OPTIMAL COVERAGE OF THEATER TARGETS WITH SMALL SATELLITE

CONSTELLATIONS

THESIS

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THESIS

Presented to the Faculty

Department of Aeronautics and Astronautics

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science (Space Systems)

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March 2006

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

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Abstract

The daylight passes of a low-Earth orbit satellite over a targeted latitude and longitude are optimized by varying the inclination and eccentricity of an orbit at different altitudes. This investigation extends the work by Emery et al, in which the optimal Right Ascension of the Ascending Node was determined for a circular, matched inclination orbit. The optimal values were determined by a numerical research method based on Emery et al.'s Matlab program. Results indicate that small increases in inclination raise the number of daylight passes up to 33%. These optimal inclinations depend on the satellite semi-major axis. Eccentricity increases also improve daylight pass numbers, but at a cost of increased range to the target.

Acknowledgments

I would like to thank my family for their continuous wisdom and support that has guided me to reach this point. I would like to express my gratitude to my faculty advisor Lt Col. Nathan Titus because without his help and guidance I would not have made it to this point. I also would like to thank my thesis committee as well for their support and guidance during this endeavor. I thank the Watson Scholars Initiative for giving myself and my fellow Scholars the opportunity to attend AFIT and earn our Masters degree.

I also appreciate the moral support that all my fellow Scholars and students gave me throughout the entire time we shared at AFIT. Finally, I am grateful for my parents and family without their constant encouragement, love, and support, I would not have the strength to endure the hard-times.

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CONSTELLATIONS

1 Introduction

1.1 Background

The use of satellites has steadily increased since the start of the space age. Although there are many reasons for using satellites, the three main uses have been Earth observation (including weather), communication, and navigation. These uses require continuous coverage of the globe in order to gather and communicate information from around the world. These missions have become increasingly important for national defense, in which planners and commanders now desire global coverage in real time. The need for real time information for an up to the minute view of an area of conflict is vital for strategic purposes. When the military needs to go into an area of conflict, the first objective is to gain visibility of that area so as to observe enemy movements and plan operations. This military need can be met by sending reconnaissance aircraft to that area in order to get this information, but missions like this put aircraft and crew at risk. Satellites can gather similar reconnaissance information as an aircraft but much more safely because of the wide difference of altitude a satellite operates compared to an aircraft. These goals are hard to accomplish with the use of one satellite, so collections of satellites (known as a satellite constellation) became a primary focus of study. At high altitudes, such as geosynchronous orbits, continual coverage is possible with just a few satellites in a constellation, but as the satellite altitude is lowered (for example, to

improve imaging resolution), the number of satellites required for continual coverage rapidly increases. Therefore, it has been proposed that constellations could be designed for limited (theater) area coverage and optimized to maximize coverage over time. Currently all satellites in orbit are national assets, this means if a field commander in a theater needs to have information provided by a satellite, that commander needs to give this request to the organization who is in charge of operating that specific satellite. The information requested by the field commander needs to be real time imaging of the specific region of interest in order to have up to date information to carry out the essential mission. With the current process used for obtaining satellite imaging information, the imagery given to the field commander would not be an accurate real time image of the specific region of interest. This is due to the time spent requesting the information and waiting to receive the information needed; this time spent could be a critical component in planning and executing a successful mission. The TACSAT program in turn wants to look at the utility of one or more satellites that are directly controlled by the commander in a theater.

1.2 Problem

A satellite able to perform the functions needed by the field commander has not yet been fielded for routine operations. The desired spacecraft need not be state-of-the-art; but rather capable only of simple tasks of taking images of the theater of operations, and sending that information to the field commander in a timely manner. The cost of building a spacecraft for this specific mission should not be too high; it is the cost of launching the satellite that would be costly. Because of the high cost of launching a satellite into space it is important to find and use the optimal configuration for a satellite's

orbit. Using one satellite may supply adequate coverage of the region but using a small

constellation of satellites, for example two satellites, can increase the coverage time.

Finding the appropriate orbit for a single satellite, as well as for a two satellite

constellation, is the primary purpose of this thesis.

As a part of their study on the military utility of TACSATs, Emery et al.

investigated using circular orbits to meet this criterion. The orbit optimization was only

one aspect of the work; they also considered the logistical impact and the analysis of the

use of satellite constellations for tactical needs. It was an AFRL sponsored thesis for a

"proof of concept development of a responsive, tactical, space based ISR system (4: 5)".

System Specifications for the TACSAT were provided; these specified different

components of the TACSAT. The specification parameters provided dealt with the

mission's life and duration as well as the orbital inclination used and area of interest for

surveillance. The TACSAT system specification for the inclination was a matched to

Theater latitude inclination. The theater in question is the Iraq Theater that has a latitude

of 33 degrees. In addition, they were provided with the design parameters for the

satellite. These included the orbit altitude and the eccentricity. The altitude of the orbit

was set at 350 km and was to be circular; in other words, it had an eccentricity of zero.

