and create an instance of each random variable present for each point in time. The process starts with eliciting the probability distributions for the static probabilistic model. This model is repeated for difference time slices, and links are drawn between these slices to represent the temporal dependencies among the nodes in the network. The approach is usually referred to as Time Sliced Bayesian Networks (TSBNs) or Dynamic Bayesian Networks (DBNs) [Murphy, 2002]. Figure 1 shows two types of TSBNs as discussed by Hanks et al., [1995]. In Figure 1 on the left, all the connections in the models are inter-slice, i.e., connections only exist among variables within different time slices. On the contrary in Figure 1 on the right, the variables in the model have both inter-slice and intra-slice connections.

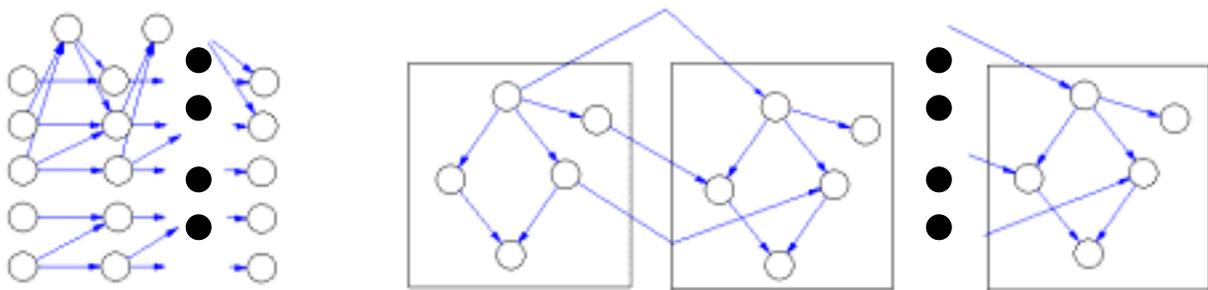


Figure 1: Examples of Time Sliced Probabilistic Networks

Despite the fact that probabilistic belief networks address the intractable problem of eliciting joint distribution of random variables in an efficient way, the number of parameters required to specify the Conditional Probability Table (CPT) of a node increases exponentially with the number of parents. Several approaches have been proposed that estimate the CPT values from parameters that are linear with the number of parents. These include but not limited to Noisy-OR [Agosta, 1991], [Drudzel and Henrion, 1993], and [Heckerman and Breese, 1996], Causal STrength (CAST) Logic [Chang et al., 1994] and [Rosen and Smith, 1996], etc. The probabilistic models that use CAST Logic as an interface for estimating the CPTs for each node in the network are referred to as Influence Nets. Influence Nets simplify knowledge elicitation by reducing the number of parameters that must be provided. They are appropriate for modeling situations in which the estimate of the conditional probability is subjective, e.g., when modeling potential human reactions and beliefs, and when subject matter experts find it difficult to fully specify all conditional probability values.

Timed Influence Nets (TINs) [Wagenhals and Levis, 1999], extend the CAST logic based interface of Influence Nets by providing a way to model uncertainty and temporal constraints present in a stochastic model from a Discrete Event System's (DES) perspective. These TINs are developed by making cause and effect or influencing relationships among variables in the domain. The links between two variables represents the temporal causal relationship between them. The impact of one variable on other variables does not necessarily occur instantaneously; rather it may occur after a specified time. This time is represented by the assignment of a delay to each link. All the nodes in the network could also have an optional time delay which

represents the information processing delay of the corresponding node. The marginal probability of a node is computed whenever there is a change in the state (the marginal probability) of any of its parents. To achieve this behavior in a computationally efficient manner, TINs propagate probabilities using independence of parents assumption, also referred to as loopy belief propagation [Murphy et al., 1999].

TINs have been used experimentally in the area of Effect Based Operations (EBO) [Wagenhals et al., 2003]. They are modeled by identifying target variables and relating them to the actions that could impact them. The purpose of creating such models is to determine how to maximize or minimize the probability of occurrence of the target variables by taking a timed sequence of actions or actionable events. Actionable events, in this context refer to the random variables that are modeled as root nodes in the corresponding TIN. The actionable events (either under the control of the decision maker or the adversary) and the variable of interest (target variable) are connected through chains of variables that represent intermediate effects. Some of these variables may be observable. This paper describes an extension to the capability of TINs by adding the provision of incorporating new evidence in the model. The algorithm presented in the paper provides an approximation scheme for updating the belief of the affected variables after observing evidence provided that certain constraints are satisfied. The algorithm first tries to reduce the net by identifying those variables which are relevant for computing the posterior probability of a target variable over an interval of time. The nodes, which do not have any impact on the variable of interest as a result of new evidence, are considered as being pruned from the net. In the second step, the algorithm computes the new beliefs on all the affected variables taking into account the time delays (communication and processing delay) present in the graph. This technique provides an initial step in the direction of integrating the impact of planned Course of Action (COA) [Wagenhals and Levis, 2000] selected by the decision maker over a time interval and the information/observations which arrive during / after the execution of that particular COA. The objective is to assess impact of the actionable events as the situation dynamically unfolds.

The rest of the paper is organized as follows. Section 2 discusses the mathematical formulation of TINs and their application. Section 3 describes the belief propagation algorithm that supports the incorporation of evidence, while Section 4 concludes the paper and points out areas for future research.

2 Modeling Uncertainty Using Timed Influence Nets

2.1 Knowledge Elicitation

TINs use CAST logic, a variant of Noisy-OR, to simplify knowledge elicitation from subject matter experts. Instead of assigning conditional probabilities, the expert first specifies the qualitative relationship between two connected nodes as either promoting or inhibiting. Figure 2 shows a simple two node influence net. In Figure 2(a), the modeler indicates that the presence of A can cause B (with some probability), and a '+' sign is attached to the arc. Similarly, the modeler indicates that the absence of A can inhibit B, therefore there is a '-' sign attached in Figure 2(b). Figure 2(c) shows the aggregated qualitative relationship between two nodes by using the double (+, -) notation. If the modeler had determined that the presence of A inhibits B

while the absence of A promotes B, then the aggregate notation would be (-, +). Qualitative relationships among variables have also been applied for Qualitative Probabilistic Networks (QPN) [Drudzel and Henrion, 1993] and Causal Maps [Nadkarni and Shenoy, 2001].

After (or during) assigning the qualitative relationships between the two nodes, the expert(s) quantify these relationships by assigning values on the scale of 0 to 1. Low values mean the promoting or inhibiting relationship is weak while values near 1 mean the relationship is strong. The CAST logic uses a heuristic to convert these qualitative relationships to form the conditional probability matrix for each non-root node. Besides reducing the number of parameters required for specifying the conditional probability matrix for each node, the CAST logic also helps in eliciting knowledge from different subject matter experts. For instance, in an international conflict, there are many dimensions of the problem, namely political, religious, ethnic, social, etc. It is difficult to find domain experts having expertise in all the above areas. The CAST logic provides a mechanism to obtain information from various experts and then combine their individual assessment in a mathematical manner. The exact details of the CAST logic algorithm are beyond the scope of this paper. The interested reader should refer to Chang et al., [1994] and Rosen and Smith, [1996].

