Fundamentals of the Human Foveal Vision System

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ABSTRACT

This paper presents a mathematical model, which includes a closed-form equation (the ABCt law) that generates the entire Blackwell-McCready (BM) threshold data set. It derives relationships among the four fundamental parameters of foveal vision: target area A, background luminance B, threshold contrast C and stimulus presentation time t. It shows that graphs of log (t) as a function of log (ABC) are hyperbolas whose asymptotic regions are related to the well known laws of Weber-Fechner and Bloch. It unifies important relationships associated with target/background scene parameters as they relate to the human foveal vision process. The constants associated with the simple empirical laws of Ricco, Blackwell, Weber-Fechner and Bloch can easily be obtained for a large range of target /background conditions and stimulus presentation times. Conditions for the most efficient stimulation of the human visual system are quantified and expressed in terms of the total energy for specific detection tasks.

1. INTRODUCTION

Along the visual axis, intersecting the retina, there is a small area on the retina, the fovea, which provides the greatest visual acuity and spatial resolution. Covering about one degree of visual angle in diameter, the fovea contains the greatest concentration of cones, but no rods. The BM data set relates to this part of the human vision system.

In an earlier paper[1,2], a hyperbolic curve fitting algorithm was derived for the BM data set. It accurately generates the threshold contrasts (C) of the circular targets for a wide range of target areas (A) and uniform background ambient luminance (B) for a given stimulus duration time (t).

Typical displays of the BM data show plots of log C as a function of log (A) or log (B) with the presentation time t held constant, as shown in Figures 1 and 2 for a presentation time t = 1/3 second. The circular markers
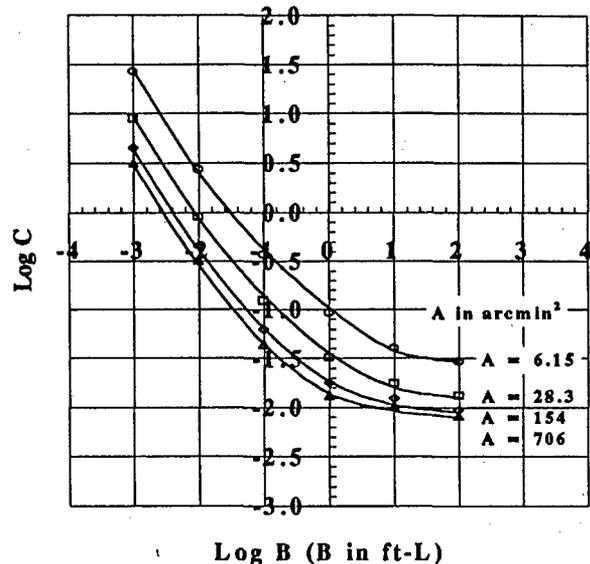
Figure 1. Log C versus Log A for t = 1/3 second.
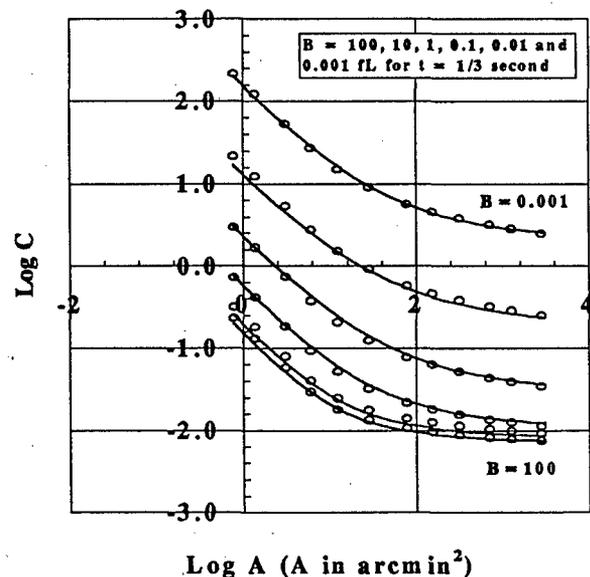
Figure 2. Log C versus Log B for t = 1/3 second.

represent points from the BM data set and the solid lines were generated from the hyperbolic curve fitting algorithm[1,2].

Previous hyperbolic curve fitting algorithms were developed for all three 2-D projections of the BM data set with constant presentation time t. Each geometric projection exhibited a number of interesting characteristic features of the human visual system. A significant accomplishment in the previous work was the recognition that the Ricco and Blackwell laws can be combined to give what we referred to as the ABC law for foveal vision. This paradigm stated that the product of A, B and C was a constant for small target areas (A< 6 arcmin^2) and low ambient light levels (B < 0.1 ft-L). Figure 3 is a graph of the ABC constant as a function of log (t). It was also shown that the largest variations in human performance, as a function of B, occurred during the time interval before dawn and after dusk. A subsequent paper[3] extrapolated these results through a redefinition of ambient luminance to real world scenarios.