One of their goals was to use the specifications and parameters given to them to simulate

the number of passes the satellite would have over the target. They needed to find an

optimal orbit configuration for the satellite so as to provide optimal coverage of the

target. A member of the group, Major David B. Smuck developed a Matlab program

specifically for this purpose. Since one of the design parameters of the TACSAT was to

be in a circular orbit, there were a number of orbital elements left as constants through

out the simulation. The elements left constant were the semi-major axis, the eccentricity, and the inclination. The right ascension of the ascending node (RAAN) and the argument of perigee were varied from 0 to 360 degrees. The RAAN and argument of perigee were varied in increments of 36 and 30 degrees, respectively. The program's main function was to quantify the total number of daylight passes over the target at different RAAN/argument of perigee combinations. The RAAN/argument of perigee combinations as well as the other orbital element values allowed Emery et al. to configure a set of orbits that would give the optimal coverage of the area of interest. The group first simulated for a single satellite. Once they acquired the optimal configuration for a single satellite, their next step was to find the optimal configuration for a constellation of satellites. Since their work only dealt with numerically simulating the coverage of a satellite constellation in a matched inclination circular orbit, the next step should be to see how non-matched inclination and elliptical orbits might affect the satellite constellation's optimal coverage of the target area.

1.3 Summary of Current Knowledge

Several researchers have studied methods for designing satellite configurations for continuous global coverage. Work done by Beste dealt with the "design of satellite constellations for optimal continuous coverage (1: 466)". Lang looked at the optimization of low-Earth orbit (LEO) constellations for continuous global coverage (5: 1199). J. G. Walker discussed different methods for a satellite constellation configuration that would give global coverage. These researchers discussed different techniques for configuring a satellite constellation that would give complete global coverage, but few have studied constellations with smaller coverage areas. One who did

was Doufer; he looked into the optimization of satellite constellations for zonal coverage (3:609). He used methods from Walker to produce his own configurations for coverage of specified latitude bands with a constellation of satellites. As discussed earlier, Emery et al. also investigated regional coverage, using a numerical approach to optimize small satellite constellations using circular orbits with inclinations matched to the target latitude. The purpose of this thesis is to expand upon the work of Emery et al. and quantify the coverage of a small satellite constellation by varying specific orbital elements.

2 Literature Review

2.1 Introduction

The literature review discusses the different techniques used to construct LEO orbit satellite constellations that provide an optimal coverage of a region on the earth's surface. Satellites in geosynchronous or geostationary orbits (GEO) are not considered hear. Although GEO satellites provide excellent Earth coverage due to their high altitude and constant longitude, the 36,000 km altitude is too high for high resolution imaging missions. The review then discusses continuous coverage of areas the size of a few hundred kilometers in radius. Continuous coverage of the above mentioned areas is desired for the purpose of this thesis. Therefore, continuous global coverage will not be discussed in much depth. It will, instead, be referenced from time to time.

There are many common orbit classes used by both military and commercial industries. They include GEO, medium Earth orbit (MEO), LEO, and highly elliptical orbits (HEO). GEO orbits have an orbital radius of over 42,000 km; these orbits provide good everyday coverage of one particular region of the earth. GEO orbits have an inclination angle near zero which means that the orbit's trajectory runs along the Equator. GEO satellites have an orbital period equivalent to one day, so that the region on Earth that the satellite's field of view covers remains the same. Typical missions for GEO satellites include communications and low resolution imaging. MEO satellites are typically either for scientific missions or navigation-related, like the global positioning system (GPS). HEO satellites are less common, but sometimes used for communications or low resolution imaging because they provide better coverage of near-polar latitudes than GEO satellites. LEO satellites provide less coverage because they have a lower

altitude than a GEO; their field of view over the earth's surface is smaller. However, lower altitude means better imaging resolution. Also LEO orbits generally have an inclination greater than zero degrees. The inclination is the angle the satellite's orbit path makes when it crosses the equator. Since a satellite on a LEO orbit can have an inclination greater than zero, the satellite makes a ground track on the Earth's surface that covers more latitude bands. In addition, a satellite in LEO orbit will have a greater angular speed compared to the Earth's rotation. This means that a satellite on a LEO orbit could pass over a certain region of the Earth many times in one day but it will not provide continuous coverage of that region. This is because once a satellite passes a certain region it may take multiple passes for the satellite to orbit over the same region again. Therefore, it is necessary to have more than one satellite to have a continuous coverage of a desired region of the Earth. This need for several satellites has led researchers to look into satellite constellations to provide what is desired.

2.2 Related Work

2.2.1 Walker Satellite Constellations

J. G. Walker has studied satellite constellations that would provide global coverage of the earth. He stated that circular orbits are preferred rather than elliptical orbits because circular orbits are more suitable for global coverage. He also discusses the importance of satellite constellation with multiple orbits by stating that "single orbit cannot provide either a regular polyhedral distribution or whole-Earth coverage (7: 559)". It is not only important to have multiple satellites in orbit but also to have them in different orbits to achieve the whole-Earth coverage desired. He goes on to say that in choosing a satellite constellation, a designer needs to meet various constraints of the

system in order to be able to achieve the required standard of coverage (7: 559). He

states that the best way to simulate a satellite constellation is to configure the

constellation relative to a polyhedron distribution. Walker pointed out that even though it

cannot be established in practice, "a satellite constellation in which the distribution of

satellites on the spherical surface containing the [circular] satellite orbits corresponds to

the vertices of a regular polyhedron (7: 560)". What he meant is that the orbit of one

satellite as well as it's relative distance from an adjacent satellite set up to relate to a

polyhedron design. The use of this geometric configuration lets him establish three

practical satellite constellations: delta patterns, sigma patterns, and omega patterns.

