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Prescribed by ANSI Std Z39-18
Estimation of Bio-Aerosol Concentration from Elastic Scattering LIDAR Data

November 11, 2003

Allen Q. Howard, Jr., George W. Lemire and Martin S. Marshall
OUTLINE

- Introduction and Application
- Relevant Theory
- Forward Model
- Noise Considerations
- Inverse Model and Simulations
- Conclusion and Future Development
Introduction

- The West Desert Test Center (WDTC) at DPG uses an elastic backscatter LIDAR system to augment the tracking and characterization of bio-aerosol clouds.
- In support of DPG’s test mission, hardware and software development for WDTC LIDAR capability is continuing.
- This talk is a progress report on our M&S project to estimate bio-aerosol concentration from LIDAR elastic backscatter data.
LIDAR Equation

A convenient form of the Lidar equation predicting the received power (in Watts), from an increment of atmosphere $\Delta R$ at range $R$, is

$$P(\lambda, R) = P_0 \frac{A_0}{R^2} \Delta R \beta_\pi(\lambda, r) e^{-2 \int_0^R \kappa_e(\lambda) dR}.$$ 

Here $\lambda$ is wave length, $A_0$ is area of the receiver telescope objective lens, $\beta_\pi$ is the backscatter coefficient and $\kappa_e$ is the extinction coefficient.
Backscatter Coefficient

The backscatter coefficient $\beta_\pi$ $(m^{-1} \text{sr}^{-1})$ is

$$\beta_\pi(\lambda, r) = \int_0^\infty N(a, r) \sigma_b(\lambda, a) \, da.$$ 

Distribution $N(a, r)$ is normalized such that

$$\rho(r) = \int_0^\infty N(a, r) \, da.$$ 

Here $\rho(r)$ is number of particles per unit volume. The Mie backscattering cross-section is

$$\sigma_b(\lambda, a) = \frac{1}{k^2} \left\{ \frac{1}{2} \sum_{m=1}^{\infty} (2m+1)(-1)^m (a_m - b_m)^2 \right\}^2.$$
The extinction coefficient \( \kappa_e(\lambda, r) \) \((\text{m}^{-1})\) is

\[
\kappa_e(\lambda, r) = \rho(r) \int_0^\infty N_1(a) \sigma_e(\lambda, a) \, da
\]

where the extinction cross section is

\[
\sigma_e(\lambda, a) = \frac{2\pi}{k^2} \Re \left[ \sum_{m=1}^\infty (2m + 1) (a_m + b_m) \right].
\]
To estimate aerosol concentration, transform the measured LIDAR data to obtain

\[ S(R, \lambda) = \log \left( \frac{R^2 P(\lambda, R)}{R_0^2 P(\lambda, R_0)} \right). \]

This definition then yields the explicit relation

\[ \frac{d}{dR} S(R, \lambda) = \frac{1}{\rho(R)} \frac{d\rho}{dR}(R) - 2 \rho(R) < \sigma_e(\lambda) >. \]

This equation is solved for the particle density function \( \rho(r) \). Necessary information includes the averaged extinction coefficient \( < \sigma_e(\lambda) > \), LIDAR data \( S(R, \lambda) \) and a boundary point \( \rho_f \).
Formal Analytic Solution to Aerosol Density

\[ \rho(R) = \left[ \frac{1}{\rho_f} + 2 \left< \sigma_e(\lambda) \right> \int_R^{R_f} \frac{dR'}{\tau(R', \lambda)} \right]^{-1}, \]

where

\[ \tau(R, \lambda) = e^{-[S(R, \lambda) - S(R_f, \lambda)]} \]
Aerosol Scattering

Lidar signals are proportional to $Q_b$ and exponentially attenuated proportional to $Q_e$

The Nation’s Chemical & Biological Defense Proving Ground
Aerosol Particle Size Distribution

Particle Size Distribution, \( \int \frac{N(a)\,da}{N_{\text{total}}} = 1 \)

- Blue: \( \nu = 5 \)
- Green: \( \nu = 6 \)
- Red: \( \nu = 7 \)

\( a, \text{ particle radius (\(\mu m\)), } N_{\text{total}} = 100 \, \text{cm}^{-3} \)

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Extinction Cross Section versus Aerosol Mode Radius $a_m$

Aerosol Averaged Extinction Cross Section

Mode radius $a_m$ ($\mu m$) for $n = 1.33$

- $\lambda = 0.266$
- 0.355
- 0.532
- 1.064

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Particle Density Function
Synthetic Density Data
Synthetic LIDAR Data

Two Frequency LIDAR data $S(R,\lambda)$ for $R_f = \frac{1}{2} R_{\text{max}}$
Noisy Synthetic LIDAR data $P(R, \lambda)$ for wavelength 1.064$\mu$. Lower panel shows effect of Kaiser window with side lobe level of 200 dB.
First Stage Inversion

