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Bekker's Terramechanics Model for Off-road Vehicle Research

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ABSTRACT

Bekker's Derived Terramechanics Model (BDTM) is an analytical tool for evaluating vehicle off-road mobility. BDTM has been developed using Bekker's equations for vehicle soil interactions. He developed the bevameter technique to measure mechanical strength characteristics for many soil and snow conditions. This procedure uses seven parameters to describe soil conditions, which differs from the conventional single parameter vehicle cone index methodology used by the NATO Reference Mobility Model (NRMM). NRMM uses the cone penetrometer technique to experimentally measure fine-grained soil mechanical characteristics.

BDTM is in a spreadsheet format, and its primary purpose is to compare mobility characteristics for robotic track and wheeled vehicles under different terrain conditions. Bekker's model is a simple, linear one degree-of- freedom (1-DOF) model, which assumes that in a perfectly cohesive soil (i.e. clay), soil thrust is only a function of contact surface area. The model also assumes that for a perfectly cohesionless or frictional soil (i.e. dry sand), soil thrust is a function of vehicular weight[1]. This paper attempts to compare the mobility characteristics of wheeled vs. track vehicles for different size, weight and terrain conditions.

INTRODUCTION

BDTM was developed as a design tool to compare different types of robotic vehicle mobility performance characteristics. No single vehicular locomotion system has optimal mobility performance under all terrain conditions. Vehicle running gear design always involves design compromises or tradeoffs over a number of mobility factors. Most future Army robotic vehicle platform concepts fall into two broad categories: wheeled and track systems.

Wheeled vehicles are typically more agile and maneuverable than tracked vehicles, but possess higher ground pressures and are therefore less trafficable. Tracked vehicles on the other hand have a lower ground pressure, superior traction and are thus more trafficable. However, they are not as agile or mechanically efficient as their wheeled counterparts due to (typically) larger mass and much larger internal motion resistance.

Both wheeled and tracked vehicles have been successful in negotiating roadways and moderately unstructured off-road terrain. Vehicles with a larger wheelbase, ground clearance and horsepower per weight ratios generally have much better intrinsic mobility performance than smaller systems. A comparison of vehicle types for equal size and weights indicates that wheeled systems are typically superior to track systems in agility, maneuverability, ride quality and terrain damage. Tracked vehicles have distinct advantages relative to stability, ground pressure, maximum vertical slope, and drawbar pull.

Selection of running gear usually becomes a choice between which mobility characteristics are most important for a vehicle's intended mission profile. Ride quality is not as important to unmanned or robotic vehicles unless equipment such as sensors exceed vibration limits or structural loading specifications are exceeded for rough terrain conditions. The vehicle need only have sufficient drawbar pull to transport itself and its payload. Low ground pressure is principally an advantage only in soft soil terrain conditions. Unmanned systems generally weigh less and have a lower ground pressure than the larger manned combat vehicles such as the main battle tanks or infantry fighting vehicles.

Agility and maneuverability are both advantageous for off-road conditions. Ground clearance, maximum slide slope angle and wheelbase are important for difficult obstacle negotiation challenges such as ditch crossings or wide vertical steps. In general a complete systems analysis is necessary to determine the optimal set of mobility characteristics for a particular mission profile. Unmanned vehicles in particular need a new set of system requirements and represent a separate set of design challenges from their traditionally manned counterparts.

BDTM is thus a modest attempt to examine tradeoffs between different mobility characteristics for wheeled and track vehicles. It is a first-order linear model, which ignores the nonlinear dynamic interactions between the vehicle and its terrain. It does, however, analyze three primary parameters essential to generic mobility: vehicle size, weight and ground pressure. Future systems will vary significantly in these parameters. Since they will also navigate over terrain with large variations in mechanical properties, BDTM provides a useful tool for determining their first-order design characteristics.

THE BEKKER MODEL

The mechanical behavior of soils varies considerably under a wide variety of environmental conditions. For example composition, moisture levels, porosity, temperature, etc., affect bulk soil mechanical behavior relative to vehicle/terrain dynamics. It is also well known that for the same amount mechanical loading, a tracked vehicle may cross soft terrain without considerable slippage, whereas wheels may slip considerably, or simply spin. The amount of slip varies with soil type.

The Bekker model uses the relationship between certain physical soil characteristics and shearing strength to predict vehicle cross-country mobility. Bekker considers wheels and tracks as simple loading surfaces having similar forms, but different lengths and widths. He extrapolates the analogy between soil shear produced by laboratory crawlers to track vehicles as shown in Fig. 1a [1]. When the blocked track is moved relative to the soil mass in the laboratory shear box, the maximum shearing force is not developed instantaneously with the initiation of relative motion. Instead the soil must be compacted to some degree before reaching the final steady state mechanical shearing stress. Thus the track grousers begin slipping before reaching the point of maximum vehicle traction. This transient condition is the basis for Bekker's simple 1-DOF model for vehicle trafficability.

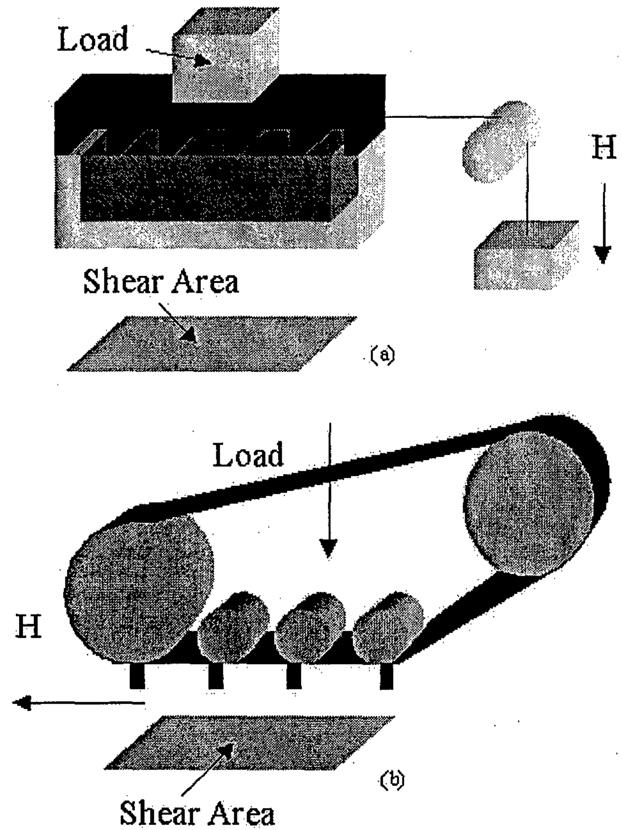
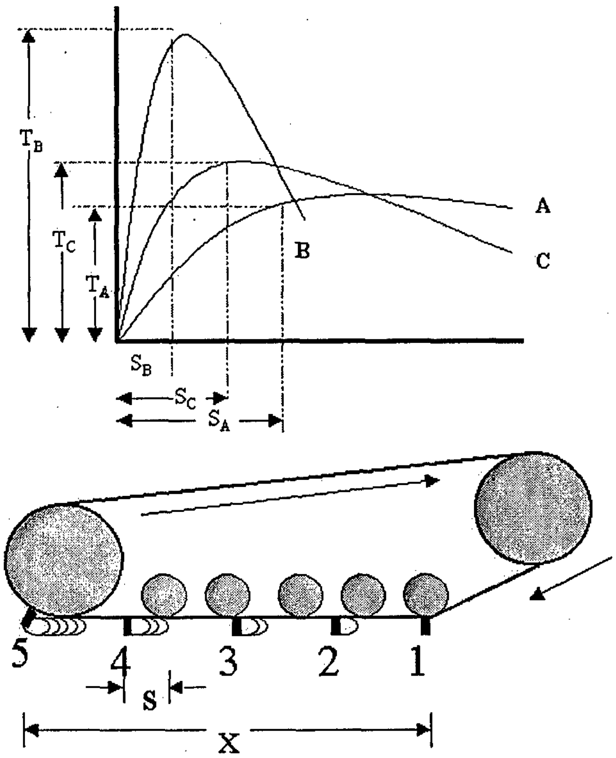


Figure 1 Soil Shear Analogy

The shear stress is the ratio between the vehicle traction force, which is parallel to the soil surface, and the area of the track normal to the surface. This tractive force is opposed by the soil resistance as the grousers slip during the shearing process. The normal loading force of the vehicle compacts the soil, which affects the resistance it exudes against the grousers as the track rotates on the vehicle. In effect the track forces, which push against the soil, generate a soil resistance that is determined by soil type and compaction. Vehicle weight generates ground pressure, which further compacts the soil and alters the soil resistance.

