ANALYTIC METHODS
FOR TACTICAL AIR WARFARE—2006

AIR CAMPAIGN AND MATHEMATICAL ANALYSIS

REPORT PA504T1

Robert V. Hemm
David A. Lee
Jeremy M. Eckhause
John A. Dukovich

LMI
GOVERNMENT CONSULTING

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Introduction

This report documents air combat analyses performed from 06 June 2005 to 28 July 2006 for the Tactical Air Division of the Program Analysis and Evaluation Office of the Office of the Secretary of Defense (OSD PA&E TACAIR). The focus of our analyses was air campaign modeling to support the Quadrennial Program Review (QDR). Specific technical efforts included improving and employing the Stochastic Lanchester Air-to-Air Combat Model (SLAACM).

To support analysis of QDR issues, we added the following features to SLAACM:

- Increased types of Red and Blue aircraft
- Selectable “smart” battle management capability for Blue
- Capability to identify each Red aircraft as fighter only, fighter or bomber, or bomber only
- Capability to identify Blue aircraft types as operating in the continuous combat air patrol (CAP) mode, in which the defensive force is managed so that Blue can maintain aircraft on station continuously
- A battle management factor for engagements between Red packages and “non-smart” Blue aircraft, reflecting the possibility that not all randomly dispatched aircraft actually find targets
- Integer programming optimization for both Red attack and “smart” Blue defense
- Automated conversion of loss (exchange) ratio data to kill-rate ratios
- Display of dispersion measures (standard deviations) of Blue losses.

To improve the general usefulness of SLAACM, we developed functionally identical classified and unclassified versions. The classified model includes weapon and scenario data, provided by PA&E TACAIR, that are necessary for QDR analyses.

During the year, under the technical direction of PA&E TACAIR, we conducted several classified analyses of QDR scenarios. We briefed the results of these analyses to TACAIR and to senior PA&E personnel. In addition, we provided examples of both classified and unclassified results in a presentation at the Military

Operations Research Society (MORS) Symposium held at the U.S. Air Force Academy in June 2006.

Mathematical analyses during the current task included development of the campaign optimization tools and analysis of phased attack and defense strategies.

This report is organized as follows:

- Chapter 2 describes the current SLAACM and presents example analyses.
- Chapter 3 contains a mathematical analysis of optimization and battle management in SLAACM.
- Chapter 4 contains a mathematical analysis of phased attack and defense.
- Chapter 5 recommends future improvements for SLAACM.

Chapter 2
SLAACM Overview and Example Analyses

This chapter updates and supplements the August 2004 *SLAACM User Guide* with descriptions of new SLAACM features and formats. We introduce the current model structure by demonstrating the battle management features of SLAACM.

The chapter begins with a discussion of loss ratio input and kill-rate ratio calculations. It then presents example analyses that include

- Red and Blue force inputs,
- Blue battle management options, and
- Campaign result plots and data displays.

LOSS RATIO INPUT AND KILL-RATE RATIO CALCULATIONS

Loss ratios for small-scale engagements are fundamental inputs for SLAACM. Assuming that air-to-air combat can be modeled as a series of Poisson events, and given the basic assumption of Lanchester modern firing logic, we derive kill-rate ratios (KRRs) from the loss ratio data and use these KRRs to model engagements of arbitrary numbers of aircraft. The loss ratios have historically been provided by PA&E TACAIR and are based on simulation results and military judgment.

KRRs corresponding to the loss ratios are obtained by iterative calculation of loss ratios given trial KRR inputs. We seed our engagement model with high- and low-bounding trial KRRs and iterate using a bisection technique until we match the target loss ratio. For KRR derivation, the engagement model must be configured to match the engagement from which the loss ratios were derived. Engagement configurations include both the basic “n Blue vs. m Red” information, and any “side conditions,” such as both sides fighting to the death or Blue leaving after two losses. Kill-rate ratio calculations were previously done offline, but as we expanded the number of Red and Blue aircraft options, it became necessary to automate the calculation of the KRRs and include them in SLAACM.

Figure 2-1 shows the nominal loss ratios for a dominant Blue force. These ratios are contained in the current unclassified version of SLAACM and in the controls for the KRR calculation.

Figure 2-1. Loss Ratio Table and KRR Calculation Controls

				Loss Ratios												
Usage*	**Name**	**payload, lb**	**cepr**	**F v. Blu1**	**F v. Blu2**	**F v. Blu3**	**F v. Blu4**	**F v. Blu5**	**F v. Blu6**	**F v. Blu7**	**F v. Blu8**	**F v. Blu9**	**B v. Blu1**	**B v. Blu2**	**B v. Blu3**	**B v. Blu4**
1	**Red F1**	1000	1.0	21	101	19	24	19	10	11	12	13	57	202	38	48
1	**Red F2**	1000	1.0	12	51	10	12	10	6	7	8	9	29	102	20	25
1	**Red F3**	2000	1.0	11	39	8	10	8	4.4	5.4	6.4	7.4	23	77	16	19
1	**Red F4**	3000	0.6	8	26	6	7	6	3.3	4.3	5.3	6.3	16	52	11	13
0	**Red F5**	6000	0.10	4.4	13.5	3.1	4.4	3.1	1.3	2.3	3.3	4.3	8.7	27.1	6.3	8.7
1	**Red F6**	3000	0.10	5.6	17.5	4.0	5.6	4.0	1.6	2.6	3.6	4.6	11.3	34.9	8.1	11.3
1	**Red F7**	2000	0.5	12	40	8	10	8	4	5	6	7	23	81	16	20
0	**Red F/B**	2000	0.6	28	101	19	24	19	10	11	12	13	57	202	38	48
-1	**Red B1**	6000	0.4	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
-1	**Red B2**	18000	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
-1	**Red B3**	12000	0.10	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

*** Usage = -1 => a/c is bomber only; 0 => bomber OR fighter; 1 => fighter only.**

Info for Calculating Kill-Rate Ratios

Blue Start	Red Start	Blue Quit	Red Quit
4	8	0	0

Get Kill Rate Ratios

The loss ratios in Figure 2-1 are based on engagements of four Blue aircraft vs. eight Red aircraft, with both sides fighting to the death. The controls for the engagement configuration and side conditions are shown at the bottom left of the figure.

The first column in Figure 2-1 contains flags that indicate whether the Red aircraft can be configured as a fighter only (1), bomber only (-1), or both (0). The "F vs. Blu" columns contain loss ratios for Red fighter configurations, and the "B vs. Blu" columns contain loss ratios for Red bomber configurations. The numbers in the columns are the Red losses compared to one Blue loss, so that the most effective Reds have the lowest ratios and the most effective Blues have the highest ratios. We see in Figure 2-1 that Red F5 is the most effective Red fighter and Blu2 is the most effective Blue fighter. The extremely high loss ratios for Red bombers indicate that they have essentially no air-to-air capability against the Blue aircraft.

Column 3 in Figure 2-1 contains the bomb payload in tons for bombers and fighter-bombers. Column 4 contains the circular error probable (CEP) ratio of the bomb payload compared to the CEP of a 500-pound gravity bomb. Small ratios indicate precision-guided munitions. The optimizer uses both the payload tonnage and CEP ratio of the munitions to determine the destructive effectiveness of the payload. Red B2, with an 18,000-pound payload of 0.10 CEP ratio munitions, is the most effective Red bomber.

Figure 2-2 shows the KRRs that result from the Figure 2-1 loss ratios. The calculation time for the complete table of KRRs is approximately 20 seconds on a 1 GHz PC.

Figure 2-2. Kill-Rate Ratios

				Kill-Rate Ratios															
Usage*	**Name**	**payload, lb**	**cepr**	**F v. B1**	**F v. B2**	**F v. B3**	**F v. B4**	**F v. B5**	**F v. B6**	**F v. B7**	**F v. B8**	**F v. B9**	**B v. B1**	**B v. B2**	**B v. B3**	**B v. B4**	**B v. B5**	**B v. B6**	**B v. B7**
1	**Red F1**	1000	1	25.7	115.5	23.7	28.8	23.7	13.5	14.6	15.8	16.9	65.6	229.1	45.2	55.4	45.2	24.8	27.1
1	**Red F2**	1000	1	15.6	59.3	13.5	16.1	13.5	8.2	9.4	10.6	11.7	35.0	116.6	24.8	29.9	24.8	14.7	16.9
1	**Red F3**	2000	1	14.8	45.3	10.9	12.9	10.9	6.9	8.1	9.3	10.4	27.4	88.5	19.8	23.6	19.8	12.1	14.4
1	**Red F4**	3000	0.6	10.9	31.3	8.2	9.6	8.2	5.4	6.7	7.9	9.1	19.8	60.4	14.7	17.2	14.7	9.5	11.8
0	**Red F5**	6000	0.1	6.8	17.3	5.2	6.8	5.2	2.6	4.0	5.4	6.6	11.9	32.5	9.0	11.9	9.0	4.4	6.9
1	**Red F6**	3000	0.1	8.3	21.7	6.4	8.3	6.4	3.1	4.5	5.8	7.1	14.8	41.2	11.1	14.8	11.1	5.3	5.3
1	**Red F7**	2000	0.5	15.2	47.5	11.1	13.2	11.1	6.9	8.1	9.2	10.4	28.2	93.0	20.2	24.2	20.2	12.1	12.1
0	**Red F/B**	2000	0.6	33.8	115.5	23.7	28.8	23.7	13.5	14.6	15.8	16.9	65.6	229.1	45.2	55.4	45.2	24.8	27.1
-1	**Red B1**	6000	0.4	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127
-1	**Red B2**	18000	0.1	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127
-1	**Red B3**	12000	0.1	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127	1127

With the KRRs established, the next task is to define the Red and Blue orders of battle using one input table for the Red force and one for the Blue force. When an aircraft can be a fighter or a bomber, the SLAACM optimizer (heuristic or integer program solver) chooses the mix of configurations that provide the highest payoff; thus, we need only specify the quantity of the basic Red type and the model determines its optimum use.

