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This report outlines the research results and activities of Margaret Cheney from 2003 to 2005.
The research results are contained in the papers titled

a) Resolution for radar and x-ray tomography
b) Synthetic-aperture imaging through a dispersive layer
c) Synthetic-aperture assessment of a dispersive surface
d) Imaging that exploits multipath scattering from point scatterers
e) Synthetic-aperture imaging from high-Doppler-resolution measurements
f) Microlocal analysis of GTD-based SAR models

which are appended to the report.

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Final Report

January 1, 2003 - September 1, 2005

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August 24, 2005

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Chapter 1

Project description

1.1 Objectives

This project deals with radar remote sensing problems, including foliage-penetrating synthetic-aperture radar (SAR), inverse synthetic-aperture radar (ISAR) and the identification of targets in a complex environment.

1.2 Personnel Supported

1.2.1 Faculty

Margaret Cheney (Principal Investigator)

1.2.2 Graduate students supported

- Charlie Beckman – Fall 2004. I arranged for Charlie to work at Rome for the summer of 2004.
- Hyoungsu Baek – Summer 2004 and Spring 2005. Hyoungsu implemented a fast ISAR imaging algorithm, related to my work with Brett Borden, based on a suggestion from Emmanuel Candès .

Chapter 2

Current Status of Effort

- **Students:**
  - In January 2005 Charlie Beckman left graduate school for other opportunities.
  - During the summer of 2005, Hyoungsu Baek transferred to Brown University because of the funding uncertainties for graduate students here at RPI.
  - Hector Morales (a citizen of Mexico) wants to work on wave propagation in dispersive media. We plan to take a computational approach to imaging through a dispersive layer.
  - Matt Ferrara (US citizen) has obtained a SMART fellowship and will be working with Greg Arnold and me on further developing the ATR approach that Brett Borden and I proposed. I arranged for him to spend the summer of 2005 working with Jamie Bergin at Information Systems Laboratories.
- Cliff Nolan and I are in the middle of work on the urban canyon imaging problem. We have found that most measurement scenarios (other than real-aperture imaging with a narrow beam) result in artifacts; in order to avoid artifacts, we need higher-dimensional data. (Initial results are given in Appendix.) We plan to consider the case of a stand-off transmitter and a swarm of receivers.
- I've been working with Birsen Yazici, a faculty member in our ECSE Dept. who is supported by Jon Sjögren's program, on incorporating noise consid-

erations into the Fourier Integral Operator formulation of Synthetic Aperture Radar processing. The ultimate goal here is to formulate properly and solve the problem of determining the best waveform to send through foliage in order to make an image with the best possible resolution. We are currently writing up our first full paper. An preliminary account of our work appeared in the Proceedings of the IEEE Radar 2005 conference.

Chapter 3

Interactions/Transitions

I am actively trying to make contacts in the radar community, by attending electromagnetic conferences and meeting with people from DoD labs and contracting companies.

One activity that has extended through multiple years of this project is my work on the planning committee for a **year-long program on Imaging at the Minnesota Institute for Mathematics and Its Applications**. I helped arrange for the Imaging year advertising poster to feature a beautiful radar image from Sandia; Sandia is one of the IMA participating instutitions. The Imaging year will begin with two 10-hour short courses: I'll lecture on "Introduction to Radar Imaging", and David Brady will lecture on "Computational Optical Imaging and Spectroscopy". Frank Natterer, Bill Symes, and I are organizing a workshop in the fall on inverse problems involving wave propagation, at which we bring together mathematicians, radar experts, and geophysicists involved in seismic exploration.

As part of the planning, I have been helping AFRL become involved with the IMA; **AFRL is now an IMA participating institution, and AFRL is funding a postdoc to begin during the year on Imaging.** I served on the postdoc selection committee. I will be spending the fall of 2005 at the IMA, along with Brett Borden, Tony Devaney, and others. I plan to run a radar "focus group" or seminar series, perhaps bringing in some outside speakers.

Most recently I have been sending out e-mail messages to many of my contacts at companies and laboratories, urging them to visit the IMA during the year.

3.1 Activities in 2003

I gave one of the 5 invited plenary talks at JBK80, the celebration of Joe Keller's 80th birthday, held at Stanford University. In this talk I pointed out a number of mathematical problems related to Air Force interests.

During the spring of 2003, **Bob Bonneau** (Rome Labs) was at RPI regularly and we had many interesting conversations about exploiting multipath scattering for imaging. Bob sent me some of his year-end money to support my work on the problem of imaging in urban canyons.

During the spring of 2003 I exchanged a number of e-mail messages with **Ed Barile** (Raytheon) regarding time-reversal imaging. I also had e-mail correspondence with **Tom Roberts** (Hanscom AFB) regarding wave propagation in dispersive media. I had some phone conversations with **Arthur Yaghjian** (Hanscom) on the connection between modeling scattering from chaff and scattering from branches.

I spent an exciting day in March 2003 visiting **Richard Albanese** and his group at Brooks AFB. Richard asked whether the SAR imaging method could be used to find the dispersive characteristics of a surface; I wrote up a draft of a short manuscript on how to do this. He sent some funding for consulting to me through Tony Devaney's company.

I made a one-week visit to Brooks in August 2003, during which I discussed with Sherwood Samn the numerical implementation of the SAR-through-a-dispersive-layer algorithm. He was planning to take the lead on the numerical implementation. I gave lectures on this method and also on the above-mentioned method for mapping the dispersive properties of a surface. Richard Albanese seemed interested in this and said he would send a copy of the manuscript to Dr. Tolimieri. He also asked about the validity of the start-stop approximation in radar imaging; since Brett and I did not make the start-stop approximation in our work, it should be an easy matter to estimate the size of the terms that would be neglected.

In April 2003 I attended the SPIE AeroSense conference and there met **Ed Zelnio** (Wright-Patterson), **Bob Hummel** (DARPA), and **Eric Keydel** (SAIC). I gave a talk about the work that Brett Borden and I had been doing on ISAR ATR. Ed Zelnio had evidently read our conference proceedings submission, and found it interesting enough to ask me to serve on a panel discussion on exploiting phase histories for target identification. Our approach is exactly what Bob Hummel advocates: do target identification directly from the data, without first making images. Ed also apparently passed our paper on to **Vince Velten** (Wright-Patterson), who in turn asked Eric Keydel and **Dennis Andersh** (SAIC) to involve me in their responses to the SAMERI BAA. Those proposals were funded; however as of August 2005, only $5k has made its way to RPI, and we have not managed to begin a true collaboration.

At the AeroSense conference I also enjoyed talking to **Armin Doerry and F. Dickey** (Sandia) about their Lynx SAR system. I chatted a bit with people I had met during my stay at China Lake: **Alan Van Nevel** (Naval Air Warfare Center, China Lake) and **Victor Chen** (Naval Research Lab). I talked with **John Gray** (Naval Surface Warfare Center, Dahlgren) about Doppler imaging; he sent me some of his papers. I also talked to people at the ERADS booth about getting their turntable data.

In May I gave two talks, one an invited plenary lecture, at the IPAM Conference on Inverse Problems at Lake Arrowhead. Both talks were about different aspects of the ISAR work I had been doing with Brett Borden.

I was invited to be a plenary speaker at the International Workshop on Wave Propagation, Scattering, and Emission: Theory, Experiments, Numerical Simulation and Inversion, which was to be held in Shanghai June 1-4, 2003. I was interested in going, mainly to meet other international figures in electromagnetics; however, the conference was cancelled because of SARS. Instead of going to Shanghai, I spent the first week in June working with **Brett Borden** at the Naval Postgraduate School on our Doppler imaging ideas.

In June 2003 I gave a talk on my SAR-through-a-dispersive-layer work at the APS/URSI meeting in Columbus. At the conference I had a chance to talk to a number of radar/electromagnetics people. In particular with **Carl Baum** (Kirtland AFB), I discussed the possibility that time-reversal methods might improve the signal-to-noise ratio for the singularity expansion method. Carl introduced me to **Tapan Sarkar** (Syracuse); I talked with **Ross Stone**, editor of *Radio Science* about resolution issues; I talked to **Glen Smith** (Georgia Tech) about modelling chaff; and I discussed wave propagation in dispersive media with **Anders Karlsson** (Lund Institute of Technology, a friend from my year there in 2000).

In late May 2003 I gave two talks at the Lake Arrowhead meeting on Inverse Problems (sponsored by IPAM). There I discussed with Andreas Kirsch the possibilities for extending the "factorization" (linear sampling) method to near-field geometry and point sources; the prospects are not promising.