Our continued efforts to characterize and further analyze the BM contrast discrimination data set has led to the discovery of a closed-form equation (the ABCt law) - which generates the entire BM smoothed threshold data set. In particular, it shows that plots of log t versus log ABC are hyperbolas (one branch) with one vertical asymptote and the other with a slope of -1, as shown in Figure 4. The asymptotic regions are related to the well known laws of Weber-Fechner and Bloch. The ABCt law unifies and quantifies many of the important interrelationships associated with the physical environment as they relate to the foveal vision process. It unifies under a single formalism the empirical laws of Ricco, Blackwell, Weber-Fechner and Bloch. The associated constants with these four laws are obtained for a wide range of target/background parameters and stimulus presentation times t. A dimensional analysis of the product ABCt shows it corresponds to the minimum energy required to detect the target. This formalism is useful for predictive models and provides quantitative estimates of human performance parameters over a wide range of conditions. It should be relevant to many applications of tactical interest and serve as a performance baseline for more advanced computational vison models. The integration of the effects of light energy over stimulus area (spatial summation) and over time (temporal summation) can also be quickly classified and quantified.
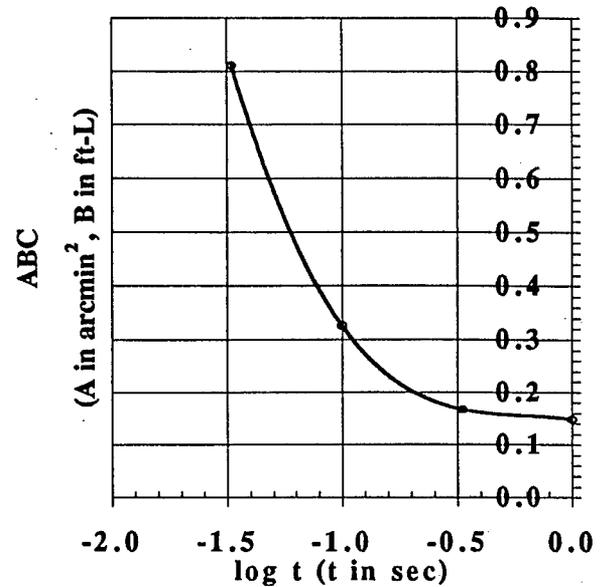

**Figure 3.** The ABC constant versus log t for B <0.1 ft-L and A < 6 arcmin^2.
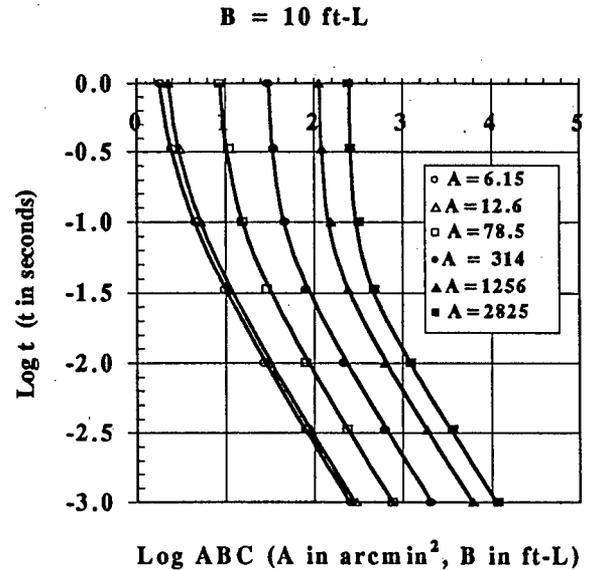
Figure 4. Log t versus Log ABC for B = 10 ft-L.

The Early Empirical Laws

A number of simple empirical laws have been formulated to describe human visual system performance. Ricco's law is relevant for small targets with large relative contrast values, and it results from the fact that diffraction effects dominate for object sizes less than some critical angle. Blackwell's law is applicable for low ambient luminance levels, and it illustrates the high, relative sensitivity of the foveal vision system during ambient lighting conditions, which typically occur before dawn and after dusk. The Weber-Fechner law applies for large, highly resolved targets when observer thresholds are nearly constant and independent of size or range. This phenomena is indicative of noise limiting performance of the human observer and relatively independent of the target characteristics A and C. Bloch's law, which applies for short stimulus presentation times, states that the product of intensity and time (ABCt) is a constant for t less than some critical value, and it is proportional to some photochemical effect in the human vision system.

In each of these simple paradigms the product of two parameters is constant for some range of asymptotic conditions. Although the ABCt law provides a means to obtain these constants, it can also provide numerical values for describing the entire range of target/background variability affecting human foveal vision.

Units

Although target areas have been expressed in arcmin^2, they can easily be converted to m^2 with the knowledge that the BM observers were 10.25 feet from the target. Background luminance levels are expressed in foot-Lamberts with 1 ft-L = 3.426 cd/m^2. Threshold contrast is the ratio of the difference between target luminance and background luminance to the background luminance that elicits a response from the vision system. Therefore, the product ABC has units of light intensity. If target area is expressed in m^2 and luminance in cd/m^2, the units for the product ABC are candelas (cd). For a point source, 1 cd = 1 lumen per steradian. Since the lumen is a unit of light power, the product ABCt represents the minimum energy required by the foveal vision system to detect the BM targets.