Figure 2b shows a tracked vehicle in motion. A grouser on the track first comes into contact with the ground at position 1. At the moment of first contact no shearing has occurred. As the vehicle moves forward, a shearing force is developed in the lateral direction. The positioning of the grouser begins to slip back pushing the soil and causing a soil distortion (S). As the vehicle continues to move, the amount of soil distortion increases[1].



A. Cohesionless Soil
B. Cohesive Soil
C. Mixture of A and B

Figure 2 Characteristics of soil deformation

Empirically generated curves in Figure 2a show the motion of soil under shear plotted for three different types of soils[1]. These curves are obtained through empirical data. The curve labeled **A** is for a loose frictional or plastic soil such as wet clay. The shearing strength τ_a of such a soil is reached after the initial period of compaction, which takes place over a distance S_a. After this point the stress remains practically the same irrespective of any slip. Soil **B** consists of a dry coherent mass: dry clay or snow at very low temperatures. This type of soil quickly reaches its maximum shearing strength and then shears off rapidly. The last curve **C** is a soil type that has intermediate properties. Upon reaching a maximum value at a certain slip distance from the origin, it starts to lose its shearing strength but not as rapidly as curve **B**[2].

For modeling purposes, it is critical to come up with a general equation for these curves. The curves in Figure 2a are identical to the displacement (x) and natural time frequency (ωt) of an aperiodic vibration:

$$x = A_1 e^{(-b+\sqrt{(b^2-1)})\omega t} + A_2 e^{(-b-\sqrt{(b^2-1)})\omega t} \quad (1)$$

where b is the coefficient of damping. To write a formula in terms of soil stress (τ) and soil deformation (S),we place $\tau = x$, $K_1 S = \omega t$, and $K_2 = b$ where K_1 and K_2 are coefficients of slippage to get the following result.

$$\tau = A_1 e^{(-K_2+\sqrt{(K_2^2-1)})K_1 S} + A_2 e^{(-K_2-\sqrt{(K_2^2-1)})K_1 S} \quad (2)$$

To determine the coefficients A_1 and A_2, for slip S = 0 and $\tau = 0$:

$$A_1 + A_2 = 0$$

Also for slip S = 0, $\tau = 0$, and $d\tau/dS = K_3$,

$$\frac{d\tau}{dS} = A_1 e^{(-K_2+\sqrt{(K_2^2-1)})K_1 S}(-K_2+\sqrt{(K_2^2-1)})K_1 + A_2 e^{(-K_2-\sqrt{(K_2^2-1)})K_1 S}(-K_2-\sqrt{(K_2^2-1)})K_1 = K_3 \quad (3)$$

A_1 and A_2,

$$A_1 = \frac{K_3}{2K_1\sqrt{(K_2^2-1)}} \quad (4)$$

$$A_2 = -\frac{K_3}{2K_1\sqrt{(K_2^2-1)}} \quad (5)$$

Substituting A_1 and A_2 into equation (2):

$$\tau = \frac{K_3}{2K_1\sqrt{(K_2^2-1)}}\left(e^{(-K_2+\sqrt{K_2^2-1})K_1 S} - e^{(-K_2-\sqrt{K_2^2-1})K_1 S}\right) \quad (6)$$

The maximum peak of the curve in Figure 2a can be calculated and is proposed by Dr. Grant Gerhart in equation (7).

$$S_m = \frac{\ln(-K_2-\sqrt{(K_2^2-1)}) - \ln(-K_2+\sqrt{(K_2^2-1)})}{2K_1\sqrt{(K_2^2-1)}} \quad (7)$$

The shear strength of soil (τ) can be defined as the maximum, or limiting, value of shear stress that may be induced within its mass before the soil yields[3]. A Mohr diagram plotting ground pressure vs. shear stress, figure 3, shows the state of the stress for any orientation of a reference axis. The Mohr circle can only expand to a critical point before failure occurs. The line tangential to where failure occurs is the Mohr-Coulomb failure line. The equation of this line is y = mx + b where b is the coefficient of cohesion, m is tan (ϕ), ϕ is the frictional angle, and x is normal stress or ground pressure. This line is the fundamental approximation to the maximum shearing strength, τ_m, of a particular type of soil and has been adopted as the definition of strength in land locomotion.

$$\tau_m = c + p\tan(\phi) \quad (8)$$

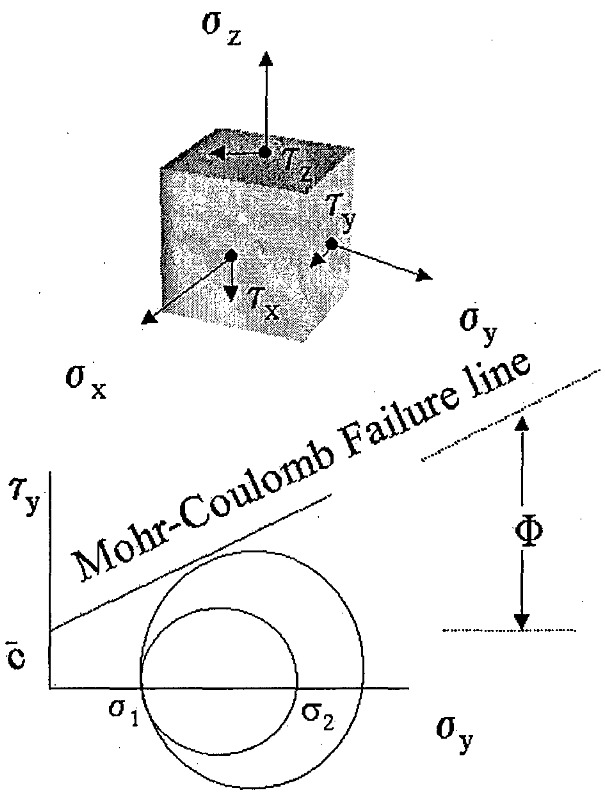


Figure 3 Mohr-Coulomb Failure Line

Since the portion contained in brackets (Eq. 6) is dimensionless, the value of $K_3 / 2K_1\sqrt{(K_2^2-1)}$ must have the units of lb/in^2 and the value of K_3 may be expressed in the following manner.

$$K_3 = \frac{2K_1\sqrt{(K_2^2-1)}(c + p\tan\phi)}{[e^{(-K_2+\sqrt{(K_2^2-1)})K_1S} - e^{(-K_2-\sqrt{(K_2^2-1)})K_1S}]_{\max}} \quad (9)$$

Now equation (6) can be simplified:

$$\tau = \frac{(c + p\tan\phi)}{y_{\max}}[e^{(-K_2+\sqrt{K_2^2-1})K_1S} - e^{(-K_2-\sqrt{K_2^2-1})K_1S}] \quad (10)$$

where y_{max} is the largest value within the brackets. The slip distortion and the amount of slip are related. The distance of shear (S_m) is equal to the speed of the slip times the time in which it occurs.

$$S_m = v_S t \quad (11)$$

However, the speed of slip is equal to the speed of the tire or track minus the actual speed:

$$v_S = v_t - v_a \quad (12)$$

$$S_m = (v_t - v_a)t \quad (13)$$

and $t = d / v_t$, where d is the distance where S_m has occurred.

$$S_m = d(1 - \frac{v_a}{v_t}) = i_o d \quad (14)$$

The amount of soil distortion that takes place at any point at a distance x from the front of the ground contact area is equal to

$$S = S_m(x/d) \quad (15)$$

So,

$$S = i_o x \quad (16)$$

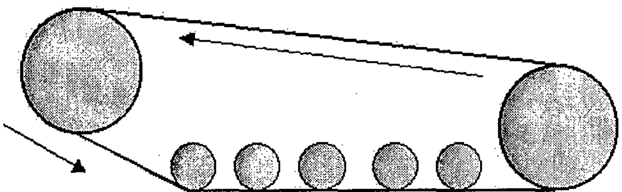
Equation (16) then allows for a relationship between tractive force and slip. Figure 4 shows the shear force of a tracked vehicle in two types of soil. The top graph is of highly frictional undisturbed firm silt. At ten- percent slip, shear is produced along the entire track; but it is clear that the front half of the track is producing the most of the force[2]. As the vehicle begins to experience more slip, most all of the shearing force is produced at the front of the tracked vehicle. In fact, the back half of the track begins to produce no shear and actually increases the resistance by creating drag.

