Two-Hydrophone Heading and Range Sensor Applied to Formation-Flying for AUVs.


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Abstract - One form of cooperative behavior for a group of AUVs is to fly in formation while performing tasks. A necessary component for formation-flying is that the vehicles must sense their relative positions. Assuming that each vehicle is capable of sensing its inertial position using an acoustic long-baseline ranging system, the relative vehicle position can be determined by exchanging this data. The penalty for this approach is that exchanging inertial position data consumes communications bandwidth. Alternately, relative position may be obtained by intercepting acoustic ranging signals used to determine inertial position, obviating the need for exchange of position data. We explore the use of a two-hydrophone sensor to measure relative heading of two vehicles in a formation. It is assumed that a broad-band navigation signal emanating from one vehicle is intercepted by another vehicle containing the sensor. Relative heading is extracted from the time delay between the two hydrophones. Cross-correlation is used to determine time delay. A model is proposed that predicts stochastic precision and bias for the sensor. For a fixed ranging waveform, precision and bias are dependent upon signal-to-noise ratio, relative range and relative heading. This dependence means that the sensor will be most useful for certain combinations of range and heading. Measurements were performed to determine the precision of the two-hydrophone arrangement as a relative heading sensor. Simulations were used to explore the performance of formation-flying controllers that employ the two-hydrophone sensor. The controller used a saturating linear output feedback control law to simultaneously follow inertial waypoints and maintained formation. The simulations showed that this controller would be able to use relative heading provided by the two-hydrophone sensor to maintain formations in which the vehicles are approximately abreast.

I. INTRODUCTION

Formation-flying is an elemental cooperative behavior that may be used by groups of Autonomous Underwater Vehicles (AUV's) while performing tasks. A requirement for maintaining formation is that each vehicle controller must have some information about the relative positions of one or more other vehicle(s) in the group. For example, it can be shown that a follower can maintain formation with a leader if the follower can sense the distance and heading to the leader [1]. In addition, if the formation must follow a specified path, at least one vehicle must have knowledge of its inertial position. Relative position can be sensed by exchanging the inertial position of each vehicle via an acoustic link [2], or by sensors that can determine the range to another vehicle directly, like those hypothesized in [3]. Because of the severe constraints on acoustic bandwidth in the underwater environment, it would be preferable for vehicle controllers to obtain relative position data by sensors that do not rely upon acoustic communications.

A hybrid control approach is to require that each vehicle maintain formation and follow inertial waypoints simultaneously [4]. Each vehicle obtains its position with an acoustic Long BaseLine (LBL) sensor. The formation is maintained by requiring a leader vehicle to broadcast its position in parallel to the other vehicles in the formation via an acoustic link. This control scheme has certain advantages. It is adaptable to 1-D, 2-D and 3-D formations. A formation can tolerate the loss of a leader, in that another vehicle in the formation can replace the leader. In the case of a total loss in communication, each vehicle can revert to the fall-back case of independent navigation. A further advantage is that the hybrid algorithm only requires that the followers know the angular heading of the leader relative to the follower, instead of both relative heading and distance.

In this paper, a potential approach to sense relative vehicle position using the navigation ping(s) from another vehicle in the formation is described. The sensor would consist of two hydrophones, located at the bow and stern of an AUV. With this arrangement, the hydrophones would be separated by a distance of approximately 1 meter. A navigation ping from another vehicle in the formation is intercepted by the two hydrophones, and the hydrophone voltage signals are used to determine the heading and range to the vehicle that issued the navigation ping. Heading would be determined by extracting the difference in arrival time by cross-correlating the hydrophone signals, and range would be determined by measuring the difference in received amplitude. Presently, we focus on the determination of relative heading and its application in the hybrid formation controller.

The use of correlation from a sensor array to determine relative heading has several advantages compared to the use of time-of-flight methods [5-7]. Interaction with transponders with known position is not required. Synchronized clocks on separate vehicles are not necessary. Determination of time-of-flight using matched filters often requires compensation for Doppler distortion caused by relative motion of the source and receiver. If heading is determined by correlation of two signals from a common source, it is thought that Doppler distortion would cancel, and compensation would not be required. For
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The original document contains color images.
determination of relative heading, the two-hydrophone sensor arrangement is similar in concept to the Relative Acoustic Tracking System (RATS) [8], which uses eight sensors to determine the angle to an acoustic source that transmits a ranging ping. The sensor described in this paper uses only two hydrophones. It is envisioned that the two hydrophones would be separated by a distance of one meter, as opposed to the approximate 20 cm size for the eight sensor RATS array.