2. BACKGROUND INFORMATION

Comprehensive experimental data sets for human observers, which relate scene luminance, visual contrast, target dimension and stimulus presentation time are relatively few in number. Blackwell published several experimental studies[4,5] beginning in the World War II time period. His earliest work[4], generally known as the Tiffany data set, was intended for use in military applications with less than maximum visibility or relative target/background contrast. The Blackwell-McCready[5] (BM) data set attempted to provide the first comprehensive body of data in which background luminance level, target size and duration were all studied over wide ranges of practical interest. The data was collected using procedures that allow the analyst to convert the threshold data to various probability of detection levels. The Tiffany data consisted of three basic studies conducted over a comprehensive range of background luminance and target size. The first and second perception experiments were forced-choice detection tasks, which required observers to search for circular targets in a uniform background over a six-second presentation time. The third experiment used bright targets in known positions for presentation times necessary to obtain maximum detection probabilities.

The basic 1958 BM data set was collected from two "highly motivated and experienced observers". The original data consists of 81,000 observations in 162 experimental sessions. Target diameters ranged from 0.802 to 51.2 minutes of arc. Background luminance values varied from 0.001 to 100 ft-L which corresponds to the range of the average pupil diameter of the human eye (2.5 mm to 7.5 mm)[6]. Unlike the Tiffany data sets, presentation times were varied from 0.001 to 1 second and more attention was given to resolving fixation concerns. The BM targets always appeared in a known location at the center of the screen where the observers initially fixated their foveal vision. Associated with each value of A, B and t was a contrast threshold value C corresponding to a detection probability of 0.5. After a smoothing process, the smoothed threshold data consisted of 546 threshold contrasts corresponding to 13 target areas, 6 background luminance values and 7 presentation times. Since the product of ABC does not change for target areas less than six arcmin^2, 126 of the BM threshold contrasts are redundant. The ABCt law refers to the remaining 420 threshold contrasts.

3. HYPERBOLIC CURVE ALGORITHM

Basically, the ABCt law is the equation of a family of hyperbolas. Its conception was prompted from an analysis of plots like those shown in Figure 4. Each plot is for a specific value of A and B. The markers represent values

from the BM data set and the solid curves were generated from the ABCt law. A few fundamentals involved in the derivation of the ABCt law are given below. Equation 1 is the simplest mathematical form for a hyperbola with foci along the y-axis, semi-axes a and b and center at the origin of the (x, y) coordinate system.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \tag{1}$$

The plots in Figure 4, however, appear to be hyperbolas described by Eq. (1) in a coordinate system which has been translated and rotated. This is illustrated in Figure 5 which shows the 2-D, geometric transformation between the (x, y) and (x', y') coordinate systems. Equations (2) and (3) represent a counterclockwise rotation (θ) and a translation of the origin to the point (h, k)

$$x' = (x-h) \cos \theta + (y-k) \sin \theta \tag{2}$$
$$y' = (y-k) \cos \theta - (x-h) \sin \theta \tag{3}$$

Equation 1 is a special case of a general equation of second degree:

$$A x^2 + B x y + C y^2 + D x + E y + F = 0 \tag{4}$$

where $B^2 - 4AC > 0$. In the (x',y') coordinate system:

$$A' x'^2 + B' x' y' + C' y'^2 + D' x' + E' y' + F' = 0 \tag{5}$$

where

$$A' = b^2 \sin^2 \theta - a^2 \cos^2 \theta \tag{6}$$
$$B' = (a^2 + b^2) \sin 2\theta \tag{7}$$
$$C' = b^2 \cos^2 \theta - a^2 \sin^2 \theta \tag{8}$$
$$D' = 2 b^2 k \sin \theta - 2 a^2 h \cos \theta \tag{9}$$
$$E' = 2 b^2 k \cos \theta + 2 a^2 h \sin \theta \tag{10}$$
$$F' = b^2 k^2 - a^2 h^2 - a^2 b^2 \tag{11}$$

The asymptotes in the (x', y') coordinate system are characterized by:

$$y' = \frac{(a \cos \theta - b \sin \theta) x' + (a h - b k)}{a \sin \theta + b \cos \theta} \tag{12}$$

$$y' = \frac{(a \cos \theta + b \sin \theta) x' + (a h + b k)}{a \sin \theta - b \cos \theta} \tag{13}$$
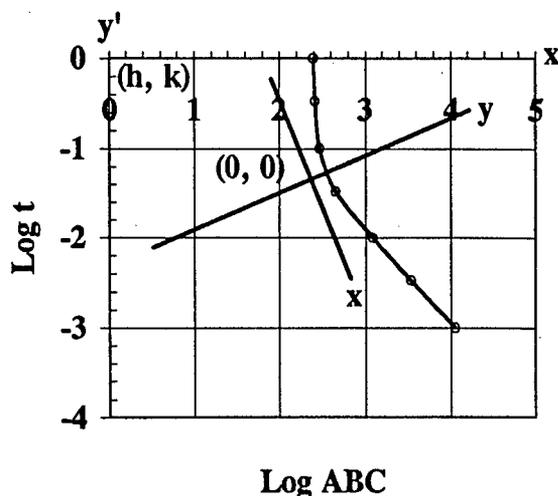

Figure 5. Transformation of coordinates.

