

Detecting Periodic Components
in a White Gaussian Time Series

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Detecting Periodic Components in a White Gaussian Time Series

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SUMMARY

A family of tests for periodic components in a white Gaussian series is proposed. The test is based on a statistic which is proportional to the ratio of the maximum periodogram to the trimmed mean of the periodograms. The asymptotic distribution of the statistic is obtained. It is shown that the test proposed and Fisher's test have the same asymptotic powers at the alternative hypotheses that the series contains a single periodic component at a non-zero Fourier frequency. The tests are applied to detect the eigenfrequencies of the Earth. The proposed test detects some peaks which Fisher's test fails to detect.

Keywords: Detecting periodic components; Fourier transform; Periodogram; Eigenvibration of the Earth.

1. Introduction

In this study, we consider the situation where we have T consecutive observations $X(t), t=0, 1, \dots, T-1$ on a time series. We are interested in detecting periodic components in the series. The periodogram has been used to search for hidden periodicities and, naturally, it became a tool for testing the presence of periodic components. Fisher (1929) introduced a test for periodicity, which is based on the periodograms at the Fourier frequencies; the test statistic is the ratio of the maximum of the periodograms to the total sum of the periodograms. Fisher also gave the exact distribution of the test statistic under the null hypothesis that the series is a white Gaussian series. The asymptotical distribution of the test statistics was given in Bloomfield (1976). Fisher's test was used in Nowroozi (1965), (1966) for detecting the eigenfrequencies of the Earth. The test was also mentioned in many books concerning time series analysis. Hartley (1949)

proposed a test based on the ratio of the maximum periodograms to the residual sum of squares, where the residuals are obtained by fitting a sinusoidal wave to the series.

In practice, we might suspect that the series consists of several periodical components; in this case, the periodogram will have multiple peaks. Multiple peaks could also be caused by the leakages from nearby peaks, by the harmonics of a fundamental frequency or by a decaying sinusoidal wave such as the eigenvibration of the Earth (Bolt and Brillinger (1979)). In these situations, we are often not satisfied just with rejecting the null hypothesis of no periodic component but are also interested in detecting all significant peaks in the periodograms. As indicated by Whittle (1954), Fisher's test is quite conservative in detecting multiple peaks. Shimshoni (1971) also observed that the method was unable to detect some periodic components in the study of the eigenvibrations of the Earth.

For detecting the presence of multiple peaks, various methods were proposed and studied. Some of the methods were developed for testing the outliers in exponential samples. We remark that these two testing problems are closely related. Whittle (1954) suggested removing the significant periodic components from the series and applying Fisher's test to the residual series. It might not be easy to do so when the periodic components are not pure sinusoid waves, such as in the case of sinusoid waves with non-constant amplitudes or phases. Shimshoni (1971) suggested using the ratio of the r -th largest periodogram to the sum of periodograms. The main difficulties of this approach are: (1) This method implicitly assumes that the frequencies of the periodic components are at the Fourier frequencies; this is usually not true in practice. (2) The number of the periodic components must be known. (3) If the number of the periodical components is smaller than r , then this test will have very small chance to reject the null hypothesis even when the series does contain some significant periodic components. Lewis and Fieller (1979) studied another test which is based on the ratio of the sum of the r biggest periodograms to the total sum of the periodograms. This test will reject the null hypothesis when there is only one huge peak in the periodograms. Therefore, the test is not able to tell us which peaks are significant. Another disadvantage of this test is that it loses significant amounts of power when the number of periodic com-

ponents contained in the series is different from r .

Likes (1966) and Kabe (1970) studied the test statistic which is the ratio of the difference between the biggest and the r -th biggest periodograms to the biggest periodogram. Siegel (1980) proposed a method based on the sum of the periodograms (normalized by the total sum of periodograms) which are bigger than a portion of the critical value of Fisher's test. One must select a proper portion in order to apply this test. The best choice is dependent on (and is sensitive to) the number of periodic components in the series. These two methods also share some of the difficulties of the tests mentioned earlier.

In the next section, we describe some basic notations and properties of the Fourier transform and periodograms. Fisher's test and its asymptotical distribution are reviewed in Section 3. In Section 4, we propose a test statistic which is proportional to the ratio of the biggest periodogram to the trimmed mean of the periodograms. The asymptotic distribution of the test statistic is also derived. In Section 5, we derive the asymptotic power functions of the tests against the alternative hypothesis that the series contains a sinusoidal wave at one of the Fourier frequencies. We show that, asymptotically, the test proposed has the same power as Fisher's test has. In Section 6, we apply the proposed test to detect the eigenfrequencies of the Earth's free vibrations. We show that our method can detect some of the peaks which Fisher's method fails to detect. The asymptotic results are supported by simulations.

