Radiated pulses decay exponentially in materials in the far fields of antennas.

There has been recent interest in using short-pulse radar to detect targets in lossy clutter. The analysis presented here shows that the energy and peak-power densities of pulses decay exponentially with depth in homogeneous, lossy, dispersive materials, provided the frequency bands of the pulses are separated from DC. Many numerical examples verify the analytical results.
Introduction: Pulse decay in Lorentz and Debye dispersion models has been studied [1-4] since 1914. Such pulses are said to decay algebraically with depth, typically as $e^{-x^2}$, $x^2$, or $x^{-2}$ as $x \to 0$ in these linear models. This is slower than the exponential decay of single-frequency signals. Several groups recognized this and recently began investigating whether algebraically decaying pulses could scatter penetrate one or other stratified, noisy media in coaxial or felds. As explained in this Letter, it is concluded that the answer is, unfortunately, no.

Algebraic decay is often claimed for pulses with DC or near-DC content. Evidence 10 of [4], e.g. shows that a pulse is divided as $x^{-2}$ in all unimpeded Lorentz models if it has DC content, but that the decay is $x^{-3/2}$ if the Fourier transform satisfies $j \omega \neq 0$ (for a, b, c, d, e, and g, see [4]), significantly near-DC content. Gauss-modulated sources $(x^2)$ and stra $(x^{-2})$ are examples. Such pulses propagate well in coaxial cables; however, it is widely known that highly conducting (e.g., copper, or, c, c, c) dipoles and loops have free-space radiation efficiencies that vanish as $e^{-x^2}$ and $x^{-2}$, respectively, at $x \to 0$. In model pulse propagation or far-field output, it is therefore assumed here that the pulse spectrum is separated from DC.

Analysis: Let $f$ be any real-valued, band-limited, incident electric-field pulse. Then $f$ propagates in any 1D dispersive half-space $z \geq 0$ as

$$R(x,t) = \int_{-\infty}^{\infty} e^{i\omega t-i\omega z} f(\omega) d\omega$$

with $t = \alpha t + \beta$, or $e^{i\omega t-i\omega z} d\omega$. Here $R(t) = 0$ except where $z$ satisfies $0 < z < \infty$. $s$ is $< z$, $s > z$. (A relevant zero point is indicated below.) The material is lossy $\mu > 0$, $\mu > 0$, and $\mu > 0$. Standard analysis and (1) yield

$$|R(x,t)| \leq \exp(-e^{-\beta \gamma t}) \int_{-\infty}^{\infty} |f(\omega)| d\omega$$

i.e., $|R|$ decays at least as fast as $\exp(-e^{-\beta \gamma t})$. The Fornescher equation similarly yields

$$\int_{-\infty}^{\infty} |f(\omega)|^2 d\omega \leq \exp(-e^{-\beta \gamma t})$$

Thus, energy densities and peak $|R|$ values decay exponentially in all lossy, dispersive media.

For example, every finite-band pulse separated from DC will decay exponentially in every Debye and damped-Lorentz model. Similar behavior occurs in the lossy band $\beta < \omega < \alpha$ of undamped-Lorentz models $\epsilon = 1 + i\alpha - (\beta - \epsilon^2)$. These examples are unlike the algebraic decay predicted [1-4] for near-DC or noise pulses in the same Debye and Lorentz models.

Regarding finite bands, the mathematics of entire functions shows that no finite-bandwidth pulse is precisely 0 over any time interval. Yet the pulses above are 0 if $z > 0$. Finite bands are also practical, and their never-greater-0 consequences suggest ordinary value. Indeed, the numerics here will omit features 15 db below peak power. "Power" and "energy" hereafter rely implicitly to densities.

Numerics: The example incident pulses have $R(t) = 0$, except for $\epsilon_{max} \leq \beta + 2\epsilon_{max}$ where $f$ is $\exp(1 + i\alpha/\omega) e^{-i\omega z}$ with $\epsilon_{max} = 1 - \epsilon_{max} - i\alpha$ and $\epsilon_{max} = 1 + \epsilon_{max} - i\alpha$. The computed inverse transforms of $f$ are truncated below -35 db. Conventionally for numerics, this is better than many other pulses with similar spectra. Fig. 1 shows the $f$ used in Fig. 2 for a Debye model. Every $f$ used here for a Lorentz model is in the spectrum of that model's so-called Brillouin zones, which are often said to decay algebraically.