If m_1 and m_2 are the slopes of the asymptotes, then we get

$$\tan 2\theta = \frac{m_1 + m_2}{m_1 m_2 - 1} = \frac{B'}{C' - A'} \tag{14}$$

4. The ABCt Law

Since plots of log t versus log ABC are hyperbolas (one branch) with one vertical asymptote and the other with a slope of -1, according to Eq. (14), $\theta = 67.5°$ and it can be shown that $b = a \tan (67.5°)$. Using these two results and Eqs. (5) to (8) yields $A' = B'$ and $C' = 0$. Hence, an equation that will generate the entire BM smoothed threshold data set is of the form

$$A' (x'^2 + x' y') + D' x' + E' y' + F' = 0$$
$$\{\text{target area } A > 6 \text{ arcmin}^2\} \tag{15}$$

where the coefficients are given by Eqs. (6) to (11) and the coordinates (x', y') correspond to (log ABC, log t). The restriction $A > 6$ arcmin^2 is founded on the fact that plots of log t versus log ABC for values below this limit are all redundant. Eq. (15) is the ABCt law.

Inspection of Eqs. (6) to (11) shows that the only parameters to identify are a, h and k. An analysis of the entire BM data set shows that each parameter, a, h and k can be expressed as a function of the target area A and the background luminance B. Plots of a, h and k versus log A are also hyperbolas; however, one asymptote is horizontal

and the other has a negative slope which depends on the ambient luminance B. A pictorial representation of this scenario is shown in Figure 6. This suggests that the a, h and k parameters can be expressed in the form of Eq. (5), where the coordinates (x', y') now correspond to (log A, a), (log A, h) or (log A, k).

For distinctness, all parameters relating to the (log A, a) hyperbolas will have the subscript "1", all parameters relating to the (log A, h) hyperbolas will have the subscript "2" and all parameters relating to the (log A, k) hyperbolas will have the subscript "3". For example, the parameters A', B', C', D', E', F', a, b, h, k, and θ of Eqs. (5) to (11) correspond to A'_1, B'_1, C'_1, D'_1, E'_1, F'_1, a_1, b_1, h_1, k_1, and θ_1 of the (log A, a) hyperbolas.

Since one asymptote of the a, h and k parameter hyperbolas is horizontal, $b_n = a_n \cot \theta_n$ and $A_n' = 0$. Hence, analogous to Eq. (5)

$$B'_n x' y' + C'_n y'^2 + D'_n x' + E'_n y' + F'_n = 0 \qquad (16)$$

where, n = 1, 2, 3; x' = log A and y' = a, h, k .

Through a comprehensive curve-fitting procedure, the parameters a_n, h_n, k_n and θ_n were adequately described as given below. Using z = log B, the a, h and k parameters are described by

$a_1 = 0.004$, $b_1 = a_1 \cot \theta_1$
$h_1 = 0.016788\, z^5 - 0.006235\, z^4 - 0.118709\, z^3 + 0.149452\, z^2$
$\quad - 0.042503\, z + 0.898705$
$k_1 = 0.000592\, z^5 - 0.007458\, z^4 - 0.034943\, z3 + 0.037570\, z^2$
$\quad + 0.186497\, z + 0.279860$
$\theta_1 = -0.082083\, z^5 - 0.320417\, z^4 + 0.177917\, z^3$
$\quad + 1.570417\, z^2 + 0.814167\, z + 180.4200$

$a_2 = 0.020$, $b_2 = a_2 \cot \theta_2$
$h_2 = 0.014778\, z^5 + 0.040227\, z^4 - 0.066438\, z^3 - 0.154833\, z^2$
$\quad + 0.048312\, z + 1.175540$
$k_2 = -0.003207\, z^5 - 0.006359\, z^4 + 0.004473\, z^3$
$\quad - 0.060491\, z^2 - 0.162769\, z - 0.179468$
$\theta_2 = -0.025000\, z^5 - 0.208333\, z^4 - 0.383333\, z^3 + 0.78333\, z^2$
$\quad + 2.633333\, z + 192.300$

$a_3 = 0.192$, $b_3 = a_3 \cot \theta_3$
$h_3 = 0.002347\, z^5 + 0.016055\, z^4 + 0.023844\, z^3$
$\quad - 0.044514\, z^2 - 0.089004\, z + 0.993574$
$k_3 = 0.001136\, z^5 + 0.001543\, z^4 - 0.020823\, z^3 - 0.105677\, z^2$
$\quad - 0.319653\, z + 1.164966$
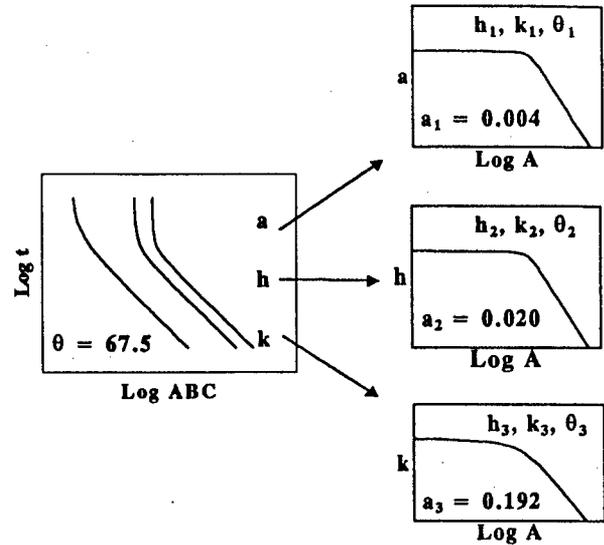
Figure 6. Components of a, h and k.