In this research, we are concerned with the case that the underlying series is a white series. The case of non-white series was considered in Hannan(1961) and Whittle(1952, 1954).

2. Basic notation and properties of periodogram

The data we consider here are a sequence of observations taken at equal time intervals. The series consists of a deterministic series and a noise series:

$$X(t) = \mu + S(t) + \epsilon(t) \quad t=0,1, \dots, T-1.. \quad (2.1)$$

Here $\epsilon(t)$ is a white Gaussian series with means zero and variances σ^2 and the deterministic part

$$S(t) = \sum_{k=1}^K A_k \cos(\gamma_k t + \phi_k) \quad (2.2)$$

consists of K sinusoidal waves at frequencies $\gamma_k \neq 0$ with amplitudes A_k and phases ϕ_k ; the frequencies are not necessarily the Fourier frequencies. Without loss of generality, we assume $\mu=0$. We are interested in testing for the existence of $S(t)$. The null hypothesis can be formulated as $H_0: S(t) \equiv 0$. $X(t)$ is a Gaussian white series under H_0 .

The Fourier transform of the series $X(t)$ at frequency ω is

$$d_X^{(T)}(\omega) = \sum_{t=0}^{T-1} X(t) \exp(-i\omega t). \quad (2.3)$$

The Fourier transforms are usually evaluated by the fast Fourier transform at the Fourier frequencies $\omega_j = 2\pi j/T$. The Fourier transforms at the Fourier frequencies are referred to as the discrete Fourier transform of $X(t)$. The Fourier transforms $d_S^{(T)}$ and $d_X^{(T)}$ have corresponding definitions. The periodogram of the series is defined by

$$I^X(\omega) = d_X^{(T)}(\omega) d_X^{(T)*}(-\omega) / (2\pi T). \quad (2.4)$$

The periodograms $I^c(\omega)$ and $I^S(\omega)$ are defined in a similar way. Under the null hypothesis, $I^X(\omega) = I^c(\omega)$ and the periodograms $I^X(\omega_j)$ at the Fourier frequencies, $\omega_j = 2\pi j/T$, $j=1, 2, \dots, (T/2)-1$, where T is an even number (for odd T , $j=1, 2, \dots, [T/2]$), are identically independently distributed according to the exponential distribution with mean $\lambda = \sigma^2 / (2\pi)$.

For the series with deterministic part $S(t) \neq 0$, $d_X^{(T)}(\omega) = d_c^{(T)}(\omega) + d_S^{(T)}$. $2I^X(\omega) / \sigma^2$ has a noncentral chi-square distribution with two degree of freedom; the noncentrality parameter is $2I^S(\omega) / \sigma^2$. For detail discussion of the properties of the Fourier transform and periodograms, the readers are referred to Brillinger (1975).

3. Fisher's test and its asymptotic distribution

For testing the existence of periodic components, Fisher (1929) proposed a test based on the test statistic

$$F_T = \max_{1 \leq j \leq n} I^X(\omega_j) / \sum_{j=1}^n I^X(\omega_j). \quad (3.1)$$

Here $n = (T/2)-1$ for even T and $n = (T-1)/2$ for odd T . In that paper, Fisher gave the exact distribution of F_T under the null hypothesis. Bloomfield (1976) derived the asymptotic

distribution of the statistic $Z_T = nF_T - \log n$,

$$P(Z_T < z) \approx \exp\{-\exp(-z)\} . \quad (3.2)$$

Since $\hat{\lambda} = n^{-1} \sum I(\omega_j)$ converges to λ in probability and $\hat{\lambda}$ has variance of order n^{-1} ,

$$P(Z_T < z) = P(Z'_T < z) + O(n^{-1/2} \log n), \quad (3.3)$$

where z is any fixed constant and $Z'_T = \max\{I^X(\omega_j)/\lambda\} - \log n$. Also

$$P(Z'_T < z) = 1 - [1 - \exp(-z)/n]^n = \exp\{-\exp(-z)\} + O(n^{-1}). \quad (3.4)$$