EXAMPLE ANALYSES

Background

To set the stage for the scenarios below, we note that the current SLAACM scenario matches a package of four Blues against Red packages containing four advanced escort fighters, four close escort fighters, and four bombers. The defenders must defeat, in turn, the advanced escorts and the close escorts to reach the bombers.

In developing the order of battle for each day, Red optimizes his attack based on the payoffs for the potential attack packages. Package payoffs are based on the destructive power of the bombers, the value of enemy aircraft killed, and the potential for success based on the kill-rate ratios of the combatants.

Blue has input options of designating individual aircraft “Smart” or “Smart Local Area Defender (LAD).” Both Smart and Smart LAD Blue aircraft are assumed to have sufficient battle management to identify and choose their target Red packages, and the Blue side performs the optimizations for those aircraft so designated. Smart Blues calculate payoff based on the destructive power of the bombers, the value of enemy aircraft killed, the value of own-side aircraft killed, and the potential for success based on the kill-rate ratios of the combatants. Smart Blue aircraft are relatively loss averse, while Smart LAD Blue aircraft, as local area defenders, are indifferent to Blue-side loss.

Blues not selected as Smart or Smart LAD engage randomly with no consideration of target value or own-side risk. The randomly assigned Blue aircraft are subject to a battle management factor, with values between 0.0 and 1.0, that defines the probability that they will be able to locate and engage any Red packages.

Note that one Blue aircraft type is labeled “LAD” and belongs to the indigenous, “Green,” force. We assume Green is defending home ground and is indifferent to losses. We typically either have the Green LAD aircraft attack randomly or select for it both Smart and LAD options.

SLAACM includes both a “greedy algorithm” heuristic optimizer that identifies, sorts, and selects the highest-payoff packages to engage, and a link to a commercial program, LINGO™, that performs a complete integer program optimization.

Scenarios

In the examples that follow, we consider a simple case with one type of Blue defender of moderate strength opposed by two types of Red fighters escorting two types of Red bombers. Table 2-1 shows quantities and performance parameters for the combatants. Blue Blu6 in the KRR table corresponds to Blue NF1 in the Blue supply figure. For clarity, we use the label “Blu6 (NF1)” in the discussions below.

Table 2-1. Scenario Input Data

Aircraft designation	Aircraft initial quantity	Blu6 (NF1) to red aircraft kill-rate ratio	Red bomber payload weight (ton)	Red bomber payload quality (CEP ratio)
Blue Blu6 (NF1)	100			
Red F1	600	13		
Red F5	100	3		
Red B1	300	>1,000	6,000	0.4
Red B2	200	>1,000	18,000	0.1

Figures 2-3 and 2-4 show the SLAACM input worksheets for Blue and Red supply. The input tables allow day-by-day replacements or phased deployments (which are not used for the current example).

Figure 2-3. Blue Supply

NOTE: A/C shown on this sheet are available for service on the indicated days. Day 0 is the initial load-out. Day

New Row

	0	1	2	3	4	5	6	7	8
Blue_F1	**0**								
Blue_F2	**0**								
Blue_F3	**0**								
Blue_F4	**0**								
Blue_F5	**0**								
Blue_NF1	**100**								
Blue_NF2	**0**								
Blue_NF3	**0**								
Blue_LAD	**0**								

Figure 2-4. Red Supply

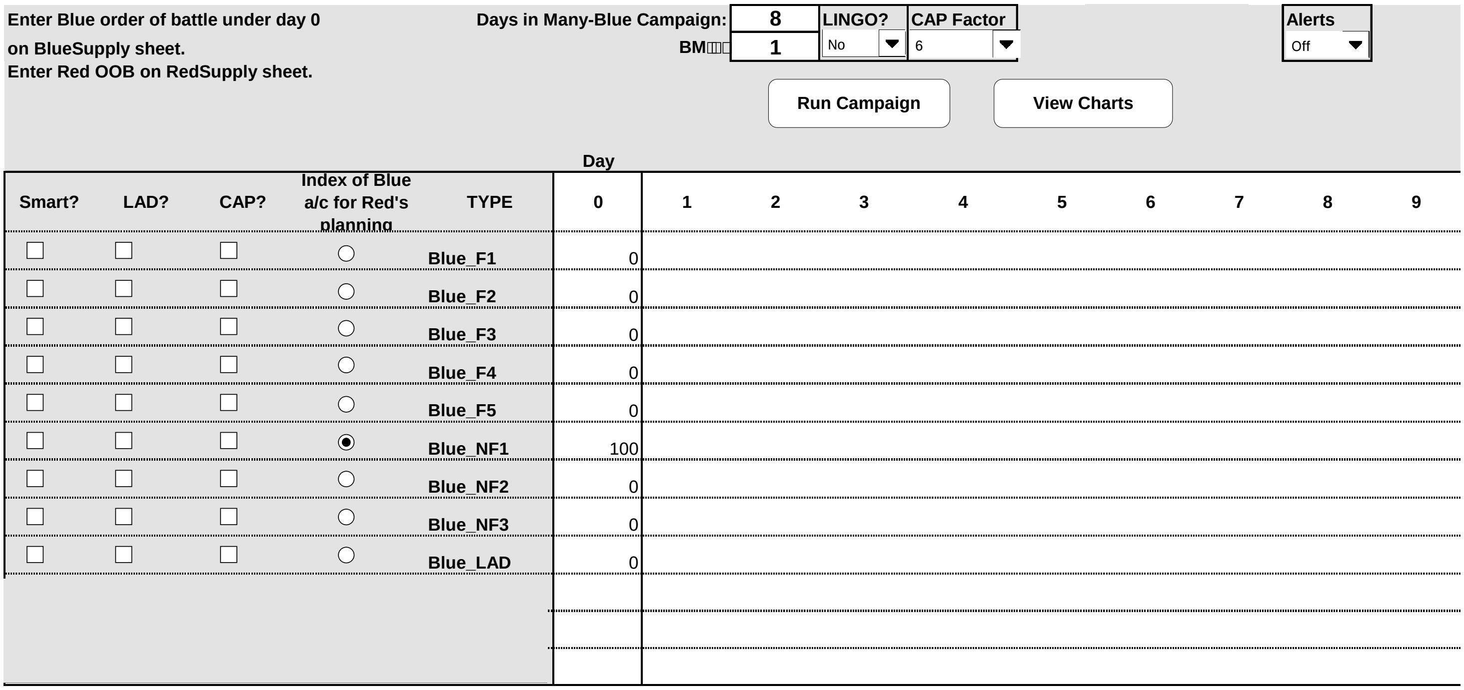
NOTE: A/C shown on this sheet are available for service on the indicated days. Day 0 is the initial load-out. Day 1

New Row

	0	1	2	3	4	5	6	7	8
Red F1	**600**								
Red F2	0								
Red F3	0								
Red F4	0								
Red F5	**100**								
Red F6	0								
Red F7	0								
Red F/B	0								
Red B1	**300**								
Red B2	**200**								
Red B3	0								

Figure 2-5 shows the Blue order of battle (OOB) worksheet that includes input, output, and run controls. Control inputs at the top of the worksheet set the number of days in the campaign, the battle management factor for random defense, and the CAP factor. ("Days" refers to individual engagement periods, between which each side has time to reorganize his forces. There could easily be more or less than one such engagement in a single day; the number of days selected should be based on the expected individual engagements.)

Figure 2-5. Blue Order of Battle Worksheet



CAP (combat air patrol) refers to cases in which Blue is compelled to stage his aircraft to maintain a continuous airborne defensive force over the battle space. If a Blue aircraft is identified as a CAP aircraft, then the Blue aircraft available for a day's engagement is equal to the Blue quantity of that aircraft divided by the CAP factor. The CAP factor is based on such information as distances of bases from the battle space, aircraft range, crew rest, and related military considerations. Currently, the same CAP factor is used for all Blue CAP aircraft; the next version of SLAACM will allow individual CAP factors. Because Blue always fights in four-ship packages, if

$$\#\,Blue\ Aircraft\,/\,CAP\ Factor < 4\,,$$

then Blue cannot maintain a full CAP and no Blues are sent. This is clearly too conservative because Blue will likely maintain a partial CAP with his remaining aircraft. Inclusion of a partial CAP in the closed-form analytic SLAACM algorithms presents a mathematical challenge that we intend to address in the future.

The remaining two controls at the top of the worksheet toggle use of the integer optimization (LINGO™) or the heuristic optimization, and toggle diagnostic program halts at intermediate stages of the calculations.

The first three columns in the table allow selection of specific aircraft as Smart, LAD, and/or CAP, as discussed above. The fourth column selects which Blue aircraft Red will use as the basis for optimizing his attack packages. With the current aircraft values and KRRs, the results are relatively indifferent to the selection in column four.

The order of battle information in the table is output information. The Day 0 column data are from the Blue supply worksheet, and the Day 2 through N column data are from campaign results.

Results and Discussion

Figures 2-6 through 2-8 show the SLAACM output charts for this campaign, and Figures 2-9 through 2-11 show the worksheet tables corresponding to the charts. Figures 2-12 through 2-14 show the SLAACM Blue loss, Red loss, and Blue loss standard deviation tables.

