Current Shortcomings of the Standard Formulations of Radar Polarization

E. Luneburg

EML Consultants
George Schmid Weg 4
82234 Wessling, Germany

Office of Naval Research International Field Office Europe

Approved for Public Release
Distribution Unlimited
Current Shortcomings of the Standard Formulations of Radar Polarization


The standard formulation of radar polarimetry is summarized in the IEEE Standard Definitions of Terms for Antennas, 145-1983 [12] and in Mott's textbook from 1992 [15]. But either formulation is unsatisfactory, because a variety of topics of interest like the separation of polarimetric basis transformations from Lie group target structures or the extraction of physical target phase differences from mathematical artifacts and the radar network performance cannot be treated.

The basic principles of radar polarimetry rely heavily on the well established formulations of Optical Polarimetry by Mueller and Jones [1, 6]; and many features have been taken over nearly word by word - by Kennaugh, Huynen, and others - for developing a standard nomenclature of radar polarimetry [4, 5; 15]. This includes in particular the notion of polarization descriptors like the polarization ellipse, Jones (Sinclair) vector representation, Sinclair and Kennaugh back-scattering matrices for coherent and incoherent scattering, coherency (covariance) matrices for polarized light and distributed scatterers.

A characteristic new feature in monostatic and bistatic scattering is the necessity to consider simultaneously the polarization properties of plane electromagnetic waves propagating in and/or in opposite directions. The standard optics based formulation is to use a local right-handed wave oriented coordinate system, and to introduce the so-called 'Forward Scattering Alignment' (FSA) convention, which was also adopted unfortunately for the IEEE Standards [12]. This formulation is logically consistent but leads to mathematical difficulties in case optimal polarizations like maximal and minimal power transfer to a receiving antenna are considered for the radar case.

The standard procedure in radar polarimetry to resolve this problem is ingenious and truly of considerable mathematical interest, for which in general the 'Back (Bistatic) Scattering Alignment' (BSA) is used [5, 12]. Let us denote by W the linear 2-dimensional complex vector space of Jones vectors. Already Kennaugh in his famed M.Sc. Thesis of 1952 [11] proposed implicitly not to use the vector space W but its dual vector space W', i.e., the vector space of linear functionals on W (it is a fact that for finite dimensional vector spaces W and W' are equivalent). This shift from W to W' was somehow disguised by the use of the voltage equation $V = (h', Sht)$ as representative symmetric bilinear form of the action of W'. Here $h'$ are the receive and transmit antenna vectors and S is the backscatter Sinclair matrix. This shift from W to W' gives the misleading impression that radar polarimetry is a clever hodgepodge of electromagnetic field theory and radar network performance. The use of the 'voltage equation' - the equivalent of the 'brightness function' in optics - allows the proper formulation of the target characteristics as best revealed by the now famous Huynen fork representation [11] by different techniques (differential calculus, Lie group techniques and geometric algebra).

The voltage equation should be considered as part of standard radar polarimetry. If one wants to avoid the use of the dual space $W'$, i.e., the voltage equation, one has to introduce new concepts for the original polarization vector space W. This has been done by the extensive use of the spinorial concept by D. Bebbington [3], the polarization and phase (PP) vectors of Z. Czyz [9] or the directional Jones and Stokes vectors of E. Lüneburg [14]. The last two approaches heavily rely on the nonlinear time reversal operation. This gives rise to the concept of the unitary con-similarity transformation, a concept implicitly used already by Azzam & Bashara [1] in optics in 1977, but not recognized as such.; and neither was it by Kennaugh ([13] 1952) nor by Huynen ([11] 1970).

It should be stressed that there is no abrupt change from conventional standard radar polarimetry to new theoretical concepts, new applications and measurement techniques. New results in one field are often quickly assimilated by the polarimetric community and become common usage without proper mathematical insight. Some aspects of modern theory of polarization are addressed in Chapter 5 of the 'Manual of Remote Sensing' [5], edited by W.-M. Boerner et al. However, the IEEE Standards dealing with polarimetry [12] urgently need updating, and perhaps need to be separated from aspects of polarimetric antenna theory all-together.

There are several topics which require the attention of all polarimetricists. These include among others:

- Precise definition and applicability of coordinate systems (FSA vs BSA);
- Change of polarization bases (unitary similarity versus con-similarity);
- Phase definition and transformation; Huynen's skip angle; Czyz's conjugate spinors;
- $3 \times 3$ and $4 \times 4$ covariance matrices versus Mueller/Kennaugh matrices; and power optimization;
• Re-definition of completely random targets;
• Coherent and incoherent target decomposition theorems (Cameron, Krogager, Huynen, Cloude);
• Wave and target dichotomy (Huynen, Cloude);
• Unique formulations of the Kennaugh and the Covariance matrices and its structures;
• Karhunen-Loève expansion (Lueneburg);
• Applicability of Lie group concepts (Pottier);
• Spinors (Bebbington) and quaternions (Pellat-Finet) for general multistatic scattering;
• Topological phase (Berry phase);
• Null polarizations (Yang [18]).

Other authors make explicit use of quaternion algebra (Pellat-Finet [16]) or of Clifford algebra (see Baylis [2], 1999) in optical polarimetry, but up to now do not address the peculiarities of backscatter radar polarimetry or the general case of multistatic polarimetric scattering arrangements.

