DEVELOPMENT OF AN ABRASIVE WATER JET OPTIMUM ABRASIVE FLOW RATE MODEL FOR TITANIUM ALLOY CUTTING (PREPRINT)

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The abrasive water jet (AWJ) is used in industry extensively, optimization of the process parameters that determine efficiency, economy and quality of the process is becoming more and more important for its successful application. However, being a complicated cutting system, an abrasive water jet is characterized by a large number of process parameters, which include water pressure, orifice diameter, traverse rate, standoff distance, impact angle, focusing tube diameter, abrasive flow rate, etc. Therefore, optimizing the process parameters involves lots of challenging efforts. This paper concentrates on investigating the optimum abrasive flow rate under different water pressures, orifice diameters and focusing tube diameters. Based on theory derivation and experimental study, an empirical model for calculating the optimal abrasive flow rate is created.
Development of an abrasive water jet optimal abrasive flow rate model for titanium alloy cutting

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1. ABSTRACT

As the abrasive water jet (AWJ) is used in industry extensively, optimization of the process parameters that determine efficiency, economy and quality of the process is becoming more and more important for its successful application. However, being a complicated cutting system, an abrasive water jet is characterized by a large number of process parameters, which include water pressure, orifice diameter, traverse rate, standoff distance, impact angle, focusing tube diameter, abrasive flow rate, etc. Therefore, optimizing the process parameters involves lots of challenging efforts. This paper concentrates on investigating the optimum abrasive flow rate under different water pressures, orifice diameters and focusing tube diameters. Based on theory derivation and experimental study, an empirical model for calculating the optimal abrasive flow rate is created.

2. INTRODUCTION

The last 20 years has seen considerable refinement in the use of abrasive water jet (AWJ) technology. As a non-contact process it offers advantages that include narrow kerfs, no heat-affected areas nor structural changes in materials, and no tool changing in a process that has flexibility in processing a wide range of thicknesses and materials.

There are two process costs that, more than any other, impact the overall system cost, and these are the amortized cost of the nozzle wear parts, and the cost of the abrasive used in cutting. Both are controlled by the amount of abrasive that passes through the nozzle, and thus a process that optimizes abrasive flow rate (AFR) is critical to productive system operation.

There have been a number of earlier attempts to optimize performance, however it should be recognized up front that, even as different nozzle designs produce different levels of cut, under equivalent nominal cutting conditions (Figure 1) [1], so the optimum value of abrasive feed rate
to the jet is going to be controlled also by the nozzle design used in the studies. Chen and Geskin [2], Miller and Archibald [3] and Himmelreich [4] measured a drop of the abrasive particle velocity with an increase in the abrasive mass flow rate. Hu et al. [5] found a linear increase in the depth of cut as the number of impacting abrasive particles increased. And other studies have shown that the optimum abrasive flow rate depended on cutting process parameters which include pump pressure [6], orifice diameter [6], traverse rate [7], focusing tube diameter [6], focusing tube length [8], abrasive particle diameter [8], abrasive particle shape [8], abrasive type [8], etc. According to Momber et al. [9], the optimum abrasive flow rate also depends on the deformation behavior of the target materials. Since these studies established relationships, then it is practical to consider that one can predict an optimum abrasive flow rate for a given set of conditions.

![Figure 1. Section showing the cut made by three different abrasive jets at the same AFR, speed and pressure.](image)

However, knowing this value is not enough to define the right abrasive flow rate for a given cutting process. Given that the cutting performance is tied to AFR, in a relationship that is not linear one must also integrate costs and other aspects of performance into assessing the most beneficial AFR that should be used. This paper considers, initially a theoretical derivation and combines this with an experimental study to build a model that can be used to select an optimum abrasive flow rate as an input into consequent studies that will also include other parameters of the cut that must also be included in controlling the cutting process to achieve an acceptable result.

3. **THEORETICAL CALCULATION OF THE OPTIMUM ABRASIVE FLOW RATE**

The abrasive mass flow rate determines the number of impacting abrasive particles as well as their kinetic energies. The higher the abrasive mass flow rate, the higher the number of particles involved in the mixing and cutting process. Without considering fragmentation of the particles in mixing and cutting, an increase in the AFR initially leads to a proportional increase in the depth of cut. However, at higher AFR, the limited kinetic energy of the water jet distributes over a greater number of particles and this decreases the kinetic energy transfer to an individual particle. Further the greater the particles, the higher the chance of inter-particle collisions, further reducing overall particle energy. These effects overcome the positive effect of the higher impact frequency [8], and performance begins to decline. In the transition, an optimum abrasive flow rate can be defined.
When the pressurized water comes out from the orifice, a high speed water jet is generated. In this case, using Bernoulli’s law,

\[
P_{at} + \frac{\rho_w}{2} \cdot V_0^2 + \rho_w \cdot g \cdot h_1 = P + \frac{\rho_{w1}}{2} \cdot V_1^2 + \rho_{w1} \cdot g \cdot h_2
\]  