So we can write (3.2) more precisely,

$$P(Z_T < z) = \exp\{-\exp(-z)\} + O(n^{-1/2} \log n). \quad (3.5)$$

Let $\tilde{I}(\omega_j) = I^X(\omega_j)/\hat{\lambda}$ be the normalized periodograms, then $nF_T = \max \tilde{I}(\omega_j)$. We see that Fisher's test statistic is proportional to the maximum of the normalized periodograms. When the series contains periodic components, the periodograms at frequencies close to the frequencies of these components have large magnitude and these periodograms can be viewed as outliers in an exponential sample. From this, it is easy to understand the weakness of Fisher's test. Fisher's test uses the sample mean to normalize the periodograms; however, the sample mean is sensitive to outliers. The sample mean tends to be larger than the true mean when the series contains periodic components; therefore, $\tilde{I}(\omega_j)$ tends to be smaller than $I(\omega_j)/\lambda$. So, the power of Fisher's test will be smaller than the power of the test based on the maximum of $I(\omega_j)/\lambda$. This suggests that we consider tests based on the maximum of periodograms normalized by a robust estimate of λ .

4. The proposed test and its asymptotical distribution

The test proposed here based on the test statistic

$$R_T(\beta) = I_n / \sum_{i=1}^{\lfloor n\beta \rfloor} I_i . \quad (4.1)$$

where n is defined in Section 3, and $I_1 < I_2 < \dots < I_n$ are the order statistics of the periodograms. β is a constant between 0 and 1 which determines the proportion of periodograms trimmed. In most case, $\beta=0.9$ or 0.95 will give good result. The performance of the test is not very sensitive to the choice of β . Letting $\beta=1$, we obtain Fisher's test as an extreme case. In the

following discussion, we assume $0 < \beta < 1$ and use the notation R_T instead of $R_T(\beta)$. The denominator in (4.1) is proportional to the trimmed mean

$$\hat{\lambda}_\beta = [n\beta]^{-1} \sum_{i=1}^{[n\beta]} I_i. \quad (4.2)$$

It is well known (cf Lehmann (1983)) that $\hat{\lambda}_\beta$ converges in mean square to

$$\int_0^{\xi(\beta)} \frac{x}{\lambda} \exp(-x/\lambda) dx = c\lambda, \quad (4.3)$$

where $\xi(\beta) = -\log(1-\beta)$ and $c = \beta + (1-\beta)\log(1-\beta)$. Therefore, $\hat{\lambda}_\beta/c$ is an asymptotically unbiased estimate of λ . The exact unbiased factor can be found in Kimber (1983).

Following the argument of the last section and letting $\tilde{Z}_T = c[n\beta]R_T - \log n$, we have

$$P(\tilde{Z}_T < z) = \exp\{-\exp(-z)\} + O\{(n\beta)^{-1/2}\log n\} \quad (4.4)$$

asymptotically. This shows that Z_T and \tilde{Z}_T have the same asymptotic distribution in the null case.

Similarly, we can obtain the asymptotic distribution of

$$U_T(r) = I_{n-r+1} / \sum_{i=1}^n I_i, \quad (4.5)$$

which is

$$P\{nU_T(r) - \log(n-r+1) < z\} = \exp\{-\exp(-z)\} + O\{n^{-1/2}\log(n-r+1)\}. \quad (4.6)$$

The test $U_T(r)$ was used for testing the existence of r periodic components Shimshoni (1971). The exact null distribution of $U_T(r)$ was derived in Grenander and Rosenblatt (1957). However, from the argument in Section 3, it is clear that we should at least exclude the $r-1$ biggest periodograms from the denominator of (4.5). This leads us to consider

$$V_T(r) = I_{n-r+1} / \sum_{i=1}^{n-r+1} I_i, \quad (4.7)$$

which has asymptotic distribution

$$P\{nV_T(r) - \log(n-r+1) < z\} = \exp\{-\exp(-z)\} + O\{(n-r+1)^{-1/2}\log(n-r+1)\}. \quad (4.8)$$

5. Asymptotic power functions of the tests

In this section we derive the asymptotic power functions of the tests described in the previous sections. Without loss of generality, we assume $\lambda=1$ in the following discussion. We first consider the alternatives

$$H_a: S(t) = a \cos(\gamma_0 t). \quad (5.1)$$

Here $\gamma_0 = 2\pi j_0 / T$, $1 \leq j_0 \leq n$ is an arbitrary integer. The discrete Fourier transform of $S(t)$ is equal to $aT/2$ at frequency γ_0 and is equal to 0 at other frequencies. This implies that the periodograms at frequencies other than γ_0 have the same distribution under both hypotheses. Under the alternative hypothesis, H_a , $2I^X(\gamma_0)$ has a noncentral chi-square distribution with 2 degrees of freedom and the noncentrality parameter $2A = 2a^2T/8\pi$. The asymptotic power functions of the tests are obtained in the following theorem. Because I_n increases at rate $\log n$, we let A increase at rate $\log n$.