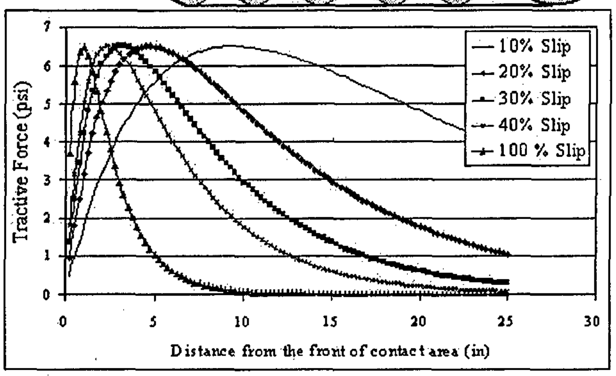
The second type of soil has a high cohesive property such as wet clay. At all values of slip, the entire length of the track is producing shear in relatively equal amounts along the length of the track.

While in motion, a track or wheel develops a force produced by the shearing strength of soil. This force H is called the gross tractive effort or soil thrust. The tractive effort is the integral of the shear produced by a tire or track. By substitution of equation (10),

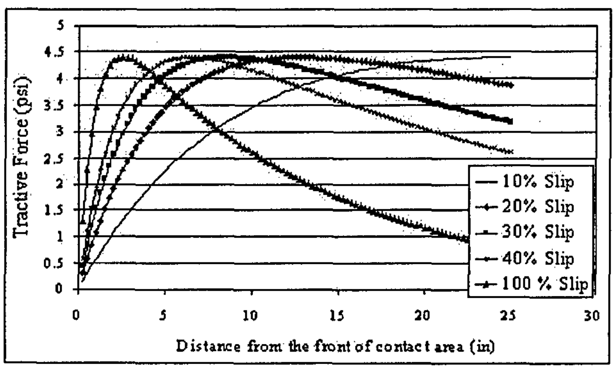
$$H = \int_0^d \tau_x dx \qquad (17)$$

$$H = \int_0^d \left\{ \frac{(c + p \tan \phi)}{y_{max}} \left(e^{(-K_2+\sqrt{K_2^2-1})K_1 ix} - e^{(-K_2-\sqrt{K_2^2-1})K_1 ix} \right) \right\} dx \qquad (18)$$





Undisturbed Firm Silt



Undisturbed Settled Sandy Loam

Figure 4 Tractive force in different soil types

Figure 4 shows the soil distortion at any distance x from the front of the ground contact area. The top graph shows tractive effort produced in undisturbed firm silt. The maximum tractive effort is quickly produced a short distance from the front of the vehicle and the rest of the track produces very little even at a very low percentage of slippage. The bottom graph shows the same track moving in an undisturbed settled sandy loam.

It is often thought of the heavier a vehicle is the greater its tractive effort. Much experience gives credibility to this statement but is it valid for all soil types? In order to answer this question, consider equation (19). Soil thrust is defined as the addition of two different soil strengths. One is from frictional properties and the second is from its cohesive properties.

$$H = A \cdot c + W \cdot \tan \phi \qquad (19)$$

If a soil type such as dry sand is chosen, a homogenous sample would contain no cohesive properties, Therefore c=0, and equation (19) is reduced to $W \cdot \tan\phi$. There is no question as the weight is increased the amount of soil thrust increases proportionally.

If the same vehicle is operated in a plastic soil such as saturated wet clay, the frictional component of the soil is equal to zero (ϕ=0). Equation (19) is reduced to $A \cdot c$ where A represents the contact surface area of the vehicle. A higher value of thrust is only obtained by an increase in contact surface area.

To answer the question in a more direct approach, vehicles that traverse in highly frictional soils benefit from an increase in payload. However, in soil types with high moisture contents or very cohesive, vehicles benefit by an increase in contact surface area. An increase in weight in this type of soil would be a liability[5].

BDTM

BDTM was established to give a first pass general evaluation of robotic vehicle mobility performance. It is a simple, linear one-degree of freedom (1-DOF) model that has been created in a spreadsheet format. The model assumes that the soil is homogenous and the loading effects on the soil are linear. A tracked vehicle and a wheeled vehicle can be simulated at one time. These vehicles are evaluated on their tractive force, tractive effort, soil sinkage, drawbar pull, and tractive coefficients (DP/W).

Inputs

The inputs into the program are divided into three categories. The first set of inputs are general vehicle information. These include the width and length of one track or wheel in contact with the ground. A corresponding code number relates to the actual shape of the print that the vehicle leaves on the ground. Other items include the number of tracks or wheels, contact area, and vehicular weight.

The second set of inputs describes the vehicle trafficability, or conversely, the vehicle performance in a given terrain. These define the strength, sinkage, and slippage that a vehicle would experience in a specific homogenous soil type. Most of these parameters are obtained from the Bevameter, which is a device created by Bekker for this purpose[1]. These include the depth of the plate sinkage, the modulus of soil deformation in cohesional and frictional soil, the exponent of soil deformation, and the coefficients of slippage. A separate section in the program provide these for different types of soil. Other parameters such as the coefficient of cohesion and the angle of friction are calculated off the Mohr-Coulomb failure line.

The third set of inputs is used for the calculations of WES mobility indexes. The purpose of such inputs is to relate the Bevameter values to the WES cone index. This then allows for the comparison of results obtained through the NRMM mobility model. WES mobility indexes are defined by equation 20 and 21 [4]. The mobility index for a tracked vehicle is calculated by:

$$MI = \left[\frac{\text{contact pressure factor} \times \text{weight factor}}{\text{track factor} \times \text{grouser factor}} + \text{bogie factor} - \text{clearance factor} \right] \times \text{engine factor} \times \text{transmission factor} \qquad (20)$$

and the mobility index for a wheeled vehicle:

$$MI = \left[\frac{\text{contact pressure factor} \times \text{weight factor}}{\text{Tire factor} \times \text{grouser factor}} + \text{wheel load factor} - \text{clearance factor} \right] \times \text{engine factor} \times \text{transmission factor} \qquad (21)$$

Outputs

The outputs are arranged into seven different categories. The first set is the theoretical soil thrust that the soil should support. This comes from the Mohr-Coulomb failure equation multiplied by contact area. It is expressed in equation (19) where $W \cdot \tan\phi$ is for the frictional composition of the soil and $A \cdot c$ is due from cohesion. Since most soil is a mixture of these two compositions, soil thrust in average soil is from the addition of these two terms.

The next output set is for strengths and pressures. The normal force exerted on the soil is due to loading from the vehicle and is referred to as the ground pressure. The maximum soil strength is Mohr-Coulomb failure equation calculated at the corresponding ground pressure. From the stress-strain curves of soil, Bekker noticed that they are identical to the displacement (x) and natural time frequency (ωt) of an aperiodic vibration[2]. The equation for tractive force was derived from this remark and is shown in equation (10). Its soil properties and the amount of slip distortion evaluate the tractive force. This is a product of the distance from the front of the track multiplied by the percentage of slippage the vehicle is experiencing. Equation (18) expresses the tractive effort in terms of soil properties, contact area, load, and slip for a given type of soil defined by its K_1 and K_2 constants.

To evaluate sinkage in frictional and cohesive soil, Bekker derived a formula from his Bevameter

$$z = \left[\frac{p}{k_c / b + k_\phi} \right]^{1/n} \qquad (22)$$

where p is the ground pressure, b is the width of the track or tire, k_c and k_ϕ are frictional and cohesive modulus of soil deformation, and n is the exponent of soil deformation. This equation answers why wider tracks or tires on vehicles with the same ground pressure sink deeper.

Not all soil thrust can be accounted for the production of useful work. Some of the soil thrust is lost in the form of energy. The energy loss that compose the external resistances are caused by compaction of soil, bulldozing, and dragging. It has been shown that the portion wasted for overcoming compaction resistance may be expressed by

$$R_c = \left(\frac{1}{(n+1)(k_c + bk_\phi)} \right) \left[\frac{W}{l} \right]^{\frac{n+1}{n}} \qquad (23)$$

where W is weight in pounds and l is the length of the tire or track in contact with the ground. It can be noted that from equation (23), the longer the contact area the smaller the compaction resistance. Bulldozing is the visible pushing of soil mass in front of a vehicle. For this model the resistances that are due from bulldozing are neglected. Also the resistances that occur from trapping the soil and dragging it are neglected.