(1)

In this equation, \( P_{at} \ll P \), \( V_0 \gg V_1 \), \( h_1 \approx h_2 \), so, the approximate velocity of the exit water jet can be expressed as,

\[
V_{0th} = \sqrt{\frac{2 \cdot P}{\rho_w}}
\]  

(2)

In practice, considering losses due to wall friction, flow disturbances and the compressibility of water, the velocity of the water jet can be modified to,

\[
V_0 = \mu \cdot V_{0th} = \mu \cdot \sqrt{\frac{2 \cdot P}{\rho_w}}
\]  

(3)

\( \mu \) is an efficiency coefficient.

The typical values for \( \mu \) lies around 0.88~0.95 [10]. In the focusing tube, particle acceleration is a momentum transfer from the high velocity water to the particles, drawn in at relatively low speed with an air carrier. According to the impulse balance equation,

\[
m_A \cdot V_{P0} + m_w \cdot V_0 + m_L \cdot V_L = (m_A + m_L + m_w) \cdot V_P
\]  

(4)

In this case, \( V_{P0} \) and \( V_L \) are negligible, and comparing with \( m_A \) and \( m_w \), \( m_L \approx 0 \), therefore,

\[
V_{Pth} = \frac{m_w V_0}{m_A + m_w} = \frac{V_0}{1 + \frac{m_A}{m_w}}
\]  

(5)

In the above equation, assume all abrasive particles have the same speed. Consider the energy losses during the acceleration process,

\[
V_P = \eta_t \cdot \frac{V_0}{1 + \frac{m_A}{m_w}}
\]  

(6)

\( \eta_t \) is a momentum transfer coefficient, \( \eta_t = 0.65~0.85 \) [10]. The energy of the abrasives can then be expressed as,

\[
E_{th} = \frac{1}{2} \cdot m_A \cdot V_P^2 = \frac{1}{2} \cdot m_A \cdot \eta_t^2 \cdot \frac{V_0^2}{(1 + \frac{m_A}{m_w})^2}
\]  

(7)

Combining equations (3) and (7) to get the abrasive energy:

\[
E_{th} = \frac{1}{2} \cdot m_A \cdot \eta_t^2 \cdot \frac{2 \cdot P \cdot \mu^2 \cdot \frac{m_w^2}{(m_w + m_A)^2}}{(m_w + m_A)^2}
\]  

(8)
The mass flow rate of the water jet is given by,

$$m_w = \rho_w \cdot \frac{1}{4} \cdot \pi \cdot d_0^2 \cdot \mu \cdot \sqrt{\frac{2P}{\rho_w}} \quad (9)$$

Therefore, the energy of the abrasives can be expressed as,

$$E_{th} = \eta_t^2 \cdot \mu^2 \cdot m_A \cdot \frac{P}{\rho_w} \cdot \frac{1}{\left(1 + \frac{4m_A}{\pi \cdot \mu \cdot d_0^2 \cdot \sqrt{2P \cdot \rho_w}}\right)^2} \quad (10)$$

From which the optimum abrasive flow rate can be deduced as:

$$\frac{\partial E_{th}}{\partial m_A} = \eta_t^2 \cdot \mu^2 \cdot P \cdot \frac{1}{\left(1 + \frac{4m_A}{\pi \cdot \mu \cdot d_0^2 \cdot \sqrt{2P \cdot \rho_w}}\right)^2} + m_A \cdot \frac{\eta_t^2 \cdot \mu^2 \cdot P}{\rho_w} \cdot (-2) \cdot \frac{\frac{4}{\pi \cdot \mu \cdot d_0^2 \cdot \sqrt{2P \cdot \rho_w}}}{\left(1 + \frac{4m_A}{\pi \cdot \mu \cdot d_0^2 \cdot \sqrt{2P \cdot \rho_w}}\right)^3} = 0 \quad (11)$$

Using equation (11), the optimum abrasive flow rate can then be calculated as,

$$m_A = \frac{\pi \cdot \mu \cdot d_0^2 \cdot \sqrt{2P \cdot \rho_w}}{4} \quad (12)$$

In practice, considering other factors would affect the optimum abrasive flow rate, equation (12) can be expressed as:

$$m_A = \frac{\pi \cdot \mu \cdot d_0^2 \cdot \sqrt{2P \cdot \rho_w}}{4} \quad (13)$$