Theorem. The power function of the test R_T and F_T against the alternatives H_a with $A = B + \log n = a^2T/8\pi$ are, asymptotically,

$$\alpha + (1-\alpha)P\{I^X(\gamma_0) > nf_0\}, \quad (5.2)$$

where α is the level of significance of the tests, $nf_0 = -\log(-\log(1-\alpha)) + \log n$ and $2I^X(\gamma_0)$ has a noncentral chi-square distribution with two degrees of freedom and noncentral parameter $2A$.

Proof. Let \tilde{f}_0 and \tilde{r}_0 be the respective critical values of the test F_T and R_T at significance level α , then $\tilde{nf}_0 = nf_0 + O(n^{-1/2}\log n)$ and $\tilde{r}_0 c[n\beta] = r_0 c[n\beta] + O\{(n\beta)^{-1/2}\log n\}$, where

$$\tilde{nf}_0 = nf_0 + O(n^{-1/2}\log n) = r_0 c[n\beta] = -\log\{-\log(1-\alpha)\}. \quad (5.3)$$

Since

$$P\{I^X(\gamma_0) < s\} \geq P\{I^c(\gamma_0) < B + \log n - s\} = 1 + O(1/n), \quad (5.4)$$

for any fixed constant s , the probability that $I(\gamma_0)$ has a rank (among all periodograms) smaller than $[n\beta]$ is of order $O(1/n)$ and

$$P\left(\sum_{i=1}^{[n\beta]} I_i^X = \sum_{i=1}^{[n\beta]} I_i^c\right) = 1 + O(1/n). \quad (5.5)$$

From the same argument in deriving the asymptotic null distributions, we have

$$P(R_T > \bar{r}_0) = P(I_n^X > r_0 c[n\beta] = nf_0) + O(n^{-1/2} \log n). \quad (5.6)$$

Since $P\{I_n^\epsilon = I^\epsilon(\gamma_0)\} = 1/n$ is negligible, (5.6) is equal to

$$P\{I_n^\epsilon > nf_0 \mid I_n^\epsilon \neq I^\epsilon(\gamma_0)\} + P\{I^X(\gamma_0) > nf_0 \text{ and } I_n^X < nf_0 \mid I_n^\epsilon \neq I^\epsilon(\gamma_0)\} + O(n^{-1/2} \log n). \quad (5.7)$$

Conditional on $I_n^\epsilon \neq I^\epsilon(\gamma_0)$, the distribution of I_n^ϵ is equal to the distribution of the maximum of $n-1$ variables. So, the first conditional probability in (5.7) is equal to $\alpha + O(1/n)$. The second conditional probability is equal to

$$\int_0^{nf_0} \int_0^z P\{I^X(\gamma_0) > nf_0 \mid I^\epsilon(\gamma_0) = r\} f_1(r \mid z) f_2(z) dr dz, \quad (5.8)$$

where $f_1(r \mid z)$ is the conditional density of $I^\epsilon(\gamma_0)$ given $I_n^\epsilon = z$ and $I^\epsilon(\gamma_0) \neq I_n^\epsilon$, and $f_2(z)$ is the density of I_n^ϵ . Now (5.8) is equal to

$$\begin{aligned} & \int_{\log n}^{nf_0} \int_0^{\log n} P\{I^X(\gamma_0) > nf_0 \mid I^\epsilon(\gamma_0) = r\} f_1(r \mid z) f_2(z) dr dz + O(1/n), \\ & = \int_{\log n}^{nf_0} \int_0^{\log n} P\{I^X(\gamma_0) > nf_0 \mid I^\epsilon(\gamma_0) = r\} f_1(r) f_2(z) dr dz + O(n^{-1/2} \log n), \end{aligned} \quad (5.9)$$

where $f_1(r)$ is the unconditional density of $I^\epsilon(\gamma_0)$. The last equation holds because $f_1(r \mid z) = f_1(r) + O(1/n)$ when $z > \log n$. Therefore, the power function of the test R_T is equal to

$$\alpha + (1-\alpha)P\{I^X(\gamma_0) > nf_0\} + O(n^{-1/2} \log n). \quad (5.10)$$

The power of Fisher's test is

$$P(F_T > f_0) = P\{I_n^X > (n-1)f_0 + f_0 I^X(\gamma_0)\} + O(n^{-1/2} \log n). \quad (5.11)$$

Since $E\{I^X(\gamma_0)\}^2 = O\{(\log n)^2\}$, (5.11) is equal to

$$P(F_T > f_0) = P(I_n^X > nf_0) + O(n^{-1/2} \log n). \quad (5.12)$$

Therefore, the test R_T and Fisher's test have the same asymptotic power indicated in the theorem.