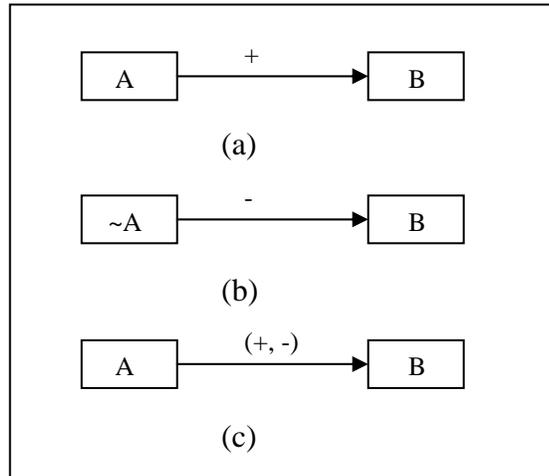


Figure 2: Qualitative Relationships in TIN

Timed Influence Nets extend the capabilities of Influence Nets by providing a mechanism to specify certain kinds of temporal constraints present in a problem domain. Wagenhals et al. [2003] have classified 4 types of temporal information that could be associated with a Timed Influence Nets. Out of them, three are part of the model itself and one is related to the input scenario. The *input scenario* can be described in terms of the actions in the Course of Action (COA) and the time at which these actions occur. Among the remaining three types of temporal information, one is related to the *duration* of an action. The second type is related to the *communication* and *processing* delay present in a problem domain. In other words, this type represents the amount of time it takes for knowledge about a change in the status of any variable to be propagated by some real world phenomenon to the node that is affected by that change. The third type of temporal information is sometimes referred to as *persistence*. This is the time interval over which an effect is manifested. Because of the complexity of this problem, the issue of modeling persistence is still an area of active research. In the sequel, when we discuss TINs

we mean Influence Nets that are capable of modeling the first three types of temporal information (without persistence). The full specification of a Timed Influence Net is as follows

1. A set of random variables that makes up the nodes of a TIN. All the variables in the TIN have binary states.
2. A set of directed links that connect pairs of nodes.
3. Each link has associated with it a pair of CAST Logic parameters that shows the causal strength of the link (usually denoted as g and h values).
4. Each non-root node has an associated CAST Logic parameter (denoted as baseline probability) while each root node has a prior probability.
5. Each link has a corresponding delay d (where $d \geq 0$) that represents the communication delay.
6. Each node has a corresponding delay e (where $e \geq 0$) that represents the information processing delay.
7. A pair (p, t) for each root node, where p is a list of real numbers representing probability values. For each probability value, a corresponding time interval is defined in t . In general, (p, t) is defined as

$$((p_1, p_2, \dots, p_n), [[t_{11}, t_{12}], [t_{21}, t_{22}], \dots, [t_{n1}, t_{n2}]]) \text{ where } t_{i1} < t_{i2} \text{ and } t_{ij} > 0 \\ \forall i = 1, 2, \dots, n \text{ and } j = 1, 2$$

The first four requirements in the above specifications are the same for static and timed Influence Nets. The last three requirements are related specifically to TINs. Once a TIN is completely specified, it can be used to observe the behavior of variables of interest over a specific period of time.

2.2 Course of Action Analysis

Figure 3 [Wagenhals and Levis, 1999] shows how a TIN model compactly represents actionable events, causal or influencing relationships between actions and effects, the strengths of those relationships, and the time delays associated with effect propagation. It illustrates the kind of analysis that could be done using TINs. The model shows the cause and effect relationship as seen by an expert on international politics.

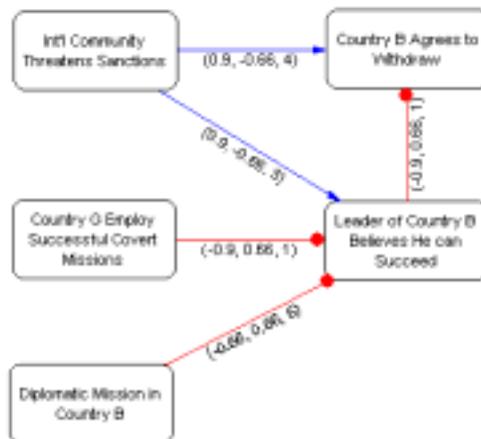


Figure 3: A Simple Timed Influence Nets

Country B has occupied portion of land of a neighboring country. The objective is to find a peaceful solution of the problem, or, in other words, the objective is to determine the probability that country B would agree to withdraw its forces. There are five variables in the Influence Net represented by the five boxes. Each arc in the net has an annotation that is a triple. The first two elements of the triple contain the influence strength of the presence and absence of the parent node on the child node. The third entry is the time delay required for the influence to reach from parent to child. The prior probabilities of each root node are also shown in the figure.

The next stage is to set the time for execution of the actions represented as root nodes. Suppose the actions are executed at the following times:

Diplomatic Mission in Country B @ 8

Int'l Community Threatens Sanctions @ 10

Country G Employs Successful Covert Mission @ 11

The influences of these actions reach the target node (Country B Agrees to Withdraw) at different times. Every time an influence arrives at the target node, the TIN updates the belief of target node. These belief updates and the time each one occurs are shown in the form of a probability profile (Figure 4).

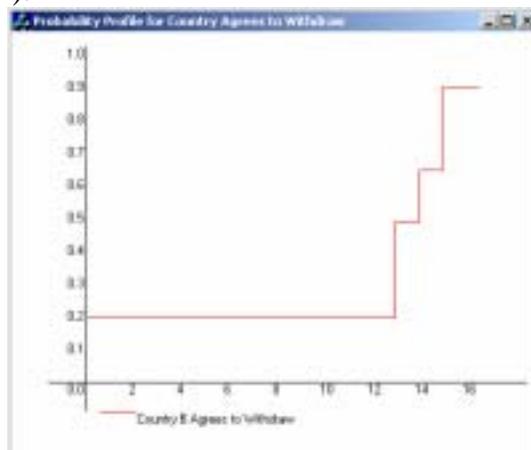


Figure 4: Probability Profile for Node "Country B Agrees to Withdraw"

3 Belief Propagation Algorithm

TINs were originally designed for the COA analysis. In the original TIN formulation it was assumed that all the actions are in fact evidence nodes and there would be no evidence on the other nodes in the networks. Thus, these TINs lacked the ability to incorporate the information/evidence coming from different sources during the execution of a COA. In a military/political scenario, this new information might come from the surveillance system observing an adversary's actions. In an economic domain, a new development in the market, e.g., bankruptcy filed by some corporation, might be taken into account before making a strategic decision. In any case, this new information results in the revision of a previously held belief about some variables in the domain. This section, which is the main theme of this paper, extends the ability of the original TINs by presenting an approach for integrating the new information with the existing beliefs on other variables in the net. The algorithm takes advantage of the fact

that while analyzing a TIN, the analyst is primarily interested in observing the behavior of the desired objectives. This feature helps in applying graph reduction techniques and simplifies the belief revision process. Instead of revising the beliefs on all the variables in the net, the belief revision process is applied to only those set of variables which impact the variable of interest in some way. The algorithm is based on the constraint *that the marginal probability of a parent node will not be updated unless all of its children which need to be updated have updated marginal probabilities.*

The algorithm has three main steps. The first step determines the sequence in which the marginal probability of nodes will be updated once evidence has been incorporated in one of the nodes of the model. Step two selects only the nodes in the sequence that are needed to update the target node. In Step 3, the updating is accomplished node by node in the sequence determined in Step one. The following sub sections describe the working of the algorithm in detail.