It is thought that the two-hydrophone sensor would be of value to a formation-flying type controller for AUV's, in that relative position could be determined without burdening the underwater communications system. However, any type of position sensor suffers from errors caused by geometry, in addition to fundamental limits set by noise. Consequently, it will be shown that the two-hydrophone sensor is valuable for certain formation geometries, but leads to poor performance in others. This fact is useful for choosing appropriate formation geometries, and may also explain why variations in formation are observed in leader-follower type experiments [9,10].

II. SENSOR MODEL

A. Heading and Range Determination

Consider the two AUV’s in Fig. 1. A top view is shown. It is assumed that vehicle C is inclined at angle $\gamma$ relative to the horizontal plane of vehicle AB, though this inclination is not shown in the figure. An omni-directional acoustic source is located at C, and omni-directional hydrophones are located at A and B. The object is to determine the relative heading $\theta$ and range $R$ of vehicle C with respect to vehicle AB.

Assuming an acoustic signal is broadcast from vehicle C, the difference in arrival time $\Delta t$ at vehicle AB will be

$$\Delta t = \frac{1}{c} \left[ \frac{R \cos \theta + \frac{d}{2}}{c} + \left( \frac{R \sin \theta}{c} \right)^2 + \left( \frac{R \sin \gamma}{c} \right)^2 \right]$$

where $d$ is the distance between the two hydrophones, $c$ is the effective speed of sound, and $R$ is the distance between vehicles AB and C projected onto the horizontal plane containing vehicle AB. If it is assumed that the vehicles are at the same depth, the inclination angle $\gamma = 0$, and, approximating the heading equation (1) in a Taylor’s series, one obtains

$$c \Delta t = d \cos \theta + \frac{d^2}{8R^2} \sin^2 \theta \cos \theta$$

For $d<5R$, the last term in (2) above contributes less than 0.5% to $\Delta t$, and (2) can be simplified to

$$\theta = \cos^{-1} \left( \frac{c \Delta t}{d} \right).$$

The range $R$ from vehicle C to vehicle AB is determined by the difference in arrival time $\Delta t$ and by the amplitude of the signals that arrive at hydrophones A and B. Let $f(t)$ be the acoustic signal broadcast by the source C. Assuming ideal hydrophones with unit sensitivity and propagation of the acoustic waves without dispersion, the hydrophone voltages $V_A(t)$ and $V_B(t)$ can be represented as

$$V_A(t) = \frac{f \left( t - \frac{r_A}{c} \right)}{r_A}, \quad V_B(t) = \frac{f \left( t - \frac{r_B}{c} \right)}{r_B}$$

where $r_A$ and $r_B$ are the distances from hydrophones A and B to the source C respectively. The amplitude of the hydrophone voltages may be quantified by the RMS values $S_A$ and $S_B$ over a specified temporal window $T$ as

$$S_A^2 = \frac{1}{T r_A} \int f^2 \left( t - \frac{r_A}{c} \right) dt, \quad S_B^2 = \frac{1}{T r_B} \int f^2 \left( t - \frac{r_B}{c} \right) dt$$

Let us define the symbol $\alpha$ such that $\alpha = S_A/S_B$. Since $r_B - r_A = c \Delta t$, it follows that

$$r_B = c \Delta t + r_A.$$ 

By substitution the range $R$ can be determined from

$$R = c \Delta t \left[ \frac{1}{\alpha - 1} + \frac{1}{2} \right].$$

In principle, it is possible to use (3) and (4) to determine relative heading $\theta$ and range $R$ from measurements of arrival time difference $\Delta t$, signal amplitudes $S_A$ and $S_B$, knowledge of the effective sound speed $c$, and hydrophone separation distance $d$.

B. Sensor Model

We derive a sensor model that includes the determination of heading and range from (3,4), and an
estimate of the uncertainty and bias that would be encountered.