$\theta_3 = -0.009167\, z^5 - 0.071667\, z^4 - 0.100833\, z^3$
$\quad + 0.441667\, z^2 + 1.240000\, z + 198.800$

$$(17)$$

Example

The following example uses the ABCt law (Eq. 15) to calculate the threshold contrast for a target area A = 78.54 arcmin², a background luminance B = 10 ft-L and a presentation time of 1/3 second. From Eq. (17):

$a_1 = 0.004$, $h_1 = 0.89750$, $k_1 = 0.46212$, $\theta_1 = 182.58$,
$b_1 = 0.089$

$a_2 = 0.020$, $h_2 = 1.05759$, $k_2 = -0.40782$, $\theta_2 = 195.10$,
$b_2 = 0.074$

$a_3 = 0.192$, $h_3 = 0.90230$, $k_3 = 0.72149$, $\theta_3 = 200.30$,
$b_3 = 0.519$.

$$(18)$$

From Eqs. (7) to (11):

$B'_1 = 0.00071$, $C'_1 = 0.00786$, $D'_1 = -0.00030$, $E'_1 = -0.00728$, $F'_1 = 0.00167$;
$B'_2 = 0.00296$, $C'_2 = 0.00509$, $D'_2 = 0.00198$, $E'_2 = 0.00411$, $F'_2 = 0.00046$;
$B'_3 = 0.1993$, $C'_3 = 0.2325$, $D'_3 = -0.07248$, $E'_3 = -0.3876$, $F'_3 = 0.1003$.

$$(19)$$

Inserting the coefficients from Eq. (19) and log A = 1.8951 into Eq. (16) produces

$$0.00786\ a^2 - 0.00593\ a + 0.00110 = 0$$
$$0.00509\ h^2 + 0.00972\ h + 0.00421 = 0$$
$$0.233\ k^2 - 0.00991\ k - 0.0371 = 0 \qquad (20)$$

Using the lower branch of the hyperbola, we now know that a = 0.329, h = -1.25, k = -0.378, θ = 67.5° and b = 0.794.

Applying the above results to Eqs. (6) to (11) yields

$$A' = 0.522,\ D' = -0.337,\ E' = -0.432\ \text{and}\ F' = -0.147 \qquad (21)$$

Inserting these coefficients into Eq. (15) with y' = log t = -0.4776 gives

$$0.522\ x'^2 - 0.586\ x' + 0.059 = 0 \qquad (22)$$

Solving for x' produces log ABC = 1.01 and C = 0.0130 or 1.3 %. The BM data set gives C = 0.014 or 1.4 %. These results are very typical of solutions obtained from the ABCt law.

Quantifying Ricco's Law and Blackwell's Law

Since the ABCt law is a closed–form equation, it can be used to quantify many of the simple empirical laws that have been formulated to describe the performance of the fovea of the human eye. Ricco's law is illustrated in Figure 2 where the plots for all A < 7 arcmin^2 are coincident and the coefficients in Eq. (15) are constant for a given value of B. Hence, for a given background luminance B and exposure time t in this region, Eq. (15) reduces to a simple quadratic equation with log (AC) as the variable when A < 7 arcmin^2. The value of the product AC is the Ricco constant for the assumed values of B and t. For example, if B = 10 ft-L and t = 1/3 second, Eq. (15) simplifies to 0.8517 (log AC)^2 + 1.863 log AC + 0.8253 = 0 and log (AC) = -0.617 or AC = 0.2415; the original BM data set gives AC = 0.247. This phenomena stems from the fact that below a minimum critical angle, diffraction effects primarily determine the apparent observer image size while the target range and physical dimensions determine the illumination and subsequent apparent threshold contrast. The end result is that the product of contrast threshold and target area is a constant whose value is a function of ambient luminance and presentation time.

Blackwell's law is illustrated in Figure 3 where plots for B < 10^{-2} ft-L are nearly coincident and the coefficients in Eq.

(15) nearly constant for a given value of A. Hence, for a given target area A and exposure time t in this region, Eq. (15) reduces to a simple quadratic equation with log (BC) as the variable when B < 10^{-2} ft-L. The value of the product BC is the Blackwell constant for the assumed values of A and t. This region describes a range where rod vision typically dominates the human vision process. Cone vision dominates for brighter ambient luminance conditions, such as direct sunlight, when B >> 0.1 ft-L which produces noise or contrast limiting conditions. The transition between B < 1 ft-L and B > 0.01 ft-L contains a mixture of both rod and cone vision such as occurs near dawn or dusk. The ABCt law generalizes Ricco's and Blackwell's laws and gives some particularly simple relationships between the relative target/background contrast, target area and ambient luminance which are useful for military and commercial applications.

Bloch's Law

Figure 4 reveals that there is an approximate straight-line segment of slope −1 associated with each plot. Figure 7 illustrates the merging of several families of hyperbolas (B = 0.01, 0.1, 1, 10 and 100 ft-L) relating to plots of log t versus log ABC produced by a horizontal co-ordinate translation.