Table 1: Algorithm for Sequencing of Nodes

<p>Let X be the evidence node, 1) NodeList = [X] 2) While NodeList has unprocessed nodes Current_Node = 1st Unprocessed node in the NodeList If there exists descendant of Current_Node unprocessed in the NodeList then Move Current_Node at the end of the NodeList Else Add parents of Current_Node in the NodeList Mark Current_Node as processed End Loop</p>
--

3.1 Sequencing of Nodes

The algorithm that determines the sequence for updating the nodes in the TIN is presented in Table 1. It creates a sequenced list of all of the ancestors of a node to which evidence is applied. The sequence is based on a breadth first protocol that ensures the closest ancestors are placed on the list before more distant ancestors. This sequence shows the order of the belief updating of nodes in the net. The node for which we have obtained hard evidence is assigned sequence number one. The parents of the node are sequenced then, and the process continues until all root nodes that have a path to the evidence node are reached. We call this process backward sequencing. The resultant sequence is then used to update the probabilities of nodes during backward propagation. It should be noted that after reaching a root node, the belief updating process continues in the forward direction and the nodes, which were not updated during the backward direction, are then updated until the algorithm reaches the target node. The example TIN in Figure 5(a) is used to explain the sequencing algorithm. Only the structure of the TIN and the time delays are shown in the figure for clarity.

Suppose evidence about node 'H' in Figure 5(a) is obtained. In the first step, the algorithm initialized the NodeList with 'H'. In the second step, the variable 'Current_Node' is assigned the first unprocessed node in 'NodeList', which in this case is 'H'. Since 'H' is the evidence node, the parents of the 'H' are entered in the list, and 'H' is considered to be a processed node. At the end of the first iteration of step 2, 'NodeList' has value [H, I, F]. Nodes 'I' and 'F' are unprocessed. The sequence of choice of parents of 'H' is arbitrary, so 'NodeList' could have the value [H, F, I].

In the next iteration, 'I' becomes the Current_Node. None of the descendants of 'I', are in the list of unprocessed nodes, so the parent of 'I' (node 'D') is added to in the 'NodeList'. At the end of this iteration 'NodeList' has the value [H, I, F, D]. The next unprocessed variable in the list is 'F' and since the descendants of 'F' have already been processed in the list, the parents of 'F' are included in the list. The 'NodeList' now has value [H, I, F, D, G, B]. Next 'D' is assigned to Current_Node but one of the descendants of 'D', i.e., 'G', is still in the list unprocessed. Thus 'D' is moved to the end of the list. The new value of 'NodeList' is [H, I, F, G, B, D]. This time node 'G' is the Current node and the only child of 'G' has already been included in the list, therefore parents of 'G' are also included in the list, making the value of NodeList be [H, I, F, G, B, D, E]. In this way, the algorithm keeps iterating, until all the nodes in the NodeList are processed. The final value of NodeList is [H, I, F, G, E, D, C, B, A, M]. In general, there can be more than one possible sequence, however, all sequences will produce the same results as far as belief updating is concerned.

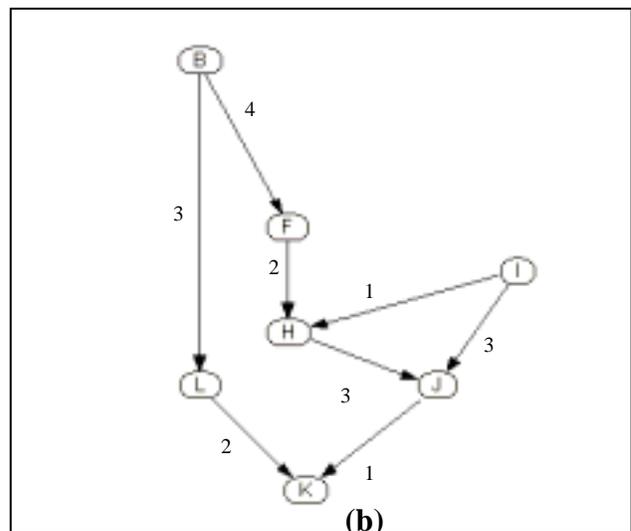
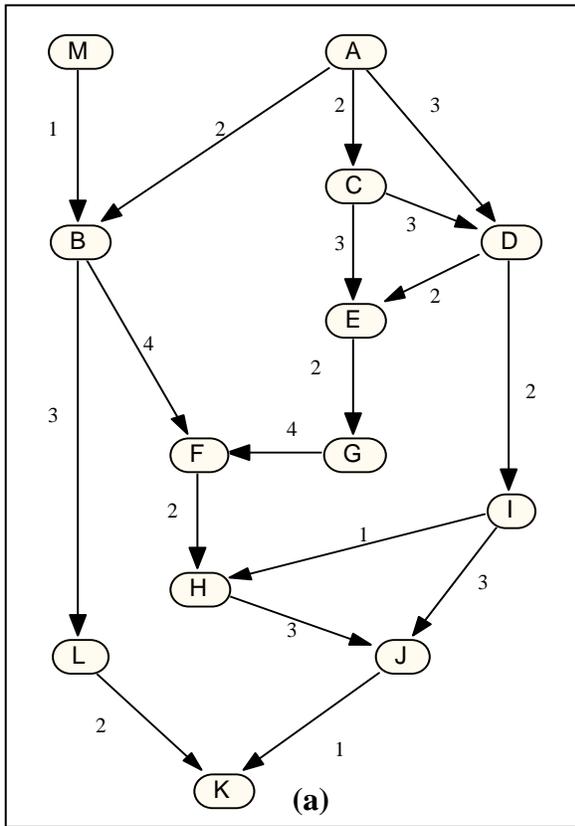


Figure 5(a): TIN for Explaining the Sequencing of Nodes Algorithm

Figure 5(b): TIN after Pruning

3.2 *Graph Reduction*

The steps described in the previous sections give the node ordering that would be used while updating the nodes in the backward direction. But not all of the nodes are required, if the objective is to only see the impact of the evidence on the target node. Considering the same example used in the previous section, nodes 'A' and 'M' represent the actions taken by the decision maker(s). If the evidence is received after the execution of these actions and the objective is to analyze the behavior of node 'K', then there is no need to update nodes G, E, D, and C. All we need to do is to update I, F, and B in the backward propagation and then update the descendants of these variables during forward propagation. The process is referred as graph pruning. The resulting TIN is shown in Figure 5(b).

The sequencing and pruning algorithms can be used when evidence is available for more than one node. When evidence is available at two or more nodes then the sequence and pruning algorithms are run multiple times. For example, if there is evidence for nodes H and L, then only nodes I and J need to be updated in order to see the effect of the evidence of both nodes on node 'K'. The remaining nodes do not need to be updated, as they could not impact 'K' through some other paths.

