Specifying and Proving Properties of Timed I/O Automata in the TIOA Toolkit *

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Abstract

Timed Input/Output Automaton (TIOA) is a mathematical framework for modeling and verification of distributed systems that involve discrete and continuous dynamics. TIOA can be used for example, to model a real-time software component controlling a physical process. The TIOA model is sufficiently general to subsume other models in use for timed systems. The TIOA toolkit, currently under development, is aimed at supporting system development based on TIOA specifications. The TIOA toolkit is an extension of the IOA toolkit, which provides a specification simulator, a code generator, and both model checking and theorem proving support for analyzing specifications. This paper focuses on modeling of timed systems with TIOA and the TAME-based theorem proving support provided in the toolkit for proving system properties, including timing properties. Several examples are provided by way of illustration.

1 Introduction

To achieve high assurance in the development of complex systems, an appropriate development framework supporting system specification, implementation, and analysis is essential. The support provided by the framework should apply not only to those systems that can be modeled as finite state machines but to those that cannot, such as many real-time embedded or hybrid systems systems involving software and/or continuous behavior. Thus an ideal general development framework should provide:

1. A mathematical model capable of capturing the range of discrete and continuous phenomena that arise in typical systems,

2. A well defined notion in the model of external (visible) behavior, and a definition of implementation of one component by another, or equivalence of two components, in terms of their visible behavior,

3. Compositionality—i.e, the ability to build larger systems by composing smaller components in a manner that respects the notion of implementation,

4. User-friendly tool support for proving the commonly encountered types of properties for the models, such as invariant properties, implementation relations, and stability, and

5. A basis supporting the use of automatic analysis and other software tools to the extent possible.

The Timed Input/Output Automaton (TIOA) toolkit [15, 9], currently under development, provides just such a framework. The TIOA toolkit, based on the TIOA model [16], is especially suited to the specification and analysis of real-time, embedded systems.

The focus of this paper is on the theorem proving support provided in the TIOA toolkit for the analysis of TIOA specifications. With a set of small examples, we illustrate how one can use the toolkit to model timed systems and specify their properties in the TIOA language, and then verify the specified properties using the theorem prover PVS [28] through the interface TAME [3].

The paper is organized as follows. Section 2 gives an overview of the Timed I/O Automaton (TIOA) model and the TIOA toolkit that supports its use. Section 3 describes how one can specify and prove properties of TIOA models and how the TIOA toolkit supports verifying (or proof checking) the properties mechanically in PVS. Section 4 presents our example TIOA specifications of automata and their properties, and shows how the properties

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\( \text{_traj_evolv e} \) and \( \text{deadline_reason} \) can be seen in Section 4.1, Figure 6 and Section 4.2, and Figure 13 respectively.

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<td>(apply_traj_evolv e ( t ))</td>
<td>Compute state time ( t ) from now</td>
</tr>
<tr>
<td>(apply_traj_stop ( t ))</td>
<td>Deduce that the stopping condition cannot hold after time ( t ) in a trajectory ( T ) unless ( T ) ends at ( t )</td>
</tr>
<tr>
<td>(apply_traj_invariant ( t ))</td>
<td>Deduce trajectory invariant holds time ( t ) from now</td>
</tr>
<tr>
<td>(deadline_reason ( t ))</td>
<td>Deduce trajectory cannot evolve more than time ( t ) if a deadline is reached time ( t ) from now</td>
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Table 1. New TAME strategies for trajectories.
3 Overview of the TIOA proof methodology

The TIOA mathematical model is useful for specifying timed distributed systems and analyzing properties of the systems as invariants and simulation relations. The model also provides a means of organizing proofs of such properties by induction over the length of the execution of an automaton into a systematic case analysis with respect to the actions and trajectories. It is therefore possible to develop PVS strategies to partially automate such proofs.