AT RPI I met with **Mike Dowe** (CEO of Information Systems Laboratories, and an RPI alumnus), who seemed eager to get me involved in collaboration with his company.

In early August 2003 Brett and I met with Eric Keydel, **Don Herrick, and Wayne Williams** of SAIC and Vince Velten, **Greg Arnold, and Devert Wicker** from Wright-Patterson in Ann Arbor. Vince expressed interest in many of the same problems outlined by Bob Bonneau.

Cliff Nolan (Math., U. of Limerick) visited RPI for a month during the summer of 2003; we discussed inversion for polarimetric data and the urban canyon imaging problem.

I spent the fall of 2003 in residence at IPAM, located on the UCLA campus. During my time there, I audited courses on stochastic processes and detection theory in the UCLA EE department.

I met several times with **Nick Marechal** (Aerospace Corp., a math Ph.D.), which indirectly led to Nick's giving a talk on SAR in the UCLA Applied Math Colloquium. I also gave an Applied Math Colloquium talk on SAR. Nick and I settled on a possible problem to collaborate on: settling the question of whether orthogonal pulses can be used to eliminate range ambiguities in SAR.

At one of the IPAM workshops I met **Yoram Bressler**, from the EE Department at U. of Illinois; with him I discussed the problem of finding the best waveform to make an image through foliage. Clearly the standard approaches don't work. I discussed this same problem with **David Donoho** (Statistics, Stanford) he made some suggestions that I later shared with Birsen Yazici when we started collaborating.

At another IPAM workshop I reconnected with **Gary Hewer** from China Lake.

While I was at UCLA, I was invited to give a talk in **Barry Simon**'s group at Caltech. I talked about Synthetic Aperture Radar. **Peter Lax** and **Oscar Bruno** were in the audience, and seemed to be more interested in my talk than the rest of the audience! I had some nice discussions with Oscar Bruno afterwards.

During my stay at UCLA, I visited JPL, where I gave a talk about my SAR work with Cliff Nolan, and also met **Scott Hensley** (who has a Ph.D. in math and runs the GeoSAR project), **Paul Siqueira, Ali Safaneli** (with whom I discussed a possible project for tomographic imaging of asteroids), and **Sasan Saatchi**. Sasan Saatchi works on foliage models relevant for wave propagation; I met a second time with him when he visited the UCLA campus. I also put him in touch with George Papanicolaou, who was doing a sabbatical at Caltech.

In November 2003 I visited Wright-Patterson for the kickoff meeting for the SAMERI 2-1 project (headed by **Eric Keydel** of SAIC). **Dennis Andersh** was also present. Since the meeting started in the afternoon but I had to fly in the night before, I arranged to spend the morning talking to **Greg Arnold** and **Vince Velten** about projects of interest to them. Greg told me about his 3DMAGI project, which has some components that are similar to the ISAR work Brett Borden and I have been doing. Vince mentioned that they're interested in SAR with long integration times (which my approach includes naturally), and told me about Ed Zelnio's interest in a project called GOTCHA which would collect rich radar data and then process it in different ways to obtain different sorts of information.

3.2 Activities in 2004

At the AFOSR Electromagnetics Workshop I learned about the Waveform Diversity conference to be held in Rome, NY later that month. Birsen Yazici and I drove over, with one of her students, for one day of the conference. There I chatted with **Eric Mokole** (Head of Surveillance Technology Branch, Radar Division at NRL), **Bill Moran** (Prometheus Inc.) and some of the Rome Labs folks.

In March 2004 I visited Lincoln Labs, where I gave a survey of my radar work and begged (unsuccessfully) for student funding.

In early April 2004 we had a kickoff conference for the new RPI Center for Inverse Problems (IPRPI). I organized a last-minute session on radar, to which I invited **Fioralba Cakoni** (Math, Delaware), **Bob Bonneau** (Rome), **Carey Rappaport** (EE, Northeastern). I also tried to get Steve Arcone (CRREL), Tony Devaney, Howard Zebker (EE, Stanford), David Colton, and Peter Monk, but they were unavailable.

In April 2004 I attended the SPIE Defense & Security conference, together with my friend and collaborator **Brett Borden**. I gave a talk about my SAR-through-a-dispersive-layer with Cliff Nolan. At the meeting I rewned my acquaintance with **Ed Zelnio** (Wright-Patterson), **Eric Keydel** (SAIC), **Greg Arnold** (Wright-Patterson), **Armin Doerry** (Sandia). In the airport I also had some nice conversations with **Randy Moses** (EE, Ohio State).

In April 2004 I also attended the IEEE Radar conference, where I gave a poster on my assessment-of-a-dispersive-surface work. There I met **Gerard Capraro** of Capraro Technologies Inc., a consulting company that contracts with AFRL Rome. I tried to send one of our RPI grad students to work with them for the summer, but evidently the (international) student didn't have the proper work permit.

At the IEEE conference I also introduced myself to **Paul Techau** (Information Systems Laboratories, Inc.), whose CEO Mike Dowe is eager to have us collaborate. The ISL radar group is currently working on Space-Time Adaptive Filtering, and told them that I need to learn more about these techniques before I could contribute usefully to their efforts.

At the IEEE conference I also listened to **Mark Stuff** (General Dynamics, Advanced Information Systems) and talked to him afterwards about the work he's doing on the 3DMAGI project for Wright-Patterson. He is thinking along lines similar to those that Brett Borden and I are pursuing, but has advanced farther in the direction of finding the relative motion between antenna and target.

At the IEEE conference I also renewed my acquaintance with **David Garren** (SAIC Chantilly), who is thinking about multiple scattering. Later, in July, I visted him at his office in Chantilly for more discussions. He showed me an interesting video of SAR reconstructions of a truck on a turntable, in which multiple scattering in the cab region causes apparent blinking on and off of scatterers.

In May 2004, **Cliff Nolan** and I met in London to work on our urban imaging paper.

During the summer of 2004 I made something like 6 or 8 trips over to Rome (NY) to work with **Bob Bonneau** and a group he has set up to validate our theory on exploiting multipath scattering from point scatterers in imaging. In particular, I had some fun discussions with **Vince Vannicola** (Rome IPA), and **Nick Currie** (Georgia Tech retiree), and I was working closely with **Adam Bojanczyk** (CS, Cornell).

While visiting Rome, I often got a chance to talk with **Mike Wicks**, who seemed quite happy to see me there. While there during the summer, I also checked in with my student Charlie Beckman, who, without much supervision, was learning about radar and coding theory and cryptography. During the early part of the summer he was being mentored by **Braham Himed**, but Braham and Mike were both on travel much of the rest of the summer.

In early June 2004, **Frank Natterer** (Applied Math., University of Münster, Germany) visited me at RPI, and we discussed point spread functions for various SAR flight paths.

At the end of June 2004, I gave a talk on my earlier time-reversal work in a workshop on Adaptive Sensing at the Minnesota Institute for Mathematics and Its Applications (IMA). In the audience I recognized **Rob Williams** (Wright-Patterson), who had been at the SPIE Defense & Security SAR sessions. I introduced myself and told him about the year on Imaging we're trying to organize for 2005-'06 at the IMA. Rob is now back at Wright-Patterson (after a stint at DARPA) and is in a position in which he is supposed to improve the interaction between AFRL labs and universities. We had a long discussion over dinner, at which he pointed out that DARPA is extremely interested in imaging in urban environments. He suggested that the result of the year at the IMA could be a proposal for a DARPA program on "The Mathematics of Urban Imaging". I put him in touch with Doug Arnold, the IMA director; Doug and I discussed ways that AFRL could participate in the IMA program. I suggested that perhaps AFRL could fund a postdoc to work on problems of interest to AFRL.

In July 2004 I was asked to give a two-hour tutorial on SAR imaging at a workshop at Delaware State University, one of the Historically Black Colleges & Universities. I also helped them invite a couple of other speakers, namely **Brett Borden** and **Victor Chen** (NRL). From Delaware I went up to Newport, RI, to visit **Bill Moran** at the Prometheus headquarters. We discussed space-time coding, and decided that there are a number of problems that my student Charlie Beckman might usefully work on.

Cliff Nolan (U. of Limerick) visited RPI for a month during the summerof 2004; we worked on the urban canyon imaging problem.