For determination of heading, an estimate of uncertainty \( U_\theta \) can be obtained from

\[
U_\theta = \left( \frac{c d}{c d} \right)^2 \left[ \left( \frac{U_c}{c c} \right)^2 + \left( \frac{U_d}{c d} \right)^2 \right],
\]

where \( U_c, U_d, \) and \( U_\theta \) are standard deviations in arrival time difference, sound speed, and hydrophone separation respectively. We will assume that any errors by an incorrect measurement in either \( c \) or \( d \) will cause not an uncertainty, but rather a repeatable bias. In this case, the uncertainty estimate reduces to

\[
U_\theta = \frac{c d}{c d} \left( \frac{1}{\left( c c \right)^2} + \left( \frac{c d}{c d} \right)^2 \right)\left( \frac{1}{\left( c d \right)^2} + \left( \frac{c d}{c d} \right)^2 \right).
\]

This expression reveals that the estimate of relative heading \( \theta \) is very poor when \( \theta \) is near 0° or 180°. If we let \( \frac{c}{d} \) be the measured value of \( \frac{c}{d} \), the bias \( B_\theta \) in heading caused by imperfect knowledge of these parameters can be estimated by

\[
B_\theta = \theta - \theta = \cos^{-1} \left( \frac{c}{d} \right) - \cos^{-1} \left( \frac{c}{d} \right)\left( \frac{c}{d} \right).
\]

An estimate of uncertainty propagated to the determination of range \( R \) can be obtained with

\[
U_R = \left( \frac{c d}{c d} \right)^2 \left[ \left( \frac{U_c}{c c} \right)^2 + \left( \frac{U_d}{c d} \right)^2 \right].
\]

If it is assumed that the effective sound speed \( c \) or signal amplitude ratio \( \alpha \) will cause a repeatable bias, then the uncertainty in range \( U_\Delta R \) reduces to

\[
\frac{U_\Delta R}{R} = \frac{U_\Delta}{\Delta R}.
\]

Since the arrival time difference \( \Delta t \) is a function of relative heading \( \theta \), it is expected that \( U_\Delta R / R \) to also vary with heading. Any noise in the water will have an effect of biasing the range measurement. If the hydrophone voltages are corrupted with uncorrelated noise of RMS level \( S_o \), then the measured standard deviations \( \hat{S}_x \) and \( \hat{S}_y \) will be

\[
\hat{S}_x = \sqrt{S_x^2 + S_y^2}, \quad \hat{S}_y = \sqrt{S_x^2 + S_y^2},
\]

and the measured amplitude ratio, \( \hat{\alpha} = \hat{S}_x / \hat{S}_y \), will be used to compute a biased range from (4). Then the bias on range will be

\[
B_R = c \Delta t \left( \frac{\hat{\alpha} - \alpha}{(\alpha - 1)(\alpha - 1)} \right).
\]

To summarize, a model of the sensor, including a prediction of uncertainty and bias, is

\[
\hat{R} = R + \Delta R + B_R, \quad \hat{\theta} = \theta + \Delta \theta + B_\theta
\]

where \( \Delta R \) and \( \Delta \theta \) are zero-mean normally distributed random variables with variance \( U_\Delta^2 \) and \( U_\theta^2 \) respectively; \( \hat{R} \) and \( \hat{\theta} \) are the range and relative heading that would be returned by the sensor.

III. EXPERIMENTS

Static tests were performed in shallow water to determine the accuracy of the bearing angle estimation techniques. In Section III.A and III.B, the apparatus and data processing techniques are described. In Section III.C, experimentally measured standard deviations in arrival time \( U_\Delta \) are presented, and the implications for heading angle \( \theta \) determination are discussed in Section III.D.