The TIOA methodology for theorem proving involves (1) writing the specification of a system and its properties in the TIOA language, (2) using the translator tool to generate the PVS equivalent of the system, and then (3) proving the properties in PVS using TAME strategies (see Figure 1). The user describes the system in the TIOA language using the state-transition structure. The user writes simple program statements to describe transitions, and specifies trajectories using differential equations. Once the TIOA description is type checked by the front end of the toolkit, the translator generates a set of PVS files. Together with the TAME library containing PVS definitions for timed I/O automata and any additional data type theories, these generated files specify the automaton and its properties. The user then uses TAME strategies developed for TIOA to prove the properties of the system in PVS.

By using this approach, the user avoids having to write the automaton description directly in PVS. Moreover, the translator also performs the task of translating program statements in TIOA into functional relations in PVS, and trajectories with differential equations into time-passage actions. An additional benefit gained from using the approach is that the user can also use other tools in the toolkit including the simulator, code generator and model checker.

4 Examples

This section provides three simple examples that together illustrate how TIOA is used to represent systems and properties, how trajectories can be used to capture desired timing behavior, and how system properties can be mechanically verified using PVS. The first example, fischer, is a timed version of Fischer’s mutual exclusion algorithm. We use this example to illustrate in some detail how various features of a TIOA specification, in particular, its trajectories, are represented in PVS. We also illustrate how its main correctness property, an invariant, can be proved using TAME. The second example, TwoTaskRace (representing, as its name suggests, a two task race), is used as an example in which the main correctness property is an abstraction property (forward simulation). The last example, timeout, representing a simple timeout system, is used to illustrate the support provided for expressing and reasoning about complex data types in the TIOA toolkit.

4.1 Fischer’s mutual exclusion algorithm

Fischer’s mutual exclusion algorithm solves the mutual exclusion problem in which multiple processes compete for a shared resource. Figure 2 shows the TIOA specification of a timed version of the Fischer algorithm.

In the Fischer algorithm, each process proceeds through different phases in order to get to the critical phase where it gains access to the shared resource. In the automaton used to model the algorithm, each phase has a corresponding action; timing is modeled in the algorithm by time bounds on the actions. The interesting action cases are test, set, and check. The action set has an upper time bound, u_set, while the action check has a lower time bound l_check, and u_set < l_check. When a process enters the test phase, it tests whether the value of a shared variable x has been set by any process; if not, the process can proceed to the next phase, set, within the upper time bound, u_set. In the set phase, the process sets a shared variable x to its index. Thereafter, the process can proceed to the next phase only after l_set amount of time has elapsed. In the check phase, the process checks to see if x contains the index of the process. If so, it proceeds to the critical phase.

The safety property we want to prove is that no two processes are simultaneously in the critical phase. We also prove simpler invariants to help us prove this main invariant. Figure 3 shows all the invariants that we have proved, the last invariant being the safety property.

To illustrate how the various elements of an automaton specification in TIOA translate into TAME, Figure 4 shows the TAME specification output by the TIOA-to-TAME translator applied to the TIOA specification in Figure 2. The TAME specification has been edited slightly to save space. In the TAME specification, automaton parameters are translated as constants, and the where clause constraining the parameters is expressed as an axiom named const_facts. The state variables are represented as a record type named states. A start predicate is defined to be true for states with the specified initial values. The actions of the automaton are declared as a subset of the actions data type in the TAME specification. A predicate enabled captures the precondition for each action, while a transition function trans captures the post-state obtained by applying the transition of an action on a given pre-state. In translating the effect of an action into the transition function, the translator performs explicit substitutions in accordance with the program statements in the specification of the effect of the action in TIOA, in order to express each state variable in the post-state explicitly in terms of the variables in the pre-state.