The drawbar pull (DP) is the total thrust minus the total resistances. It is customary to view the difference as the vehicle's ability to move. If the total is zero or negative, then the locomotion of the vehicle will stop. In BDTM, there are three different values of DP. The first is considering soil thrust developed purely off of soil parameters. The second DP value is including the additional thrust that is created by the action of grousers or treads. The Mohr-Coulomb line equation is then modified for this result:

$$H = blc(1 + 2h/b) + W \tan\phi\left(1 + 0.64\left[(h/b)\cot^{-1}(h/b)\right]\right) \quad (24)$$

where b is the width, l is the length, h is the height of the grouser or tire tread, c is the coefficient of cohesion, and ϕ is the angle of friction. The last value of DP that is in BDTM is the value of the total tractive force evaluated at a certain slippage at a specific distance from the front of the contact area. A common comparison used to evaluate vehicles is to normalize these DP by there weight. This is often called the traction coefficient and should not be used as a stand-alone measure in evaluating vehicles.

The final set of outputs is devoted to cone index (CI) and mobility index (MI) conversions. Waterways Experiment Station (WES) came up with a way to measure soil parameters. The cone index is the parameter that is obtained by using their cone pentrometer device. The CI values are obtained by converting Bevameter values into CI values from equation (25)[6]. The conversion was proposed by Janosi and tested by WES in 1964. It was shown to be consistent within the limits of accuracy.

$$CI = 1.625\left(\begin{array}{l}\dfrac{k_c}{(n+1)}\left((z+1.5)^{n+1} - z^{n+1}\right) \\ +0.517k_\phi\left(\dfrac{(z+1.5)^{n+2}}{(n+1)(n+2)} + \dfrac{z^{n+2}}{n+2} - \dfrac{(z+1.5)z^{n+1}}{n+1}\right)\end{array}\right) \quad (25)$$

Charts

In BDTM, there are four charts that provide useful information. The first chart is traction coefficients versus k values. This provides curves for both the tracked and wheeled vehicle for the traction coefficient in different strengths of soil. Figure 5 shows a tracked vehicle and the same vehicle with tires in a mostly frictional soil type. It can be seen that the tracked vehicle can easily traverse soil with less consistency than the same vehicle with tires on.

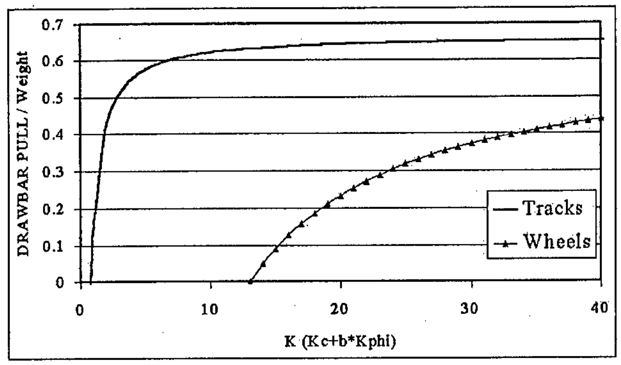


Figure 5 DP/W vs. K values

The next two charts show the tractive force produced under the contact area of the track or tire.

These curves are made at 10, 20, 30, 40, and 100 percent slip. Figure 6 shows a tracked vehicle in an undisturbed settled sandy loam. It is shown that the track produces force constantly down the contact area of the track. Even at various levels of slip.

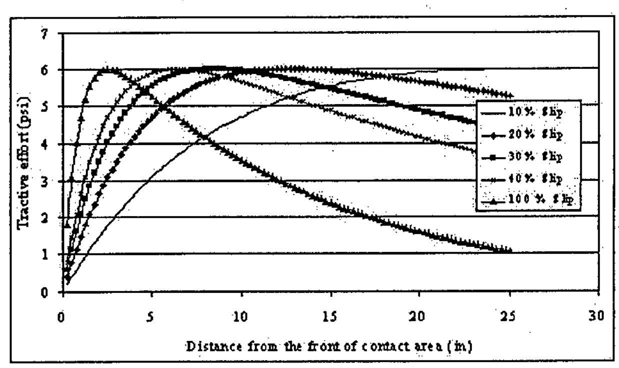
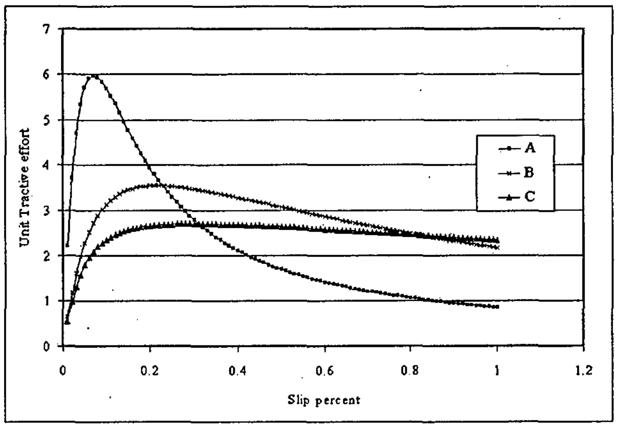


Figure 6 Tractive Force vs. Distance Under Track

The last chart displays tractive effort per unit area with soil distortion. The amount of work that is accomplished as the amount of soil distortion occurs. This is evaluated as the slippage increases. Figure 7 shows tractive effort versus slip at three different soil types. Soil type A is a highly frictional soil type and can be seen that almost all the tractive effort is produced when the vehicle experiences less than ten percent slippage. On the other-hand, soil type C is a plastic or cohesive soil type. Tractive effort is produced relatively uniform regardless of the soil distortion or slippage experienced.



A. Cohesionless or loose frictional soil
B. Mixture of A and C
C. Cohesive or plastic soil

Figure 7 Tractive Effort vs. Slip

EXAMPLES

Two examples have been provided to demonstrate the ability of the model.

Example 1

The track vs. tire case has been argued quite extensively. It has been slated by some that a low-pressure pneumatic tire can perform as well as a track. What does the BDTM predict for small robotic platforms?

To address this question, lets look at an example of a robotic vehicle traversing in a highly cohesive soil type such as wet clay. A small four-wheeled robotic platform with 12" diameter tires as shown in figure 8a leaves a rectangular print 3"x4". The weight of the platform is 1000 lb. and the tires are located a distance of 36" apart. Each tire has a total contact area of 12" and an overall surface contact area of 48". The ground pressure of the vehicle is 21 psi.

The model shows the vehicle sinks to a level of 2.3". At this depth, the resistance to motion created by compacting and bulldozing is greater than the maximum soil thrust generated. The Drawbar-Pull is a negative value indicating that the vehicle is incapable of moving.

It is often though that an increase in payload could help in this situation. When a 200-lb payload is added to the robotic vehicle, the vehicle begins to sink deeper. The resistance to motion increases. Drawbar-Pull remains a negative number and the vehicle still is incapable of moving.

When the diameter of the tire is increased to allow for a 3"x6" print as shown in figure 8b, the total surface area is increased to 72" squared and the vehicle only sinks to a level of 1.5". The amount of resistance to motion has decreased to a level that the vehicle is capable of moving. This is indicated by a positive Drawbar-Pull; however, the amount of DP that is produced is minimal.

It is seen that an increase in tire diameter, which is an increase in surface contact area, leads to an increase in DP. The next logical step would then be to continually increase the diameter of the tire until the desired amount of DP is obtained. This approach leads to other problems such as turning radius and for our purpose is not practical.

A possible solution to this is to add another set of wheels. Figure 8d displays the robotic vehicle with six wheels. When the 12" diameter wheels are used leaving a 3"x4" print, the amount of surface contact area is equivalent to the four-wheeled vehicle with enlarged tires. Therefore, the same results may be obtained by using six smaller wheels than with four enlarged. By increasing each of the six tires to allow for a 3"x6" print for each, the total surface area is increased to 108" squared. The vehicle's ground pressure has decreased to 9.1 psi and sinks 1" in the ground. The amount of Drawbar-Pull doubled.