A. Apparatus

Fig. 1 shows the basic configuration. Source and receivers were suspended from two stationary barges separated by open shallow water to produce realistic multipath effects. Tests were conducted at the Acoustic Research Detachment [11] in freshwater having an approximate depth of 10 m. A Woods Hole acoustic micro-modem [12] was used to drive the source. Two ITT-8140 hydrophones, used as receivers, were separated by a fixed distance of 0.457 m (18 in). The source transducer was an ITC-1032 which is omni-directional, broadband, with a resonant frequency of 32 kHz. The source amplitude level was 183 dB (re 1 Pa @ 1yd). The source emitted a BPSK navigation ping generated by the Woods Hole modem having a carrier frequency of 26 kHz, bandwidth of 4 kHz, and duration of 7 ms. The hydrophones were omni-directional and had a flat frequency response from 1 kHz to 40 kHz. The hydrophone voltage signals were anti-alias filtered and sampled simultaneously at a rate of 131,072 Hz with 16-bit resolution.

B. Data Processing

The difference in arrival time \( \Delta t \) from the two hydrophone signals was extracted using a cross-correlation method. Prior to correlation, leading-edge detection was first used to generally locate the transmitted acoustic signal within each hydrophone data stream. Next, each hydrophone signal was heterodyned to baseband to remove the carrier. The resulting complex analytic signals were then cross-correlated producing a sampled complex correlation output. The desired delay time \( \Delta t \) was extracted by maximizing the cross-correlation amplitude. Interpolation techniques were used to avoid sample-interval round-off. This procedure is similar to that used to extract time-of-flight using matched filter processing [11], except that two measured signals were used in this procedure, as opposed to a measured signal and a noise-free replica. In addition, no compensation for Doppler shift was performed.

C. Experimental Measurement of Arrival Time Difference Uncertainty

The standard deviation \( U_\Delta \) in experimentally measured arrival time difference \( \Delta t \) between the two sensors are shown in Table 1. Each row corresponds to a summary of tests at a particular geometry. The first four columns contain the planar range \( R \), source and receiver depths, and the relative heading \( \theta \). The fifth column contains the standard deviation in arrival times that was obtained from 20 pairs of hydrophone waveforms. Outliers, defined as arrival time difference \( \Delta t \) such that \( |c \Delta t/d| > 1 \), were removed from the sample set when computing standard deviations. It was found that the standard deviation in time arrival difference ranged from 5 μs to 30 μs, with the exception of one geometrical configuration. In this geometrical configuration (\( R=21.0 \) m, \( z_s=1.8 \) m, \( z=2.1 \) m, \( \theta=0^\circ \)), the
observed standard deviation was $U_{\Delta t}=113\mu s$.

It was our observation that all outliers in measured arrival time difference $\Delta t$ were caused by correlations with the first surface bounce. This correlation occurred at the time difference $\Delta t_b$ given by

$$\Delta t_b = \pm R + 2 \sqrt{\left(\frac{R^2}{Z_s^2} + Z_r^2\right) - \frac{Z_r^2}{Z_s^2}}, \quad (10)$$

where $Z$ is the depth of the source and hydrophones below the surface. An example of this phenomenon is shown in Fig. 2, which contains the arrival time differences $\Delta t$ that were extracted from 20 waveforms for the geometry $(R=12.8m, Z_s=4.6m, Z_r=4.6m, \theta=135^\circ)$. It can be seen that six outliers were caused by correlation with surface bounce at a time arrival difference of $\Delta t_b=\pm 20\mu s$. Correlation with surface bounce was the cause of the abnormally large standard deviation in arrival time $U_{\Delta t}=113\mu s$ that was observed for the geometrical configuration $(R=21.0m, Z_s=1.8m, Z_r=2.1m, \theta=0^\circ)$. In this configuration the time difference correlated with surface bounce was $218\mu s$, and was not rejected as an outlier because it inferred a physically realizable heading angle $\theta$.

D. Uncertainty in Relative Heading

The standard deviation in arrival time difference $U_{\Delta t}$ directly determines the uncertainty in determination of relative heading $\theta$ from (3). Shown in Fig. 3 is a plot of the uncertainty in heading $U_{\theta}$ that would be obtained with an uncertainty of $\sigma_{\Delta t}=20\mu s$, and sensor separation of $d=1m$. It can be seen that an uncertainty in heading of somewhat less than $2^\circ$ can be expected for headings in the range $45^\circ<\theta<135^\circ$. For angles outside this range, the uncertainty in heading could become very large.

IV. FORMATION-FLYING SIMULATIONS

Simulations were performed to explore the feasibility of using a two-hydrophone arrangement to sense relative position for formation-flying control algorithms. The control algorithm used for the simulations is described in Section IV.A, and the simulation results are described in Section IV.B.