3.3 *Computation of Posterior Probability*

Once the sequence is obtained, the iterative application of Bayes' rule is used for computing the posterior distribution of the affected variables in the net. In our example, suppose the decision maker has taken actions 'M' and 'A' at time 6 and 8, respectively. The conditional probability tables associated with each non-root node are not shown in the figure to enhance the readability of the figure. The reader should be able to trace the flow of information as the actions take place. For instance, node 'B' would be updated at time 7 and 9. Similarly, node 'D' would be updated at times 11 and 13. The times of update for other nodes could be computed in the similar manner. Once we finish the update in the forward direction, the findings could be entered in the system. Suppose we get evidence about the presence of 'H' at time 24. Mathematically, $P'(H) = 1.0 @ 24$, where the notation $P'(H)$ means that this is an updated marginal probability. Before getting this evidence, the marginal probabilities and their times of update for node 'H' and its parents 'F' and 'I' are as follows:

$$P(H) = 0.44 @ 23$$

$$P(F) = 0.21 @ 21$$

$$P(I) = 0.12 @ 15$$

After updating node 'H' at time 24, the belief updating process selects the next element in 'NodeList' which is 'I' in our example. The time delay on the arc between 'H' and 'I' is 1. Hence the probability of 'I' at time 23 should be revised. The probability is computed as

$$P'(I) = P(I | H) P'(H) + P(I | \sim H) P'(\sim H) \quad (1)$$

where $P(I | H)$ and $P(I | \sim H)$ are computed using Bayes' rule. $P'(H)$ and $P'(\sim H)$ represent the new probability of H and $\sim H$ as a result of the new evidence.

$$P(I | H) = P(H, I) / P(H) \quad (2)$$

$$P(I | \sim H) = P(\sim H, I) / P(\sim H) \quad (3)$$

The numerator of Eq. (2) can be expanded as

$$P(H, I) = P(H, I, F) + P(H, I, \sim F) = P(H | I, F) P(I, F) + P(H | I, \sim F) P(I, \sim F) \quad (4)$$

As discussed earlier, TINs assume that both parents of 'H', i.e., 'I' and 'F', are independent, which results in the simplification of equation (4),

$$P(H, I) = P(H | I, F) P(I) P(F) + P(H | I, \sim F) P(I) P(\sim F) \quad (5)$$

Suppose the Conditional Probability Matrix for node 'H' is given as

$$\begin{aligned} P(H | I, F) &= 0.15 & P(H | I, \sim F) &= 0.98 \\ P(H | \sim I, F) &= 0.005 & P(H | \sim I, \sim F) &= 0.5 \end{aligned}$$

Using these conditional probabilities, equation (5) becomes

$$P(H, I) = (0.15)(0.12)(0.21) + (0.98)(0.12)(0.79) = 0.097$$

The numerator of equation (3) can be computed in a similar manner

$$P(\sim H, I) = (0.85)(0.12)(0.21) + (0.02)(0.12)(0.79) = 0.023$$

From the above two equations, we can compute equations (2) and (3)

$$\begin{aligned} P(I | H) &= 0.097 / 0.44 = 0.22 \\ P(I | \sim H) &= 0.023 / 0.56 = 0.04 \end{aligned}$$

Equation (1) thus becomes:

$$P'(I) = (0.22)(1.0) + (0.04)(0) = 0.22$$

The probability of F is updated in a similar manner at time 22, as the time delay on the arc between 'F' and 'H' is 2. After these updates, the probabilities of H, F, and I become:

$$\begin{aligned} P'(H) &= 1.0 @ 24 \\ P'(F) &= 0.01 @ 22 \\ P'(I) &= 0.22 @ 23 \end{aligned}$$

The next node in the 'NodeList' is 'B'. The time delay between 'F' and 'B' is 4. Hence the probability of B is revised at time 18. It has been discussed earlier that there is no need to update the probability of nodes 'G', 'E', 'D', and 'C' if the only objective is to observe the impact of evidence on the target node 'K'. But in order to show how the constraint of not updating the parent unless all the children are being updated works, we could continue the process of backward propagation till we reach node 'D'. The impact of evidence arrives at 'D'

through both of its children, 'I' and 'E' at time 21 and 14, respectively. If we update 'D' at 21 based on the new probability of 'I' at time 23 then the probability of 'E' will also be updated at 23 during forward propagation. Further, the probability of 'F' would be updated at time 29. Since the child of 'F', i.e., 'H' is already an evidence node, the new probability at 'F' results in the update of the other parent of 'H', namely 'I' at time 30. Thus, as a result of updating the probability of 'D' based on the new value of 'I' we have obtained a new probability of 'I'. This cycle of update would continue forever. In order to avoid falling into the problem of infinite loop which would result if we consider the impacts of 'I' and 'E' on node 'D' individually, the earliest time will be used for the update. Hence node 'D' will be updated at time 14. In general, if the impact of evidence reaches node Y through multiple paths then the update time is computed as

$$t_Y = \min(t_{X1} - \alpha_{YX1}, t_{X2} - \alpha_{YX2}, \dots, t_{XN} - \alpha_{YXN})$$

where $X1, X2, \dots, XN$ represent the children of Y that are already updated. $\alpha_{YX1}, \alpha_{YX2}, \dots, \alpha_{YXN}$ represent the time delay on the links between Y and its children $X1, X2, \dots, XN$, respectively while $t_{X1}, t_{X2}, \dots, t_{XN}$ represent the time of update of $X1, X2, \dots, XN$, respectively.

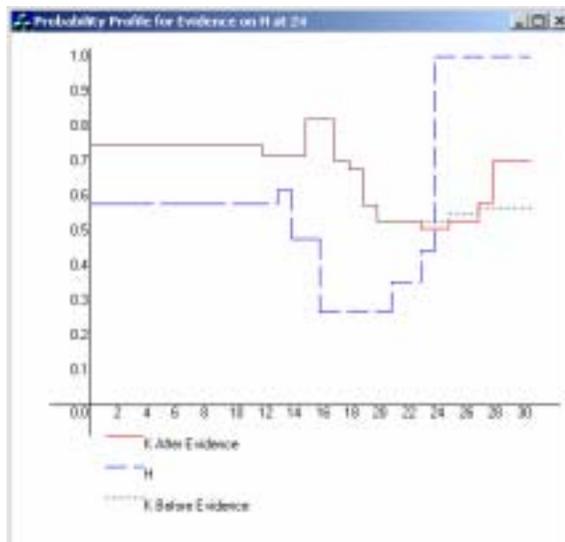


Figure 6: Probability Profiles of Nodes 'H' and 'K' After Evidence on 'H'

Once the backward propagation is finished, the algorithm starts in the forward direction. The probability of node 'L' is updated at time 21 (the time of last update of B plus the time delay on the arc between 'B' and 'L'). The impact of evidence on node 'J' arrives through two paths: H-I-J, and H-J. The last update of I occurred at time 23 and the arc delay has value 3. Hence there is a change in the marginal probability of 'J' at time 26. The impact of evidence at 'H' reaches 'J', through the direct path between 'H' and 'J', in 3 time units. Thus, there is another update at time 27 (time of evidence plus delay on arc from H to J). The changes in the marginal probabilities of the parents of the target node 'K' would result in the computation of new beliefs in node 'K' at time 23, 27, and 28. These changes are shown in the probability profile of Figure 6. The figure shows that incorporating the evidence about node H at time 24 changes the probability of node K from 0.57 to 0.70 at time 28. This example assumes that the time delays

are associated only with the arc. The same technique can be applied for computing the posterior probability if the time delays are associated with both nodes and arcs.

The ability to enter evidence in the model allows the possibility to compute the value of information. In the planning domain, the decision makers are confronted with the task of the placement of evidence gathering resources that may be limited in number. Having the ability of finding the impact of certain evidence on the desired objective, the planners would be in a better position to decide how to use these scarce resources based on the contribution each evidence node makes in reducing the uncertainty in the objective node.

4 Conclusions

The paper presented a computationally efficient technique for belief updating in Timed Influence Nets. The proposed technique updates the nodes in the sequential manner using the constraint that all the children of a node affected by the new evidence will be updated first before updating the belief in that particular parent node. The algorithm also takes advantage of the fact that in TINs, the focus is on observing the behavior of few nodes in the network. Hence there is no need to update all the nodes of the network. The nodes that receive impact of evidence and have a path to the target nodes only need to be updated. This relaxation helps in applying graph reduction techniques on TINs.