First stage inversion results using two-frequency method.
Second Stage Inversion with $S/N = 10$ dB noise using equations for $q_{nm}$ and point detector calibration.
Linear Scale Inversion Results

Linear scale particle density inversion in S/N = 10 dB noise. Upper sub-plot is exact density.
Figure 11: Inversion error analysis as a function of azimuth angle fraction $f_\phi$ with five values of $f_R$ as shown in the legend. Upper left-hand panel is the relative percent error in the extinction coefficient estimate $\alpha^{(1)} = 2 < \sigma_\varepsilon(\lambda_1) >$. The upper right panel displays the analogous percent relative errors in averaged boundary values $q_f$. Note the condition number is independent of $f_\phi$. 
Discrete Forward Model

\[ q_{nm}^{(i)} = [V_{nm}^i + q_{fm}^{(i)}] \tau_{nm}^i, \]

where

\[ \tau_{nm}^i = \tau(R_n, \phi_m, \lambda_i), \]

\[ \tau(R, \phi, \lambda_i) = \exp[S(R, \phi, \lambda_i) - S(R_f, \phi, \lambda_i)], \]

\[ q_{nm}^{(i)} = q(R_n, \phi_m, \lambda_i), \]

\[ q_{fm}^{(i)} = q^{(i)}(R_f, \phi_m), \]

\[ q(R, \phi, \lambda_i) = 1/(2 < \sigma_e(\lambda) > \rho(R, \phi)), \]

\[ V_{nm}^i = \int_{R_n}^{R_f} dR' / \tau(R', \phi_m, \lambda_i). \]
Density Calibration

\[ \hat{\rho}_{nm} = \rho_{cal} \frac{q^i_{cal}}{q^i_{nm}} \]
The Nation’s Chemical & Biological Defense Proving Ground

Single-Ray, Two-Frequency Algorithm

\[
\begin{pmatrix}
T_{m,a}^{(2,1)} & v_{m,a}^{(2)} \\ T_{m,b}^{(2,1)} & v_{m,b}^{(2)}
\end{pmatrix}
\begin{pmatrix}
q_{fm}^{(1)} \\ C^{(1,2)}
\end{pmatrix} =
\begin{pmatrix}
v_{m,a}^{(1)} \\ v_{m,b}^{(1)}
\end{pmatrix},
\]

where

\[
T_{m,a}^{(i,j)} = \frac{1}{N_a} \sum_{n=1}^{N_a} \left[ \tau_{nm}^i - \tau_{nm}^j \right],
\]

\[
T_{m,b}^{(i,j)} = \frac{1}{N_b} \sum_{n=N_a+1}^{N_R} \left[ \tau_{nm}^i - \tau_{nm}^j \right].
\]
where

\[ V_{m,a}^{(i)} = \frac{1}{N_a} \sum_{n=1}^{N_a} V_{nm}^i \tau_{nm}^i, \]

\[ V_{m,b}^{(i)} = \frac{1}{N_b} \sum_{n=N_a+1}^{N_R} V_{nm}^i \tau_{nm}^i, \]

\[ N_a + N_b = N_R. \]
Single-Ray, Three-Frequency Algorithm

\[
\begin{pmatrix}
T_{m,a}^{(2,1)} & 0 & v_{m,a}^{(2)} & 0 & 0 \\
T_{m,b}^{(2,1)} & 0 & v_{m,b}^{(2)} & 0 & 0 \\
T_{m,a}^{(3,1)} & 0 & 0 & v_{m,a}^{(3)} & 0 \\
T_{m,b}^{(3,1)} & 0 & 0 & v_{m,b}^{(3)} & 0 \\
0 & T_{m,a}^{(2,3)} & 0 & 0 & v_{m,a}^{(2)} \\
0 & T_{m,b}^{(2,3)} & 0 & 0 & v_{m,b}^{(2)}
\end{pmatrix}
\begin{pmatrix}
q_{f,m}^{(1)} \\
q_{f,m}^{(3)} \\
C_{(1,2)}^{(1)} \\
C_{(1,3)}^{(1)} \\
C_{(3,2)}^{(3)}
\end{pmatrix}
= 
\begin{pmatrix}
v_{m,a}^{(1)} \\
v_{m,b}^{(1)} \\
v_{m,a}^{(1)} \\
v_{m,b}^{(1)} \\
v_{m,a}^{(3)} \\
v_{m,b}^{(3)}
\end{pmatrix}
\]
Conclusion & Future Work

- 2 and 3 frequency elastic scattering algorithms for particle density have been developed.
- At least 2 frequencies are necessary to determine reference arc values $\rho_f$.
- Ongoing work includes background removal, first stage signal-to-noise enhancement, and system conditioning.