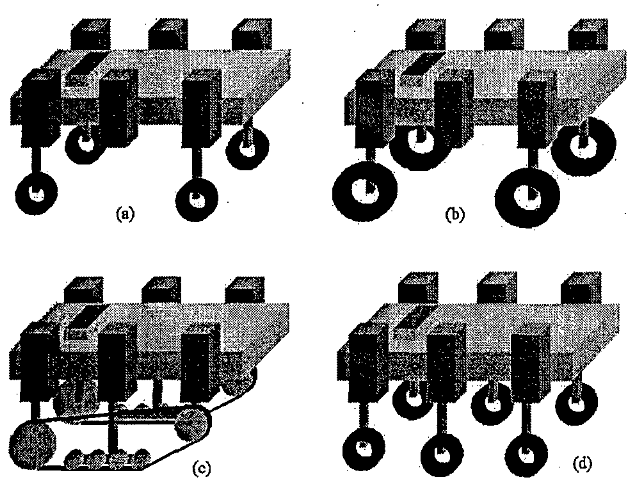


Figure 8 Robotic Vehicles

When the vehicle is outfitted with a track that is 25" long and 3" wide as shown in Figure 8c. The ground pressure has decreased to a level of 6 psi and sinks .6" in the soil. The drawbar pull is 14 times greater than the 4-wheeled vehicle, 4 times greater than the vehicle with enlarged tires, 4 times greater than the six-wheeled vehicle with 12" diameter wheels, and 2 times greater than the six-wheeled vehicle with enlarged wheels. When an additional 200-lb payload is placed on the tracked vehicle, the tractive effort remains unchanged, and the DP actually decreases due to sinkage.

This example shows that in a plastic soil type such as wet clay. The vehicles ability to traverse is dependent on the amount of contact surface area.

Example 2

For this example, we will look at the question in example 1 about track vs. tire but in a highly frictional type of soil such as dry sand. The same robotic vehicle platforms as shown in Figure 8 has been selected.

When the vehicle operates with tires that leave a 3"x4" rectangular print, the tractive force and soil thrust produced are very comparable to the vehicle outfitted with a 3"x25" track. The track outperforms the tire only 1.5 to 1. If the 4-wheeled vehicle is outfitted with the oversized tires leaving a 3"x6" surface contact print. The ratio is decreased to 1.2 to 1. The six-wheeled vehicle with the 3"6" print tires are almost 1 to 1.

It is quite interesting to note that when the vehicles are experiencing more slip. The 4-wheeled vehicles actually start to outperform the tracked vehicle. This begins to occur at around 33% slip for the 3"x4" print and 24% for the oversized tire.

Another thing that is fascinating is when the payload is increased for the tracked vehicle; the tractive force, soils thrust and drawbar increased respectively. When the 4-wheeled vehicle payload increased, the tractive force and soil thrust increased; but the drawbar pull decreased. By decreasing the weight of the 4-wheeled vehicles by 200-lb, the track only outperformed by 1.3 to 1 for the 3"x4" print and 1.1 to 1 for the oversized tire.

It can clearly be seen by this example that a lower weighted-wheeled vehicle can perform as well if not better than a tracked vehicle in highly frictional soil types such as dry sand.

SUMMARY

BDTM was developed as a design tool to compare different types of robotic vehicle mobility performance characteristics. BDTM was established to give a first pass general evaluation of robotic vehicle mobility performance. It is a simple, linear one-degree of freedom (1-DOF) model that has been created in a spreadsheet format.

REFERENCES

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TACOM

Mobility and Firepower for America's Army

Bekker's Derived Terramechanics Model (BDTM)

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Outline of Presentation

- Background of Materiel
- Definition of Terms
- BDTM spreadsheet
- Examples

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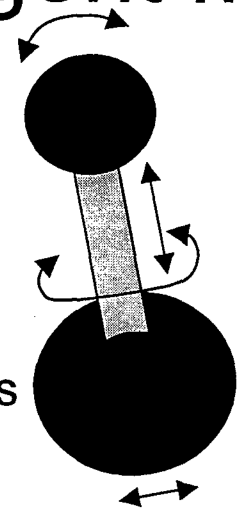
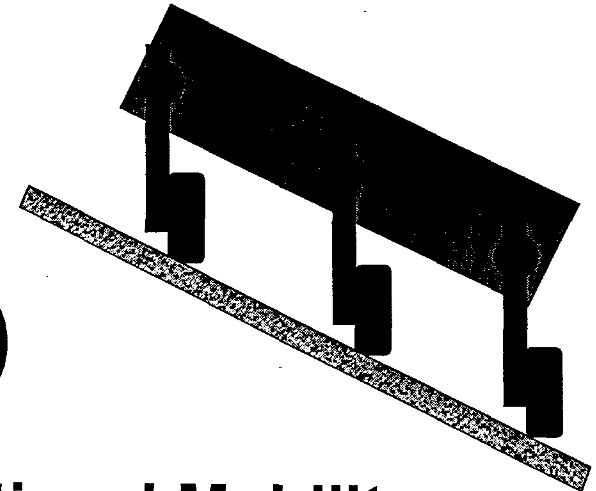
What Is Intelligent Mobility?

Inherent/Intrinsic Mobility

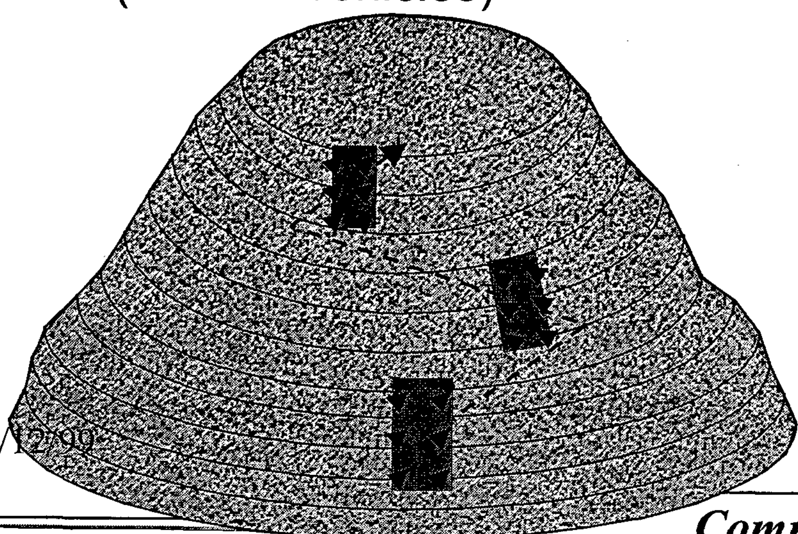
- Basic physical capability
- Ability to adjust the configuration and performance characteristics
- Governs the vehicle to execute commanded maneuvers and trajectories
- Advanced running gear, drive, control technologies and dynamic coupling (tandem vehicles)





Operational Mobility

- Applied mobility
- Governs and directs inherent mobility
- Selects the driving mode and route/velocity trajectory
- Advanced trajectory planning, navigation, learning and reactive behaviors



Bekker's Derived Terramechanics Model (BDTM)

- An analytical tool for evaluating vehicle off-road mobility
- Has been developed using Bekker's equations for vehicle soil interactions
- Purpose is to compare mobility characteristics for robotic track and wheeled vehicles under different terrain conditions

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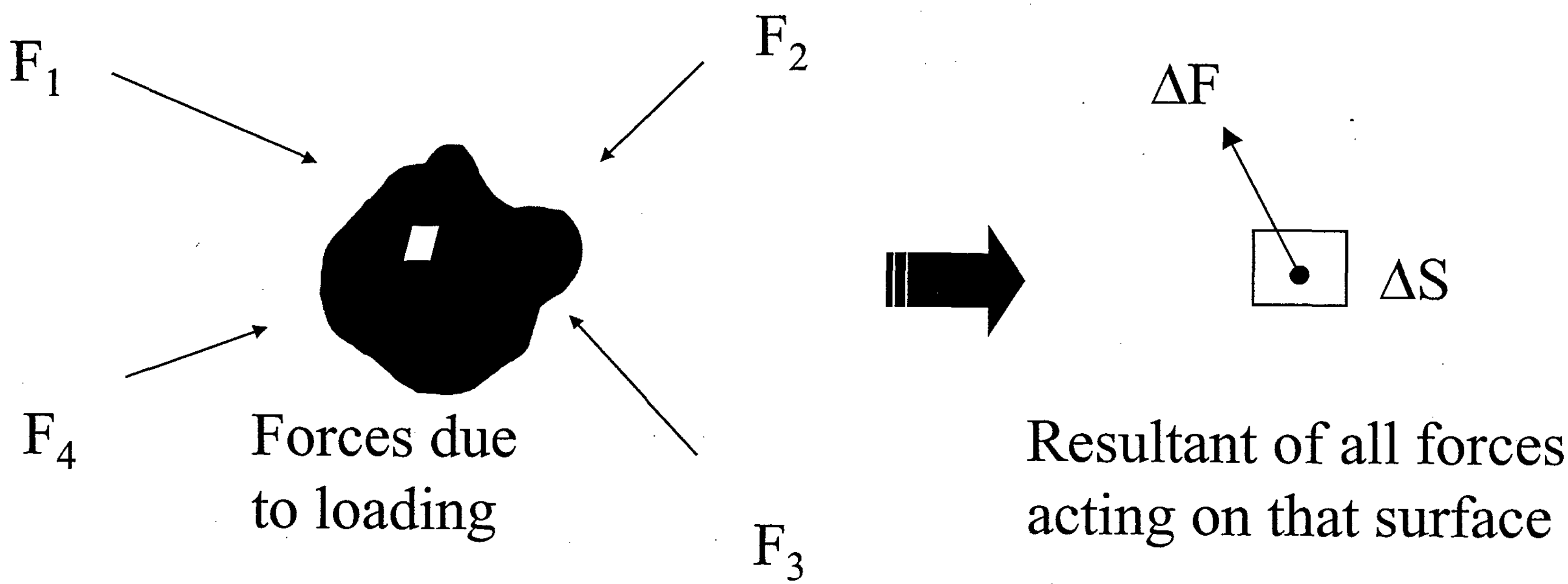
Loading Forces