Figure 2-6. Blue Order of Battle

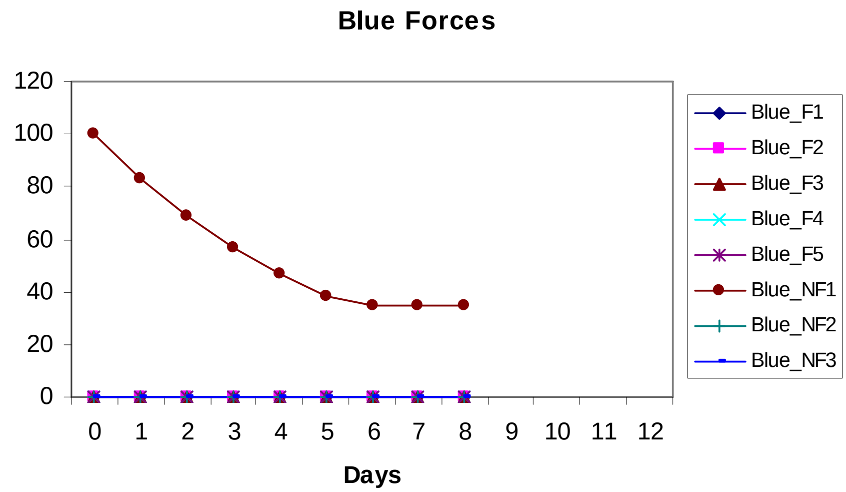


Figure 2-7. Red Order of Battle

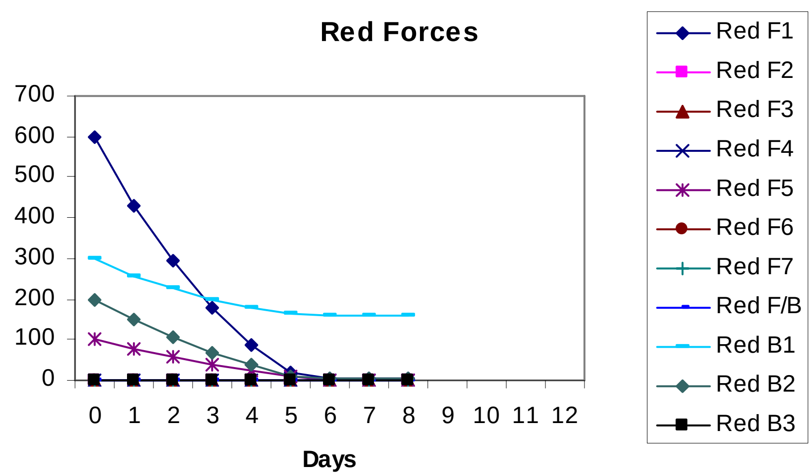


Figure 2-8. Bomb Tonnage Dropped

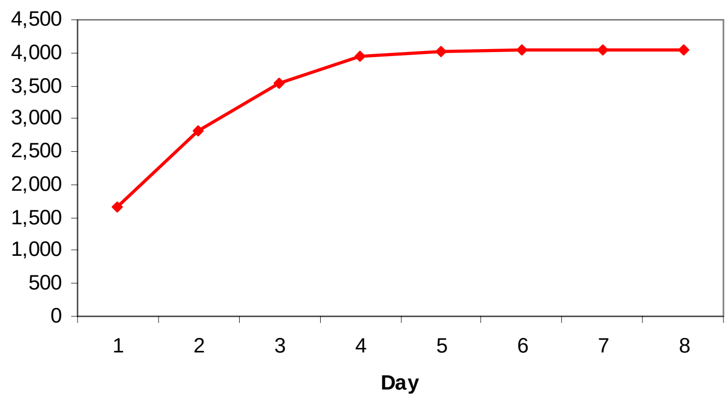


Figure 2-9. Blue Tabular Order of Battle

Smart?	LAD?	CAP?	Index of Blue a/c for Red's planning	TYPE	Day 0	1	2	3	4	5	6	7	8
☐	☐	☐	○	Blue_F1	0	0	0	0	0	0	0	0	0
☐	☐	☐	○	Blue_F2	0	0	0	0	0	0	0	0	0
☐	☐	☐	○	Blue_F3	0	0	0	0	0	0	0	0	0
☐	☐	☐	○	Blue_F4	0	0	0	0	0	0	0	0	0
☐	☐	☐	○	Blue_F5	0	0	0	0	0	0	0	0	0
☐	☑	☐	◉	Blue_NF1	100	83	69	57	47	38	35	35	35
☐	☐	☐	○	Blue_NF2	0	0	0	0	0	0	0	0	0
☐	☐	☐	○	Blue_NF3	0	0	0	0	0	0	0	0	0
☐	☐	☐	○	Blue_LAD	0	0	0	0	0	0	0	0	0

Figure 2-10. Red Tabular Order of Battle

Red OOB AC TYPE	DAY:	0 QTY	1	2	3	4	5	6	7	8
1	Red F1	600	429	294	180	87	20	4	4	4
2	Red F2	0	0	0	0	0	0	0	0	0
3	Red F3	0	0	0	0	0	0	0	0	0
4	Red F4	0	0	0	0	0	0	0	0	0
5	Red F5	100	77	57	39	24	8	2	2	2
6	Red F6	0	0	0	0	0	0	0	0	0
7	Red F7	0	0	0	0	0	0	0	0	0
8	Red F/B	0	0	0	0	0	0	0	0	0
9	Red B1	300	258	225	198	177	164	160	160	160
10	Red B2	200	148	106	69	37	11	5	5	5
11	Red B3	0	0	0	0	0	0	0	0	0
	TOTAL	**1,200**	**912**	**682**	**486**	**325**	**203**	**171**	**171**	**171**

Figure 2-11. Tabular Bomb Tonnage

	day:	1	2	3	4	5	6	7	8
Bombs dropped									
	lbs	3,305,005	2,333,251	1,460,675	772,712	188,265	28,475	0	0
	tons	1,653	1,167	730	386	94	14	0	0
	cumulative	1,653	2,819	3,549	3,936	4,030	4,044	4,044	4,044

Figure 2-12. Blue Loss Table

Blue Losses

	1	2	3	4	5	6	7	8	Total	Std. Dev.
Blue_F1	0	0	0	0	0	0	0	0	0	0.00
Blue_F2	0	0	0	0	0	0	0	0	0	0.00
Blue_F3	0	0	0	0	0	0	0	0	0	0.00
Blue_F4	0	0	0	0	0	0	0	0	0	0.00
Blue_F5	0	0	0	0	0	0	0	0	0	0.00
Blue_NF1	17	14	12	10	9	3	0	0	65	8.76
Blue_NF2	0	0	0	0	0	0	0	0	0	0.00
Blue_NF3	0	0	0	0	0	0	0	0	0	0.00
Blue_LAD	0	0	0	0	0	0	0	0	0	0.00
									0	0.00
									0	0.00
									0	0.00
TOTAL	**17**	**14**	**12**	**10**	**9**	**3**	**0**	**0**		
cumulative	17	31	43	53	62	65	65	65		

Figure 2-13. Red Loss Table

Red Losses

Red F1	171	135	114	93	67	16	0	0
Red F2	0	0	0	0	0	0	0	0
Red F3	0	0	0	0	0	0	0	0
Red F4	0	0	0	0	0	0	0	0
Red F5	23	20	18	15	16	6	0	0
Red F6	0	0	0	0	0	0	0	0
Red F7	0	0	0	0	0	0	0	0
Red F/B	0	0	0	0	0	0	0	0
Red B1	42	33	27	21	13	4	0	0
Red B2	52	42	37	32	26	6	0	0
Red B3	0	0	0	0	0	0	0	0
TOTAL	**288**	**230**	**196**	**161**	**122**	**32**	**0**	**0**
cumulative	288	518	714	875	997	1,029	1,029	1,029

Figure 2-14. Blue Loss Standard Deviation Table

Blue Standard Deviations

	1	2	3	4	5	6	7	8
Blue_F1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Blue_F2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Blue_F3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Blue_F4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Blue_F5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Blue_NF1	4.48	4.05	3.76	3.47	3.26	1.87	0.00	0.00
Blue_NF2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Blue_NF3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Blue_LAD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

In the figures above, the losses occur monotonically for all aircraft types, which is typical of random engagement logic. The last day of the campaign was Day 6. At the end of Day 6, Red still has 160 RB1 bombers and 5 RB2 bombers, but because Red has only 4 RF1 and 2 RF5 fighters, he cannot provide the 8 fighter escorts required to generate an attack package.

The dispersion information contained in Table 2-14 is not currently used in our campaign analyses. We hope, in future work, to use this information to develop confidence intervals for campaign outcomes.

To demonstrate the impact of Blue battle management, we run the same case as above, first with the Smart option selected and second with both the Smart and LAD options selected. Figures 2-15 and 2-16 show the Blue order of battle and bomb tonnage results for these cases, along with the results for the case above. In the case of the Smart Blue, just one fewer Blue aircraft is lost (64 vs. 65), but the delivered bomb tonnage is dramatically reduced (2,931 tons vs. 4,044 tons) compared to the random case. In the case of Smart LAD Blue, one additional Blue aircraft is lost (66 vs. 65) and delivered bomb tonnage is further reduced (2,709 tons vs. 4,404 tons) compared to the random case.