During the fall of 2004, I was very happy to be asked by **Randy Moses** (Electroscience, Ohio State) to join a team putting in a response to one of the MURI BAAs. Other team members were **Lee Potter** (Ohio State) , **Müjdat Çetin** (MIT), bf Alan Willsky (MIT), **Todd Hale** (AFIT), **Brian Rigling** (Wright State U., a former student of Randy Moses). On the basis of our white paper, we were invited to submit a full proposal, but did not win the MURI award.

In October 2004, I gave a colloquium on SAR in the Mathematics Department at the University of Delaware, where I talked with David Colton, Fioralba Cakoni, and Rakesh.

In November 2004, I flew to Dayton to participate in the **Industry Day** put together by the ATR directorate at Wright-Patterson. There I was pleased to find Doug Arnold, director of the IMA, whom I introduced to **Ed Zelnio**. Doug and I also talked with **Mark Stuff** (General Dynamics) about Mark's work with Greg Arnold on the use of group invariants in ATR. I renewed my acquaintance with **Mike Minardi** (Wright-Patterson), Greg Arnold, and **Aaron Lanterman** (ECE, Georgia Tech), and I finally met Lee Potter. I talked also with **Ron Dilsavor** about the Doppler-only imaging that Brett and I were doing. I also talked with **Eric Keydel**.

3.3 Activities in 2005

In Febraury 2005, I gave a talk on radar imaging at the IMA workshop Career Options for Women in the mathematical Sciences.

Early in 2005, Birsen Yazici, Brett Borden, and I submitted a planning letter to Doug Cochran, who was putting together a DARPA project on waveform diversity. Doug Cochran subsequently put together a team consisting of the 3 of us, **Edwin Chong and Louis Scharf** (both from Colorado State ECE), **Robert Calder-**

bank (Applied Math, Princeton), **Bill Moran and Stephen Howard** (Australian DSTO).

Also early in 2005, Matt Ferrara, one of our best math graduate students, applied for a SMART fellowship, and I arranged for **Greg Arnold** to be his DoD mentor.

In mid-March of 2005, I gave a talk at the Waveform Diversity meeting in Huntsville. My talk, on my work with Brett Borden about Doppler-only imaging, attracted a lot of interest. **Bob McMillan**, one of the organizers, thanked me afterwards for a very interesting talk. Birsen, Bill Moran, Stephen Howard, and I also talked there with Bob Bonneau, who conveyed information from Doug Cochran about what he wanted to see in the DARPA proposal. I also introduced myself to **Jamie Bergin** (ISL), who gave a talk on a topic closely related to what Birsen and I are working on. We made plans to meet later in Troy. At the conference I also met **Todd Hale** and **Marshall Greenspan** (Norden Systems, Northrup Grumman); the latter turned out to be on the same departing flight out of Huntsville. In the Cincinnati airport Marshall kindly sat down with me and explained his views on the relationship between the Doppler-only SAR ideas I had talked about and his single-frequency tomographic approach to through-the-wall imaging. (He told me later that he nearly missed his connecting flight!)

At the end of March 2005, I gave two talks at the SPIE Defense & Security Conference. The first talk was on my multipath work with Bob Bonneau; the second established a connection between my work with Brett and the Moses-Potter model for scattering centers. At the end of my second talk, I showed some rudimentary reconstructions, produced by my student Hyoungsu Baek, from the backhoe data set released the previous year by Wright-Patterson. **Brett Borden** was also there, and we skipped some of the sessions to sit in the lobby and do calculations together. At the meeeting **Dennis Braunreiter** (SAIC) introduced himself to me as a potential industry partner for our DARPA team.

In April Birsen Yazici and I traveled down to Princeton to join Edwin Chong, Louis Scharf, Robert Claderbank, Bill Moran, and Stephen Howard for a DARPA-proposal-writing session.

In early May 2005, on my way to the IEEE Radar conference, I visited **Eric Mokole's group** at NRL, where I gave a talk outlining both the work with Bob Bonneau and the Doppler work with Brett Borden. Eric is interested in wave propagation in dispersive media, and gave me reprints of some of his papers on the topic. During that visit I talked also with **Victor Chen**.

Later that week I attended the IEEE Radar conference, where Birsen Yazici and I presented a poster, "Synthetic-aperture inversion in the presence of noise and clutter", which showed our prelimiinary results. There I saw again Eric Mokole, Jamie Bergin, and Marshall Greenspan. I had lunch with **Lars Ulander** (FOI, Sweden), whom I had met during my year in Sweden in 2000. I met **Elaine Chapin** of JPL, who gave a nice talk about along-track radar interferometry.

From the Radar conference, I flew to Mississippi State University, where I gave a plenary talk on SAR at the 6th Mississippi State - UAB Conference on Differential Equations and Computational Simulations, a conference being held to celebrate the birthdays of Louis Nirenberg and Klaus Schmitt.
(See `http://www.msstate.edu/dept/math/de2005/`.) There I talked with Tuncay Aktosun and Paul Sacks.

In mid-May, **Jamie Bergin** stopped off at RPI on his way home from Rome, and he, Birsen, and I spent the afternoon discussing ideas and possible joint projects.

In late May, Birsen Yazici and I hosted a one-day workshop on Radar Waveform Design and Imaging, held under the auspices of the RPI Center for Inverse Problems (IPRPI). **Mike Wicks** opened the meeting with a plenary talk; we had also talks from **Jeff Goldman** (ARL, Adelphi), **Brian Rigling**, **Bob Bonneau**, **Marshall Greenspan**, and **Bill Moran**, with posters by **Jamie Bergin**, my student Matt Ferrara, **Capraro Technologies**, and myself and Birsen Yazici. We ended the day with a panel discussion, the panel consisting of Michael Wicks, Marshall Greenspan, Bob Bonneau, and Bill Moran. One of the students in the audience asked "What courses should I take so I can work in this area?"

At the end of May 2005 I gave two general-audience talks in the math department at the University of Washington, where I had some discussions with **Gunther Uhlmann** on developing the theory of Fourier Integral Operators with complex phase. This theory needs to be developed to handle wave propagation in dispersive media. Gunther plans to suggest the problem to one of his bright postdocs.

I flew from Seattle to Monterey, where I spent a week working with **Brett Borden**. At the end of the week Brett flew out to Troy for another week of collaboration.

About this time, since Matt Ferrara's name wasn't on the list of SMART awardees, I arranged for him to spend the summer working at ISL. A few weeks later, after he had started his job in Virginia, he was notified that he DID win a SMART fellowship. Consequently **Greg Arnold** has agreed to travel to Troy in late August to discuss possible topics for Matt's dissertation.

On June 24, Brett and I attended the *Inverse Problems* editorial board meeting in London. We are planning a special section of the journal, edited by myself and some other person, on radar imaging.

The last week in June 2005 I spent at the Applied Inverse Problems conference (See `http://www.cs.ucl.ac.uk/staff/S.Arridge/AIP2005/`.) where Frank Natterer and I had organized a session on microwave imaging and radar. Unfortunately the "real" radar people I had invited were unable to come at the last minute because both their wives were ill; the speakers in the session were Cliff Nolan, Thorsten Hohage, Dominique Lesselier, Oliver Dorn, Frank Natterer, and myself. Cliff and I spent some time there working on our joint project, and Oliver and I decided to organize a session at the PIERS meeting in 2006.

The last week in August I will spend 3 days at Colorado State, where I will give a talk on SAR in the Applied Math Colloquium, work with Edwin Chong and Louis Scharf, and participate in a team meeting for our DARPA project.

Chapter 4

Specific Technical Findings

The following technical work was funded by this contract.

4.1 Journal Publications

4.1.1 Resolution for radar and X-ray tomography

This is by F. Natterer, M. Cheney, and B. Borden, and appeared in Inverse Problems 19 (Dec. 2003) S55-S64. This paper arose from the fact that whenever Brett lectured on ISAR to a mathematical audience, someone would invariably ask: when both radar and x-ray computed tomography are based on the Radon transform, why is it that radar can make images from an aperture of 2°, whereas x-ray computed tomography requires at least 120°? This paper shows that the ability of radar to make images from such a small aperture is due to its use of a high carrier frequency. In addition, an image showing corners as isolated points would be considered acceptable as a radar image, but not as an x-ray image.

4.1.2 SAR through a dispersive layer

This is by M. Cheney and C.J. Nolan, and appeared in Inverse Problems 20 (2004) 507-532. This paper develops a synthetic-aperture imaging algorithm for looking though a dispersive medium. This approach positions the high-frequency information correctly, and results in an image in which the singularities (edges) appear in the correct location and orientation. For the dispersive-medium models for foliage, for which the amplitude of the high-frequency information does not decay too rapidly, the algorithm in this paper is probably useful. For other media such as human tissue, it is probably less useful. There are many open questions related to this work, including the question of whether energy carried by precursors is positioned correctly in the image.