4. EXPERIMENTAL VERIFICATION

In order to verify the derivation, a series of experiments was carried out using the titanium alloy Ti6Al4V. The experiments were carried out using a PAR Vector 5-axis robotic cutting system with a 100-hp KMT pump. SURFCAM and CIMSYS-XM were used as CAD and CAM software for this 5-axis system respectively. In these experiments, an abrasive metering and delivery system by which the abrasive flow rate can be adjusted freely from 0 b/min to 3 b/min was used. 80 mesh Barton garnet was used in these tests. Three KMT nozzle combinations with 0.010:0.030, 0.012:0.030 and 0.014:0.043 inch diameter for the orifice and focusing tube were used in the tests. In order to locate the optimum abrasive flow rate for the different cutting conditions, three kinds of experiments, including static piercing tests, dynamic piercing tests and line cutting tests, were carried out.
4.1 Static piercing tests
Tests were carried out using a factorial experiment in which the three nozzle combinations were run at pressures from 30,000 psi to 55,000 psi in 5,000 psi increments, and at 6 AFR rates that were adjusted to give the same abrasive concentrations in the water at the different flow rates achieved for the different nozzle combinations. Tests were carried out at a standoff distance of 0.05 inch.

In these tests a video record was made of the jet as it initially penetrated a 0.5-inch thick plate of titanium. The time from which the jet began to penetrate (the angle of reflection of the jet changed (Figure 2), and the scene became foggy) to the time that it penetrated the target (rebound ceased and mist disappeared (Figure 3)) could be accurately measured in this way to within 0.03 seconds.

Figure 2. Initial impact start frame.  
Figure 3. Penetration frame.

Figure 4. Static piercing test sample.
4.1.1 Results and discussion

Figure 5. Static piercing time as a function of jet pressure for three nozzles.

Figure 6. Static piercing time as a function of abrasive feed rate for 3 nozzles.
It is interesting to note that the data, when initially summarized, show that there is less benefit in going to the high pressures (45,000 psi seems perhaps to be optimal) and that there are clear optima in the curves for AFR for the three nozzles, with the value increasing (as the theory predicts) as the water volume flow increases with the larger orifice diameters. When individual test data were evaluated it was found that for the two smaller orifices (0.010 and 0.0120 the optimal ARR was around 1.35 lb/gal, but that with the largest nozzle the optimum AFR was in excess of 2 lb/min.

4.2 Dynamic piercing tests  
The dynamic piercing tests used one nozzle set (the 0.014/0.043 inch) at four pressures (35000/45000/50000/55000 psi) and four abrasive feed rates, with the other cutting conditions similar to those of the earlier test – i.e. 0.5-inch thick titanium plate cut at a 0.05-inch standoff.

In many cases, it is now common to start the nozzle moving relative to the target at the instant that cutting starts, and thus to do a dynamic cut into the part. This has the advantage of moving the jet stream so that it does not have to overcome the rebounding energy of the previous slug of water before reaching the target. The result, if the cut can be started off-line is that the jet reaches full cutting depth at the time that it also reaches the final cut line required, and there is a significant saving in time and abrasive.

In these tests, a sample piece with four sets of 6 differing diameter holes were cut in a set pattern in each plate. As a secondary purpose, the angle that the jet made to the vertical was slightly changed as was the hole diameter, during the course of the tests. The sample was then separated from the main body of metal by a relieving cut all around the sample section. This design pattern was prepared for another project [12]. As with the previous test, dynamic piercing time was obtained from a video record. In this paper, only the dynamic piercing time for the holes that were cut with a 0° inclination nozzle are considered.
4.2.1 Results and discussion

![Dynamic Piercing Time VS Abrasive Flowrate With 0.014:0.043 Nozzle](image)

**Figure 9.** Dynamic piercing time as a function of abrasive flowrate.

![Dynamic Piercing Time VS Water Pressure With 0.014:0.043 Nozzle](image)

**Figure 10.** Dynamic piercing time as a function of water pressure.

As expected, optimum abrasive flow rates could be found, though these were at a lower value, around 1.0 lb, than the values, almost double this, for static piercing. It is also interesting to note that the piercing time reached a minimum at a pressure of around 45,000 psi. This is more consistent with the data from the static tests, but suggests that there are unknown events occurring that make the higher pressure systems less productive. These could well include a higher level of particle fragmentation, which would reduce the effective cutting ability if particle size distribution contained a greater portion of particles reduced below 100 microns in size.