We now consider the problem of detecting r periodic components. We define the critical bound of a test (F_T , R_T or V_T) at significance level α as the product of the denominator of the test and the critical value of the test at significance value α . We say that a test detects a peak at

significance level α if the periodogram at the peak is bigger than the critical bound of the test at significance level α . According to these definitions, F_T (or R_T) will reject the null hypothesis if and only if the test detect any peak in the periodogram. The test $V_T(r)$ will reject the null hypothesis if and only if $V_T(r)$ can detect at least r peaks. The alternatives we now consider are

$$H: S(t) = \sum_{i=1}^r a_i \cos(\gamma_i t), \quad (5.13)$$

where r is a fixed number and $\gamma_i, i=1, \dots, r$ are distinct non-zero Fourier frequencies. Without loss of generality, we can assume $a_1 > \dots > a_r$. Similar to the discussion in the proof, it is easy to see that the probability of detecting the r -th peak is $\alpha + (1-\alpha)P\{I^X(\gamma_r) > n f_0\}$, which is not affected by the $r-1$ biggest peaks. In order to compare the probability with the power of the test $V_T(r)$, we assume a_1, \dots, a_{r-1} are big enough, such that the probability of missing the $r-1$ peaks is negligible. Supposing $a_r^2 T / 8\pi = B_r + \log n$, we can show that the power of the test $V_T(r)$ is also asymptotically equal to $\alpha + (1-\alpha)P\{I^X(\gamma_r) > n f_0\}$.

Remark 1. For the sake of comparison, we only consider the alternatives of pure sinusoidal waves at Fourier frequencies. However, as shown in the next section, the test R_T is much better than the tests F_T and V_T in situations where the periodic components are not at Fourier frequencies and that the amplitudes or phases are not constant.

Remark 2. $V_T(r)$ might accept the null hypothesis while there are some ($< r$) peaks bigger than the critical values. This will cause a lot of confusion in interpreting the result.

6. Detecting the Eigenvibrations of the Earth

The example we used here is the time series obtained by the long-period pendulums at the Grotta Gigante, Trieste, following the 1960 Chile earthquake (Bolt & Marussi (1962)). The sampling interval is 2 minutes and the length of the series is 2548. Bolt and Brillinger (1979) modeled the periodic components as exponentially decaying sinusoidals. 16 sets of estimates of the eigenfrequencies, damping constants, amplitudes and phases were obtained. The first 1200 values of the series are plotted in Figure 1. It shows that most of the vibrations die out after 20 hours. We

compute the periodograms of the first 600 observations. The critical bounds at significance level 0.01 of the tests F_T , R_T and V_T are also computed. The results are shown in Figure 2. The heights of the four horizontal lines in the plot correspond (from top to bottom) to the critical bounds of the tests F_T , $V_T(8)$, $R_T(0.8)$ and $R_T(0.9)$. We see that the tests $R_T(0.8)$ and $R_T(0.9)$ both detect 8 peaks, while F_T only detects 4 peaks. The 8 peaks detected by R_T were also confirmed in Bolt and Brillinger (1979). There are only 6 peaks bigger than the critical value of $V_T(8)$, so the null hypothesis of no periodic components is accepted at significant level 0.01, and we are not able to make inference based on $V_T(8)$ about the significances of the peaks which are bigger than the critical bound of $V_T(8)$. The nice feature we mentioned earlier that the test R_T is not sensitive to the value β used is also demonstrated in Figure 2. We see that the critical bounds of $R_T(0.8)$ and $R_T(0.9)$ are very close.

We also compute the periodograms of the first 1024 observations and plot them in Figure 3. The periodograms at very low frequencies become dominant; this might be expected from Figure 1. The low frequency components are primarily caused by tidal effects. The top and bottom lines in Figure 3 correspond to the critical bounds of F_T and $R_T(0.9)$, respectively. In this case, both tests detect the peak at very low frequency. However, R_T still detects the same 8 peaks detected earlier while F_T fails to detect any of these peaks. This shows a big advantage of the test R_T in that the ability of detecting a particular peak is not affected much by the presence of other peaks.