Log ABC (A in arcmin^2, B in ft-L)
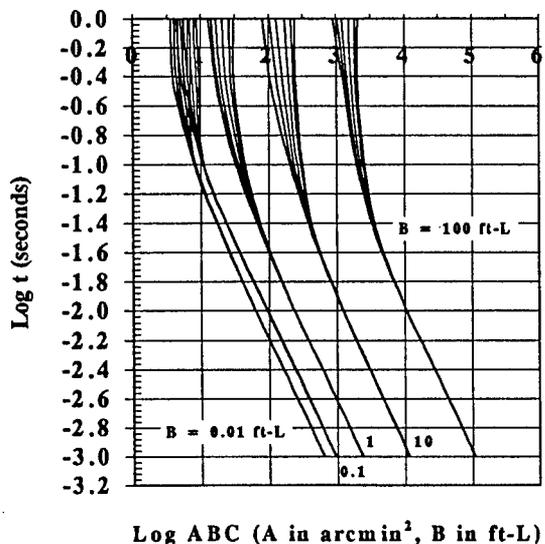
Figure 7. The background luminance family of log t versus log ABC plots after a horizontal co-ordinate translation.

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Those plots with the same background luminance have been horizontally translated to a point of coincidence with the plot corresponding to A = 2825 arcmin². The interest in these mergers is that it can now be seen that all plots of log t versus log ABC have an extended section that is an approximate straight-line segment with a slope of -1. For these regions the ABCt law simplifies to

$$\log t = -\log ABC + \log N \qquad (23)$$

where log N is the y-intercept of the approximate straight-line segment. Equation (23) can also be written as

$$ABCt = N \qquad (24)$$

where N is a constant for a given A and B. The parameter N represents the threshold energy required to elicit excitations from the visual system. The duration for which Eq.(24) is valid is called the critical duration and varies with background luminance. It is somewhat affected by target area. It ranges from approximately 30 msec at high background luminance to approximately 100 msec for low background luminance. It is within these durations that the visual system is most efficient. An equation for N can be obtained from the equations of a straight-line and Eq. (15). Since the line of slope -1 passes through the point (x*, -3), using (x', y') = (x*, -3) in Eq. (15) yields

$$A' x^{*2} + (D' -3 A') x^* + (F' -3 E') = 0 \qquad (25)$$

Solving for x* gives

$$x^* = \frac{(3 A' - D') + \sqrt{(D' - 3 A')^2 - 4 A' (F' - 3 E')}}{2 A'}$$

$$(26)$$

From the equation of a line with slope -1 passing through the points (x*, -3) and (0, log N)

$$\log N = x^* -3. \qquad (27)$$

Since A', D', E' and F' are all functions of target area and background luminance, log N is also a function of these two parameters. Figure 8 shows the relationship between threshold energy (log N), target area (log A) and background luminance during the critical duration when ABCt = N. An analysis of Figure 8 reveals a number of important correlations. For any target diameter less than
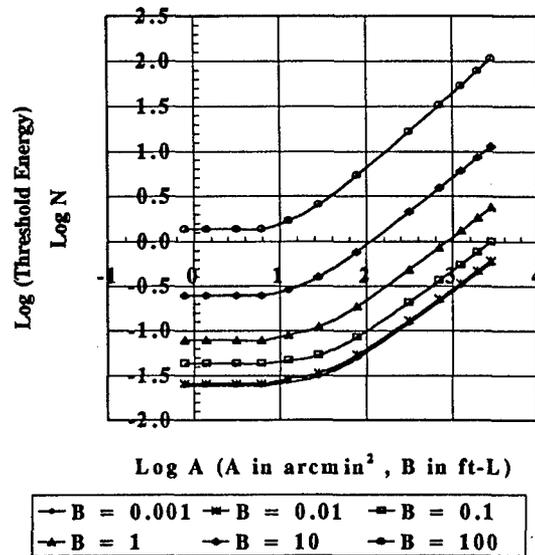
Figure 8. Temporal summation of light energy. Y-intercept (log N) versus log A.

1 degree, the threshold energy increases with background luminance. However, there is very little difference in the threshold energies at very low background luminance levels (B < 0.01 ft-L). In the Ricco's law regime (A < 6 min²), threshold energy is independent of target area; threshold energy is constant for a given background luminance. As target area increases beyond the Ricco's law regime, threshold energy is also increased. For target areas A > 100 min², plots of log N versus log A appear to be straight, parallel lines with slopes roughly equal to 0.75 (the slopes range from 0.72 to 0.85). This implies

$$\log N = 0.75 \log A + \log K \qquad (28)$$

where log K is the y-intercept and K is a function of background luminance B. Equation (28) can also be written as

$$N = K A^{0.75} \qquad (29)$$

where A is in arcmin², B in ft-L and t in seconds.

In other words, for large targets, the threshold energy is proportional to $A^{0.75}$.

The following example finds the value of N for A = 78.54 arcmin² and B = 10 ft-L:

Inserting the values from Eq. (20) into Eq. (26) gives x* = 2.88. From Eq. (27) we get log N = -0.12 and N = 0.76. At t = 0.001 seconds, the BM data set gives C = 0.982; hence, the product ABCt = 0.77.

The above observations relate to Bloch's law which can be expressed as follows: the product of threshold intensity and target duration is a constant within the critical duration. During the critical duration we have what is commonly referred to as to as temporal summation of light energy. Total temporal summation of light energy occurs when ABCt = constant. It is important to remember that the value of N in Eq. (24) is a function of both A and B. It is not appropriate to substitute values for A and B without adjusting the value of N. Figure 9 illustrates the effect of using different values for A and B that produce the same product for AB. The plots of log t versus log ABC for the same product AB are not the same.