One of the possible limitations of the approach is that the algorithm works only if the time stamp of the evidence is later than the time stamp of the last update of the evidence node caused by the forward propagation of the effects from all of the action nodes. This constraint might be very strong in some cases. It is quite possible that the evidence could be observed earlier than expected by the model. There are few possible approaches to relax this constraint. Either the expert should revise the communication and processing delays in the network, or the portion of the graph which is in conflict with this new evidence should be pruned before starting the backward propagation. The other alternative is to convert the Timed Influence Nets into a Time Sliced Bayesian Network (TSBN) and use a vast variety of algorithm available for TSBN. The transformation from TINs to TSBNs is addressed in a future paper. It should be mentioned, though, that the problem of inference in TSBNs (or even in static Bayesian Networks) is computationally intractable. Thus, there is a trade off between the use of approximate algorithms and the exact algorithms in terms of accuracy and the time to compute the posterior probability of the variable of interest. An efficient algorithm for belief updating in TSBNs is still an area of active research.

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A Heuristic Approach for Belief Revision in Timed Influence Nets

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Alexander H. Levis

**The 2004 Command and Control Research and
Technology Symposium**

June 15 – 17, 2004

San Diego, CA

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Outline

- Introduction of Probabilistic Graphical Models
 - Bayesian Networks
 - Influence Nets
- Specification of Timed Influence Nets
- Belief updating in Probabilistic Graphical Models
- An algorithm for approximate belief revision in Timed Influence Nets
- Conclusions and future research directions

Important Probability Concepts

$P(A, B) = P(A) P(B)$ (A and B are independent)

$P(A | B, C) = P(A | C)$ (A and B are conditionally independent given C)

Bayes Theorem: $P(A | B) = \frac{P(B | A) P(A)}{P(B)}$

Advantages of Probabilistic Belief Networks

64 probability values are required to represent the joint distribution of 6 binary state variables, i.e., $2^6 = 64$

Probabilistic Network representations can reduce this number significantly

The joint distribution is computed as

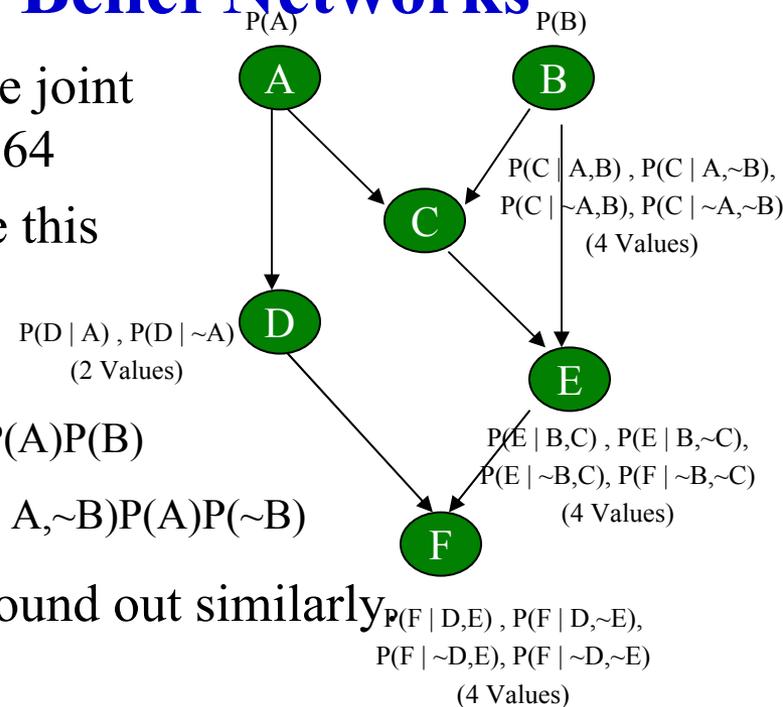
$$P(A,B,C,D,E,F) = P(F | D,E)P(D | A)P(E | B,C)P(C | A,B)P(A)P(B)$$

$$P(A,B,\sim C,\sim D, E,F) = P(F | \sim D,E)P(\sim D | A)P(E | \sim B,C)P(C | A,\sim B)P(A)P(\sim B)$$

Probabilities for other 62 combinations can be found out similarly

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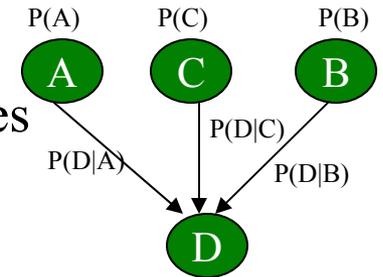
Total Values = 16

Bayesian Network with Noisy OR (BN2O)

- Based on Independence of Causal Influences Assumption
- Required n parameters to estimate 2^n conditional probabilities
- Given $P(D | A)$, $P(D | B)$, and $P(D | C)$

$$P(D | A,B,C) = 1 - P(\sim D | A,B,C)$$

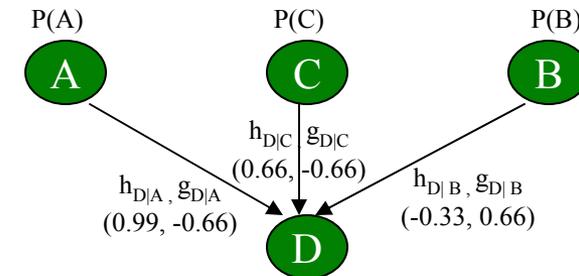
$$= 1 - P(\sim D | A)P(\sim D | B)P(\sim D | C)$$



$P(D | A,B,C)$, $P(D | A,B,\sim C)$,
 $P(D | A,\sim B,C)$, $P(D | A,\sim B,\sim C)$,
 $P(D | \sim A,B,C)$, $P(D | \sim A,B,\sim C)$,
 $P(D | \sim A,\sim B,C)$, $P(D | \sim A,\sim B,\sim C)$

Causal Strength (CAST) Logic

- Extension of Bayesian Network with Noisy OR (BN2O)
- Inputs have ranges from -1 to 1 .
- $h_{D|A}$ is analogous (but not equal) to $P(D | A)$ while $g_{D|A}$ is analogous (but not equal) to $P(D | \sim A)$.
- If all g values are zero and all h values are positive then **CAST Logic = BN2O**



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Influence Nets

Probabilistic Belief Networks that use CAST Logic for model specification are termed as **Influence Nets**.

The current implementation of Influence Nets assume that the parents of a node are marginally independent.