Figure 2-15. Blue Order of Battle by Day

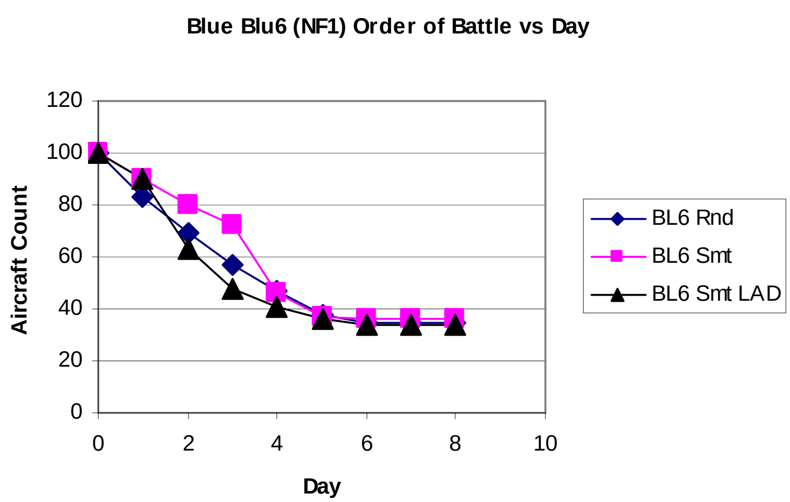
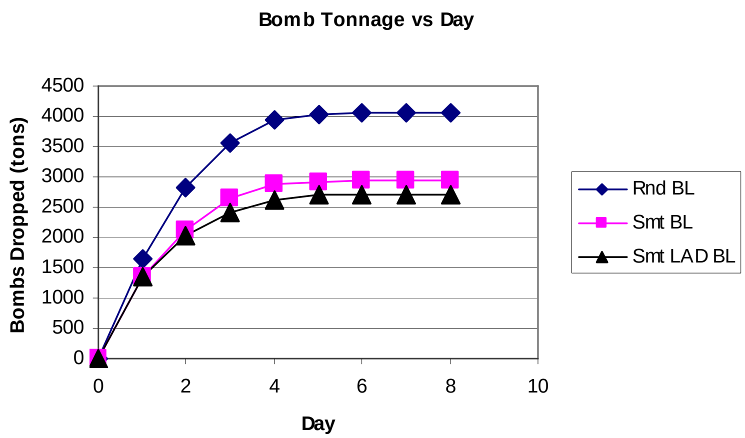


Figure 2-16. Bomb Tonnage Dropped by Day



In both the Smart and Smart LAD cases, the defenders focus on the Red packages containing Red RB2 bombers. On Day 1, both Smart and Smart LAD Blues attack RB2 packages escorted by Red RF1 fighters because they offer the highest probability of success. On Days 2 and 3, the Smart Blues and Smart LAD Blues continue to intercept RB2 packages, but the loss-averse Smart Blues switch much of their defense to Red RB1 packages to avoid the RB2 packages escorted by Red RF5 fighters. On Day 4, the Smart Blues finally must engage Red RF5 escorted RB2 packages.

Figure 2-17 shows the order of battle for Red's most valuable bomber, the RB2. We see that RB2 is preferentially intercepted in both the Smart Blue and Smart LAD Blue scenarios. The Red RF5 order of battle shows that the Smart Blues avoid combat with Red RF5s until all lower risk, high-payoff options are exhausted. The Blue Blu6 (NF1) and Red RF5 losses shown Figures 2-18 and 2-19 provide another display of the Blue strategies.

Figure 2-17. Red RB2 Bomber Order of Battle

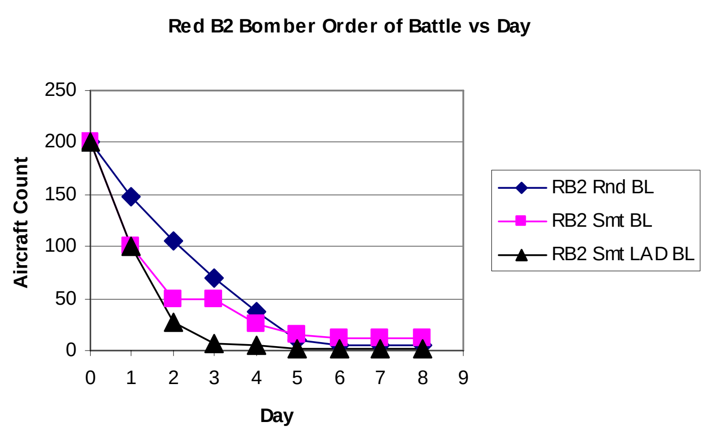


Figure 2-18. Blue Losses by Day

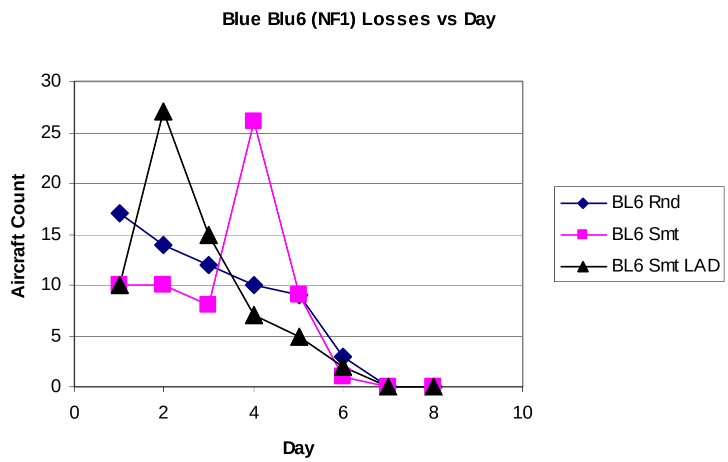
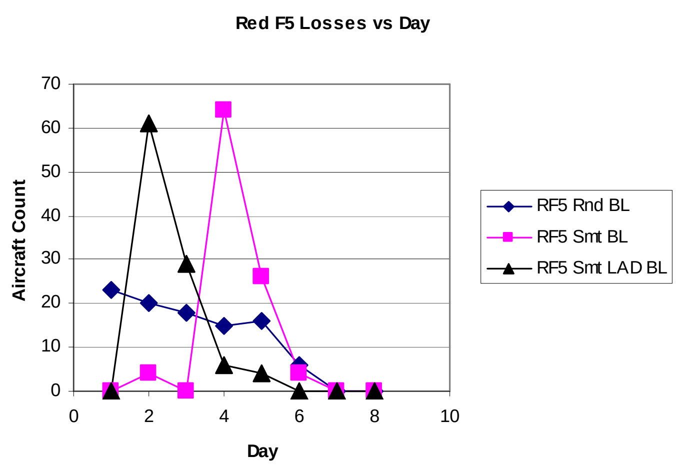


Figure 2-19. Red RF5 Losses by Day



In the results above, we see that both Smart Blue and Smart LAD Blue battle management resulted in a 30 percent reduction in bomb tonnage compared to the random case.[1] By being loss-averse, the Smart Blues lost one fewer aircraft than the random case and two fewer aircraft than the Smart LAD case at the cost of 200 additional tons of bombs delivered.

SUMMARY

The discussion and simple examples in this chapter demonstrate the basic operation and the capabilities of the current version of SLAACM. Consideration of the input sheets shows that SLAACM is capable of analyzing real-world scenarios of

[1] The reduction in destructive power is greater than the tonnage difference because the optimizer considers the type of munition in valuing the packages for intercept, but it is not currently plotted. Plots of destructive power will be added in future versions.

high complexity with provision for true integer programming optimization by both attackers and defenders. Even complex cases run in seconds, so analysts can efficiently examine wide ranges of alternatives. These features make SLAACM a valuable tool for the Department of Defense.

Chapter 3
Mathematical Analysis: Battle Management, Optimization, and Dispersion in SLAACM

This chapter explains the mathematical methods used in three features of the current version of SLAACM: accounting for imperfect battle management, treating Red and Blue forces' optimization problems, and dealing with dispersion in the two sides' daily losses.

BATTLE MANAGEMENT

Battle management determines the ability of defensive counter air forces to engage targets. SLAACM provides two kinds of battle management for defensive aircraft, determined by whether or not the user identifies an aircraft type as "smart."

Smart aircraft are able to locate advancing Red attack packages and identify the types of aircraft in them and, by sharing targeting information, to conduct optimized interceptions. Smart defenders always intercept their targets.

Blue aircraft not identified as smart encounter Red packages randomly. They do not conduct optimized defensive encounters. There is still an element of battle management for these aircraft, however, and that is the efficiency with which they encounter Red attack packages.

With perfect battle management, if the number of defending flights is at least as large as the number of attack packages, every attack package would be intercepted. However, the intercepting aircraft types would be random, rather than optimal. If the number of attack packages is larger than the number of defending flights, every defending flight would engage an attack package.

With less-than-perfect battle management, not every attack package would be intercepted in the former case, and not every defending flight would be engaged in the latter case. SLAACM's battle management feature allows the user to enter a "goodness" parameter that characterizes the effectiveness of battle management for Blue aircraft that are not smart. The following paragraphs explain how that parameter affects SLAACM's calculations.

Let K distinct types of defender flights deal with an attack by J distinct types of attack packages. The defenders are assumed not to know the makeup of individual attack packages before interception, so that the type of attack package engaged by a given defending flight is the result of random selection from the set of attack packages.

Let m_i be the number of defender flights of type i, and let n_j be the number of attack packages of type j. Then the total number of attack packages, N, and the total number of defending flights, M, are given respectively by

$$N = \sum_{1}^{J} n_j ; \quad M = \sum_{1}^{K} m_i \qquad \text{[Eq. 3-1]}$$

First we consider the case of perfect battle management. If $M \geq N$, then every attack package will be intercepted. Not all defending flights engage; in this simple analysis, we assume that the fraction N/M of each defending flight type engages.[1] Then, a measure of $\overline{E}_{ij}$, the central tendency of the number E_{ij} of m_i vs. n_j engagements, is

$$\overline{E}_{ij} = \frac{N}{M} m_i \frac{n_j}{N} = \frac{m_i n_j}{M} \qquad \text{[Eq. 3-2]}$$

For this simple analysis, we take $\overline{E}_{ij}$ to be the number of m_i vs. n_j engagements. Summing $\overline{E}_{ij}$ over j shows that the fraction of each defending flight type engaged is N/M, as it should be; summing over i shows that all attacking flights of each type are engaged.

When $M < N$, every defending flight engages, but not all attack packages can be engaged. The estimate for $\overline{E}_{ij}$ analogous to the one given in (2) is

$$\overline{E}_{ij} = m_i \frac{n_j}{N} = \frac{m_i n_j}{N} \qquad \text{[Eq. 3-3]}$$

Summing this estimate for $\overline{E}_{ij}$ over i shows that the fraction M/N of each attack package type is engaged; summing over j shows that every defending flight of each type is engaged.