The trajectory definition traj in the TIOA specification is translated as a time passage action nu_traj in the TAME specification which has two parameters:
fischer_types

types process, PcValue enumeration { pc_rem, pc_test, pc_set, pc_check, pc_leavetry, pc_crit, pc_leaveexit, pc_reset, pc_crit

program (fischer1) l_check := 0; program (fischer2) l_check := 0; program (fischer3) l_check := 0

imports fischer_types

signature (i: process) l_check := 0; (i: process) l_check := 0; (i: process) l_check := 0

output try(i: process) internal test(i: process) output crit(i: process) internal set(i: process) output exit(i: process) internal check(i: process) internal reset(i: process)


transitions internal test(i) internal reset(i) output try(i) output crit(i) output exit(i) internal check(i) internal set(i)

fi

if turn := nil then pc[i] := pc_set; last_set[i] := now + u_set; fi

if turn := embed(i) then pc[i] := pc_leavetry; else pc[i] := pc_test; fi; first_check[i] := 0;

trajectories traj[delta_t] stop when s[i] := process (now := last_set[i]) evolve
d(now) := 1

Figure 2. TIOA specification for fischer.

invariant of fischer:
2 ∀ i: process (pc[i] = pc_set ⇒ last_set[k] ≤ (now + u_set))

invariant of fischer:
6 ∀ i: process (now ≤ last_set[k])

invariant of fischer:
10 (pc[k] = pc_set ⇒ last_set[k] ≠ \infty)

invariant of fischer:
12 ∀ i: process ∀ j: process (pc[i] = pc_check ∧ turn := embed(i) ∧ pc[j] = pc_set ⇒ first_check[i] > last_set[j])

invariant of fischer:
20 ∀ i: process ∀ j: process (pc[i] = pc_leavetry ∨ pc[i] = pc_crit) ⇒ turn := embed(i) ∧ pc[j] ≠ pc_set

invariant of fischer:
26 ∀ i: process ∀ j: process (i ≠ j ⇒ pc[i] ≠ pc_crit ∨ pc[j] ≠ pc_crit)

fischer_Decls : THEORY BEGIN
...
1_l_check := real; u_set := real
const_facts: AXIOM U_set < 1_l_check AND U_set >= 0 AND 1_l_check >= 0
start(s: states) := s WITH { turn := bottom, now := 0.0, pc := lambda(i_0: process): pc_rem, last_set := lambda(i_0: process): fintime(u_set), first_check := lambda(i_0: process): 0 |
Figure 4. TAME representation of fischer

Vocabulary for fischer:
variables
fischer

Figure 3. TIOA invariants for fischer.
inv_5(s:states):bool = 
  forall (i: process, j: process):
  i /= j => pc(s)(i) /= pc_crit OR pc(s)(j) /= pc_crit

lemma_5: lemma forall (s:states): reachable(s) => inv_5(s);

Figure 5. TAME proof of lemma_5 for fischer

/* Proof lemma_5-1 for formula fischer_invariants.lemma_5 */
/* developed with shostak decision procedures */
(auto_induct)

/* Forall (t: (interval(zero, delta_t_action))):
   nu_invariant(nu_traj(delta_t_action, F_action))
   (F_action(t))
   => turn(prestate) = up(i) AND pc(prestate)(j) /= pc_set
   pc(prestate)(i) = pc_crit OR pc(prestate)(i) = pc_reset
   pc(prestate)(i) = pc_leave
   FORALL (i: process, j: process):
   [now := 1 * dur(delta_t_action) + now(prestate)]
   (try_simp)
 */

/* Case crit(i_action) */
apply_specific_precond
/* Applying the precondition */
apply_inv_lemma "4" "j_theorem" "i_theorem"
/* Applying the lemma */
apply_inv_lemma "4" "j_theorem" "i_theorem"
/* Applying the precondition */
apply_specific_precond
/* Applying the precondition */
apply_specific_precond
/* Applying the lemma */
apply_specific_precond
apply_specific_precond
apply_specific_precond
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Figure 6. TAME proof of lemma_5 in fischer
delta_t, the duration of the trajectory, and F, a function representing the trajectory, which maps time values to states. The definitions traj_invariant, traj_stop, and traj_evolve capture the invariant, stopping condition, and evolve clause of the trajectory definition respectively. The effect of the “trajectory action” nu_traj is constrained—and thus, effectively, captured—by the precondition of nu_traj, which asserts that (1) the invariant holds throughout the duration of the trajectory, (2) the stopping condition holds only in the last state of the trajectory, and (3) the evolution of the state variables satisfies the evolve clause. The transition function for nu_traj simply returns the post-state obtained by applying the trajectory function F after an elapsed time of delta_t. This method of representation, adapted from a technique of Luchangco [21], allows trans to be represented as a function from states and actions to states while allowing the result of a nu_traj “action” to be nondeterministic.