A. Controller Algorithm

The control algorithm used for the simulations presented in this paper was based on the hybrid leader-follower algorithm given in [4]. The controller consists of two parts: a trajectory following controller and a formation controller.

A diagram of the geometry used for the trajectory following part of the controller is shown in Fig. 4. The trajectory following controller calculated and minimized a deviation of heading, $\delta_{\theta}$, and perpendicular distance, $\delta_z$, from the trajectory. The steering control law was determined by a linear combination of these variables as

$$\delta_z = K_\theta \delta_{\theta} + K_z \delta_z, \quad (11)$$

where $\delta_z$ is the rudder angle, $\delta_z$ is a perpendicular distance from the trajectory, $\delta_{\theta}$ is a deviation of heading from the trajectory; $K_\theta$ and $K_z$ are controller gains. The first term on the right hand side of the equation implies that the vehicle tries to be parallel with the trajectory. On the other hand, the second term implies that if the vehicle is far from a
The formation controller can be described as a combination of a velocity regulator and a formation controller. Velocity was regulated at a predetermined reference while the formation controller either increases or decreases the velocity to maintain a desired distance to the leader. The velocity control law was a linear combination of the regulator and formation controller, i.e.,

$$F_{th} = K_v \left( v_{ref} - v \right) + K_{FC} \left( r_{ref} - r \right),$$

where $F_{th}$ is the thrust force, $v$ and $v_{ref}$ are the vehicle and reference velocities, $r$ and $r_{ref}$ are the actual and a desired distance to the leader in a local coordinates; $K_v$ and $K_{FC}$ are velocity and formation control gains. The actual distance to the leader $r$ is the only variable which couples the follower with the leader. This variable is illustrated in Fig. 5. The distance is $r$ defined as a magnitude of a distance vector $\mathbf{r}$. An identical velocity controller for all vehicles was used, except the leader. The controller for the leader used a gain of $K_{FC}=0$ to disable the formation control.

Because the controller follows a trajectory and maintains formation simultaneously, a follower only requires knowledge of the relative heading of the leader to maintain formation. Since the trajectory offset $\Delta y$ is known 'a priori' by the follower, the distance $r$ contained in the controller law (10) can be calculated from the trajectory offset $\Delta y$ and the relative heading $\theta$.

### B. Formation-Flying Performance

Simulations were performed to investigate the use of the two-hydrophone sensor in various formation geometries. Two factors were considered, the nominal relative heading $\theta_0$ and separation distance $r_0$ in the formation. Three different formation shapes were used for the simulations: 40yd-abreast formation ($\theta_0 = 90^o$, $r_0 = 40$yd), 10yd-abreast formation ($\theta_0 = 90^o$, $r_0 = 10$yd), and 10yd-wedge formation ($\theta_0 = 45^o$, $r_0 = 14.1$yd). The sensor was modeled with bias $B_0=0$, standard deviation in arrival time difference $\sigma_{\Delta t}=20\mu s$, $d=0.914$ m, and $c=1371.6$ m/s.

The leader was given a lawn-mower waypoint trajectory, and the follower’s waypoint trajectory was obtained by shifting the leader’s trajectory by a desired distance. For the 40yd-abreast formation and the 10yd-abreast formation, the leader’s lawn-mower path was shifted by 40yd and 10yd respectively. For the wedge formation, they still followed the same lawn-mower path as the previous one, but the follower was assigned to be 45° behind the leader. The vehicles were assigned to maintain formation and perform a lawn-mower search in a rectangular field.

A two degree-of-freedom model was used for the simulations. The coefficients for the model were taken from the six degree-of-freedom REMUS model given in [13]. The two degree-of-freedom model had one translational and one rotational motion so that the direction of vehicle’s motion and vehicle’s heading were assumed to be the same.

The performance of a controller that used the two-hydrophone sensor was compared with a controller that received the leader’s position through communication. It was assumed that all vehicles determined their inertial position using an LBL navigation system, and that the leader was capable of broadcasting its position in parallel to the followers. From previous experience, we know that it takes 2sec for one vehicle to receive LBL position update, and 4 sec for the leader to broadcast its position [4]. If there were five vehicles in a formation, it would take a total of 16 sec to complete one cycle. In this paper, a formation of two vehicles is considered, and it is assumed that both vehicles receive their LBL position update and the two-hydrophone sensor measurement at every 16sec simultaneously.