In some cases of interest, M and N are both $O(10^2)$. For such large M and N, providing this "perfect" defender's battle management could exceed the capabilities of available systems. We account for the limitations of the defender's battle management with a simple adaptation of the "perfect" case. We make the adaptation by introducing "ghost" attack packages or defender flights. A defender flight that engages a ghost attack package does not actually engage; an attack package engaged by a ghost defender flight is not actually intercepted.

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[1] We fully appreciate the fact that this simple analysis treats a complex combinatoric problem crudely.

When $M \geq N$, we introduce m_g defending flights of type "ghost," and proceed as in the above analysis (where the defender has perfect battle management). This gives us a new estimate for $\overline{E}_{ij}$,

$$\overline{E}_{ij} = \frac{m_i n_j}{M + m_g} = \frac{m_i n_j}{M} b_1$$ [Eq. 3-4]

where

$$b_1 \equiv \frac{M}{M + m_g}$$ [Eq. 3-5]

When $M < N$, we introduce n_g attack packages of type "ghost," and find

$$\overline{E}_{ij} = \frac{m_i n_j}{N + n_g} = \frac{m_i n_j}{M} b_2$$ [Eq. 3-6]

where

$$b_2 \equiv \frac{N}{N + n_g}$$ [Eq. 3-7]

Noting the similarity of (3-4) and (3-6), we introduced into SLAACM the simple model

$$\overline{E}_{ij} = \frac{m_i n_j}{M} b$$ [Eq. 3-8]

and allow the user to choose the "battle management efficiency factor" b, $0 < b \leq 1$.

Once seen, equation (3-8) is so simple, and seemingly obvious, that it is fair to ask why we did not simply introduce it into SLAACM without analysis. The reason is that we wanted to understand what that simple choice implied about how the battle proceeded. The analysis given here supplies that understanding.

OPTIMIZATION

This section discusses a formal integer programming optimization process that replaces (or supplements) our original optimization heuristic. (The results of both these analyses have been incorporated into the latest versions of SLAACM.) Here we formalize the integer programming problems performed in the LINGO model within

LMI's SLAACM. LINGO is a general linear, nonlinear, and integer programming solver tool.[2] The version embedded in SLAACM is Extended LINGO 9.0.

The main idea behind using an integer programming tool is to solve the following optimization problem. Red wishes to come with optimal attack packages based on a determined payoff function (this is the first optimization). Then, the Blue aircraft that are sophisticated enough to have a priori knowledge of what Red attack packages are coming may optimize their defending packages accordingly (the second optimization). The remaining Blue aircraft that do not have advanced knowledge of Red's attack compositions will encounter the attackers randomly, often with less than perfect battle management. The random encounters were described mathematically in the previous section.

A Red attack package consists of four advanced escorts, four close-in escorts, and four bombers. Since some fighters may be bombers, we could have a set of possible attack packages T, where, given R types of fighters and bombers, we could have that $|T| = R^3$. In reality, however, most mathematically feasible combinations of packages are unrealistic for warfare (one would not, for example, dispatch a package of heavy bombers escorting fighters). Since the number of reasonable attack packages is much smaller than the complete enumeration, we assume those undesirable packages are removed in advance of the optimization; then, we limit ourselves to those remaining, reasonable possibilities in the integer program. The degree to which one wishes to limit the number of potential packages depends primarily on the size of the integer program, which is a function of the size of R.

We define a set of *a* attack packages for Red as $I \subseteq T$. We define variable r_{ij} as the number of Red aircraft of type j used in attack package i. To calculate its payoff, Red plans its attack assuming each package will be confronted by one specific Blue defender type, usually the most numerous Blues. Based on the payoff functions described previously, each attack package for Red has a certain payoff, denoted as p_i. We denote the number of Red aircraft of type j as n_j. At this point, Red solves for its optimal attack strategy. We have the following integer optimization problem:

$$\max \sum_i p_i x_i$$

$$\text{s.t.} \sum_i r_{ij} x_i \le n_j \quad \forall j \in J$$

$$x_i \in \{0,1,2,\ldots\} \qquad \text{[Eq. 3-9]}$$

[2] Lindo Systems, Inc., http://www.lindo.com/.

This problem is a variant of the classic knapsack problem. The solution to this integer programming problem provides the optimal set of attack packages dispatched by Red.

As stated previously, Blue responds with optimal defenses of Red's attack if a given aircraft is "smart" enough to have a priori knowledge of Red's attack package composition. We assume Red has obtained its solution for (3-9) before Blue does its optimal response. We denote the optimal solution for Red—that is, $x^* = (x_1^*, x_2^*, \ldots, x_I^*) = (a_1, a_2, \ldots, a_I)$—as constraints for Blue's optimization. In other words, a_i is the number of Red attack packages of type i.

Blue has its own payoff function for each type of potential Red attack package. We denote $\hat{p}_{ik}$ as the payoff to Blue for intercepting a Red package i with the Blue four-ship package of aircraft type k. Again, the components of the payoff function have been described previously; they include a loss-aversion factor for Blue, along with positive payout for expected Red kills and bombs stopped. We denote the set of Blue aircraft types with a priori knowledge of Red's attack packages as $K' \subseteq K$. We denote the number of Blue aircraft of type $k \in K'$ as m_k. We then formulate and solve the following integer programming problem for Blue. In short, we want to solve for the optimal number of Blue four-ship package of type k that intercepts Red attack package i, denoted by y_{ik}.

$$\begin{aligned} &\max \sum_{i,k} \hat{p}_{ik} y_{ik} \\ &\text{s.t. } \sum_{i} 4y_{ik} \le m_k \quad \forall k \in K' \\ &\qquad \sum_{k} y_{ik} \le a_i \quad \forall i \in I \\ &y_{ik} \in \{0,1,2,\ldots\} \end{aligned} \qquad \text{[Eq. 3-10]}$$

The integer programming model written in LINGO is embedded into SLAACM. LINGO solves the integer programming problem and returns the solution to SLAACM. At this point, the remaining "dumb" Blue aircraft, denoted by $K'' \subseteq K$, where $K' \cup K'' = K$ and $K' \cap K'' = \emptyset$, engage the Red packages that were not selected by the "smart" Blues. That is, the Blue aircraft without a priori knowledge engage randomly the Red packages that remain after (3-10) is solved, i.e., those Blue aircraft engage a set of attack packages $(a_1', a_2', \ldots, a_I')$, where $a_i' \equiv \max\left(a_i - \sum_k y_{ik}, 0\right)$. Those encounters were described in the previous section on battle management. Since the previous section described a slightly different problem, the notation from this section and the previous is not to be considered interchangeable.

DISPERSION

SLAACM engagements between a four-ship defender flight and a 12-ship attacking package have 16 possible outcomes.[3] They may be described as absorbing boundary states (a, c, b, d), where a is the number of advanced escorts, c the number of close escorts, b the number of bombers, and d the number of defenders. In four outcomes, the advanced escorts defeat the defenders, and the system state is (i, 4, 4, 0), $1 \le i \le 4$. In four other outcomes, the defenders defeat the advanced escorts, but are themselves defeated by the close escorts; the corresponding system states are (0, i, 4, 0), $1 \le i \le 4$. The defenders may defeat both advanced and close escorts but lose to the bombers (some bombers have substantial defensive capability). Then the system states are (0, 0, i, 0), $1 \le i \le 4$. Finally, the defenders may prevail, with system states (0, 0, 0, i), $1 \le i \le 4$.

SLAACM presently calculates the probabilities of all 16 outcome states, for every Blue vs. Red engagement that takes place. Part of this year's work was to review these statistics, and upgrade SLAACM to display appropriate measures of dispersion. Table 3-1 shows an example of the probabilities of the 16 outcome states.

Table 3-1. Outcome State Probabilities

a	c	b	d	Probability
1	4	4	0	7.52E-08
2	4	4	0	1.23E-07
3	4	4	0	1.14E-07
4	4	4	0	5.59E-08
0	1	4	0	0.000121
0	2	4	0	0.000203
0	3	4	0	0.0002
0	4	4	0	0.000112
0	0	1	0	1.91E-06
0	0	2	0	3.82E-06
0	0	3	0	5.69E-06
0	0	4	0	7.54E-06
0	0	0	1	0.002155
0	0	0	2	0.01635
0	0	0	3	0.132935
0	0	0	4	0.847904

In this example, which is representative of many cases considered in this year's studies for OSD/PA&E/TACAIR, the defenders are significantly stronger than the

[3] LMI, *Stochastic Models of Air Superiority Engagements and Campaigns*, Report PA104S1, David Lee, Scott Houser, Robert Hemm, and Jeremy Eckhause, June 2003.

Red aircraft in the attack package. Consequently, probabilities of cases in which Red aircraft prevail over the Blue defenders are all much less than one. The central tendencies of the losses for all Red aircraft—advanced escorts, close escorts, and bombers—are quite close to four, and the standard deviations for losses of all the Red aircraft are small compared to one (the probability that fewer than four advanced escorts are lost is less than 10^{-6}; the probability that fewer than four close escorts or fewer than four bombers are lost is less than 10^{-3}).

The situation is somewhat different for the Blue defenders. While the probability that the defenders prevail is very nearly one (it is larger than 0.999), there is considerable dispersion in the number of Blue losses. The probability of no Blue losses is 0.85; the probability of one Blue loss is 0.13; the probability of two Blue losses is roughly 0.02; and the probability of three Blue losses is less than 0.01. Table 3-2 shows the marginal distribution functions of losses for all four combatants: advanced escorts, close escorts, bombers, and defenders.