Root Nodes

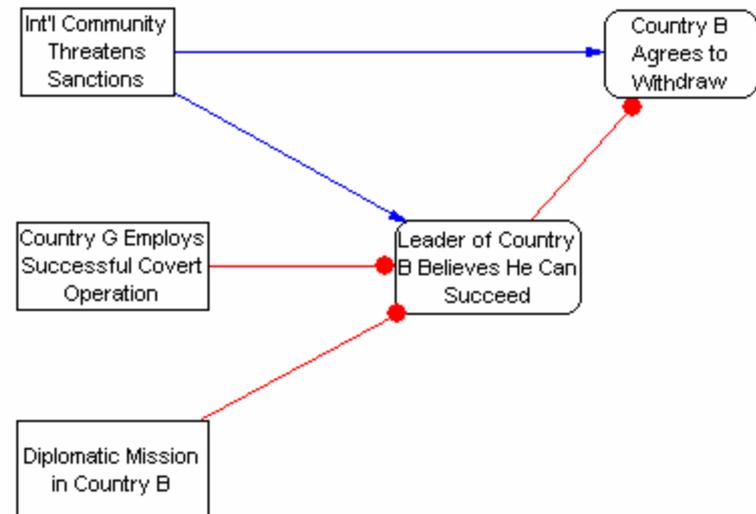


Non-root Nodes

Roots Nodes typically represents actionable events

 Positive Impact

 Negative Impact



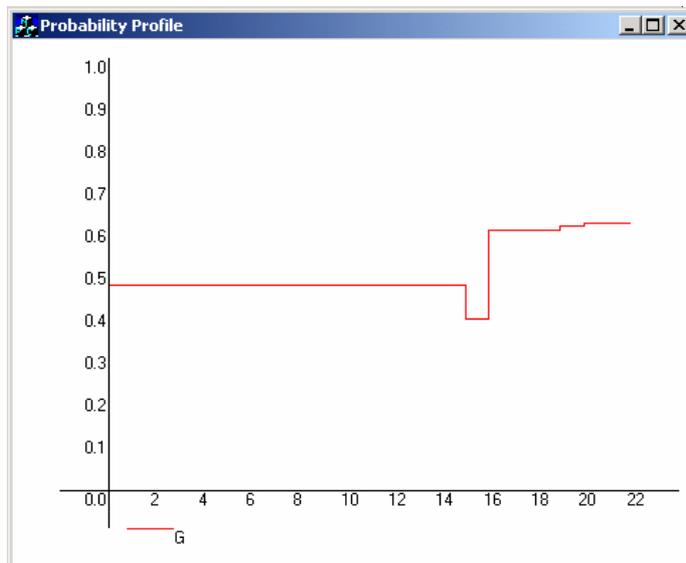
Timed Influence Nets

Timed Influence Nets have following additional parameters

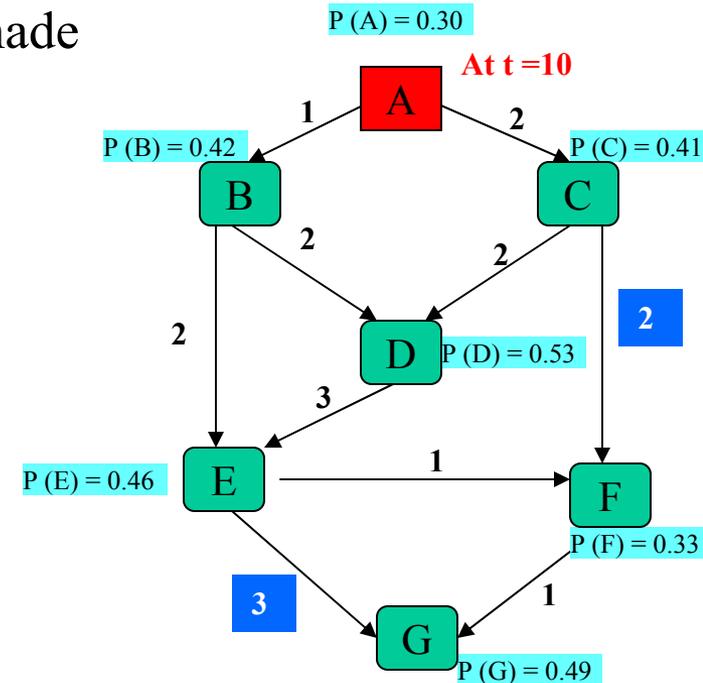
A time delay is associated with each arc.

A time delay is associated with each node.

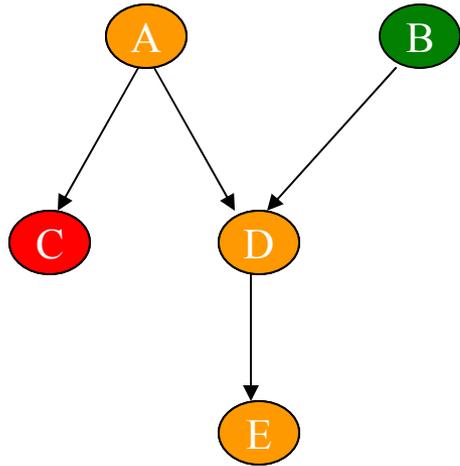
Each actionable event is assigned time stamp(s) at which the decision(s) regarding the state of that action is(are) made



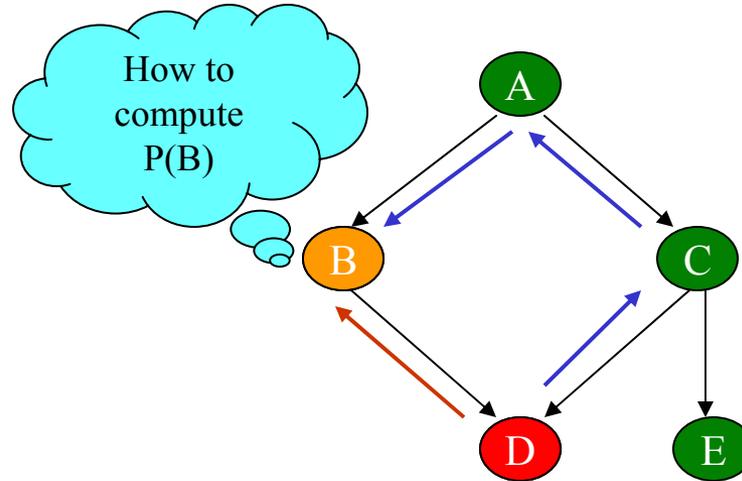
0	0.488
15	0.408
16	0.616
18	0.617
19	0.625
20	0.632



Belief Updating in Bayesian Networks



Singly Connected Network(SCN)

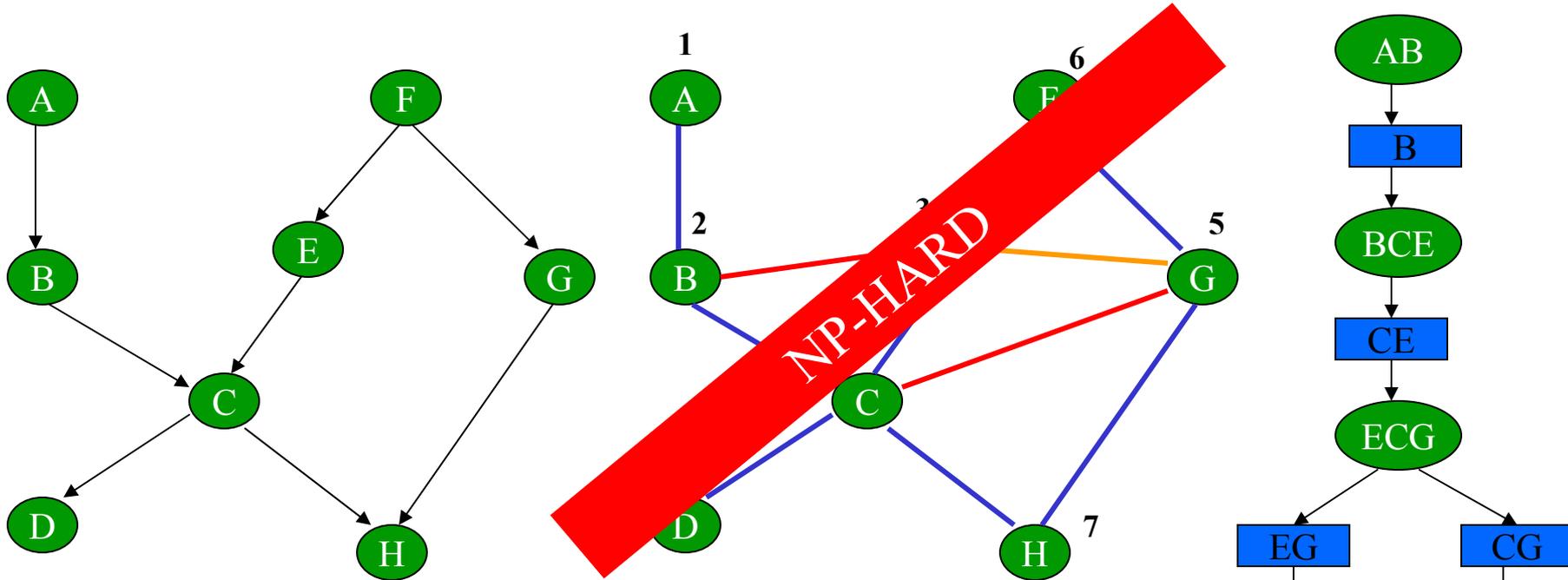


Multiply Connected Network(MCN)