Fig. 6 through Fig. 8 show the follower’s ability to maintain a specified formation with the leader. Trajectory of vehicles is shown in Fig. 6a, 7a, and 8a, and the distances between vehicles are given in Fig. 6b, 7b, and 8b. The numbers in the figures indicate the distances between the leader and the follower at each segment. The segment 1, 3, and 5 are search legs, and 2 and 4 are turns. The desired distances between the vehicles are $\Delta X_d = 0$yd and $\Delta Y_d = 40$yd for 40yd-abreast formation, $\Delta X_d = 0$yd and $\Delta Y_d = 10$yd for 10yd-abreast formation, and $\Delta X_d = 10$yd and $\Delta Y_d = 10$yd (or -10yd at 3) for 10yd-wedge formation. In the figures the solid lines are simulation results with the two-hydrophone sensor, and the dashed lines are these with communication.

As far as the formation shape is concerned, the 10yd-abreast and the 40yd-abreast formations worked much better than the wedge formation since they used the sweet spot of the sensor. Furthermore, for these abreast formations a multi-vehicle search with the two-hydrophone sensor and a multi-vehicle search with communication had almost the same search performance. When the same formation shapes are concerned, a smaller formation worked better with the sensor as predicted.
Fig. 6a: Trajectory of vehicles, 40yd-abreast formation ($\theta_0 = 90^\circ$, $r_0 = 40$yd), solid: with two-hydrophone sensor, dashed: with communication.

Fig. 6b: Distance between vehicles, 40yd-abreast formation ($\theta_0 = 90^\circ$, $r_0 = 40$yd), solid: with two-hydrophone sensor, dashed: with communication.

Fig. 7a: Trajectory of vehicles, 10yd-abreast formation ($\theta_0 = 90^\circ$, $r_0 = 10$yd), solid: with two-hydrophone sensor, dashed: with communication.

Fig. 7b: Distance between vehicles, 10yd-abreast formation ($\theta_0 = 90^\circ$, $r_0 = 10$yd), solid: with two-hydrophone sensor, dashed: with communication.

Fig. 8a: Trajectory of vehicles, 10yd-wedge formation ($\theta_0 = 45^\circ$, $r_0 = 14.1$yd), solid: with two-hydrophone sensor, dashed: with communication.

Fig. 8b: Distance between vehicles, 10yd-wedge formation ($\theta_0 = 45^\circ$, $r_0 = 14.1$yd), solid: with two-hydrophone sensor, dashed: with communication.
Comparing performance of the formation controllers, the controller which used the two-hydrophone sensor worked as well as the controller which used the leader’s broadcast except during and recovering from the turns. Because the follower was in the dead spot of the sensor during the turns, the formation control was disabled. Consequently, the follower spent some time to catch up the leader after coming out of the turns. On the other hand, when the leader’s broadcast was used for formation control, there was virtually no dead spot. As a result, the formation error converged even when the vehicles were changing lanes, resulting in better performance.

Another observation is that the formation error decayed slower with the two-hydrophone sensor. It might be due to the assumption to compensate for the range measurements, where the lateral distance to the leader is always the desired value, \( d \).

V. CONCLUSIONS

Experimental measurements in shallow freshwater approximately 10m in depth show that relative heading can be measured within \( 2^\circ \) at distances of 20m using cross correlated signals from two hydrophones separated by 1 m (39.4 in). This precision can be obtained over a range of heading angles \( 45^\circ < \theta < 135^\circ \). Angles outside this range would suffer from severe uncertainty.

Simulations show that a hybrid formation-flying controller can perform as well using the two-hydrophone sensor to determine relative position as would be the case if this information were derived from exchange of inertial data with and acoustic link. This conclusion is valid for formations that attempt to maintain a configuration with the vehicles abreast. If the formation requires follower vehicles to maintain a position behind the formation leader, use of the two-hydrophone sensor would lead to poor formation-flying performance.

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