Table 3-2. Marginal Loss Distributions

Losses	Probability (advanced escorts)	Probability (close escorts)	Probability (bombers)	Probability (defenders)
0	5.5879E-08	0.000112	0.000645	0.847904
1	1.1365E-07	0.0002	5.69E-06	0.132935
2	1.22778E-07	0.000203	3.82E-06	0.01635
3	7.52281E-08	0.000121	1.91E-06	0.002155
4	0.999999632	0.999363	0.999344	0.000656

In view of these types of results, which were common in the cases studied this year, we decided to track the dispersion of Blue losses in SLAACM outputs, but not to track dispersion in Red losses.

Also, these typical results show that it is reasonable to propagate only central tendencies (expected values) of Red losses to obtain day-to-day Red orders of battle. In addition, while it is not unreasonable to propagate only central tendencies of Blue losses to obtain day-to-day Blue orders of battle, SLAACM will give better understanding of overall dispersion if a few representative values of Blue losses are propagated day to day.

We report dispersion (standard deviation) of Blue losses in the present version of SLAACM. We intend to propagate three representative values of Blue losses day-to-day.

Chapter 4
Mathematical Analysis: Phased Attack and Defense

In air defense, it is sometimes necessary to provide continuous combat air patrol coverage of the battle space. The current SLAACM includes the option for Blue defense aircraft to be time phased over the battle space in order to provide continuous defense CAP. The inclusion of a phased Blue defense led to discussions with TACAIR about whether Red could phase its attack to take advantage of Blue's phasing. In response to these discussions, we addressed the problem in two substantially different ways, and include both in the sections that follow. Approach 1 demonstrates a method for generating probabilistic results for a well-defined, specific scenario. Note that in this analysis the postulated parameters are selected solely to demonstrate the approach. While these parameters are generally reasonable, they do not represent any known operational scenario. Approach 2 is a generalized combinatorial analysis that explores the probability space for random selections of phasing strategies for both Blue and Red.

APPROACH 1

We are interested in the way CAP attrition is treated in SLAACM. Presently, SLAACM assumes that all the packages Red sends in a "day" are simultaneously observable by Blue's battle management, which may include target identification onboard certain aircraft. This allows certain Blue aircraft to selectively engage high-threat packages. This assumption of high-threat Red attacks seems plausible when saturating Blue's defenses is an attractive option for Red.

But for a scenario in which Blue's target identification/battle management, and the two sides' available forces, would allow Red to saturate Blue's defense with relatively low-value attack packages while Blue remained unaware of upcoming attacks by high-value packages, the SLAACM assumptions are optimistic for Blue. This is particularly true when target identification is onboard aircraft that fly combat air patrols from bases far from the theater.

Full exploration of the options available to each side by time phasing of attacks and defenses will involve considerations of target and basing geographies and may well require case-by-case analysis. Here we consider some specific, simple examples to illustrate the potential significance of these effects.

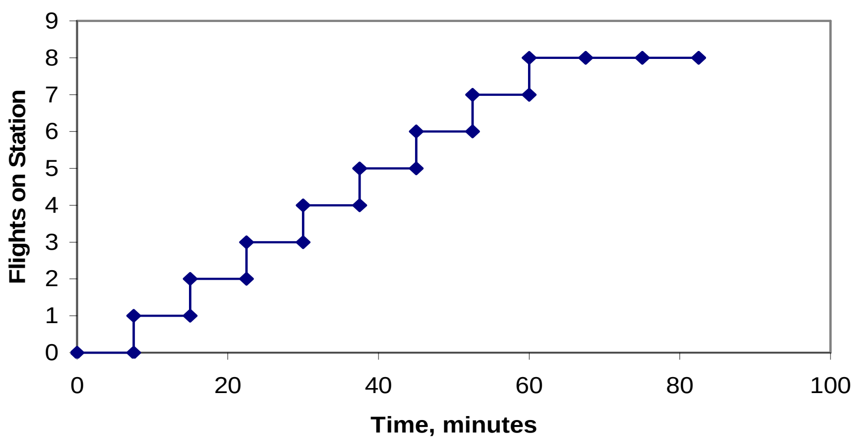
Suppose that Blue has a fleet of 192 aircraft, which must accomplish a 2-hour flight to the theater. Suppose also that their aircraft can remain on station for 1 hour, that they carry six missiles, that they will keep one missile for selfprotection while returning to base, and that their turnaround time is 1 hour. This gives the Blue aircraft 1 hour out of six on station, so that six flights must be available to keep one flight continuously on station. Suppose that Blue aircraft always fight in four-ship packages.

Suppose that Red attacks always come in 12-ship packages, made up of advanced escorts, close escorts, and bombers, and that the Blue aircraft must defeat the escorts in order to engage the bombers. Then each 4-ship flight, having 20 usable missiles, can defeat only one attacking package before rearming.

Now, Blue's 192 aircraft provide eight, six-flight groups, so that Blue can keep eight flights on station continuously. In this simple example, Blue controls only the time phasing of the eight groups' CAPs.

Figure 4-1 shows an example of the build-up of Blue forces when the CAPs relief times are evenly spaced.

Figure 4-1. Arrival Process for Evenly Spaced CAPs



By sending eight attacking packages, Red can at any time "reset" Blue's strength to the start of the buildup, because defeating eight packages exhausts the missiles available to the eight Blue flights on station.[1] In this way the Reds can give themselves intervals of time in which there are fewer than eight Blue flights.

Let us look first at a bad Blue option for the CAPs' relief times. Suppose that, rather than the uniform spacing of Figure 4-1, the eight groups' CAPs are exactly in phase. That is, the flights of all eight groups relieve their predecessors at the same time. Then, by sending eight low-value packages, Red can exhaust the missiles available to the Blue force, and there will be no more Blue aircraft for roughly 1 hour. This gives Red considerable scope to dispatch higher-value packages unopposed.

[1] Blue flights might regroup so that the remaining missiles available to two or three flights could be used against another Red package, but this appears to call for complicated reorganization and to give only a second-order effect anyway.

Presumably, Blue will not arrange the groups' CAPs in this way. To consider other Blue options and Red tactics in more detail, we must be more specific about the time required for Red to carry out missions and about the area in which Red packages are vulnerable to Blue's flights.

Let us say, then, that Red's packages are vulnerable for 10 minutes ingress, 5 minutes delivering bombs, and 10 minutes egress. To focus strictly on time phasing, suppose that the Blue defenders are invincible, so that any Blue flight in Red's vulnerable zone, with at least as many Red packages as Blue flights are also in the zone, destroys a Red package.

For further simplification, let us suppose that Red has eight low-value packages and eight high-value packages to dispatch. For specificity, suppose that bombers in the high-value packages carry three times the weight of bombs as those in the low-value packages.

Let us also assume that all eight Blue CAPs are in place, and that the Blue aircraft unfailingly attack high-value packages in preference to low-value packages.

Now, if Red dispatches all 16 packages at once, the eight defending flights will eliminate all eight high-value packages, and Red will deliver eight low-value bomber loads (we'll call this eight "units") of bombs. No aircraft in the low-value packages will be lost.

For completeness, let us treat the above "bad" Blue option in detail. If all eight CAPs were in phase, it seems incredible that Red would not know the times at which the CAPs were relieved. Then Red can send eight low-value packages to arrive just after a relief time. All eight will be destroyed. But the missiles of all eight CAPs will be exhausted too. If the eight high-value packages were undetectable while, say, 20 minutes behind the low-value packages, they could ingress, bomb, and be out of the vulnerable area 15 minutes before the relieving CAPs arrived.

Blue would, however, want to arrange the CAPs' phases to avoid such an outcome. One option would be to space the CAPs' relief points evenly as shown in Figure 4-1.

If Red sends the eight low-value packages to arrive just at one CAP's relief time, and delays the eight high-value packages for, say, 20 minutes to make sure that Blue does not know they are coming, the high-value packages will face only two defending flights, with a third defending flight joining after 2.5 minutes. One more Blue flight will arrive just as the five Red packages start bombing; it will destroy a Red package, and four high-value Red packages will drop their bombs.

Two defending flights will arrive before the four packages that survive ingress and bombing leave the vulnerable area (the second arrives just as they leave, but it seems reasonable to give Blue the benefit of a tie), so that the outcome of the attack is six high-value and eight low-value packages destroyed, with four high-value bomber loads of bombs delivered.

Thus, spacing the CAPs' relief times evenly allows Blue to destroy six more high-value packages, and allows four fewer bomber flights to "leak" through Blue's defenses, than would have been the case if the relief times coincided. Red, on the other hand, gets four high-value bomber flights through, delivering 50 percent greater weight of bombs, and loses two fewer high-value packages, than he would have done by sending all the packages simultaneously (as SLAACM now assumes he would do) against the evenly spaced CAPs.

As a final example, we suppose that the eight CAPs' relief times are randomly shifted in time and that Red sends eight low-value packages at a random time. Let us generalize the discussion, assuming that the defending flights' time-on-station is S minutes and that Red follows up eight high-value packages D minutes later. Further suppose that ingress and egress require I and E minutes, respectively, and that bombing takes B minutes.

Then, the number j of Blue defending flights arriving while the high-value packages ingress and drop bombs has the binomial distribution B(j, 8, [D + I + B]/S). The number p of flights that do *not* arrive during that period is, of course, distributed as B(p, 8, [S − D − I − B]/S).

When k defending flights arrive during ingress and bombing, then 8 − k flights arrive in the interval S − I − B, and their arrival times are uniformly distributed over that interval. Thus, the number m of defending flights arriving during egress has the binomial distribution B(m, 8 − k, E/[S − I − B]), for $0 \le m \le 8 - k$.