Exact Computation of Posterior Probability is

- Possible when the graph is singly-connected
- NP-Hard when the graph is multiply-connected

Computation in Multiply-Connected Networks (MCN)



Step 1: Make the graph unidirectional

Step 2: Moralize the graph by adding a link between common parents

Step 3: Triangulate the graph

Step 4: Order the nodes by using Maximum Cardinality Search

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Sequencing Approach

A node will not be updated during the backward propagation until all of its descendants that are affected by the evidence are updated first.

Step1: [H, F, I]

Step2: [H, E, I, G, B]

Step3: [H, F, I, G, B, D]

Step4: [H, F, I, G, B, D, E]

Step5: [H, F, I, G, B, D, E, M, A]

Step6: [H, F, I, G, B, D, E, M, A]

D cannot be updated as E is not updated yet.

Step7: [H, F, I, G, B, E, M, A, D, C]

Step8: [H, F, I, G, B, E, M, A, D, C]

Step9: [H, F, I, G, B, E, M, A, D, C]

A cannot be updated as C and D are not updated yet.

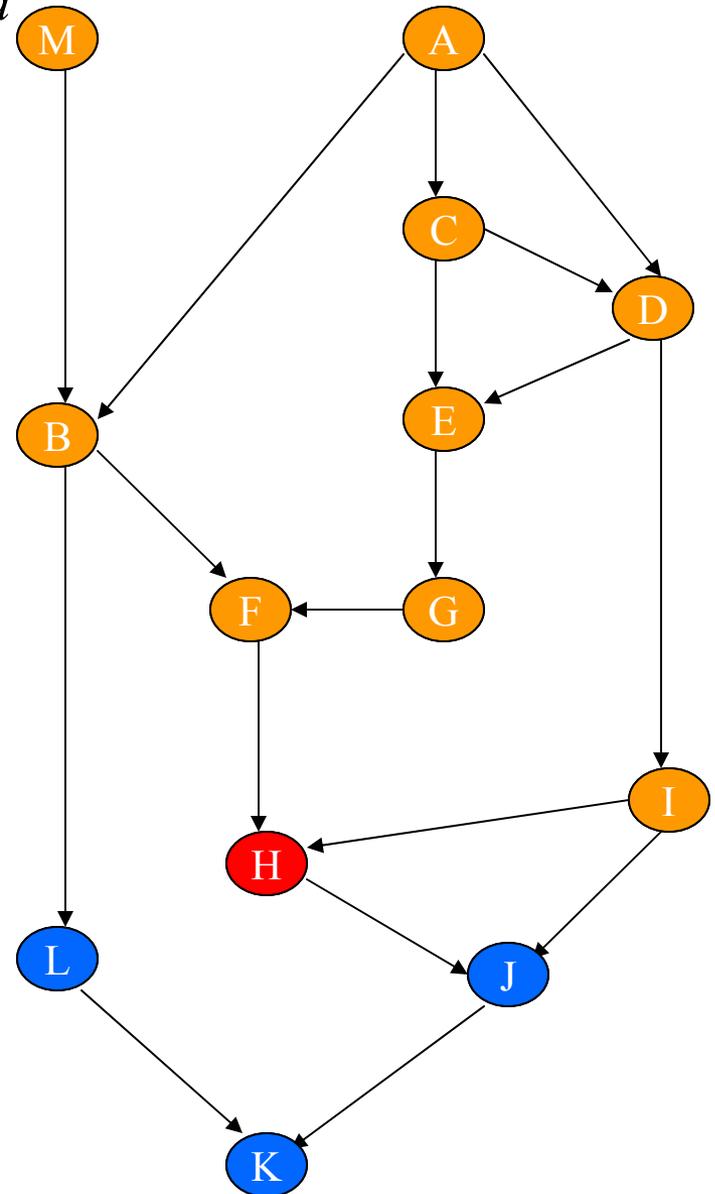
Step10: [H, F, I, G, B, E, M, D, C, A]

Step11: [H, F, I, G, B, E, M, D, C, A]

Step12: [H, F, I, G, B, E, M, D, C, A]

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Belief Propagation in Singly Connected Network

Let E = (Engine Status = Normal), T = (Temperature Status = Normal)

L = (Light On = True), D = (Product Defective = True)

C = (Plant Closed = True)

$$P(E | D) = \frac{P(D | E) P(E)}{P(D | E) P(E) + P(D | \sim E) P(\sim E)}$$

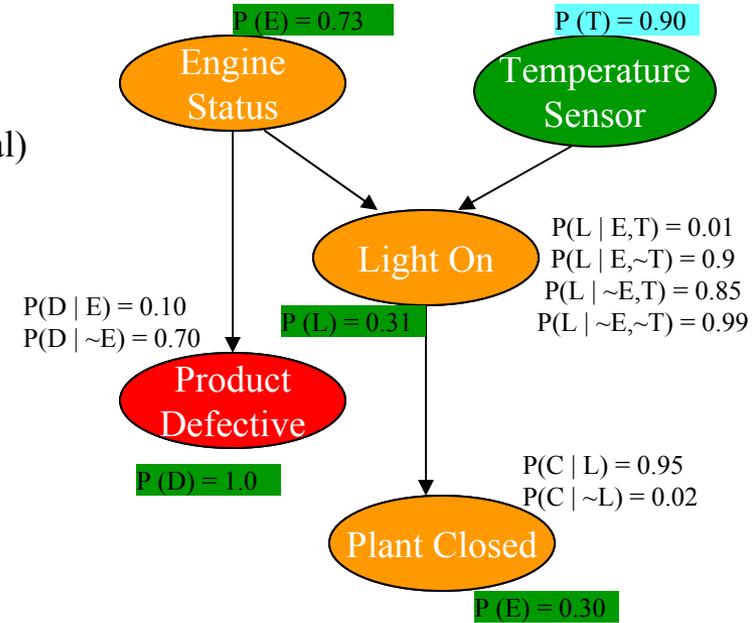
$$P(E | D) = 0.73$$

$$P(L | D) = P(L | E, T)P(E)P(T) + P(L | E, \sim T)P(E)P(\sim T) + P(L | \sim E, T)P(\sim E)P(T) + P(L | \sim E, \sim T)P(\sim E)P(\sim T)$$

$$P(L | D) = 0.31$$

$$P(C | D) = P(C | L) P(L) + P(C | \sim L) P(\sim L)$$

$$P(C | D) = 0.30$$



Belief Propagation in Multiply Connected Network

$$P(E | F) = \frac{P(F | E) P(E)}{P(F | E) P(E) + P(F | \sim E) P(\sim E)}$$

$$P(D') = P(D | E, F)P(E, F) + P(D | E, \sim F)P(E, \sim F) + P(D | \sim E, F)P(\sim E, F) + P(D | \sim E, \sim F)P(\sim E, \sim F)$$