4.1.3 SAR assessment of a dispersive surface

This paper arose from my discussion with Richard Albanese in March 2003. He asked "Can your SAR imaging technique handle frequency-dependent scatterers?" This paper shows that the answer is "yes", assuming that the scattering takes place at a known surface and assuming that backscattered data can be measured over a two-dimensional surface (i.e., multiple passes can be made). I am the sole author of this paper; it appeared in the International Journal of Imaging Systems and Technology 14 (2004) 28-34.

4.1.4 Imaging that exploits multipath scattering from point scatterers

The authors of this paper are M. Cheney and R.J. Bonneau; the paper was published in Inverse Problems 20 (2004) 1691-1711 . This paper shows that when known multipath scattering can be incorporated into the imaging process, it can increase the effective aperture and thus potentially improve the resolution. We modeled the multipath scatterers as point scatterers. Ed Zelnio suggested that this work might be useful in a scenario in which microwave repeaters are scattered throughout a scene.

4.1.5 Doppler-only SAR

The authors of this paper are B. Borden and M. Cheney; it appeared in Inverse Problems 21 (2005) 1-11. This paper suggests an approach to synthetic-aperture imaging based on measuring only the Doppler shift. These ideas could be useful in a scenario in which there are severe limitations on the spectrum that can be used for transmissions.

4.2 Conference Proceedings and other articles

4.2.1 Microlocal structure of HRR ISAR data

The authors of this paper are M. Cheney and B. Borden; the paper appeared in Wave Propagation, Scattering and Emission in Complex Media, ed. Ya-Qiu Jin, Science Press and World Scientific, 2003. (This was for the conference in Shanghai that was cancelled due to SARS.) This paper is a simplified summary of some of the work that Brett Borden and I did in 2002. The present grant funded the writing of this paper.

4.2.2 Microlocal ISAR for Low Signal-to-Noise Environments

The authors of this paper are M. Cheney and B. Borden; the paper appeared in Algorithms for Synthetic Aperture Radar Imagery X, ed. E.G. Zelnio and F.D. Garber, SPIE Proceedings series vol. 5095 (SPIE, Bellingham, WA, 2003). This is the proceedings of the SPIE AeroSense Conference, Orlando, April 2003. This is also a simplfied version of our earlier work; the present grant funded the writing of this paper.

4.2.3 Some problems in Electromagnetics

This is by myself alone, and appeared in the proceedings of the conference in honor of Joe Keller's 80th birthday. In this paper, and the talk on which it is based, I pointed out some of the open mathematical problems in areas of interest to the Air Force.

4.2.4 Featured book review, "Scattering"

This is a book review I wrote of the book *Scattering* ed. R. Pike and P. Sabatier, SIAM Review, vol 45, no. 3 (2003) 591 - 600. This review is similar to the above paper in the Keller proceedings.

4.2.5 SAR through a dispersive layer

This is by M. Cheney and C.J. Nolan, and appeared in Algorithms for Synthetic Aperture Radar Imagery XI, ed. E.G. Zelnio and F.D. Garber, SPIE Proceedings series vol. 5427 (SPIE, Bellingham, WA, 2004). This is the Proceedings of SPIE

Defense & Security conference, Orlando, April 2004. This is a summary of the above paper Cliff Nolan and I wrote.

4.2.6 Microlocal analysis of GTD-based SAR models

The authors of this paper are M. Cheney and B. Borden; it appeared in Algorithms for Synthetic Aperture Radar Imagery XII, ed. E.G. Zelnio and F.D. Garber, SPIE Proceedings series vol. 5808 (SPIE, Bellingham, WA, 2005), pp. 15-23. This is the proceedings of the SPIE Defense & Security (formerly AeroSense) Conference in Orlando, March 2005. This paper establishes the connection between a) the GTD-based SAR models developed by Moses and Potter (models of interest to the Wright-Patterson ATR group) and b) the SAR models based on microlocal analysis that Brett and I developed. This paper also shows how a multiple-scattering-from-point-scatterers solution could include angular dependence (meant to model some shadowing effects).

4.2.7 Imaging that exploits multipath scattering from point scatterers

This is by M. Cheney and R.J. Bonneau, and appeared in Algorithms for Synthetic Aperture Radar Imagery XI, ed. E.G. Zelnio and F.D. Garber, SPIE Proceedings series vol. 5808 (SPIE, Bellingham, WA, 2003), pp. 142-155. This is a summary of the full paper (described above) that I wrote with Bob Bonneau.

4.2.8 Doppler-only imaging of a stationary target

This is by B. Borden and M. Cheney, and appeared in Algorithms for Synthetic Aperture Radar Imagery XII, ed. E.G. Zelnio and F.D. Garber, SPIE Proceedings series vol. 5808 (SPIE, Bellingham, WA, 2005), pp. 132–141. This is a summary of the full paper (described above) that Brett Borden and I wrote.

4.2.9 Synthetic aperture inversion for arbitrary flight paths in the presence of noise and clutter

This is by B. Yazici and M. Cheney, and appeared in the proceedings of the IEEE Radar Conference 2005. This is a summary of our preliminary findings on how to incorporate statistical considerations into inversion techniques based on microlocal ana;ysis.

Appendix A

Appendix: Artifacts for SAR in a waveguide

The work below was funded under this contract but has not yet been published. The results below are negative ones in the sense that they show that SAR imaging in a waveguide will produce artifacts. We plan to publish this work together with the positive result that imaging from a swarm of receivers will provide artifact-free images.

A.1 Introduction

The problem of active sensing within a waveguide can arise in a number of different applications, including acoustic or electromagnetic sensing in areas such as a shallow ocean regions, tunnels, or other structures.

The main difficulty in studying wave propagation in a waveguide is the presence of multipath scattering, which confuses standard imaging methods. However, the presence of multiply-reflecting waves also provides an opportunity for imaging, because such waves can illuminate the object from a variety of directions. If these waves can be accounted for and used in the imaging process, they can improve the image of the object.

This paper considers two different scenarios for using backscattered waves in order to image objects in a waveguide. These two scenarios correspond to different assumptions about the measurement equipment. In the first scenario, we assume that the antennas or transducers are small relative to the wavelength, so that they cannot produce a focussed beam. We show that in this case, a backprojection imaging algorithm results in an image with many artifacts.

In the second scenario, we assume that the antenna array can produce a beam sufficiently narrow to distinguish different ray paths. In this case, backprojection can produce an image without artifacts. Moreover, the imaging algorithm in this situation is able to exploit the multipath scattering to produce an image with better resolution than in the free-space case.

In this paper we consider only the simplest possible waveguide, namely the region between two parallel plates. We assume also that any objects of interest are positioned at a known distance into the waveguide; the imaging algorithm produces an image of objects on a known cross-sectional plane. This geometry might be appropriate, for example, for imaging the region of the earth between two buildings, or imaging underwater objects at a known range. We refer to this cross-sectional plane as the "scene".

For scattering from the target or scene of interest, we use the Born (single-scattering) approximation. This approximation neglects multiple scattering within the target and multiple scattering between the target and its environment and is commonly used.

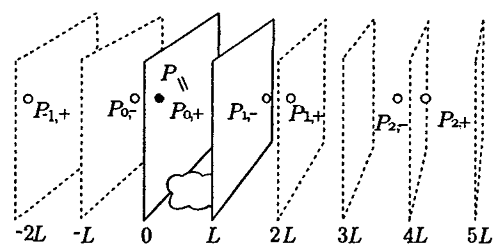

Figure A.1: Scattering geometry. The darker walls and dot denote the true walls and a point source; the lighter ones represent virtual walls and corresponding virtual sources.

The paper is organized as follows. In section A.2 we develop a mathematical model for the measured signal. We show that this signal is of the form of a Fourier integral operator applied to the scene. In section A.3 we explore various measurement geometries to determine which give rise to image artifacts and which do not. In all cases we form the image with a filtered backprojection method.

For the measurement geometries without artifacts, it is straightforward to determine the appropriate backprojection filters and show that the resulting image faithfully reproduces the visible edges appearing in the scene. Moreover, for measurement geometries without artifacts, the resolution of the imaging method can be quantified. This analysis we leave for the future.