These results give the distribution of the number p of bomber-loads of bombs dropped as B(p, 8, [S − D − I − B]/S), and the distribution P(n) of the total number n of attack packages destroyed as

$$P(n) = \sum_{j=0}^{n} B(n-j, 8, p_1)\, B(j, 8-n+j, p_2)$$

where $p_1 = (D + I + B)/S$, and $p_2 = E/(S - D - I - B)$. With this information, one can plot the statistics of bomb units dropped and Red packages destroyed (each high-value package delivers three bomb units). Figures 4-2 and 4-3 show the results for S = 60, D = 20, I = E = 10, and B = 5.

Figure 4-2. Probability Distribution of Bomb Units Dropped

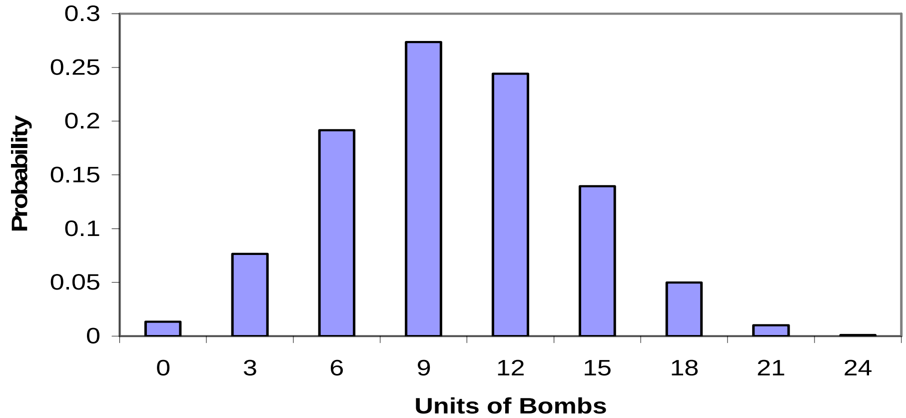
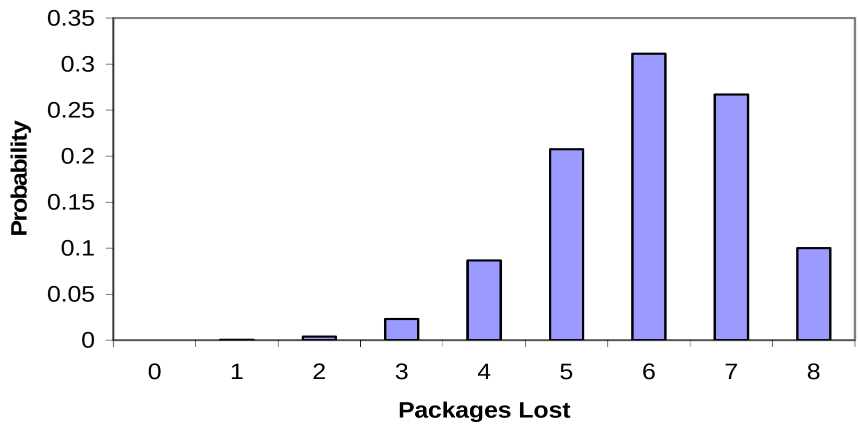


Figure 4-3. Probability Distribution of Red Packages Lost



By sacrificing eight low-value packages, Red has 72 percent confidence of delivering more bomb units than with the simultaneous assault now assumed in SLAACM, and a 90 percent confidence of losing fewer high-value packages (these marginal statistics may not, of course, occur together).

APPROACH 2

As discussed above, we are interested in determining if Red can systematically phase his attack to maximize the success of his high-value attack packages. We assume here, as above, that both the Red attack and the Blue defense can be time phased within the engagement period, and each four-ship Blue flight can engage and defeat only one 12-ship Red package. We further assume that the Red attack is numerous, and Red is actually time phasing his high-value packages within a large number of low-value packages, such that Blue defense flights will engage the high-value Red packages available immediately on arrival and will engage the lower value packages if no high-value packages are available. All Blue defense flights will be absorbed by low-value Red packages whenever high-value packages are not available.

Consider the example in which eight Blue flights can arrive during a single engagement. We assume the engagement is divided into equal periods corresponding to the number of Blue flights, eight in this example. Blue can distribute his flights, and Red can distribute his high-value packages arbitrarily among the eight periods. Because each Blue flight engages only one Red package, Blue can kill (and Red can lose) a maximum of eight high-value Red packages.

We wish to determine if there is a way to phase the Blue arrivals to ensure maximum kills of high-value Red packages. Our assumptions are as follows:

- Blue is invincible and kills Red packages with 100 percent probability.
- Blue flights can preferentially detect and attack high-value Red packages.
- All Blue flights will be committed (or consumed) in each period either by high- or low-value Red packages.
- Red must commit all of his high-value packages during the engagement.

Analysis

By inspection we can deduce that Blue can guarantee at least one high-value package kill by distributing his arrivals equally. Beyond that, the results are not obvious.

To address the Blue and Red options, we need to determine the number of ways Blue and Red can distribute eight flights over eight time slots. This is a multinomial, combination-permutation in which order is important among different quantities, e.g., $(1, 2) \neq (2,1)$, but not important among multiple occurrences of the same quantities, e.g., $(2_1, 2_2) = (2_2, 2_1)$. We first demonstrate a calculation method using four balls in four bins, and later apply the method to eight flights in eight time slots.

Four Balls Distributed Among Four Bins

First, we note that there are 4! ways to distribute four distinguishable items, such as 4 colored balls among 4 bins, 1 to a bin. In our problem, however, the 0, 1, 2, 3, and 4 counts of balls in a bin are distinguishable, but the balls themselves and multiple occurrences of the counts are indistinguishable. The choice of placing 0, 1, 2, 3, or 4 balls in a given bin restricts the options for the remaining bins. For example, if 4 balls are placed in any 1 bin, the rest of the bins must hold 0. (This is same as having 1 red ball and 3 blue balls.) The order of the bin containing the 4 balls is significant, so there are 4 ways to distribute the 4-ball set among the 4 bins. There is only 1! way to arrange the single bin containing 4 indistinguishable balls. If the 0s were distinguishable, there would be 3! ways to distribute them among 3 bins, but we do not distinguish differences in order among the bins containing 0s. When we reduce the

4! maximum options to account for the indistinguishable bins containing the same counts of balls, i.e., the three 0s, we get 4!/(3! * 1!) = 4 unique distributions. Table 4-1 confirms this by showing the complete set of 4 unique states.

Table 4-1. State Count for Four Balls in a Given Bin

Distribution count	Bins			
	1	2	3	4
1	4	0	0	0
2	0	4	0	0
3	0	0	4	0
4	0	0	0	4

To determine the total number of unique distributions for 4 balls in 4 bins, we need to identify all the positional distributions of unique ball counts. This we do by hand. Table 4-2 shows the unique distributions for 4 balls in 4 bins and the corresponding calculations of state counts.

Table 4-2. Unique Distributions and State Counts for Four Balls in Four Bins

	Bins				Distribution counts	Distribution counts
	1	2	3	4		
Unique distributions	1	1	1	1	4!/4!	1
	2	1	1	0	4!/(1!*2!*1!)	12
	2	2	0	0	4!/(2!*2!)	6
	3	1	0	0	4!/(1!*1!*2!)	12
	4	0	0	0	4!/(1!*3!)	4
					Total	35

As a check on the method, Table 4-3 shows the detailed state enumeration for 4 balls in 4 bins.

Table 4-3. State Enumeration for Four Balls in Four Bins

	Bins				Distribution counts
	1	2	3	4	
States	1	1	1	1	1
	2	1	1	0	12
	2	1	0	1	
	2	0	1	1	
	1	2	1	0	
	1	2	0	1	
	0	2	1	1	
	1	1	2	0	
	1	0	2	1	
	0	1	2	1	
	1	1	0	2	
	1	0	1	2	
	0	1	1	2	
	2	2	0	0	6
	2	0	2	0	
	2	0	0	2	
	0	2	2	0	
	0	2	0	2	
	0	0	2	2	
	3	1	0	0	12
	3	0	1	0	
	3	0	0	1	
	1	3	0	0	
	0	3	1	0	
	0	3	0	1	
	1	0	3	0	
	0	1	3	0	
	0	0	3	1	
	1	0	0	3	
	0	1	0	3	
	0	0	1	3	
	4	0	0	0	4
	0	4	0	0	
	0	0	4	0	
	0	0	0	4	
	Total				35

EIGHT FLIGHTS IN EIGHT TIME SLOTS

Now we consider eight flights distributed among eight time slots. The unique distributions and distribution counts are shown in Table 4-4.

Table 4-4. Unique Distributions and State Counts for Eight Flights in Eight Time Slots

	Time slot								Distribution counts	Distribution counts
	1	2	3	4	5	6	7	8		
Unique distributions	1	1	1	1	1	1	1	1	8!/8!	1
	2	1	1	1	1	1	1	0	8!/(6! 1! 1!)	56
	2	2	1	1	1	1	0	0	8!/(4! 2! 2!)	420
	2	2	2	1	1	0	0	0	8!/(3! 3! 2!)	560
	2	2	2	2	0	0	0	0	8!/(4! 4!)	70
	3	1	1	1	1	1	0	0	8!/(5! 2! 1!)	168
	3	2	1	1	1	0	0	0	8!/(3! 3! 1! 1!)	1,120
	3	2	2	1	0	0	0	0	8!/(4! 2! 1! 1!)	840
	3	3	1	1	0	0	0	0	8!/(4! 2! 2!)	420
	3	3	2	0	0	0	0	0	8!/(5! 2! 1!)	168
	4	1	1	1	1	0	0	0	8!/(4! 3! 1!)	280
	4	2	1	1	0	0	0	0	8!/(4! 2! 1! 1!)	840
	4	2	2	0	0	0	0	0	8!/(5! 2! 1!)	168
	4	3	1	0	0	0	0	0	8!/(5! 1! 1! 1!)	336
	4	4	0	0	0	0	0	0	8!/(6! 2!)	28
	5	1	1	1	0	0	0	0	8!/(4! 3! 1!)	280
	5	2	1	0	0	0	0	0	8!/(5! 1! 1! 1!)	336
	5	3	0	0	0	0	0	0	8!/(6! 1! 1!)	56
	6	1	1	0	0	0	0	0	8!/(5! 2! 1!)	168
	6	2	0	0	0	0	0	0	8!/(6! 1! 1!)	56
	7	1	0	0	0	0	0	0	8!/(6! 1! 1!)	56
	8	0	0	0	0	0	0	0	8!/(7! 1!)	8
									Total	6,435

From Table 4-4 we see that there are 6,435 ways to distribute eight Blue flights (and eight Red packages) among eight time slots. Now we want to see the potential impact of this on Blue payoff.