The Debye model $\epsilon_{max} = 1 + i(1 + u) = \cosh 1 + i\sinh 1$ approximate rates. Let $\alpha = 1 - \alpha$. The damped-Lorentz model $\epsilon_{max} = 1 + 39 x^2$ $\epsilon_{max} = 1 + i \alpha$ (in $\cosh 1 + i\sinh 1)$ is used on $\epsilon_{max}$ and $\epsilon_{max}$ have compatible $\epsilon_{max}$ curves in Fig. 3. Indeed, Fig. 3 shows that the bands 13-18 GHz in $\epsilon_{max}$ and 40-50 GHz in $\epsilon_{max}$ have the same $\epsilon_{max}$ and $\epsilon_{max}$. Propagating the corresponding pulses $\epsilon_{max}$ and $\epsilon_{max}$ yields normalized energies $\epsilon_{max} = s_{max} e^{-2\gamma t}$ $\epsilon_{max} = s_{max} e^{-2\gamma t}$ where $\epsilon_{max}$ and $\epsilon_{max}$ are split within a line width $\delta$ due to the formula of Fig. 2. Normalized peak powers $P_{max}(\epsilon_{max}) = \epsilon_{max} e^{-2\gamma t} / \epsilon_{max} e^{-2\gamma t}$ in $\epsilon_{max}$ and $\epsilon_{max}$ in Fig. 2 verify the analytical results.

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T.M. Roberts

There has been recent interest in using short-pulse rate to detect targets in noisy clutter. The analysis presented here shows that the energy and near-ground density of pulses decay exponentially with depth in homogeneous, lossy, dispersive media, provided the frequency bands of the pulses are separated from DC. Many numerical examples verify the analytical results.

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Going beyond the analysis, a more sensitive measure of exponential decay is: \( P(x,t) = \frac{1}{P(0)} \). This is -1 times the independent slope (in dB/m/cm) of \( P(x,t) \) when graphed in dB. Thus, \( -\frac{dx}{dt} \) is the local exponential decay rate of \( P(x,t) \). For the peaks of Figs. 1 and 2, the computed \( -\frac{dx}{dt} \) varies from 1185-1257 m/s (20-30 dB/m) for the first 60 dB of energy attenuation. This is in the range [20, 250] dB of 10^14-10^15 for the first 60 dB of peak power. This is not yet predictable by theory.

Fig. 3 shows ranges of local exponential decay rates /P/P of peaks P for the first 60 dB of peak-power attenuation. The large, marked rectangle, e.g., signifies that the incident /P with spectrum 40-70 GHz has a peak that decays in the Lorentz model \( \alpha_L \) as an exponential rate that varies from 700-1000 m/s. The dashed curve shows this is within the range [10, 20] of 10^15-10^16 m/s for c_L. This example is the seven others in Fig. 3 verify the \( n \exp(-20 \text{ dB}) \) prediction for peaks. The spectrum for the small, marked triangle is 6-8 GHz. Linearity and Fig. 2 thereby yield near four-decades function of exponential of peaks that decay faster than \( 10^{15} \text{ dB} \), with bandwidths over three and four decades.

Conclusions: In this Letter a practical model of pulses in free-field, lossy materials is used to show that exponential decay is typical. This is verified by many numerical examples, with bandwidths up to four decades. Two other numerical observations have not yet been explicated by analysis. First, analysis has not explained why \( \exp(-20 \text{ dB}) \) is a bound as in Fig. 2. Secondly, the local exponential decay rates of examples and peaks are within bounds suggested, but not yet predicted, by analysis. The analytical results in this Letter, however, are all numerically verified.

A consequence of most practical interest is that algebraic or decay no longer seems to be a useful design principle for pulse generation of free-field, lossy clutter. Although algebraic decay might be recovered from exponential decay in a mathematical limit \( \alpha_L \rightarrow 0 \), this would not escape the real difficulties of low radiation efficiency and low resolution posed by near-GC signals.


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Electronics Letters Online No: 20020212

DOI: 10.1049/el:20020212

T.M. Roberts (Antenna Technology Branch, Air Force Research Laboratory/SNHA, 80 Scott Drive, Hanscom AFB, MA 01731, USA)

E-mail: roberts@maxwell.nrl.navy.mil

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