Remark 3. We should point out that Nowroozi (1966) has removed tidal effects before computing the periodograms. So, Fisher's test was not affected by the low frequency component. The low frequency components might not be easy to remove in some cases.

Remark 4. The test R_T failed to detect some peaks which were found in Bolt and Brillinger (1979). The failure is caused by the fact that periodic components are decaying. Currently, there is no test designed for detecting decaying sinusoidal waves. A test procedure which takes the damping factor into consideration might be more powerful in detecting decaying sinusoidal waves.

7. Discussion

In this paper, we proposed a method for testing the existence of periodic components in a white Gaussian series. This test procedure can also be used to detect the outliers in an exponential sample. Some of the advantages of the proposed test were discussed in previous sections. One small disadvantage is that we only obtained an approximation for the the exact null distribution. We have shown that the error caused by using the asymptotic distribution is of order $n^{-1/2} \log n$. The size of the series encountered in practice is often quite large and the approximation becomes very accurate. Asymptotic approach has an advantage in that the asymptotic distribution is easy to find and it is easy to compute critical values and p-values.

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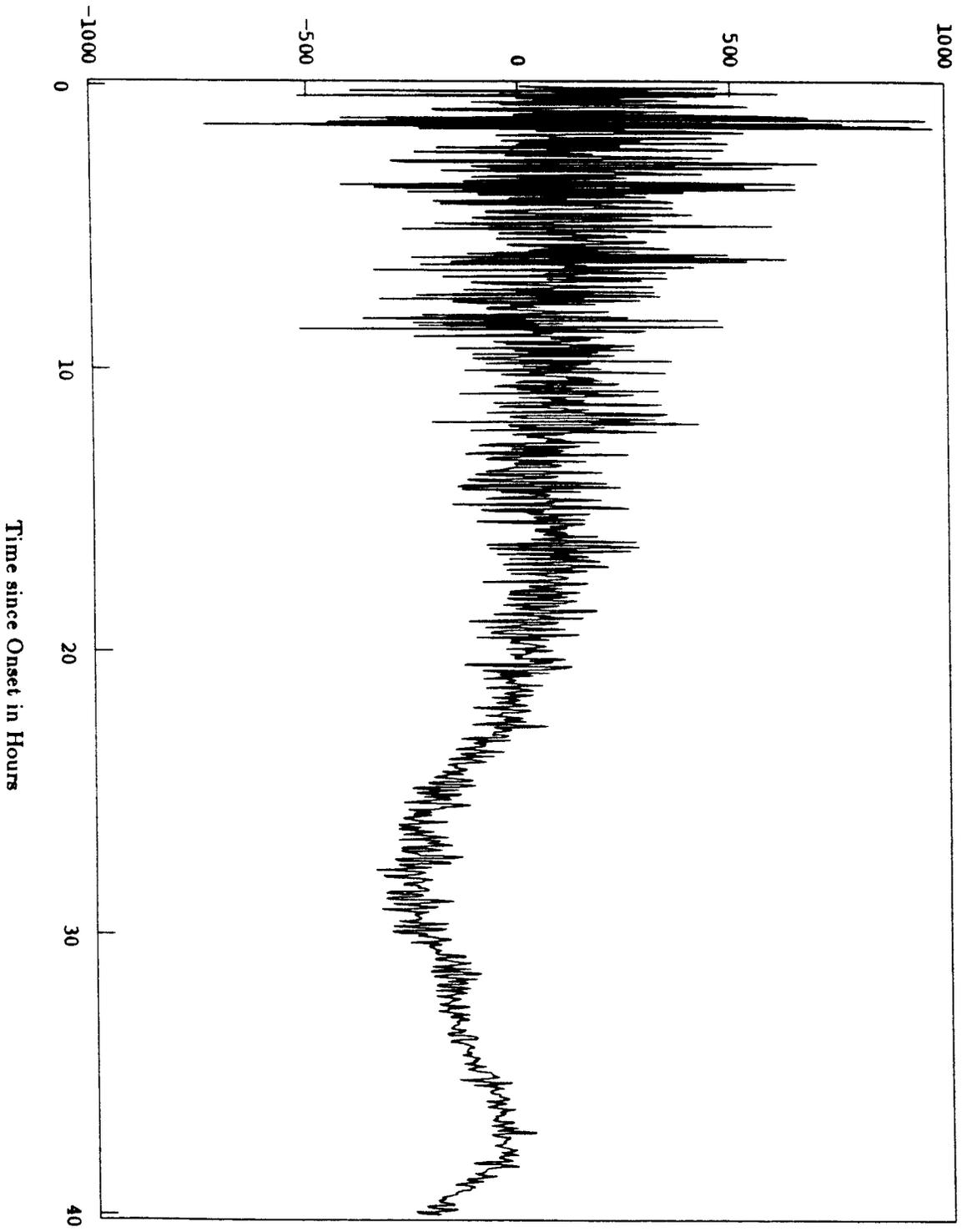
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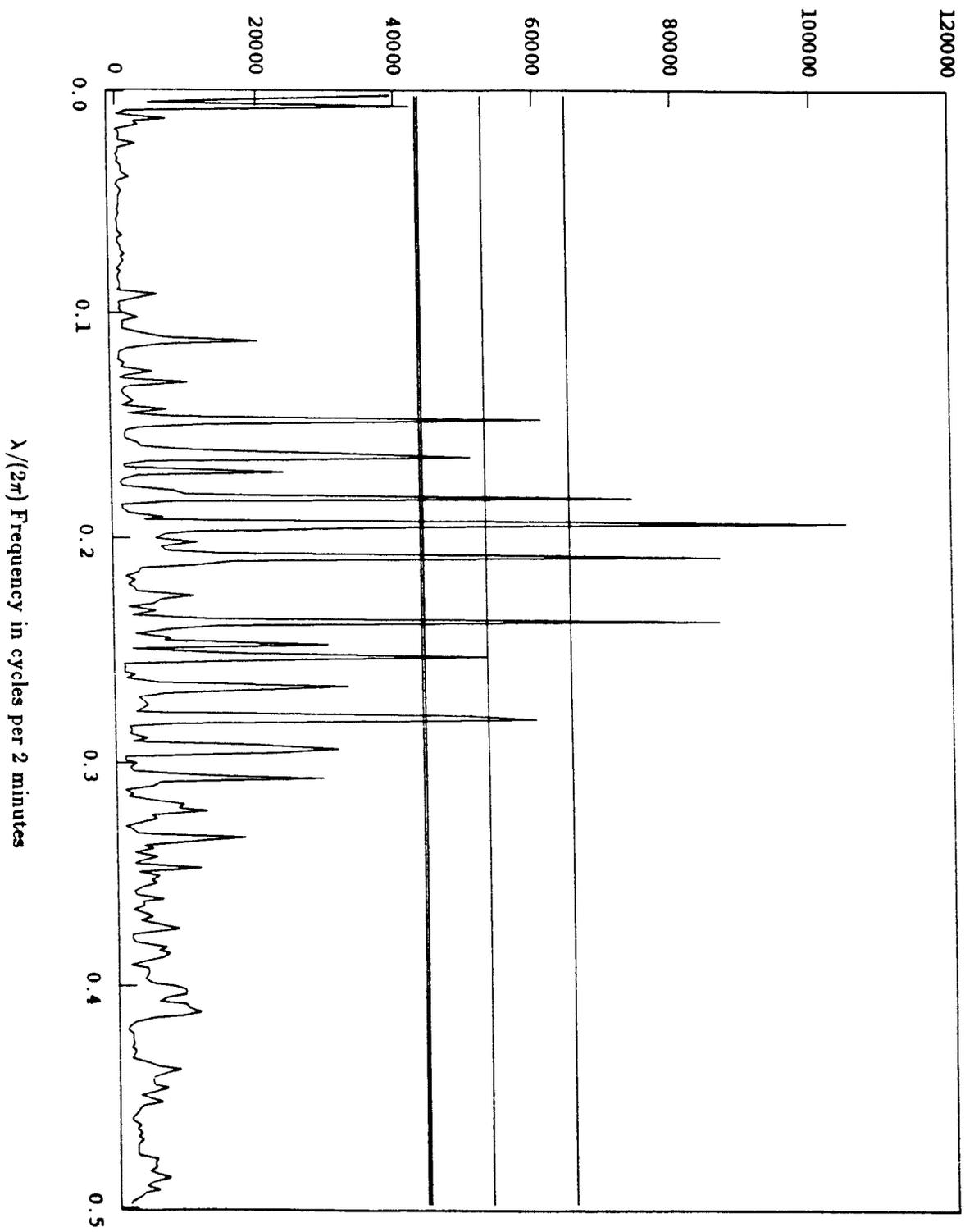
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Figure 1. The first 1200 observations of the series, the sampling interval is 2 minutes.