References

Current Shortcomings of the Standard Formulations of Radar Polarization


The standard formulation of radar polarimetry is summarized in the IEEE Standard Definitions of Terms for Antennas, 145-1983 [12] and in Mott's textbook from 1992 [15]. But either formulation is unsatisfactory, because a variety of topics of interest like the separation of polarimetric basis transformations from Lie group target structures or the extraction of physical target phase differences from mathematical artifacts and the radar network performance cannot be treated.

The basic principles of radar polarimetry rely heavily on the well established formulations of Optical Polarimetry by Mueller and Jones [1, 6], and many features have been taken over nearly word by word - by Kennaugh, Huynen, and others - for developing a standard nomenclature of radar polarimetry [4,5, 13]. This includes in particular the notion of polarization descriptors like the polarization ellipse, Jones (Sinclair) vector representation, Sinclair and Kennaugh back-scattering matrices for coherent and incoherent scattering, coherency (covariance) matrices for polarized light and distributed scatterers.

A characteristic new feature in monostatic and bistatic scattering is the necessity to consider simultaneously the polarization properties of plane electromagnetic waves propagating in and/or in opposite directions. The standard optics based formulation is to use a local right-handed wave oriented coordinate system, and to introduce the so-called 'Forward Scattering Alignment' (FSA) convention, which was also adopted unfortunately for the IEEE Standards [12]. This formulation is logically consistent but leads to mathematical difficulties in case optimal polarizations like maximal and minimal power transfer to a receiving antenna are considered for the radar case.

The standard procedure in radar polarimetry to resolve this problem is ingenious and truly of considerable mathematical interest, for which in general the 'Back (Bistatic) Scattering Alignment' (BSA) is used [5, 12]. Let us denote by W the linear 2-dimensional complex vector space of Jones vectors. Already Kennaugh in his famed M.Sc. Thesis of 1952 [11] proposed implicitly not to use the vector space W but its dual vector space W', i.e., the vector space of linear functionals on W (it is a fact that for finite dimensional vector spaces W and W' are equivalent). This shift from W to W' was somehow disguised by the use of the voltage equation $V = (h', Sh')$ as representative symmetric bilinear form of the action of W'. Here $h'^T$ are the receive and transmit antenna vectors and S is the backscatter Sinclair matrix. This shift from W to W' gives the misleading impression that radar polarimetry is a clever hodgepodge of electromagnetic field theory and radar network performance. The use of the 'voltage equation' - the equivalent of the 'brightness function' in optics - allows the proper formulation of the target characteristics as best revealed by the now famous Huynen fork representation [11] by different techniques (differential calculus, Lie group techniques and geometric algebra).

The voltage equation should be considered as part of standard radar polarimetry. If one wants to avoid the use of the dual space W', i.e., the voltage equation, one has to introduce new concepts for the original polarization vector space W. This has been done by the extensive use of the spinorial concept by D. Bebbington [3], the polarization and phase (PP) vectors of Z. Czyz [9] or the directional Jones and Stokes vectors of E. Luneburg [14]. The last two approaches heavily rely on the nonlinear time reversal operation. This gives rise to the concept of the unitary con-similarity transformation, a concept implicitly used already by Azzam & Bashara [1] in optics in 1977, but not recognized as such.; and neither was it by Kennaugh ([13] 1952) nor by Huynen ([11] 1970).

It should be stressed that there is no abrupt change from conventional standard radar polarimetry to new theoretical concepts, new applications and measurement techniques. New results in one field are often quickly assimilated by the polarimetric community and become common usage without proper mathematical insight. Some aspects of modern theory of polarization are addressed in Chapter 5 of the 'Manual of Remote Sensing' [5], edited by W.-M. Boerner et al. However, the IEEE Standards dealing with polarimetry [12] urgently need updating, and perhaps need to be separated from aspects of polarimetric antenna theory all-together.

There are several topics which require the attention of all polarimetrists. These include among others

- Precise definition and applicability of coordinate systems (FSA vs BSA);
- Change of polarization bases (unitary similarity versus con-similarity);
- Phase definition and transformation; Huynen's skip angle; Czyz's conjugate spinors;
- $3 \times 3$ and $4 \times 4$ covariance matrices versus Mueller/Kennaugh matrices; and power optimization;
• Re-definition of completely random targets;
• Coherent and incoherent target decomposition theorems (Cameron, Krogager, Huynen, Cloude);
• Wave and target dichotomy (Huynen, Cloude);
• Unique formulations of the Kennaugh and the Covariance matrices and its structures;
• Karhunen-Loeve expansion (Lueneburg);
• Applicability of Lie group concepts (Pottier);
• Spinors (Bebbington) and quaternions (Pellat-Finet) for general multistatic scattering;
• Topological phase (Berry phase);
• Null polarizations (Yang [18]).

Other authors make explicit use of quaternion algebra (Pellat-Finet [16]) or of Clifford algebra (see Baylis [2], 1999) in optical polarimetry, but up to now do not address the peculiarities of backscatter radar polarimetry or the general case of multistatic polarimetric scattering arrangements.

References