4.3 Linear cutting tests

A full factorial experiment was now carried out using the same combination of AFR and nozzle sizes as in the static testing. Tests were carried out at traverse speeds of 20 and 40 in/min and the sample thickness increased to 1.0 inches. This was chosen since the intent with the linear cuts was to determine the effective depth of cut, rather than piercing time, and thus the sample had to be thicker than the anticipated cut. Sample material, abrasive size and standoff distance were as earlier.
Because the nozzle accelerates at the beginning of the cut and at the end, only data from the center of the cut was recorded. After all the cuts were complete the sample was sectioned into four pieces, and three depths of cut measured, and the average depth was recorded. The volume rate of removal was then calculated, as the product of depth, traverse speed and slot width. As an original indication of performance the individual data points were plotted, and it was found that the optimal abrasive flow rates at both traverse speeds were the same for each nozzle at a designated pressure. The two sets of data were therefore combined in the subsequent initial data plotting. Similarly there was relatively little variation in the optimal AFR as a function of jet pressure for each nozzle (for e.g. Figure 11), and thus the data for each was initially averaged (Figure 12).

Figure 11. Depth of cut as a function of abrasive concentration for the 0.012 -0.030 nozzle combination.

Figure 12. Depth of cut as a function of abrasive feed rate for the three nozzles tested.
To this point it appeared that the AFR was performing as anticipated, and the optimal values for the different conditions could be determined for each curve. To verify equation (13), the results calculated from the equation were compared with the experimental results. In equation (13), \( \mu_f \) is an unknown coefficient. The experimental results obtained with the 0.014:0.043 nozzle at 40 in/min were used to derive a value for \( \mu_f \). With a calculated transfer coefficient of 0.2027, the theoretical results with 0.010:0.030 nozzle and 40 in/min traverse speed were then solved. These results were compared with the actual experimental results.

![Figure 13. A comparison of actual optimum abrasive flow rate and theoretical optimum abrasive flow rate.](image)

However, the numbers alone are a little deceptive, since it is actually the abrasive concentration (the amount of abrasive per unit volume of water) that is the more critical measure, since this changes both with nozzle geometry and with pressure. When this plot was then made of the data, a somewhat unexpected result was obtained (Figure 14). It can be seen that the largest nozzle size required the lowest abrasive concentration, while the smaller nozzles required a larger abrasive feed rate to achieve a maximum cutting depth. This was originally a surprise, since the initial thought was that the higher flow rates would have, if anything, supported a greater AFR. But this is not AFR it is concentration, and here there is a different phenomenon at work. It is suggested that the reason for the higher concentrations being required to give optimal results has to do with the survivability of the particles in the system.

Experiments at UMR have shown [11] that abrasives lose cutting performance where they fall below 100 micron in size. It is conjectured (but not yet validated) that the smaller nozzles are creating greater fragmentation of the particles during mixing, and thus, being dispersed more throughout the jet, a greater load can be carried, and thus be more effective, than is the case with the larger particles and the larger nozzle geometry.
Regardless if this were the case it did suggest that there were a considerable number of other considerations that need to be brought to bear, as the program moved to more sophisticated levels of prediction. However, rather than spend considerable time developing equations that would ultimately be specific only to a single nozzle geometry, it was decided, instead, to move forward more rapidly, by developing a set of empirical equations to cover the less-than-ideal case. It is anticipated that an opportunity to revisit this problem will occur later in the program.

In seeking to develop a more general, empirical equation, it was recognized that the momentum transfer process is controlled by many factors, including focusing tube diameter, focusing tube length, and the structure of the mixing chamber, abrasive grain size, abrasive type, etc. A number of these factors, abrasive type and size, for example, have become readily standardized. Others vary considerably between manufacturers. For this study it was decided to standardize on one manufacturer, and, at this time, on a single target material, a titanium alloy.

The nozzle size combinations were separated by orifice and focusing tube diameters, and a model developed. Based on the above discussion, the empirical model was:

$$m_A = n_0 \cdot d_0^{n_1} \cdot P^{n_2} \cdot d_f^{n_3}$$  \hspace{2cm} (14)

Where \( n_0, n_1, n_2, n_3 \) are regression coefficients. After performing a regression analysis, the regression coefficients were obtained as 
\( n_0 = 5948.1, \ n_1 = 0.3941, \ n_2 = 1.5132, \ \text{and} \ n_3 = 0.9433 \).

Within the above regression analysis, the empirical model could then be expressed as:

$$m_A = 5948.1 \cdot P^{0.3941} \cdot d_0^{1.5132} \cdot d_f^{0.9433}$$  \hspace{2cm} (15)
5. CONCLUSIONS

The above theoretical derivation and experimental evaluation led to the following conclusions:
1) Under defined cutting conditions, an optimum abrasive flow rate exists;
2) Using traditional fluid laws, a theoretical equation could be derived to calculate the optimum abrasive flow rate, and the validation of the theoretical equation has been evaluated by the experimental results;
3) Considering the actual situation, an empirical model was proposed, and the regression analysis was performed which provided the coefficients for the model.

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7. REFERENCES