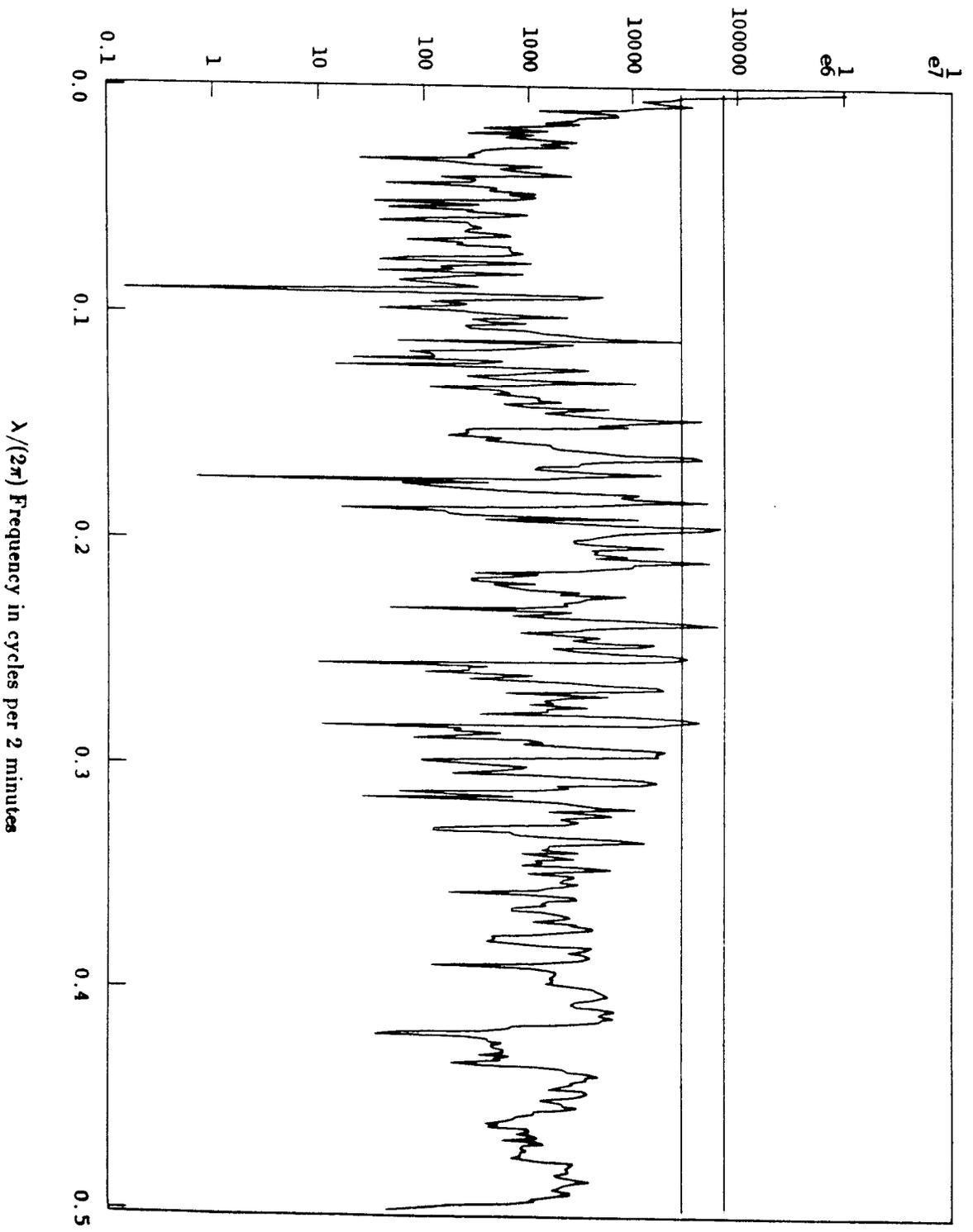
Figure 2. The Periodograms of the first 600 observations, the heights of the horizontal lines correspond to the critical bounds of the tests F_T , $V_T(8)$, $R_T(0.8)$ and $R_T(0.9)$.

Figure 3. The periodograms of the first 1024 observations (in log10 scale), the horizontal lines correspond to the critical bounds of the tests F_T and $R_T(0.9)$.





$\lambda/(2\pi)$ Frequency in cycles per 2 minutes



$\lambda/(2\pi)$ Frequency in cycles per 2 minutes