A.2 The Mathematical model

The mathematical model involves a number of ingredients: 1) a model for wave propagation in free space, 2) a model for the array of sources, 3) a model for multiple scattering from buildings, and 4) a (linearized) model for scattering from the scene,

A.2.1 A model for the wave propagation

Model for wave propagation

We assume that waves propagate within the waveguide according to the scalar wave equation:

$$\nabla^2 u - c_0^{-2}\ddot{u} = 0, \tag{A.1}$$

where the dots denote differentiation with respect to t and c_0 is the speed of light.

The field due to a point source at $\boldsymbol{P} = 0$ at time $t = 0$ is

$$g_0(t, |\boldsymbol{P}|) = \frac{\delta(t - |\boldsymbol{P}|/c_0)}{4\pi|\boldsymbol{P}|} = \int \frac{e^{-i\omega(t-|\boldsymbol{P}|/c_0)}}{4\pi|\boldsymbol{P}|} d\omega \tag{A.2}$$

We will use capital letters for frequency-domain quantities, which are related to time-domain quantities by the Fourier transform

$$U(\omega, \boldsymbol{P}) = \frac{1}{2\pi} \int e^{i\omega t} u(t, \boldsymbol{P}) dt. \tag{A.3}$$

We write $k = \omega / c_0$.

Wave propagation in the waveguide

We consider the waveguide formed by two infinite parallel walls. We take one wall to be the plane $x = 0$ and the other to be the plane $x = L$. For perfectly conducting walls, a Green's function for the waveguide can be constructed by the method of images:

$$G(\omega, \boldsymbol{P}, \boldsymbol{P}') = \sum_{\alpha\in\{\pm\}} \sum_{n=-\infty}^{\infty} (-1)^n \frac{e^{ik|\boldsymbol{P}-\boldsymbol{P}'_{n,\alpha}|}}{4\pi|\boldsymbol{P}-\boldsymbol{P}'_{n,\alpha}|} \tag{A.4}$$

where the $\boldsymbol{P}'_{n,\pm}$ are the *virtual images* of $\boldsymbol{P}'$: if $\boldsymbol{P}' = (x', y', z')$, then $\boldsymbol{P}'_{n,+} = (x'+2nL, y', z')$ and $\boldsymbol{P}'_{n,-} = (-x'+2nL, y', z')$ (See Fig. A.1). The sum in (A.4) is over all the virtual images.

For simplicity of notation, we drop the factor of $(-1)^n$; this corresponds to taking Neumann boundary conditions on the walls rather than Dirichlet conditions.

We note that the virtual images satisfy certain symmetry relations:

$$\begin{aligned} |\boldsymbol{P}-\boldsymbol{P}'_{n,+}| &= \sqrt{[x-(x'+2nL)]^2+\cdots} &= |\boldsymbol{P}_{-n,+}-\boldsymbol{P}'| \\ |\boldsymbol{P}-\boldsymbol{P}'_{n,-}| &= \sqrt{[x-(-x'+2nL)]^2+\cdots} & \\ &= \sqrt{[-x+2nL-x')]^2+\cdots} &= |\boldsymbol{P}_{n,-}-\boldsymbol{P}'| \end{aligned} \tag{A.5}$$

and that

$$(\boldsymbol{P}+\boldsymbol{p})_{n,\alpha} = \boldsymbol{P}_{n,\alpha} + \boldsymbol{p}_{0,\alpha}. \tag{A.6}$$

We use tildes to indicate reflection of the x-coordinate about the y-z-plane: $\tilde{\boldsymbol{p}} = \boldsymbol{p}_{0,-}$ and $\tilde{\boldsymbol{p}}_{0,-} = \boldsymbol{p} = \boldsymbol{p}_{0,+}$.

Within the region $-L < x < L$, G satisfies

$$\nabla^2 G(\omega, \boldsymbol{P}, \boldsymbol{P}') + k^2 G(\omega, \boldsymbol{P}, \boldsymbol{P}') = -\delta(\boldsymbol{P}, \boldsymbol{P}') \tag{A.7}$$

To model a finite waveguide, say one extending up to height $z = H$, we include only certain of the virtual sources.

A.2.2 The model for scattering from the target

We model the target as a change in wave speed $V(\boldsymbol{P}) = c^{-2}(\boldsymbol{P}) - c_0^{-2}$. The scattered field $U^{sc}(\omega, \boldsymbol{P}, \boldsymbol{P}')$ at $\boldsymbol{P}$ due to an incident field $U^{in}(\omega, \boldsymbol{P}, \boldsymbol{P}') = G(\omega, \boldsymbol{P}, \boldsymbol{P}')$ is given by [?] the Lippmann-Schwinger equation:

$$U^{sc}(\omega, \boldsymbol{P}, \boldsymbol{P}') = \int G(\omega, \boldsymbol{P}, \boldsymbol{P}'')\omega^2 V(\boldsymbol{P}'')[G(\omega, \boldsymbol{P}'', \boldsymbol{P}') + U^{sc}(\omega, \boldsymbol{P}'', \boldsymbol{P}')]d\boldsymbol{P}'' \tag{A.8}$$

We wish to reconstruct V from knowledge of G^{sc}.

Unfortunately (A.8) is an integral equation whose solution U^{sc} depends in a nonlinear way on V. To simplify the problem, we make the *Born* or *single-scattering* approximation: we neglect the term involving U^{sc} on the right side of (A.8). This gives us

$$U_{\mathrm{B}}^{\mathrm{sc}}(\omega, \boldsymbol{P}, \boldsymbol{P}') = \int G(\omega, \boldsymbol{P}, \boldsymbol{P}'')\omega^2 V(\boldsymbol{P}'')G(\omega, \boldsymbol{P}'', \boldsymbol{P}')d\boldsymbol{P}'' \tag{A.9}$$

The Born approximation makes the mapping from V to U^{sc} linear, but it is not necessarily a good approximation. Another linearizing approximation that can be used for reflection from smooth surfaces is the *Kirchhoff approximation*, in which the scattered field is replaced by its geometrical optics approximation. Here, however, we consider only the Born approximation.

The corresponding time-domain field is

$$u_B^{\mathrm{sc}}(t, \boldsymbol{P}, \boldsymbol{P}') = \int \mathrm{e}^{-\mathrm{i}\omega t} G(\omega, \boldsymbol{P}, \boldsymbol{P}'')V(\boldsymbol{P}'')G(\omega, \boldsymbol{P}'', \boldsymbol{P}')\omega^2 d\omega d\boldsymbol{P}'' \tag{A.10}$$

We note that since G is given by a sum (A.4), (A.10) is of the form

$$u_B^{\mathrm{sc}}(t, \boldsymbol{P}, \boldsymbol{P}') = \sum_{\alpha,\beta\in\{\pm\}} \sum_{m,n} F_{n,\alpha,m,\beta}[V](t, \boldsymbol{P}, \boldsymbol{P}') \tag{A.11}$$

where

$$F_{n,\alpha,m,\beta}[V](t, \boldsymbol{P}, \boldsymbol{P}') = \int \frac{\mathrm{e}^{-\mathrm{i}\omega(t-|\boldsymbol{P}-\boldsymbol{P}''_{n,\alpha}|/c_0)}}{4\pi|\boldsymbol{P} - \boldsymbol{P}''_{n,\alpha}|} V(\boldsymbol{P}'') \frac{\mathrm{e}^{\mathrm{i}\omega|\boldsymbol{P}''-\boldsymbol{P}'_{m,\beta}|/c_0}}{4\pi|\boldsymbol{P}'' - \boldsymbol{P}'_{m,\beta}|}\omega^2 d\omega d\boldsymbol{P}'' \tag{A.12}$$

To the initial factors in (A.12) we apply the symmetry relations (A.5) and then we re-label the indices of (A.11) to write u^{sc} in the same form (A.11) with

$$F_{n,\alpha,m,\beta}[V](t, \boldsymbol{P}, \boldsymbol{P}') = \int \frac{\mathrm{e}^{-\mathrm{i}\omega(t-|\boldsymbol{P}_{n,\alpha}-\boldsymbol{P}''|/c_0)}}{4\pi|\boldsymbol{P}_{n,\alpha} - \boldsymbol{P}''|} V(\boldsymbol{P}'') \frac{\mathrm{e}^{\mathrm{i}\omega|\boldsymbol{P}''-\boldsymbol{P}'_{m,\beta}|/c_0}}{4\pi|\boldsymbol{P}'' - \boldsymbol{P}'_{m,\beta}|}\omega^2 d\omega d\boldsymbol{P}'' \tag{A.13}$$

A.3 Imaging for different measurement scenarios

We consider a variety of different measurement scenarios. One of the key features of the imaging problem is the number of degrees of freedom in the measured data, which determines whether we can form a three-dimensional image of a volume or merely a two-dimensional image of a surface. The number of degrees of freedom also has a big influence on whether artifacts will be present in images formed from the data.