As target duration increases beyond the critical duration, the slopes of the tangents to the log t versus log ABC curve approach infinity; the visual system becomes increasingly inefficient as noise begins to dominate the system. As the slopes approach infinity, the threshold intensity ABC approaches some minimum value related to the Weber-Fechner law that depends on the target area and the background luminance. After a critical duration, target duration no longer has any influence and brightness discrimination is determined entirely by the incremental luminance of the target. This minimum is easily obtained from the ABCt law by differentiating Eq. (15) with respect to x and allowing the derivative to approach infinity. This procedure yields

$$\log (ABC)_{min} = -E'/A' \qquad (30)$$

where A' and E' are given by Eqs. (6), (10), (16) and (17). Figure 10 shows how the minimum intensity varies with target area and background luminance. In the Ricco's law regime, the minimum threshold intensity is a constant for a given background luminance; smaller values are associated with higher luminance levels . As the target area increases beyond the Ricco area, the minimum threshold intensity also increases; more light energy is required to elicit excitation. It can also be seen from Figure 9 that for a given target area the minimum threshold intensity increases as the background luminance increases. In fact, for target areas greater than approximately 100 arcmin^2, the minimum threshold intensity is directly proportional to the target area. Using the approximation
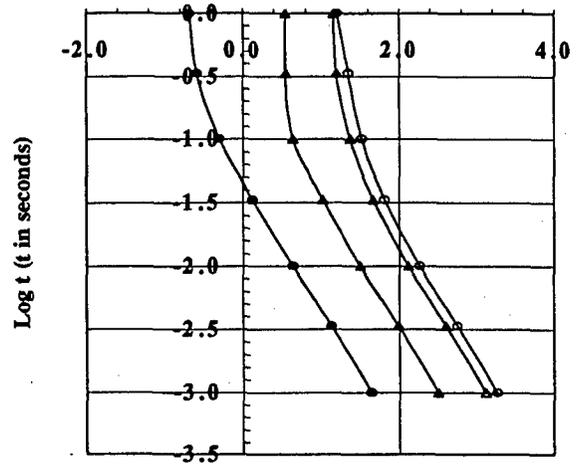
Log ABC (A in arcmin^2, B in ft-L)

-●-A = 12.56, B = 100	-▲-A = 1256, B = 1
-●-A = 12.56, B = 0.1	-▲-A = 1256, B = 0.001

Figure 9. Plots of log ABC versus log t are not unique for the same product AB.
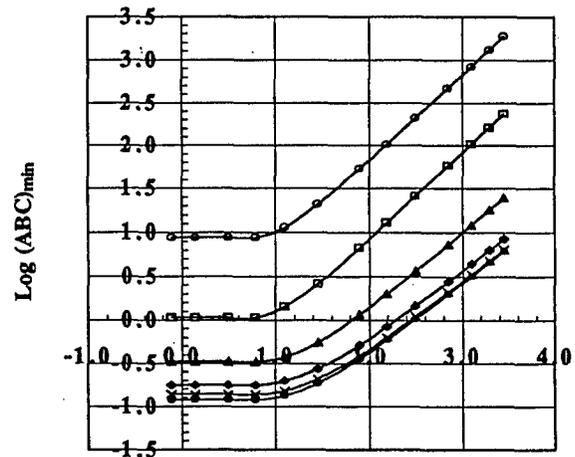
Log A (A in arcmin^2, B in ft-L)

| -●-B = 0.001	-+-B = 0.01	-●-B = 0.1
-▲-B = 1	-■-B = 10	-●-B = 100

Figure 10. Minimum log ABC versus log A.

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$$-(B_2\, E_2 - 2\, C_2\, D_2) \cong B_2\, \sqrt{E_2{}^2 - 4\, C_2\, F_2} \qquad (31)$$

in Eq. (16), it can easily be shown that the plots in Figure 10 are hyperbolas with asymptotic slopes of zero and +1.

5. TWO PART FORMALISM OF THE ABCt LAW

Interestingly, the critical duration seems to be related to the response time of the cone cells in the foveal region of the retina. Studies performed on monkeys[7], using measurements of membrane currents from monkey cones generated by brief flashes of light of varying strength, show that their response times correspond closely to the critical duration for humans. An alternate formalism of the ABCt law has also been pursued that closely relates to the critical duration and the response mechanism of the human eye. It considers the threshold presentation time t to consist of the product of two functions, Q and t^*, where t^* represents the total integration of light energy (ABCt) over time. This is referred to as temporal summation. Total temporal summation occurs when ABCt = constant (during the critical duration). The Q parameter is a measure of the amount that the vision system deviates from total temporal summation of light energy.

If $t = Q\, t^*$ then $\log t = \log Q + \log t^*$. Using $y = \log t$, $y_1 = \log Q$ and $y_2 = \log t^*$, Eq. (15) becomes

$$A'\, x^2 + A'\, x\, (y_1 + y_2) + D'\, x + E'\, (y_1 + y_2) + F' = 0 \qquad (32)$$

If y_2 is the y-coordinate of a straight line through the point $(x^*, -3)$ with slope = -1, then

$$y_2 = -x + x^* -3 \qquad (33)$$

and Eq. (32) can be written as

$$A'\, x\, y_1 + E'\, y_1 - (3\, A' - D' - A'\, x^* + E')\, x$$
$$- (3\, E' - E'\, x^* -F') = 0$$
$$\qquad (34)$$

Since closed-form equations have been derived for the y-intercepts ($y_0 = \log N$), Eq. (34) represents a closed-form solution for the multiplicative function Q. Also, since the coefficients of second degree vanish, Eq. (34) is the equation of a hyperbola with one asymptote vertical and the other horizontal.