The new TAME strategies in Table 1, combined with the existing TAME strategies, provide a set of proof steps that allow the fischer invariants shown in Figure 3 to be proved interactively in PVS in a clear, high-level fashion. The TIOA-to-TAME translator transforms the six invariants in Figure 3 into TAME invariants and lemmas numbered starting from 0. Thus, the goal safety property, the last invariant in Figure 3, becomes the TAME invariant/lemma pair shown in Figure 5.

Figure 6 shows a verbose TAME proof of lemma_5 in Figure 5. To create this proof, which can be rerun in PVS, the user simply types in the eight TAME proof steps in the proof script—(auto_induct), (apply_specific_precond), and so on. The comments in this proof (which appear as text after semicolons) are generated by the TAME strategies, and serve to label the proof branches and document the facts introduced by the proof steps in these branches. Because TAME automatically handles “trivial” cases, only the proof steps requiring human guidance need to be recorded. This proof can be understood as follows: The proof step auto_induct automates as far as possible the standard initial steps of a proof by induction on the reachable states, including skolemization. The values with names ending in “_theorem” or “_action” are skolem constants standing for variables in the lemma and parameters in the current action, respectively. The name prestate refers to the prestate of the current action, and the values of state variables in any state s are represented as functions of s. The base case and all the action cases except nu_traj(delta_t_action, F_action) and crit(i_action) are trivial. The nu_traj(delta_t_action, F_action) case is proved by recalling the full precondition with apply_specific_precond, and then using the new TAME step traj_evolve in Table 1 to compute what the current state will be after time delta_t_action. Once this is done, only “obvious” reasoning is needed, which is performed by try_simp. The proof in the crit(i_action) case first recalls the precondition and then uses apply_inv_lemma to apply two earlier invariant lemmas to appropriate instances of their quantified variables. Then, only “obvious” reasoning with try_simp is needed to complete the proof.

4.2 A two task race

The two-task race system (see Figure 7 for its TIOA description) increments a variable count repeatedly, within a1 and a2 time, a1 < a2, until it is interrupted by a set action. This set action can occur between b1 and b2 time from the start, where b1 ≤ b2. After set, the value of count is decremented (every [a1, a2] time) and a report action is triggered when count reaches 0.
TwoTaskRace\((a_1, a_2, b_1, b_2: \text{Real})\) where
\[ a_1 \geq 0 \land a_2 \geq 0 \land b_1 \geq 0 \land b_2 \geq 0 \land a_1 \land b_2 \geq b_1 \]

signature
internal increment
internal decrement
internal set
output report
states
count: Int := 0,
flag: Bool := false,
reported: Bool := false,
now: Real := 0,
first_main: Real := a_1,
last_main: AugmentedReal := a_2,
first_set: Real := b_1,
last_set: AugmentedReal := b_2

transitions
internal increment
pre ~flag \land now \geq first_main
iff count := count + 1;
last_main := now + a_1;
22
internal decrement
pre ~flag \land now \geq first_set
iff flag := true;
first_set := 0;
last_set := \infty
internal set
dec
pre ~flag \land count > 0 \land now \geq first_main
iff count := count - 1;
first_main := now + a_1;
lst_last := now + a_2;
output report
pre ~flag \land count = 0 \land ~reported \land now \geq first_main
iff reported := true;
first_main := now + a_1;
lst_last := \infty
trajectories
trajdef traj*
stop when \(now = \text{last_main} \lor now = \text{last_set}\)
evolve
d\(\text{now}\) = 1