- State of stress at a point in an object



Soil Strength

- Any arbitrary surface will have a resultant force ΔF from the summation of forces acting on that surface ΔA
- The limit of the ratio $\Delta F/\Delta S$ as $\Delta A \to 0$ is equal to the stress on that point
- Soil strength is then defined as the maximum, or limiting value of stress that may be induced within its mass before the soil yields

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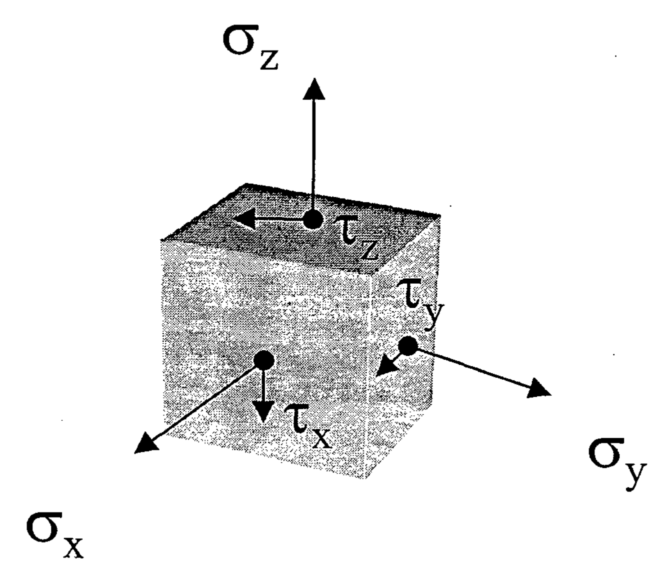
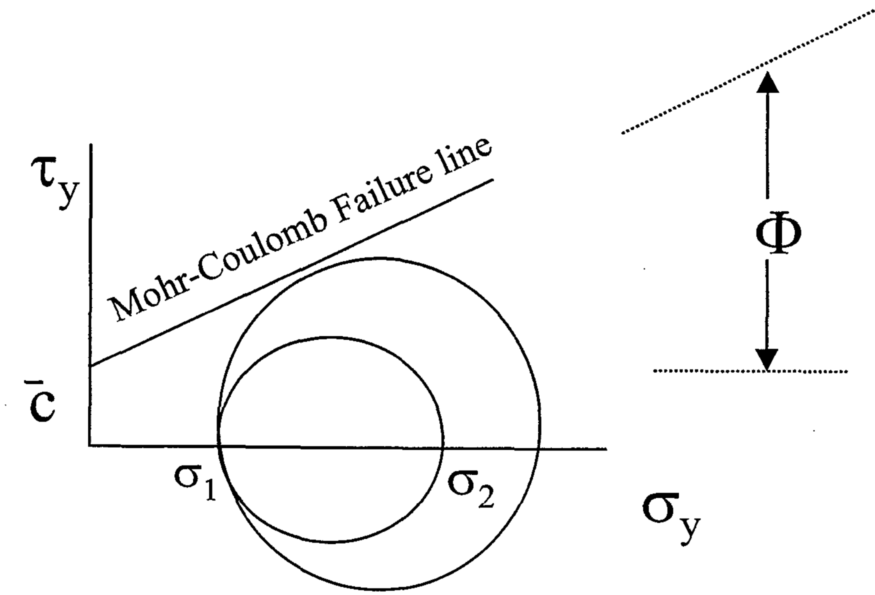
Take the ratio of the resulting forces $\frac{\Delta F}{\Delta A} \Rightarrow \lim_{\Delta A \to 0}\left(\frac{\Delta F}{\Delta A}\right)$

Normal Stress and Shear Stress





Mohr-Coulomb Failure line

$$\tau = c + \sigma \tan \Phi$$

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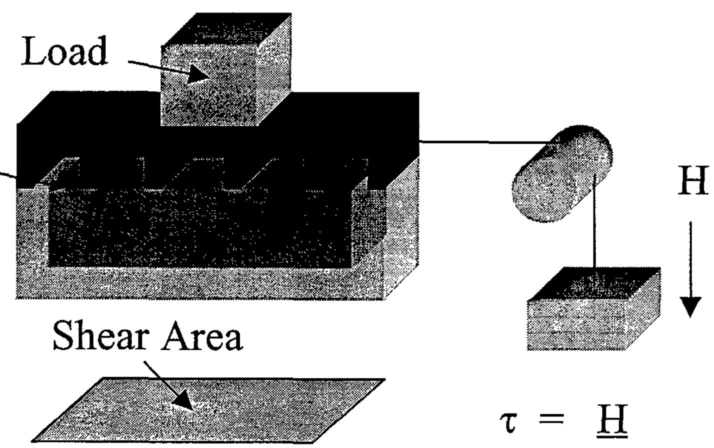
BDTM

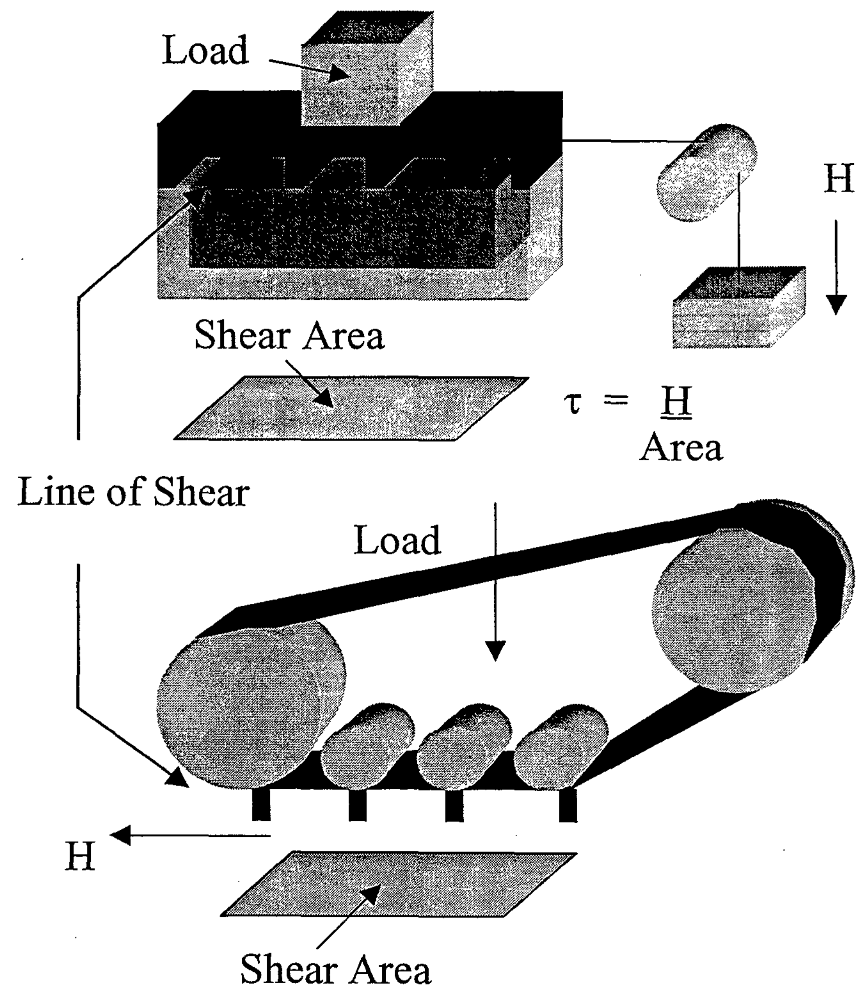
- BDTM is a simple, linear one-degree of freedom (1-DOF) model
- Based off the analogy between the soil produced by a laboratory apparatus and the shear caused by a track



$$\tau = \frac{H}{Area}$$

Apparatus and Blocked Track

- When the blocked track is moved relative to the soil mass in the laboratory shear box, the maximum shearing force is not developed instantaneously with the initiation of relative motion. Instead the soil must be compacted to some degree before reaching the final steady state mechanical shearing stress. Thus the track grousers begin slipping before reaching the point of maximum vehicle traction. This transient condition is the basis for Bekker's simple 1-DoF model for vehicle trafficability.



