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6. AUTHOR(S)
William Moran
Department of Electrical and Electronics Engineering
University of Melbourne, VIC Australia

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
UC San Diego
La Jolla CA 92093-0111

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9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Dr. Jon Sjogren
AFOSR/NM
875 N Randolph Str, Suite 325
Arlington, VA 22203

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14. ABSTRACT
The discrete Heisenberg-Weyl group provides a unifying framework for a number of important sequences significant in the construction of phase coded radar waveforms, in communications as spreading sequences, and in the theory of error correcting codes . Among the sequences which can be associated with the Heisenberg-Weyl group are the first and second order Reed-Muller codes, the Golay or Welti sequences, and the Kerdock and Preparata codes , which are non-linear binary error correcting codes containing more codewords for a given minimum distance than any linear code. The Kerdock codes are associated with decomposition of the Heisenberg-Weyl group into disjoint maximally commutative subgroups. It is a surprising fact that a certain general class of Golay sequences exist within the Kerdock codes. This had had previously been noted by Davis and Jedwab. Golay sequences are pairs of sequences of unimodular complex numbers with the property that the sum of their individual auto- correlation functions forms delta spike or thumb tack. These sequences have found application in the construction of radar waveforms and in modulation schemes for communications.

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Final Report

AFOSR FA9550-04-1-0431
DARPA S-505

Investigator: **William Moran and Arthur Robert Calderbank**
Project Title: **Coding Theory Information Theory and Radar**
Location of Project: **Univ. of Melbourne and Princeton Univ.**

Source of Support: **Defense Advanced Research Projects Agency and Air Force Office of Scientific Research**
Total Award period Covered: **07/01/04 to 09/30/05**
Total Award Amount: **$117,800**

1. Participants

- **Bill Moran (University of Melbourne) and Stephen Howard (DSTO, Australia)** have been involved in the area of electronic surveillance and radar for the past ten years. Within the Defence Science and Technology Organisation Dr. Howard has led the research effort into the development of algorithms in all areas of electronic surveillance, including radar pulse train deinterleaving, precision radar parameter estimation and tracking, estimation of radar intra-pulse modulation and advanced geolocation techniques. Since 2003, he has led the DSTO long range research program in radar resource management and waveform design. In collaboration with Dr. Moran, he has provided rigorous methods for the design of libraries of waveforms, or more generally libraries of radar modalities, for detection, identification and tracking application.
- **Robert Calderbank** is an expert on methods of limiting the end to end complexity of signal processing for multiple antenna systems so that it is close to that of single antenna systems. In collaboration with Dr. Howard and Dr. Moran, he has provided rigorous methods for the design of libraries of waveforms, or more generally libraries of radar modalities, for detection, identification and tracking application. When different waveforms can be transmitted independently and coherently from different array elements, waveform selection is based on a generalization of the standard radar ambiguity function that describes the response of the radar to a point target at a fixed range and Doppler.

2. Activities and Findings:

- The major research focus was the relationship between new ideas in classical and quantum error correcting codes and the development of an information theory for radar.

- We found that the m-dimensional discrete Heisenberg-Weyl group provides a unifying framework for a number of important sequences significant in the construction of phase coded radar waveforms, in communications as spreading sequences, and in the theory of error correcting codes [1, 2] . Among the sequences which can be associated with the Heisenberg-Weyl group are the first and second order Reed-Muller codes, the Golay or Welti sequences, and the Kerdock and Preparata codes [7], which are non-linear binary error correcting codes containing more codewords for a given minimum distance than any linear code. The Kerdock codes are associated with decomposition of the Heisenberg-Weyl group into disjoint maximally commutative subgroups [1, 2, 6, and 7]. It is a surprising fact that a certain general class of Golay sequences exist within the Kerdock codes. This had had previously been noted by Davis and Jedwab [8].

Golay sequences [3–5, 8] are pairs of sequences of unimodular complex numbers with the property that the sum of their individual auto-correlation functions forms delta spike or thumb tack. These sequences have found application in the construction of radar waveforms and in modulation schemes for communications.

- This project led to the creation of a new cross-functional team (Robert Calderbank, Stephen Howard (DSTO, Australia, and Bill Moran (Melbourne) spanning the application domain, classical harmonic analysis, algebra and combinatorics. The capabilities developed by this team as part of this project have been integrated into a larger multi-institutional response to the task specific DARPA BAA 05-19.

- The results of this project were shared with the mainstream signal processing community at ICASSP'05 (Philadelphia) and with the defense signal processing community at DASP'05 (Homestead, Utah).

References

[1] S. D. Howard, A. R. Calderbank, W. Moran, H. A. Schmitt and C. O. Savage. Relationships Between Radar Ambiguity and Coding Theory, Proc. IEEE International Conference on Acoustics, Speech and Signal Processing, 2005, March 2005.

[2] S. D. Howard, A. R. Calderbank and W. Moran. The Discrete Finite Heisenberg Group in Radar and Communications, to appear in EURASIP special issue, 2005.

[3] M. J. E. Golay, Complementary Series IRE Transactions on Information Theory, 7(12):82–87, April 1961.

[4] G. R. Welti Quaternary codes for pulsed radar IRE Trans. Inf. Theory, 6:400-408, 1960.

[5] S. Z. Budisin, B. M. Popovic and I. M. Indjin, Designing Radar Signals Using Complementary Sequences. IEEE Transactions on Aerospace and Electronic Systems 21(2):170–9, March 1985.

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[7] A. R. Calderbank, P. J. Cameron, W. M. Kantor and J. J. Seidel, Z4-Kerdock Codes, Orthogonal Spreads, and Extremal Euclidian Line-Sets Proc. London Math. Soc., 3(75):436–480, 1997.
[8] J. A. Davis and J. Jedwab Peak-to-Mean Power Control in OFDM, Golay Complementary Sequences, and Reed-Muller Codes IEEE Trans. Info. Theory, 45(7):2397-2417, November 1999.

3. Publications and Products

[1] S. D. Howard, A. R. Calderbank, W. Moran, H. A. Schmitt and C. O. Savage. Relationships Between Radar Ambiguity and Coding Theory, Proc. IEEE International Conference on Acoustics, Speech and Signal Processing, 2005, March 2005.

[2] S. D. Howard, A. R. Calderbank and W. Moran. The Discrete Finite Heisenberg Group in Radar and Communications, to appear in EURASIP special issue, 2005.

4. Application Value

This project is the starting point for the development of algorithms to significantly expand existing radar functionality. The appropriate advanced hardware functionality exists in the NRL Chesapeake Bay Detachment radar, and at other sites in the US and in Australia. This facility is capable of high levels of temporal and spatial diversity of waveforms and polarization, and the mode of operation can be rapidly scheduled. This project supports development of techniques to fully exploit the diversity and flexibility provided by this advanced functionality through adaptive multidimensional waveform and polarization scheduling based on environment modeling and tracking, as well as multi-dimensional adaptive processing of the returns.