For each of the measurement scenarios, we give a Born-approximated expression for the data and write down the natural filtered-backprojection imaging formula. We then analyze this formula to determine whether there are artifacts present in the image; this is done by a stationary phase analysis of the point spread function.

A.3.1 Strip-map synthetic-aperture geometry

In the strip-map synthetic-aperture case, we measure the time-varying backscattered wave from each point along a one-dimensional curve γ, which we parametrize as $\{\gamma(s) : s \in (s_{min}, s_{max})\}$.

The data then contains two degrees of freedom and we expect to make only a two-dimensional image of objects sitting on a known surface. In this case, we denote the known surface by $\{P = \psi(p) : p \in \mathbb{R}^2\}$, where $p = (p_1, p_2)$, and we write the wave speed perturbation as $V(p)\delta(P - \psi(p))$.

The synthetic-aperture data is then

$$D^{\mathrm{SA}}(t, s) = \int \mathrm{e}^{-\mathrm{i}\omega t} G(\omega, \gamma(s), \psi(p''))V(p'')G(\omega, \psi(p''), \gamma(s))\omega^2 d\omega dp'' \quad \text{(A.14)}$$

Since G is given by a sum (A.4), (A.14) is of the form

$$D^{\mathrm{SA}}(t, s) = \sum_{\alpha,\beta\in\{\pm\}} \sum_{m,n} F^{SA}_{n,\alpha,m,\beta}[V](t, s) \quad \text{(A.15)}$$

where

$$F^{\mathrm{SA}}_{n,\alpha,m,\beta}[V](t, s) = \int \frac{\mathrm{e}^{-\mathrm{i}\omega(t-|\gamma(s)-\psi_{n,\alpha}(p'')|/c_0)}}{4\pi|\gamma(s) - \psi_{n,\alpha}(p'')|} V(p'') \frac{\mathrm{e}^{\mathrm{i}\omega|\psi(p'')-\gamma_{m,\beta}(s)|/c_0}}{4\pi|\psi(p'') - \gamma_{m,\beta}(s)|} \omega^2 d\omega dp'' \quad \text{(A.16)}$$

As before, we apply we apply the symmetry relations (A.5) to (A.16) and re-label the indices in (A.15) to write D^{SA} in the same form (A.15) with

$$F^{\mathrm{SA}}_{n,\alpha,m,\beta}[V](t, s) = \int \frac{\mathrm{e}^{-\mathrm{i}\omega(t-|\gamma_{n,\alpha}(s)-\psi(p'')|/c_0)}}{4\pi|\gamma_{n,\alpha}(s) - \psi(p'')|} V(p'') \frac{\mathrm{e}^{\mathrm{i}\omega|\psi(p'')-\gamma_{m,\beta}(s)|/c_0}}{4\pi|\psi(p'') - \gamma_{m,\beta}(s)|} \omega^2 d\omega dp''$$

(A.17)

We note that $F^{\mathrm{SA}}_{n,\alpha,m,\beta}$ is of the form

$$F^{\mathrm{SA}}_{n,\alpha,m,\beta}[V](t,s) = \int \mathrm{e}^{\mathrm{i}\omega[t-(|\gamma_{n,\alpha}(s)-\psi(p'')+|\psi(p'')-\gamma_{m,\beta}(s)|)/c_0]} A^{\mathrm{SA}}_{n,\alpha,m,\beta}(\omega,s,\boldsymbol{p}'') V(\boldsymbol{p}'') d\omega d\boldsymbol{p}'' \tag{A.18}$$

Image formation

We form an image by filtered backprojection:

$$I(\boldsymbol{r}) = \sum_{\alpha,\beta\in\{\pm\}} \sum_{m,n} B^{\mathrm{SA}}_{n,\alpha,m,\beta}[D^{\mathrm{SA}}](\boldsymbol{r}) \tag{A.19}$$

where

$$B^{\mathrm{SA}}_{n,\alpha,m,\beta}[D^{\mathrm{SA}}](\boldsymbol{r}) = \int \mathrm{e}^{\mathrm{i}\omega[t-(|\gamma_{n,\alpha}(s)-\psi(\boldsymbol{r})+|\psi(\boldsymbol{r})-\gamma_{m,\beta}(s)|)/c_0]} M^{\mathrm{SA}}_{n,\alpha,m,\beta}(\boldsymbol{r},s,\omega) D^{\mathrm{SA}}(t,s) d\omega ds dt \tag{A.20}$$

for some filter $M^{\mathrm{SA}}_{n,\alpha,m,\beta}$ to be determined. Some of the M's may be chosen to be zero to restrict the appearance of artifacts [?].

Analysis of the image

Using (A.15) and (A.12) in (A.19) results in an equation of the form

$$I(\boldsymbol{r}) = \int K(\boldsymbol{r},\boldsymbol{p}) V(\boldsymbol{p}) d\boldsymbol{p}, \tag{A.21}$$

where the kernel K is the imaging *point-spread function*. It is given by

$$K(\boldsymbol{r},\boldsymbol{p}) = \sum_{\alpha,\beta\in\{\pm\}} \sum_{m,n} \sum_{\alpha',\beta'\in\{\pm\}} \sum_{m',n'} K^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\boldsymbol{r},\boldsymbol{p}) \tag{A.22}$$

with

$$\begin{aligned} K^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\boldsymbol{r},\boldsymbol{p}) &= B^{\mathrm{SA}}_{n,\alpha,m,\beta}[F^{\mathrm{SA}}_{n',\alpha',m',\beta'}[V]](\boldsymbol{r}) \\ &= \int \mathrm{e}^{\mathrm{i}\omega[t-(|\gamma_{n,\alpha}(s)-\psi(\boldsymbol{r})+|\psi(\boldsymbol{r})-\gamma_{m,\beta}(s)|)/c_0]} \end{aligned}$$

$$M^{\mathrm{SA}}_{n,\alpha,m,\beta}(\boldsymbol{r},s,\omega)$$
$$\int \mathrm{e}^{\mathrm{i}\omega'[t-(|\gamma_{n',\alpha'}(s)-\psi(\boldsymbol{p})+|\psi(\boldsymbol{p})-\gamma_{m',\beta'}(s)|)/c_0]}$$
$$A^{\mathrm{SA}}_{n',\alpha',m',\beta'}(\omega',s,\boldsymbol{p})d\omega d\omega' ds dt \qquad \text{(A.23)}$$

If we had $K(\boldsymbol{r},\boldsymbol{p}) = \delta(\boldsymbol{r}-\boldsymbol{p})$, then the image I would be perfect; we want to determine the M's so that K comes as close as possible to being a delta function.

In (A.23) we carry out the t and ω' integrations, using $\int \mathrm{e}^{\mathrm{i}(\omega-\omega')t}dt = 2\pi\delta(\omega-\omega')$, and obtaining

$$K^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\boldsymbol{r},\boldsymbol{p}) = 2\pi \int \exp\left[\mathrm{i}\phi^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\omega,s,\boldsymbol{r},\boldsymbol{p})\right] M^{\mathrm{SA}}_{n,\alpha,m,\beta}(\boldsymbol{r},s,\omega)A^{\mathrm{SA}}_{n',\alpha',m',\beta'}(\omega,s,\boldsymbol{p})d\omega ds \qquad \text{(A.24)}$$

where t is evaluated at $t = (|\gamma_{n',\alpha'}(s)-\psi(\boldsymbol{p})+|\psi(\boldsymbol{p})-\gamma_{m',\beta'}(s)|)/c_0$ and where

$$\phi^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\omega,s,\boldsymbol{r},\boldsymbol{p}) = k(|\gamma_{n',\alpha'}(s)-\psi(\boldsymbol{p})|+|\psi(\boldsymbol{p})-\gamma_{m',\beta'}(s)| -(|\gamma_{n,\alpha}(s)-\psi(\boldsymbol{r})|+|\psi(\boldsymbol{r})-\gamma_{m,\beta}(s)|) \qquad \text{(A.25)}$$

In order for the K to be a close approximation to a delta function, we would like each diagonal terms, for which $n = n', \alpha = \alpha', m = m'$, and $\beta = \beta'$, to itself be a good approximation to a multiple of a delta function, and we would like the off-diagonal terms to be zero or to contribute only higher-order terms.