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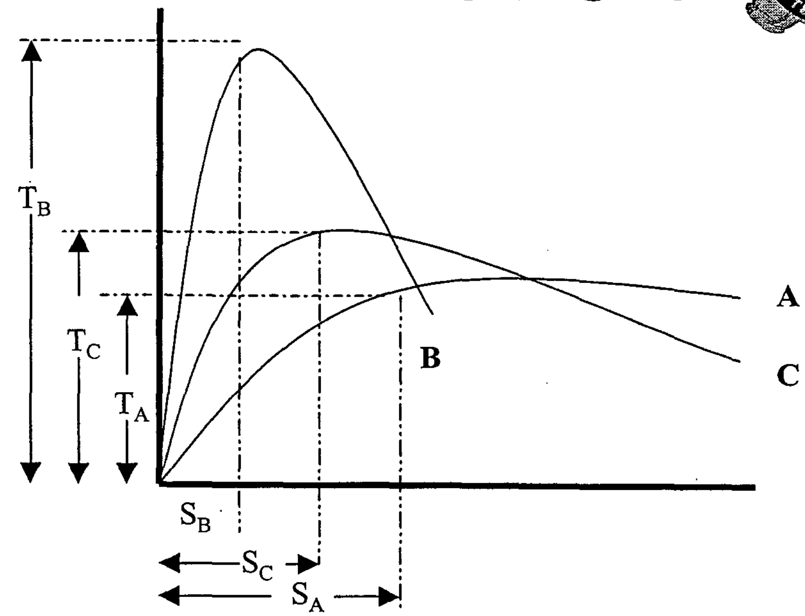
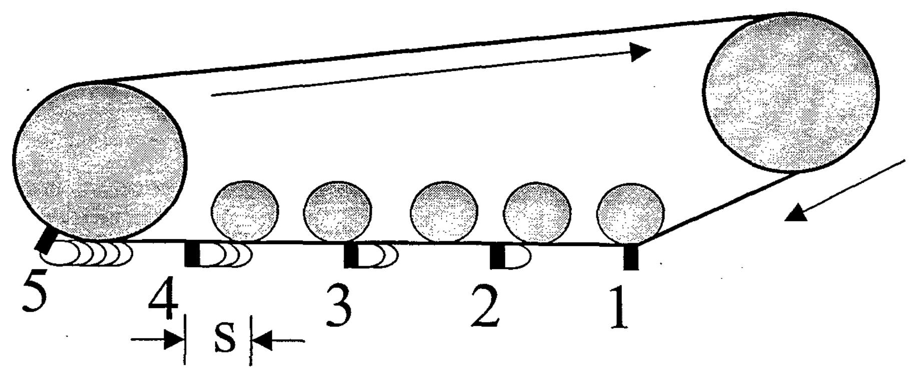
Tracked Vehicle in Motion

- Relative motion of the soil under shear is plotted as a function of corresponding stress
- A is for loose frictional or plastic soil
- B is not loose but a solid coherent mass such as dry clay
- C has intermediate properties





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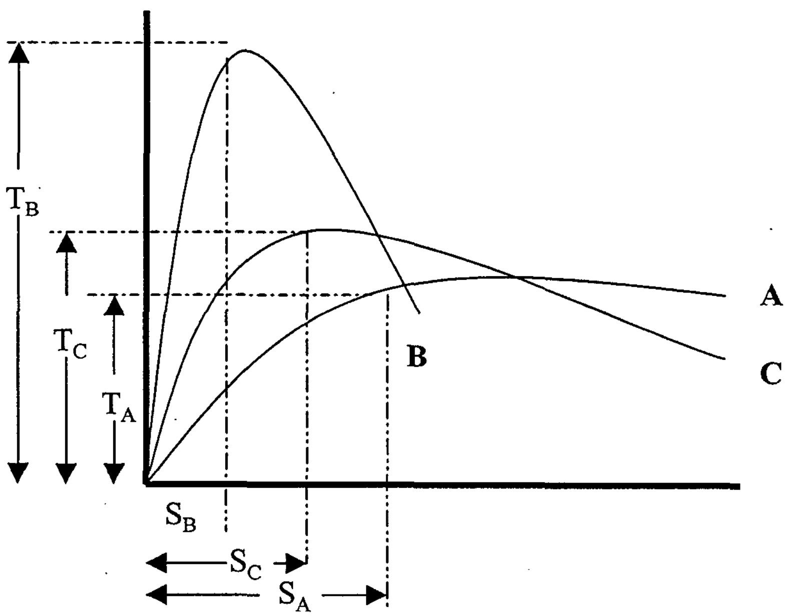
Shear Vs. Soil Distortion



- These curves are identical to the displacement, x, and natural frequency time of an aperiodic mass damper vibration.
- The equation of such motion is:

$$x = A_1 e^{(-b+\sqrt{(b^2-1)})\omega t} + A_2 e^{(-b-\sqrt{(b^2-1)})\omega t}$$

Tractive Force

- In order to come up with an equation in terms of soil deformation(S), replace the $\omega_n t$ with $K_1 S$,τ for x, and K_2 for the damping coefficient b.

$$\tau = A_1 e^{(-K_2+\sqrt{(K_2^2-1)})K_1 S} + A_2 e^{(-K_2-\sqrt{(K_2^2-1)})K_1 S}$$

- After solving for A_1 and A_2, the tractive force can be found using this equation

$$\tau = \frac{(c + p\tan\phi)}{y_{max}}[e^{(-K_2+\sqrt{K_2^2-1})K_1 S} - e^{(-K_2-\sqrt{K_2^2-1})K_1 S}]$$

- where c + ptanϕ is the Mohr-Coulomb failure line

Vehicle Slip

- The distance of shear (S_m) is equal to the speed of the slip times the time in which it occurs.

$$S_m = v_S t$$

- However, the speed of slip is equal to the speed of the tire or track minus the actual speed.

$$v_S = v_t - v_a$$

So,

$$S_m = (v_t - v_a)t$$

and , where d is the distance where S_m has occurred.

$$S_m = d(1 - \frac{v_a}{v_t}) = i_o d$$

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- The amount of soil distortion that takes place at any point at a distance x from the front of the ground contact area is equal to:

$$S = S_m(x/d) \qquad S = i_o x$$

Tractive Effort

- Horizontal propelling forces produced by the shearing strength of the ground under vehicle weight is called tractive effort.
- A part of the thrust is wasted by overcoming resistance, but the remaining portion which is referred to as the **Drawbar-Pull** is useful to accelerate the vehicle, climb slopes, pull-loads, etc.
- The BDTM uses two approaches for figuring out Tractive effort and Draw-bar pull.
  - 1. Tractive effort is Mohr-Coulomb failure line times area plus a small additional shearing force produced by tire tread or track spuds (H′)

$$H = A \cdot c + W \cdot \tan(\phi) + H'$$

  - 2. Tractive effort is the integral of tractive force

$$H = \int_0^d \{\frac{(c + p\tan\phi)}{y_{\max}}(e^{(-K_2+\sqrt{K_2^2-1})K_1 ix} - e^{(-K_2-\sqrt{K_2^2-1})K_1 ix}\}dx$$