$A = 78.5\ \mathrm{arcmin}^2$
$B = 10\ \mathrm{ft\text{-}L}$

Log ABC (A in arcmin^2, B in ft-L)
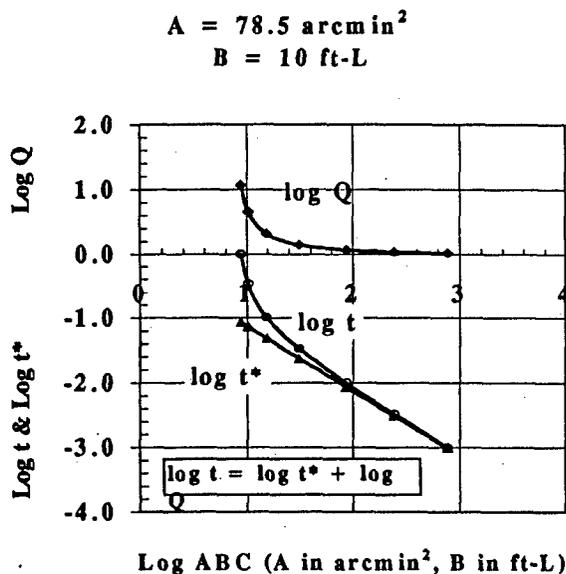
Figure 11. Two part formalism of the ABCt law.

Figure 11 illustrates the two-part formalism of the ABCt law. Evidently, the process of detecting a BM target, using foveal vision, can be equated to a linear process associated with total temporal summation of light energy modified by a quadratic multiplicative function.

6. RESULTS

The validity of the ABCt law can be quickly determined by comparing BM threshold contrasts with those calculated using the ABCt law. Of the 420 unique BM threshold contrasts, 21 threshold contrasts obtained from the ABCt law produce percent differences greater than 10 percent. These contrasts are identified in Table 1 along with the conditions which produced them. Seven of the Table 1 entries differ from the BM threshold contrasts by less than 0.10. Also, the largest percent difference is less than 17 % with the majority of the errors in the 11-13 % range. Hence, only three percent of the unique BM threshold contrasts are reproduced with any appreciable error.

Table 1. Conditions producing a percent difference greater than 10% between BM threshold contrasts and those calculated from the ABCt law.

#	A (min²)	BM C	Model C	C % Diff	Δ C	t (sec)	B (ft-L)
1	314.12	0.020	0.022	10.90	0.00	1/30	100
2	706.46	0.017	0.019	10.65	0.00	1/30	100
3	28.25	0.320	0.285	-10.94	0.03	1/300	100
4	28.25	1.035	0.900	-13.03	0.13	1/1000	100
5	78.54	0.764	0.683	-10.54	0.08	1/1000	100
6	153.85	0.082	0.092	11.35	-0.01	1/100	10
7	314.12	0.068	0.075	10.39	-0.01	1/100	10
8	706.46	0.113	0.132	16.57	-0.02	1/100	1
9	6.15	1.941	2.186	12.62	-0.24	1/30	0.1
10	6.15	5.433	4.560	-16.05	0.87	1/10	0.01
11	6.15	13.740	12.207	-11.16	1.53	1/30	0.01
12	6.15	134.276	115.931	-13.66	18.35	1/300	0.01
13	314.12	13.274	11.700	-11.86	1.57	1/300	0.01
14	706.46	10.257	9.071	-11.55	1.19	1/300	0.01
15	6.15	434.510	382.474	-11.98	52.04	1/1000	0.01
16	314.12	42.954	38.637	-10.05	4.32	1/1000	0.01
17	6.15	24.099	21.337	-11.46	2.76	1	0.001
18	6.15	52.723	47.186	-10.50	5.54	1/10	0.001
19	12.56	205.589	232.681	13.18	-27.09	1/100	0.001
20	28.25	114.815	130.145	13.35	-15.33	1/100	0.001
21	2825.46	20.370	22.685	11.36	-2.31	1/100	0.001

REFERENCES

1. R.M. Matchko and G.R. Gerhart, "Parametric analysis of the Blackwell-McCready data," Opt. Eng. 37 (7) 1937-1944 (1998).
2. R.M. Matchko and G.R. Gerhart, Errata, Opt. Eng. 38 (3) 564 (1999).
3. R.M. Matchko and G.R. Gerhart, "Target detection for low ambient backgrounds," Opt. Eng. 37 (7) 1945-1950 (1998).
4. H.R. Blackwell, "Contrast thresholds of the human eye," J. Opt. Soc. Am. 36, 624-643 (1946).
5. H.R. Blackwell and D.W. McCready, " Foveal contrast thresholds for various durations of single pulses," Report # 2455-13-F, U. Michigan Visual Research Laboratories (1958).
6. R.J. Farrell and J.M. Booth, *Design Handbook for Imagery Interpretation Equipment*, Boeing Aerospace., Seattle, Sec. 3.2, p. 8 (1984)
7. J.L. Schnapf and D.A. Baylor, "How the photoreceptor cells respond to light," Sc. Am., 256 (4): 40-47 (1987).

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