We use the method of stationary phase (see appendix) to determine the leading-order contributions. Analysis of the critical points of the phase determines the locus of points $\boldsymbol{r}$ that will appear in the image due to a scatterer located at $\boldsymbol{p}$. We would like the critical conditions to imply that $\boldsymbol{r} = \boldsymbol{p}$; if this is not the case, the other possible solutions $\boldsymbol{r}$ tell us what artifacts that will appear in the image due to a scatterer at $\boldsymbol{p}$.

The critical conditions we write as

$$0 = \frac{\partial\phi}{\partial\omega} \propto T_{n',\alpha',m',\beta'}(s,\boldsymbol{p}) - (|\gamma_{n,\alpha}-\psi(\boldsymbol{r})|+|\psi(\boldsymbol{r})-\gamma_{m,\beta}| \qquad \text{(A.26)}$$
$$0 = \frac{\partial\phi}{\partial s} \propto \nu_{n',\alpha',m',\beta'}(s,\boldsymbol{p}) - \widehat{\gamma_{n,\alpha}-\psi}(\boldsymbol{r})\cdot\dot{\gamma}_{n,\alpha} - \widehat{\gamma_{m,\beta}-\psi}(\boldsymbol{r})\cdot\dot{\gamma}_{m,\beta} \qquad \text{(A.27)}$$

where we have temporarily dropped some of the s's and where

$$T_{n',\alpha',m',\beta'}(s,\boldsymbol{p}) = |\gamma_{n',\alpha'}(s)-\psi(\boldsymbol{p})|+|\psi(\boldsymbol{p})-\gamma_{m',\beta'}(s)| \qquad \text{(A.28)}$$

$$\nu_{n',\alpha',m',\beta'}(s,\boldsymbol{p}) = \widehat{\gamma_{n',\alpha'}(s) - \psi(\boldsymbol{p})} \cdot \dot{\gamma}_{n',\alpha'}(s) + \widehat{\gamma_{m',\beta'}(s) - \psi(\boldsymbol{p})} \cdot \dot{\gamma}_{m',\beta'}(s); \tag{A.29}$$

In (A.26) and (A.27), we imagine the scatterer at $\boldsymbol{p}$ to be fixed; (A.26) and (A.27) are then equations that determine the location of $\boldsymbol{r}$.

Equation (A.26) says that $\psi(\boldsymbol{r})$ must lie on an ellipsoid whose foci are the virtual sources and receivers $\gamma_{n,\alpha}$ and $\gamma_{m,\beta}$. We note that this equation has no solution if $T_{n',\alpha',m',\beta'}(s,\boldsymbol{p})$ is sufficiently small relative to $|\gamma_{n,\alpha}(s) - \gamma_{m,\beta}(s)|$.

Equation (A.27) says that $\psi(\boldsymbol{r})$ lies on a certain Doppler surface; for the case $n = m, \alpha = \beta$, this surface is a constant-Doppler cone. (In other words, points on this surface see the same down-range component of the relative velocity and would thus exhibit the same Doppler frequency shift.) We note that $|\nu_{n',\alpha',m',\beta'}(s,\boldsymbol{p})| \leq 2|\dot{\gamma}(s)|$.

The intersection of the surfaces (A.26), (A.27), and the surface $\{\psi(\boldsymbol{r}) : \boldsymbol{r} \in \mathbb{R}^2\}$ determines the location of the point $\boldsymbol{r}$ in the image due to a scatterer at location $\boldsymbol{p}$. For some values of T and ν, the curves formed by the intersection of (A.26) and (A.27) with the earth surface do not intersect at all, but in general these curves intersect at two points, one to the right of the flight path and one to the left. If $(n',\alpha',m',\beta') = (n,\alpha,m,\beta)$, one of these points is at $\boldsymbol{r} = \boldsymbol{p}$ (i.e., the image of $\boldsymbol{p}$ appears in the correct location); the other intersection causes a familiar [?] artifact in the image that can be avoided by using an array of sources whose beam illuminates points only to one side of the flight path. For many values of $(n',\alpha',m',\beta') \neq (n,\alpha,m,\beta)$, however, the intersections occur for $\boldsymbol{r} \neq \boldsymbol{p}$, and these intersections give rise to artifacts in the image. Essentially the backprojection operator misinterprets multiply scattering paths as paths with the same total travel time but a different number of bounces, coming from a different direction.

It is possible that some or all of these artifacts can be avoided by a combination of a) using certain flight paths, b) using an antenna beam that does not illuminate the region where the artifacts occur, and c) taking some of the filters $M^{\mathrm{SA}}_{n,\alpha,m,\beta}$ of (A.20) to be zero, so that backprojection is carried out only along certain paths. A more detailed investigation of these possibilities is left for the future.

Examples. In Figures A.2 and A.3 we show some Maple plots of the locus of points on a flat earth satisfying (A.26) and (A.27). In both cases, the waveguide is the region from $x = 0$ to $x = L = 1$. The flight path is a level straight line at height $H = 3$. We consider the case when the sensor is positioned at $\gamma(s) = (.4, 0, 3)$ and we consider artifacts produced by a scatterer located at $\boldsymbol{P} = (.2, 1, 0)$.

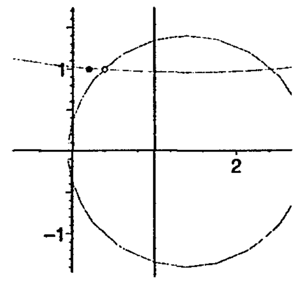

Figure A.2: This shows the intersection of (A.26) and (A.27) for the strip-map SAR $K^{1,+0,+}_{1,-,1,-}((x,y),(.2,1))$. The true image point $(.2,1)$ is marked with a black dot; the artifact, which appears at the intersection point at approximately (.3686, .9925), is marked with a grey dot. The dark vertical lines denote the waveguide walls. Here the flight path is upward.

Figure A.2 shows the case of $K^{1,+0,+}_{1,-,1,-}((x,y),(.2,1))$ when the flight velocity vector is $(0,1,0)$. The intersection of the ellipsoid (A.26) and the curve (A.27) identifies a point [approximately (.3686, .9925)] within the waveguide where an image of $(.2,1)$ will appear; since this point is not at $(.2,1)$, it will be an artifact. This intersection point, however, is sufficiently near $(.2,1)$ that it is likely to be within the sensor footprint.

Figure A.3 shows the case of $K^{1,+0,+}_{0,-,2,-}((x,y),(.2,1))$ when the flight velocity vector is $(1,0,0)$. The intersection of the ellipsoid (A.26) and the curve (A.27) identifies two points within the waveguide where an image of $(.2,1)$ will appear; these points are both artifacts.

A.3.2 Single fixed source, single moving receiver

In the case of a source fixed at $\boldsymbol{P}'$ and receiver moving along the curve $\gamma(s)$, again the data is a function of two variables (t and s); again we attempt only to image objects on a known surface ψ.

The data is then

$$D^{\mathrm{MR}}(t,s) = \int \mathrm{e}^{-\mathrm{i}\omega t} G(\omega,\gamma(s),\psi(\boldsymbol{p}''))V(\boldsymbol{p}'')G(\omega,\psi(\boldsymbol{p}''),\boldsymbol{P}')\omega^2 d\omega d\boldsymbol{p}'' \tag{A.30}$$

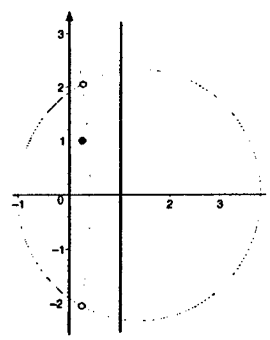

Figure A.3: This shows the intersection of (A.26) and (A.27) for the strip-map SAR $K^{1,+0,+}_{0,-,2,-}((x,y),(.2,1))$. The true image point (.2, 1) is marked with a black dot; the intersections marked with grey dots, give rise to artifacts. The dark vertical lines denote the waveguide walls. Here the flight path is to the right.