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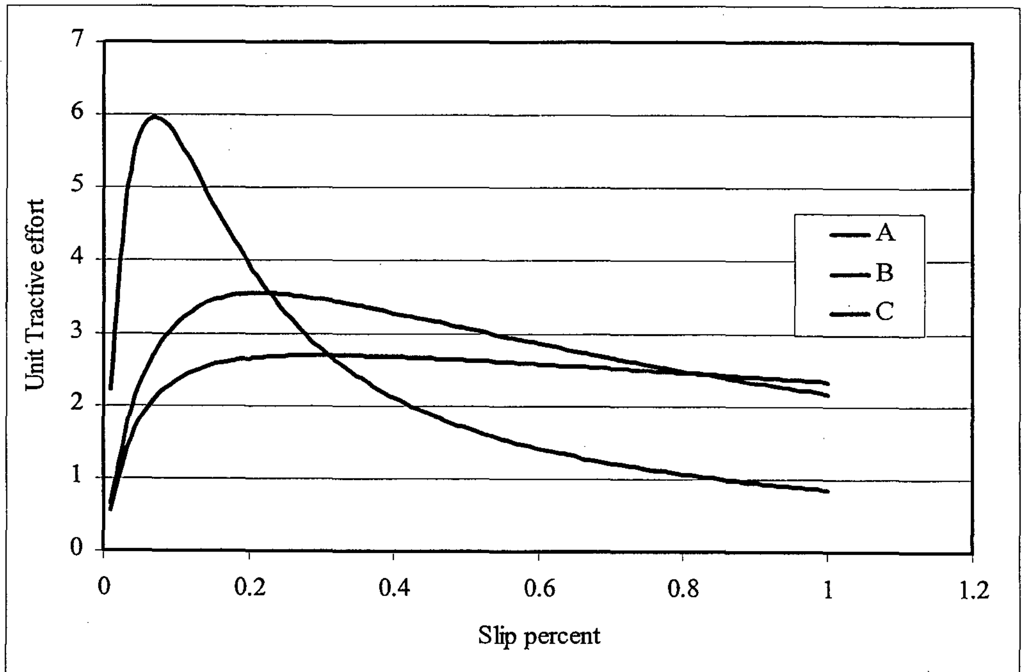
Tractive Effort Example

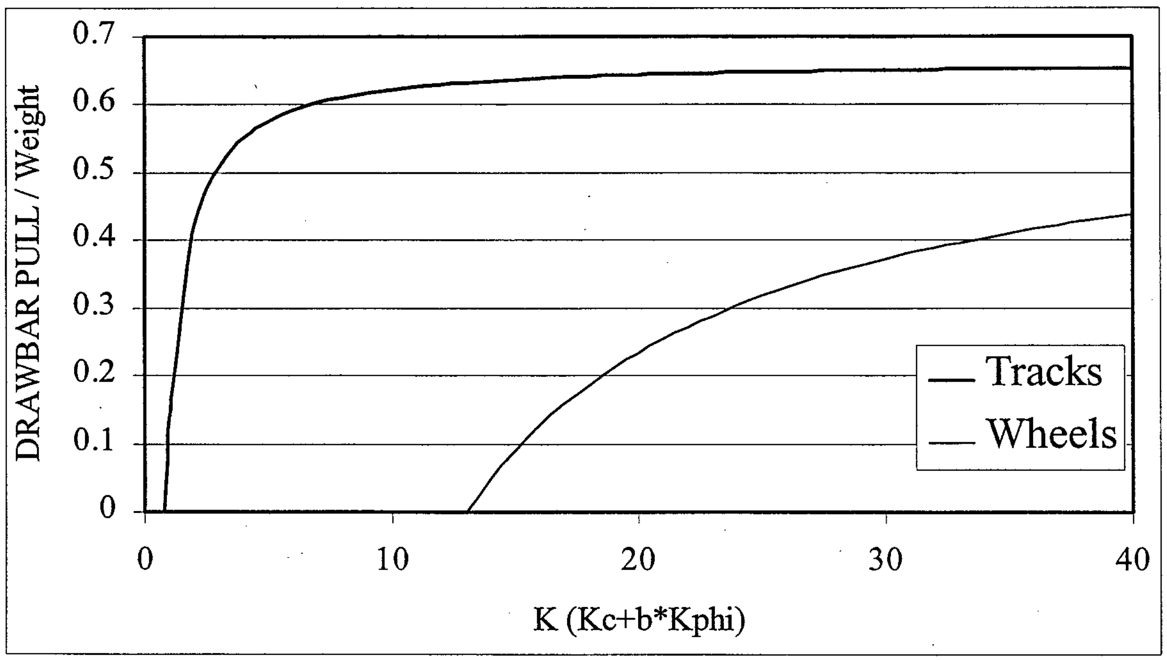


- A.Sand
- B.Mixture of A and C
- C.Cohesive or plastic soil

- Tractive Effort vs. Slip in three different types of soils

Traction Coefficient

- Traction coefficient is the Drawbar Pull / Weight
- Traction vs. K values, determines trafficability in soft terrain



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BDTM Spreadsheet

❖ Input Section

Vehicle Surface Contact

Vehicle Inputs		
Width	3	4
Length	25	6
Code	2	2
Number	2	4
Area	150	96
Weight	900	900

Soil Properties

Soil Properties		
c	0.2	0.2
Phi	30	30
Gamma	0.06	0.06
z	1	1
Kf	3	3
Kc	20	20
n	0.9	0.9
K1	0.3	0.3
K2	2	2
h	2	0.9
K	2	2
slippage	0.2	0.2
X	9	12

WES Vehicle Info

WES		
Bogie #	0	
Area of Shoe	12	
Clearance	5	5
Horsepower	15	15
Transmissio	1	1
Radius		10
# Axels		2

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BDTM Spreadsheet

- Output

Soil Thrust H (lb)		
Cohesion	30	19.2
Friction	519.615	519.615
Avg Soil	549.615	538.815
Strength & Pressure		
Ground Pressure	6	9.375
Soil Strength	3.6641	5.61266
Tractive Force	3.54279	5.60814
Tractive effort	3.12405	4.17118
Sinkage		
Veh Sinkage	0.58865	1.19271

Resistance		
Compaction	24.0928	101.7
Drawbar-Pull		
DP	525.522	437.115
DP/Weight	0.58391	0.48568
(+)Grouser/Tread	572.103	451.656
DP+g/t / Weight	0.63567	0.50184
DP from Tractive	507.325	436.682
DP/Weight	0.56369	0.4852

RCI Conversion		
CI	84.516	84.516
MI	181.32	9.4723
VCI (1)	43.054	10.408
VCI (50)	96.572	25.252
CI-VCI(1)	41.462	74.109
CI-VCI(50)	-12.06	59.265

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Tractive Effort Example

- It is often thought of the heavier a vehicle is the greater its drawbar pull
- Soil thrust is defined as the addition of two different soil strengths. One is from frictional properties and the second is from its cohesive properties

$$H = A \cdot c + W \cdot \tan \phi$$

- If a soil type such as dry sand is chosen, a homogenous sample would contain no cohesive properties, Therefore c=0, and equation 20 is reduced to $W\tan\phi$. There is no question as the weight is increased the amount of soil thrust increases proportionally
- If the same vehicle is operated in a plastic soil such as saturated wet clay, the frictional component of the soil is equal to zero ($\phi=0$). Therefore equation 20 is reduced to $A \cdot c$ where A represents the contact surface area of the vehicle. A higher value of thrust is only obtained by an increase in contact surface area.

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Example 1

- The track vs. tire case has been argued quite extensively. It has been slated by some that a low-pressure pneumatic tire can perform as well as a track. What does the BDTM predict for small robotic platforms?
- To address this question, lets look at an example of a robotic vehicle traversing in a highly cohesive soil type. A small four-wheeled platform with 12” diameter tires leaves a rectangular print 3”x4”. The weight of the platform is 1000 lb. and the tires are located a distance of 25” apart. Each tire has a total contact area of 12” and an overall surface contact area of 48”. The ground pressure of the vehicle is 21 psi

Run Example 1

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Example 2

- For this example, we will look at the question in example 2 about track vs. tire but in a highly frictional type of soil. The same robotic vehicle platform has been selected.

Run Example 2

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OPSEC REVIEW CERTIFICATION

(AR 530-1, Operations Security)

I am aware that there is foreign intelligence interest in open source publications. I have sufficient technical expertise in the subject matter of this paper to make a determination that the net benefit of this public release outweighs any potential damage.

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Robt E. Karlsen (Signature) 12AUG99 (Date)

Description of Information Reviewed:

Title: BEKKER'S TERRAMECHANICS MODEL FOR OFF-ROAD VEHICLE RESEARCH

Author/Originator(s): SEAN LAUGHERY, RICH GOETZ, GRANT GERHART

Publication/Presentation/Release Date: 8/18/99

Purpose of Release: GROUND TARGET MODELING AND VALIDATION CONFERENCE

An abstract, summary, or copy of the information reviewed is available for review.

Reviewer's Determination (circle one):

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