Figure 7. TwoTaskRace in TIOA

We want to show that the time bounds on the occurrence of the report action are: lower bound: if \(a_2 < b_1\) then \(\min(b_1, a_1) + (b_1 - a_2) \cdot a_1\) else \(a_1\), and upper bound: \(b_2 + a_2 + a_1\). This property is proved by specifying an abstract automaton TwoTaskRaceSpec which performs a report action within these bounds (see Figure 8) and defining a forward simulation relation from TwoTaskRace to TwoTaskRaceSpec (see Figure 10).
The PVS formulation of the forward simulation property is ported in Figure 11 just before the statement of the theorem for formulating abstraction relations between automata described in [26]. The theory forward_simulation imported in Figure 11 just before the statement of the theorem provides the generic definition in PVS of the property forward_simulation stating what it means for a relation between two automata to be a forward simulation. The PVS formulation of the forward simulation property is

based on the definition in [24]. The proof of this property for TwoTaskRace and TwoTaskRaceSpec uses invariants of both automata.

The invariants of TwoTaskRace and TwoTaskRaceSpec needed for the forward simulation proof have all been proved in TAME. The proofs of these invariants are all quite simple; in fact, all of the invariants needed for TwoTaskRaceSpec are proved automatically by the TAME induction strategy auto_induct. The proofs of a few of the invariants for TwoTaskRace are interesting because they illustrate the use of the new TAME strategy deadline_reason, which was not used in the invariant proofs for fischer. One such invariant is invariant 4 in Figure 12, whose TAME proof is shown in Figure 13. Invariant 4 essentially says that in the TIOA model of TwoTaskRace, the current time now cannot pass beyond the deadline last_main. In this proof, auto_induct has determined that the base case and four of the five possible action cases are nontrivial. The crux of this proof.

Figure 10. Forward simulation from TwoTaskRace to TwoTaskRaceSpec

Figure 11. Simulation relation in TAME

Figure 12. TwoTaskRace invariants 0–4.
is the reasoning in the single time passage case, namely, the action case nu\text{\_}traj(delta\text{\_}action). After using apply\text{\_}specific\text{\_}precond and apply\text{\_}traj\text{\_}evolve to compute the state after time delta\text{\_}action and using apply\text{\_}inv\text{\_}lemma to use invariant \text{\_}i to establish that now \geq 0 at the beginning of the trajectory, the new TAME step deadline\text{\_}reason argues that now \leq last\_main at the end of the trajectory. The step try\text{\_}simp then completes the proof with “obvious reasoning”. The remaining cases are easily proved using “obvious reasoning” following, in some cases, the use of const\_facts to introduce facts about the constants in the specification.

TAME also provides strategies for establishing abstraction relations between automata, including forward simulation. Forward simulation proofs have a high-level structure similar to the structure of induction proofs of invariants; however, rather than beginning with the proof step auto\_induct, they begin with the proof step prove\_fwd\_sim. For more details, see [26].

4.3 A simple timeout system

A simple timeout system consists of a sender, a delay prone channel, and a receiver (see Figure 14 for its TIOA description). The sender sends messages to the receiver, within \text{\_}u1 time after the previous message has been sent. A timed\_message\_Queue delays the delivery of each message by at most \text{\_}b time. A failure can occur at any time, after which the sender stops sending. The receiver times out after not receiving a message for at least \text{\_}u2 time.