To find the potential payoff for Blue, we want to find the probabilities of Blue (and Red) experiencing 0 through eight flights in a given slot. Table 4-5 shows the flight count occurrences for each unique distribution.

Table 4-5. Flight Count Occurrences per Unique Distribution

	Unique state distributions, by time slot								Flight count occurrences per unique distribution, by time slot									Distribution counts
	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8	
	1	1	1	1	1	1	1	1		8								1
	2	1	1	1	1	1	1	0	1	6	1							56
	2	2	1	1	1	1	0	0	2	4	2							420
	2	2	2	1	1	0	0	0	3	2	3							560
	2	2	2	2	0	0	0	0	4		4							70
	3	1	1	1	1	1	0	0	2	5		1						168
	3	2	1	1	1	0	0	0	3	3	1	1						1120
	3	2	2	1	0	0	0	0	4	1	2	1						840
	3	3	1	1	0	0	0	0	4	2		2						420
	3	3	2	0	0	0	0	0	5		1	2						168
Unique distributions	4	1	1	1	1	0	0	0	3	4			1					280
	4	2	1	1	0	0	0	0	4	2	1		1					840
	4	2	2	0	0	0	0	0	5		2		1					168
	4	3	1	0	0	0	0	0	5	1		1	1					336
	4	4	0	0	0	0	0	0	6				2					28
	5	1	1	1	0	0	0	0	4	3				1				280
	5	2	1	0	0	0	0	0	5	1	1			1				336
	5	3	0	0	0	0	0	0	6			1		1				56
	6	1	1	0	0	0	0	0	5	2					1			168
	6	2	0	0	0	0	0	0	6		1				1			56
	7	1	0	0	0	0	0	0	6	1						1		56
	8	0	0	0	0	0	0	0	7								1	8
																		6,435

Dividing the flight count occurrences in a given row of Table 4-5 by 8 gives the conditional probability of the count occurrences given the distribution corresponding to the row. Dividing the distribution counts by the total count gives the probability of each distribution. Multiplying the distribution probabilities by the conditional count occurrence probabilities generates the probabilities for count occurrences shown in Table 4-6.

Table 4-6. Flight Occurrences Probabilities for Eight Flights in Eight Time Slots

Distribution probabilities	Flight count probabilities								
	0	1	2	3	4	5	6	7	8
0.00016	0.0000	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.00870	0.0011	0.0065	0.0011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.06527	0.0163	0.0326	0.0163	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.08702	0.0326	0.0218	0.0326	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.01088	0.0054	0.0000	0.0054	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.02611	0.0065	0.0163	0.0000	0.0033	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4-6. Flight Occurrences Probabilities for Eight Flights in Eight Time Slots

Distribution probabilities	Flight count probabilities								
	0	1	2	3	4	5	6	7	8
0.17405	0.0653	0.0653	0.0218	0.0218	0.0000	0.0000	0.0000	0.0000	0.0000
0.13054	0.0653	0.0163	0.0326	0.0163	0.0000	0.0000	0.0000	0.0000	0.0000
0.06527	0.0326	0.0163	0.0000	0.0163	0.0000	0.0000	0.0000	0.0000	0.0000
0.02611	0.0163	0.0000	0.0033	0.0065	0.0000	0.0000	0.0000	0.0000	0.0000
0.04351	0.0163	0.0218	0.0000	0.0000	0.0054	0.0000	0.0000	0.0000	0.0000
0.13054	0.0653	0.0326	0.0163	0.0000	0.0163	0.0000	0.0000	0.0000	0.0000
0.02611	0.0163	0.0000	0.0065	0.0000	0.0033	0.0000	0.0000	0.0000	0.0000
0.05221	0.0326	0.0065	0.0000	0.0065	0.0065	0.0000	0.0000	0.0000	0.0000
0.00435	0.0033	0.0000	0.0000	0.0000	0.0011	0.0000	0.0000	0.0000	0.0000
0.04351	0.0218	0.0163	0.0000	0.0000	0.0000	0.0054	0.0000	0.0000	0.0000
0.05221	0.0326	0.0065	0.0065	0.0000	0.0000	0.0065	0.0000	0.0000	0.0000
0.00870	0.0065	0.0000	0.0000	0.0011	0.0000	0.0011	0.0000	0.0000	0.0000
0.02611	0.0163	0.0065	0.0000	0.0000	0.0000	0.0000	0.0033	0.0000	0.0000
0.00870	0.0065	0.0000	0.0011	0.0000	0.0000	0.0000	0.0011	0.0000	0.0000
0.00870	0.0065	0.0011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0011	0.0000
0.00124	0.0011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002
Totals	0.4667	0.2667	0.1436	0.0718	0.0326	0.0131	0.0044	0.0011	0.0002

Table 4-7 repeats the occurrence probabilities from Table 4-6 and adds the Blue payoff probabilities, which are calculated by multiplying the probabilities by their corresponding flight counts. The expected payoff shown in Table 4-7 is 1 compared to the maximum payoff of 8.

Table 4-7. Probabilities and Payoff for Eight Flights in Eight Time Slots

Probability per state									
0	1	2	3	4	5	6	7	8	Total
0.4667	0.2667	0.1436	0.0718	0.0326	0.0131	0.0044	0.0011	0.0002	1.000

Blue payoff/Red loss probability									
0	1	2	3	4	5	6	7	8	Total
0	0.2667	0.2872	0.2154	0.1305	0.0653	0.0261	0.0076	0.0012	1.000

LOOK-AHEAD TARGET IDENTIFICATION

The case in which Blue can see beyond the current period, and thus avoid wasting flights on low-value packages, can be modeled simply by reducing the number of time slots. Using the same approach followed above, we find that the expected payoff for eight flights in four time slots is 2, and the expected value for eight flights in two time slots is 4. The simplicity of these numbers suggests that there may be a more fundamental way to derive them than we have applied. That said, the results demonstrate the value of long-range identification to Blue and the value of preventing such identification to Red.

Approach 2 Summary

We note that these results represent the case in which Blue and Red each randomly select one of an exhaustive set of strategies. *It does not represent the case in which Blues and Reds arrive randomly*. The case of random phasing of Blue arrivals, and random arrival of the Red attack, is considered in Approach 1.

At this time, we do not know of any gaming strategy that can reliably improve the results for either Blue or Red.

Chapter 5

Planned SLAACM Development

This chapter addresses future development for SLAACM. Specifically, we recommend the following:

- *Implement a unique CAP factor for every Blue aircraft type.* Currently, we use a single combat air patrol (CAP) factor for all the Blue aircraft selected as CAP aircraft. Because basing, aircraft range, and other physical and military considerations vary among aircraft types, we recommend adding to SLAACM individual type-based CAP factors.

- *Automate the addition of Red and Blue aircraft types.* The current SLAACM spreadsheets are configured for fixed maximum numbers of aircraft types. Changing the maximum number of types currently requires reprogramming of the model. We recommend implementing programming to automate the additional of Blue or Red aircraft types easily.

- *Provide additional output charts and data displays.* Although almost all the output statistics a user of SLAACM would want to see are calculated within the model, the tabular and graphical outputs are currently limited to the order of battle of Blue, Red, and Green aircraft types on each day; the number of bombs dropped; and losses on both sides by aircraft type. Based on experience gained through conducting analyses and on feedback from reviewers of model results, it has become clear that additional output would be beneficial. Specifically, we recommend adding the Red engagement packages sent each "day" and the corresponding Blue packages that intercept them, as well as improving displays of delivered munitions to include the explosive power of the munitions, identification of "smart" and "dumb" bombs dropped, and counts of cruise missiles and other identifiable payloads.

- *Further analyze dispersion in campaigns.* At the end of this year's task, we began analyzing the dispersion of Blue and Red losses day-by-day and showing a campaign total loss and total standard deviation for each Blue type. In general, Blue losses are relatively dispersive, with coefficients of determination 25 percent or more in many cases. Red losses typically are not at all dispersive due the Blue kill rate dominance in cases studied, but they will become more dispersive as the forces approach more equal strength. Propagating engagement dispersions through a campaign typically results in computational difficulties due to the explosive growth of analytical states (i.e., follow-on engagement scenarios). We believe methods may be found to bound the results and recommend conducting research to investigate tractable methods to use the dispersion data to generate confidence intervals on campaign results.

- *Explore the acquisition of more detailed information from other sources in order to generate richer engagement models.* We recommend continuing efforts to engage combat modelers and analysts within organizations such as Air Force Studies and Analysis to obtain results that can better calibrate our model inputs. Those data can be helpful in obtaining better insight into things such as two-phase kill models.

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The report describes analytical tasks performed for OSD/PA&E TACAIR during the period from June 2005 through July 2006. The report describes use of and extension to the stochastic Lanchester Air-to-Air Campaign model (SLAACM) probabilistic model of campaigns for air superiority between two opponents, described in previous reports. The current report describes the current model structure and battle management analysis features. In a separate section, the report discusses the mathematics of battle management and SLAACM integer programming capability. The report also contains mathematical analyses of strategies for defensive combat air patrol time phasing. The final section discusses proposed future model development.

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