Where

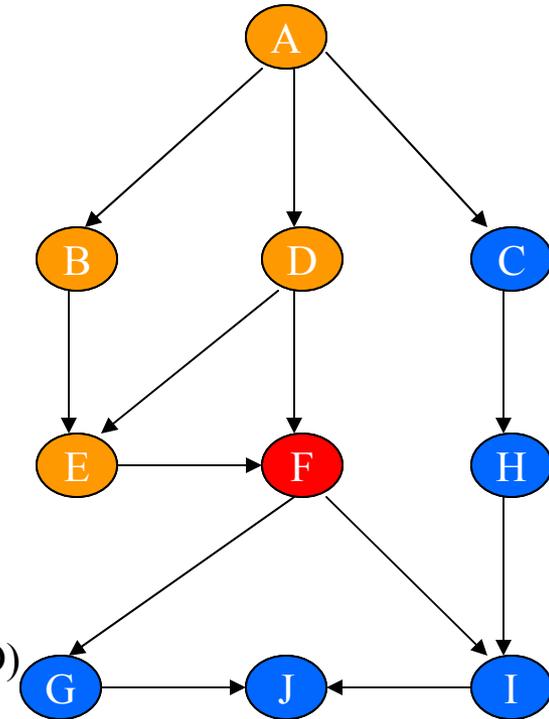
$$P(D | E, F) = \frac{P(F | E, D) P(E)P(D)}{P(F | E, D) P(E)P(D) + P(F | E, \sim D) P(E)P(\sim D)}$$

Similarly,

$$P(A') = P(A | B, D)P(B, D) + P(A | B, \sim D)P(B, \sim D) + P(A | \sim B, D)P(\sim B, D) + P(A | \sim B, \sim D)P(\sim B, \sim D)$$

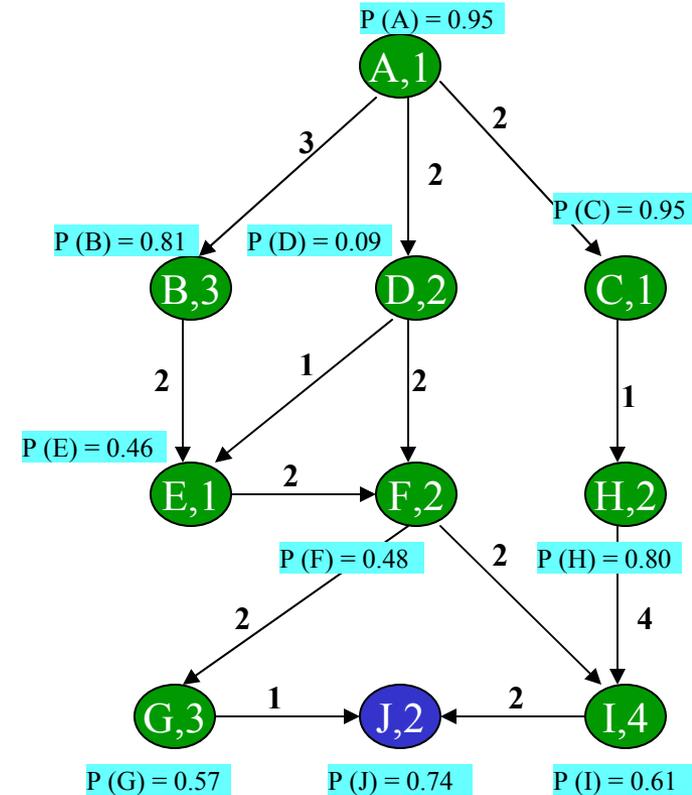
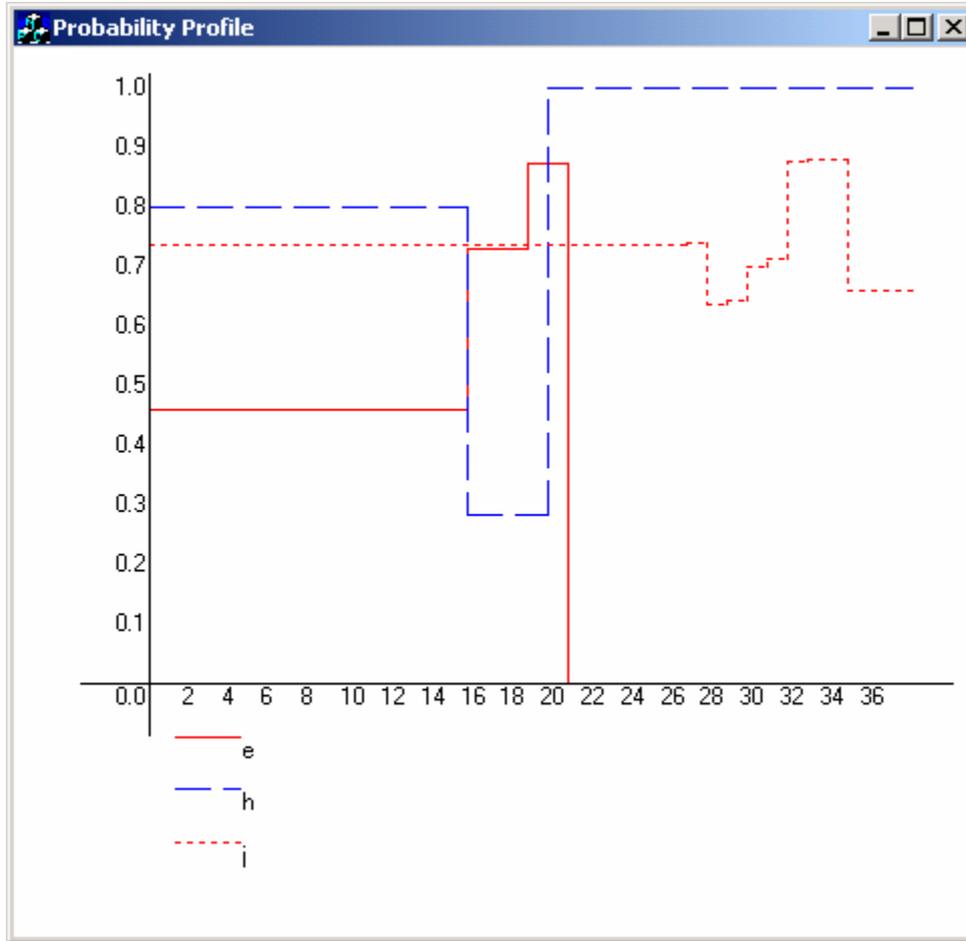
Where

$$P(A | B, D) = \frac{P(D | A) P(B | A)P(A)}{P(D | A) P(B | A)P(A) + P(D | \sim A) P(B | \sim A)P(\sim A)}$$



Belief Updating in Timed Influence Net

$P(E) = 0$ at time $t = 20$



Conclusions

- A heuristic approach of belief revision for Timed Influence Nets is presented.
- The approach updates the nodes in the sequential manner during the backward propagation.
- Limitations: The algorithm works only if the time stamp of the evidence is later than the time stamp of the last update of the evidence node caused by the forward propagation of the effects from all of the action nodes.
- One alternative approach is to convert a Timed Influence Net into a Time Sliced Bayesian Network.