We are interested in proving the two following properties for this system: (1) Safety: A timeout occurs only after a failure has occurred; (2) Timeliness: A timeout occurs within \text{\_}u2 + \text{\_}b time after a failure. The safety property can be captured by an invariant of the system. As in the two-task race example, to show the timeliness, we first create an abstract automaton that times out within \text{\_}u2 + \text{\_}b time of occurrence of a failure, and then we prove a forward simulation from the system to its abstraction. Both the safety and timeliness properties have been proved using the TAME strategies in a manner analogous to the invariant and for-

```
automaton timeout(u1, u2, b: Real) 2  where u1 >= 0 & u2 >= 0 & b >= 0 & u2 > (u1 + b) imports timed\_queue 4  signature internal send(m: M) 6 internal receive(m: M) output fail 8 output timeout states p\_Clock: AugmentedReal = 0, t\_Clock: AugmentedReal = u2, suspected: Bool = false, failed: Bool = false, 10 now: Real = 0, queue: timed\_message\_Queue = mtQ 12 transitions internal send(m) 14 pre now >= 0 & ~failed & p\_Clock = now 16 eff if (now + u1) >= 0 then p\_Clock := now + u1 fi; 18 if (now + b) >= latest\_deadline(queue) then queue := enQ(MK\_timed\_message(a, now + b), queue) 20 end 22 internal receive(m) 24 pre now >= 0 & enQ\_qn(queue) & m = earliest\_msg(queue) 26 eff if (now + u2) >= 0 then t\_Clock := now + u2 fi; 28 output if enQ\_qn(queue) then queue := deQ(queue) fi 30 output fail 32 pre ~failed 34 eff failed := true; 36 p\_Clock := infinity 38 output timeout 40 pre now >= 0 & ~suspected & t\_Clock = now 42 eff suspected := true; 44 t\_Clock := infinity 46 trajectories trajdef traj 48 step when now >= 0 (now = p\_Clock v now = t\_Clock 50 v now = earliest\_deadline(queue)) 52 evolve d(now) = 1
```