As before, since G is given by a sum (A.4), (A.30) is of the form

$$D^{\mathrm{MR}}(t,s) = \sum_{\alpha,\beta\in\{\pm\}} \sum_{m,n} F^{MR}_{n,\alpha,m,\beta}[V](t,s) \tag{A.31}$$

where

$$F^{\mathrm{MR}}_{n,\alpha,m,\beta}[V](t,s) = \int \frac{\mathrm{e}^{-\mathrm{i}\omega(t-|\gamma(s)-\boldsymbol{\psi}_{n,\alpha}(\boldsymbol{p}'')|/c_0)}}{4\pi|\gamma(s)-\boldsymbol{\psi}_{n,\alpha}(\boldsymbol{p}'')|} V(\boldsymbol{p}'') \frac{\mathrm{e}^{\mathrm{i}\omega|\boldsymbol{\psi}_{m,\beta}(\boldsymbol{p}'')-\boldsymbol{P}'|/c_0}}{4\pi|\boldsymbol{\psi}_{m,\beta}(\boldsymbol{p}'')-\boldsymbol{P}'|} \omega^2 d\omega d\boldsymbol{p}'' \tag{A.32}$$

As before, we apply we apply the symmetry relations (A.5) to (A.32) and re-label the indices in (A.31) to write D^{MR} in the same form (A.31) with

$$F^{\mathrm{MR}}_{n,\alpha,m,\beta}[V](t,s) = \int \frac{\mathrm{e}^{-\mathrm{i}\omega(t-|\gamma_{n,\alpha}(s)-\boldsymbol{\psi}(\boldsymbol{p}'')|/c_0)}}{4\pi|\gamma_{n,\alpha}(s)-\boldsymbol{\psi}(\boldsymbol{p}'')|} V(\boldsymbol{p}'') \frac{\mathrm{e}^{\mathrm{i}\omega|\boldsymbol{\psi}(\boldsymbol{p}'')-\boldsymbol{P}'_{m,\beta}(s)|/c_0}}{4\pi|\boldsymbol{\psi}(\boldsymbol{p}'')-\boldsymbol{P}'_{m,\beta}(s)|} \omega^2 d\omega d\boldsymbol{p}'' \tag{A.33}$$

We note that $F^{\mathrm{MR}}_{n,\alpha,m,\beta}$ is of the form

$$F^{\mathrm{MR}}_{n,\alpha,m,\beta}[V](t,s) = \int \mathrm{e}^{\mathrm{i}\omega[t-(|\gamma_{n,\alpha}(s)-\boldsymbol{\psi}(\boldsymbol{p}'')+|\boldsymbol{\psi}(\boldsymbol{p}'')-\boldsymbol{P}'_{m,\beta}(s)|)/c_0]} a_{n,\alpha,m,\beta}(\omega,s,\boldsymbol{p}'') \, V(\boldsymbol{p}'') d\omega d\boldsymbol{p}'' \tag{A.34}$$

Image formation

We form an image by filtered backprojection:

$$I(\boldsymbol{r}) = \sum_{\alpha,\beta\in\{\pm\}} \sum_{m,n} B^{\mathrm{MR}}_{n,\alpha,m,\beta}[D^{\mathrm{MR}}](\boldsymbol{r}) \tag{A.35}$$

where

$$B^{\mathrm{MR}}_{n,\alpha,m,\beta}[D^{\mathrm{MR}}](\boldsymbol{r}) = \int \mathrm{e}^{\mathrm{i}\omega[t-(|\gamma_{n,\alpha}(s)-\boldsymbol{\psi}(\boldsymbol{r})+|\boldsymbol{\psi}(\boldsymbol{r})-\boldsymbol{P}'_{m,\beta}|)/c_0]} \, b^{\mathrm{MR}}_{n,\alpha,m,\beta}(\boldsymbol{r},t,s,\omega) D^{\mathrm{MR}}(t,s)\,d\omega\, ds\, dt \tag{A.36}$$

for some filter $b^{\mathrm{MR}}_{n,\alpha,m,\beta}$ to be determined.

Analysis of the image

We follow the same procedure: use (A.31) and (A.12) in (A.35) to obtain an equation of the form (A.21) where now the kernel K is given by

$$K(\boldsymbol{r},\boldsymbol{p}) = \sum_{\alpha,\beta\in\{\pm\}} \sum_{m,n} \sum_{\alpha',\beta'\in\{\pm\}} \sum_{m',n'} K^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\boldsymbol{r},\boldsymbol{p}) \tag{A.37}$$

with

$$\begin{aligned} K^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\boldsymbol{r},\boldsymbol{p}) &= B^{\mathrm{MR}}_{n,\alpha,m,\beta}[F^{\mathrm{MR}}_{n',\alpha',m',\beta'}[V]](\boldsymbol{r}) \\ &= \int \mathrm{e}^{\mathrm{i}\omega[t-(|\gamma_{n,\alpha}(s)-\boldsymbol{\psi}(\boldsymbol{r})+|\boldsymbol{\psi}(\boldsymbol{r})-\boldsymbol{P}'_{m,\beta}|)/c_0]} \\ &\qquad b^{\mathrm{MR}}_{n,\alpha,m,\beta}(\boldsymbol{r},t,s,\omega) \\ &\qquad \int \mathrm{e}^{\mathrm{i}\omega'[t-(|\gamma_{n',\alpha'}(s)-\boldsymbol{\psi}(\boldsymbol{p})+|\boldsymbol{\psi}(\boldsymbol{p})-\boldsymbol{P}'_{m',\beta'}|)/c_0]} \\ &\qquad a_{n',\alpha',m',\beta'}(\omega',t,s,\boldsymbol{p})\,d\omega\, d\omega'\, ds\, dt \end{aligned} \tag{A.38}$$

In (A.38) we carry out the t and ω' integrations as before, obtaining

$$K^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\boldsymbol{r},\boldsymbol{p}) = \int \exp\left[\mathrm{i}\phi^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\omega,s,\boldsymbol{r},\boldsymbol{p})\right] b_{n,\alpha,m,\beta}(\boldsymbol{r},t,s,\omega) a_{n',\alpha',m',\beta'}(\omega,t,s,\boldsymbol{p})\,d\omega\, ds \tag{A.39}$$

where t is evaluated at $t = (|\gamma_{n',\alpha'}(s) - \boldsymbol{\psi}(\boldsymbol{p}) + |\boldsymbol{\psi}(\boldsymbol{p}) - \gamma_{m',\beta'}(s)|)/c_0$ and where

$$\phi^{n,\alpha,m,\beta}_{n',\alpha',m',\beta'}(\omega,s,\boldsymbol{r},\boldsymbol{p}) = k(|\gamma_{n',\alpha'}(s) - \boldsymbol{\psi}(\boldsymbol{p})| + |\boldsymbol{\psi}(\boldsymbol{p}) - \boldsymbol{P}'_{m',\beta'}|$$

$$-(|\gamma_{n,\alpha}(s) - \psi(r)| + |\psi(r) - P'_{m,\beta}|) \quad \text{(A.40)}$$

The critical conditions are

$$0 = \frac{\partial \phi}{\partial \omega} \propto T_{n',\alpha',m',\beta'}(s,p) - (|\gamma_{n,\alpha} - \psi(r)| + |\psi(r) - P'_{m,\beta}| \quad \text{(A.41)}$$

$$0 = \frac{\partial \phi}{\partial s} \propto \nu_{n',\alpha',m',\beta'}(s,p) - \gamma_{n,\alpha} \widehat{- \psi}(r) \cdot \dot{\gamma}_{n,\alpha} \quad \text{(A.42)}$$

where we have temporarily dropped some of the s's and where

$$T_{n',\alpha',m',\beta'}(s,p) = |\gamma_{n',\alpha'}(s) - \psi(p)| + |\psi(p) - P'_{m',\beta'}| \quad \text{(A.43)}$$

$$\nu_{n',\alpha',m',\beta'}(s,p) = \gamma_{n',\alpha'}\widehat{(s) - \psi}(p) \cdot \dot{\gamma}_{n',\alpha'}(s); \quad \text{(A.44)}$$

In (A.41) and (A.42), we imagine the scatterer at p to be fixed; (A.41) and (A.42) are then equations that determine the location of r.

Equation (A.41) says that $\psi(r)$ must lie on an ellipsoid whose foci are the virtual sources and receivers $\gamma_{n,\alpha}$ and $P'_{m,\beta}$. We note that this equation has no solution if $T_{n',\alpha',m',\beta'}(s,p)$ is sufficiently small relative to $|\gamma_{n,\alpha}(s) - P'_{m,\beta}(s)|$.

Equation (A.42) says that $\psi(r)$ lies on a certain Doppler cone whose axis is the flight velocity vector $\dot{\gamma}(s)$. We note that $|\nu_{n',\alpha',m',\beta'}(s,p)| \leq 2|\dot{\gamma}(s)|$.

As in the monostatic strip-map synthetic-aperture case, equations (A.41) and (A.42) have many solutions with $r \neq p$; this gives rise to artifacts in the image.