Figure 14. TIOA description of timeout

vocabulary timed\_queue
4 types M, timed\_message\_Queue, timed\_message
operators m\_t: M, MK\_timed\_message(\text{\_}m, now + b), queue
IMPORTING Queue\_t\_TYPE, timed\_message\_thy\_M
BEGIN
mt\_t: M, mk\_t\_Q:Q\_t\_TYPE
length\_Q(q:Queue): RECURSIVE nat =
IF m\_t\_Q(q) THEN 0
ELSE length\_Q(before\_last\_Q(q)) + 1 ENDIF
MEASURE length\_Q(q), (\lambda q:Q\_t\_TYPE. (\text{\_}\{n:nat| n >= i & n <= length\_Q(q)-1\}))
MEASURE length\_Q(q), (\lambda q:Q\_t\_TYPE. (\text{\_}\{n:nat| n >= i & n <= length\_Q(q)-1\}))
MEASURE length\_Q(q), (\lambda q:Q\_t\_TYPE. (\text{\_}\{n:nat| n >= i & n <= length\_Q(q)-1\}))
```

Figure 15. TIOA declaration of custom data types and operators used in timeout.

Figure 16. Sample PVS definitions of custom data types and operators used in timeout.
ward simulation proofs in the previous examples, with one extra complication: the need to introduce knowledge about special data types referred to in the TIOA specifications.

The timeout system makes use of a custom data type timed_message_queue. TIOA provides a vocabulary syntax to allow the user to declare custom data types and operators. Figure 15 shows how the data type for timed_message_queue and the associated operators are declared in TIOA. The actual PVS definitions of these types and operators are provided as part of a TIOA library of data type theories; Figure 16 shows a sample of these definitions. Aside from the PVS operator enQ? (which implements the TIOA operator enQ? for querying whether a timed_message_queue is a nonempty queue), the PVS vocabulary is identical to the TIOA vocabulary. Properties of these data types have been proved in PVS, and have been used in proofs of the specification properties.

5 Discussion

Developing theorem proving support. Our approach to developing appropriate theorem proving support for TIOA is to study many examples of TIOA specifications and their properties and identify what is needed for implementing a standard, straightforward set of proof steps sufficient to mechanize proofs of the properties. One lesson we have learned is that the details of the specification template that a translator to PVS targets, if chosen carefully, can greatly facilitate the implementation of PVS strategies. Details of the TAME template for TIOA that have proved helpful for strategy development include the overall scheme for representing trajectories illustrated in Figure 9 and the scheme for representing the start state predicate start(s) as an equality of the form s = ..., possibly in conjunction with additional restrictions (see, for example, Figure 4). Another detail of our translation scheme is the use of symbolic computation, if necessary, to permit the effects of transitions, which are defined in TIOA as the effect of a sequence of computations, to be represented in trans by explicit updates to state variables. This allows the theorem prover to reason directly about new state values of individual variables with less effort.

One goal in developing support for interactive theorem proving is to find a minimal set of proof steps that are natural to use in high level reasoning and that are sufficient (or nearly so) for mechanizing proofs of properties. Studying many examples has helped us in this regard. For example, we observed that many proofs included the observation that time cannot pass beyond a given deadline unless some discrete action occurs. This observation led us to include deadline_reason among our set of proof steps.

Mechanizing proofs. The theorem proving support we are developing for TIOA does not make mechanizing proofs of properties automatic, but it does make it simpler. A user who wishes to prove properties of a TIOA specification using TAME must in general be a domain expert for the system modeled in TIOA. To prove the desired safety or simulation properties, the user often must first find an appropriate set of supporting lemmas. Doing this may require some creativity; some guidance on how to go about it can be found in [24]. The user must also be able to sketch out at a high level why, based on the set of supporting lemmas, a given property is expected to hold. To produce a mechanical proof of the property, the user then can apply TAME reasoning steps that match this high level reasoning. Typically, this can be done using steps such as const_facts, apply_inv_lemma, apply_specific_precond, deadline_reason, and so on, to introduce the facts appealed to in each nontrivial case in the proof sketch, and then invoking try_simp to do the “obvious” reasoning based on these facts.

While it is good to have a mechanical check of a proof’s validity, it is equally important to have some feedback on what went wrong if the mechanical check fails. For failed proofs, TAME provides some useful feedback: the saved TAME proof script can be used to detect the place in the proof where the proof breaks down. The user can then review the high level reasoning to see whether there is an error or if introducing additional facts can complete the proof.

Scalability. We have begun experimentation with using the TAME support for TIOA on larger examples. Our first larger example is the Small Aircraft Traffic System protocol SATS developed at NASA Langley. An abstract model of this system has been defined in [8]. An IOA version of this model has been represented and verified in PVS [29]. We have used the TIOA-to-TAME translator to represent the IOA model in TAME, and have begun redoing the proofs using the TAME strategies.

The SATS example has raised an issue that is likely to arise in many large examples: the use by specifiers of multi-layered definitions of application-specific functions and predicates. One way to manage the many definition expansions for proof efficiency would be to expand them in layers to allow reasoning to proceed at the highest possible layer. A goal for the translator is to generate “local strategies” for a specific application that group definitions by layer. A scheme of this sort is used in the SCR-to-TAME translator to increase the efficiency of the TAME strategies that support reasoning about SCR automata [3].

6 Related work

Previous work has been performed to develop tools to translate specifications written in the IOA language to the language of various theorem provers, for example, Larch [6, 10], PVS [7], and Isabelle [30, 27]. Our implementation of the TIOA to PVS translator described in [20] builds upon [6]. The target PVS specifications of this trans-
lator strongly resemble TAME specifications. In addition, an early version of TAME’s deadline_reason strategy was implemented as the PVS strategy deadline_check described in [20]. The TIOA-to-TAME translator is essentially a version of the TIOA-to-PVS translator of [20] with modifications that allow the straightforward implementation of new TAME strategies for TIOA and the most effective use of existing TAME strategies. A more complete description of the recent improvements made to the translation scheme and strategies described in [20] can be found in [19]. In [12], a slightly different approach using urgency predicates instead of stopping conditions or invariants to limit trajectories is used to describe timed I/O automata. An approach to proving invariant properties of timed I/O automata using urgency predicates is described, but no tool support. A proposed design for supporting urgency predicates in the TIOA toolkit is given in [4].

7 Conclusion

The TIOA framework is ultimately intended to support all phases of system development from specification, through verification and validation, to implementation. In this paper, we have focused on the usability of the TIOA framework for modeling and mechanical verification of properties of timed systems with both discrete and continuous transitions. We have described the theorem proving support provided, and illustrated how it is used in examples where the properties of interest are invariant properties or simulation properties, and where the models involve non-trivial data types.

Our plan for the future is experiment with more complex examples, such as SATS or the Dynamic Host Configuration Protocol DHCP (using models based on the work described in [13]), to explore extensions and improvements to our proof support.

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References