Delta patterns contain a total number of satellites obtained by multiplying the total

number of orbits being used by the number of evenly space satellites in each orbit (7:

563). He states that delta patterns have superior coverage characteristics. In regular

satellite patterns the relative position of the satellites changes during an orbital revolution

around the earth, but "the highly uniform nature of a delta pattern ensures that similar

configurations recur frequently during one orbital period (7: 563)". Another advantage of

delta patterns is that the method of description is "independent of satellite altitude or

orbital period (7: 563)". Thus the pattern of the satellite constellation will remain

unchanged regardless of the altitude or period (7: 563). The second practical satellite

constellation follows the sigma patterns which is a subset of delta patterns with more

attention to the path the satellite follows along the earth. Sigma patterns consist of

patterns which "follow a single Earth-track which [does] not cross itself and [is]

repetitive after [a certain amount of] days (7: 565) ". It is simply trying to simulate a

sinusoidal pattern with that of the satellite ground track. Omega pattern on the other hand

takes into account that satellites will fail for some reason. Unlike the first two patterns which were uniform, omega patterns is used for non-uniform constellations. It is a sub-set of delta patterns by having a certain number of satellites that are needed for the configuration but actually using fewer satellites for the coverage needed. It will allow you to have extra satellites already in the satellite constellation at your disposal whenever there is a need to use them for whatever reason.

2.2.2 Beste Continuous Coverage Design

Dr. David C. Beste discussed two satellite constellation designs that provide continual coverage of certain regions on earth.   One satellite constellation design involved polar orbits; these are orbits that generally have a North to South trajectory with an inclination of 90 degrees. He stated that this polar orbit arrangement "clusters the satellites in an optimal manner at the equator (1: 467) ". In other words the maximum coverage of polar orbit satellite constellations is centered on the equator. Dr. Beste derived his results for single coverage which meant he wanted to find the maximum coverage area of a particular region using the least amount of satellites and orbits. He then follows this first design with "Full Coverage Beyond Latitude λ (1: 468)". This second design was used to show the maximum regions these polar orbits covered between certain latitude, the north and south poles. These regions are beyond latitudes of positive and negative 30 degrees up to the north and south poles respectively. He later examined non-polar orbits, orbits that have an inclination of less than 90 degrees. From his results he concluded that the "polar-orbit configuration was superior (1: 469)". Dr. Beste then derived a configuration that involved ideas of both polar and non-polar orbits by seeking a "three mutually orthogonal orbital planes (1: 469)". This configuration

consisted of four orbital planes and a total of 12 satellites; the results showed that the coverage area of this configuration was higher than a polar configuration of three orbits and 12 satellites (1: 469). He stated that in order to have continuous coverage of the earth there needs to be a considerable amount of overlap (1: 469). This means that the coverage areas of each satellite in each configuration need to have a section of their region also covered by an adjacent satellite.

2.2.3 Lang's LEO global coverage

Dr. Thomas J. Lang discusses optimal LEO constellations for continuous global coverage (3: 1199). He states that even though a satellite constellation for this purpose in a LEO orbit would require a significant amount of satellites, it is a cost effective solution. Like the other researchers mentioned above, Dr. Lang suggested it was "important to optimize the constellation so as to find the minimum number of satellites required performing the mission (5: 1200)". He also stated that in many cases "non-polar constellations outperformed similarly sized polar constellations (5: 1200)". Because Dr. Lang focused on the continual global coverage of satellite constellations, his work reinforces the work by both Dr. Walker and Dr. Beste work.

2.2.4 Doufer's zonal coverage optimization

Dr. F. Doufer deviates from the previous works by working on optimizing zonal coverage of satellite constellations instead of global coverage (3: 609). He states that many methods have been developed to determine the coverage of constellation patterns, but that all involved global coverage (3: 609). He says that constellations that are used for continual global coverage are not cost effective because of the uselessness of covering

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the entire planet (3: 609). He categorizes the many methods for evaluating coverage into two categories, the semi-analytical methods and the numerical methods (3: 609). A semi-analytical method is like "Walker's satellite triplets or Rider's streets of coverage (3: 609)". He states that the numerical method are "normally more flexible and can usually deal with [evaluating more complex coverage objectives, which the previous method is inadequate of doing], but to the detriment of computation times (3: 609)". For his work on zonal coverage, Doufer uses Walker's triplets' method for global coverage analysis and extends it for evaluating and optimizing zonal coverage (3: 610). His objective is to "precisely assess zonal coverage properties of any constellation pattern where all satellites are using circular orbits with identical orbital periods (3: 610)". He first begins by discussing the coverage of a single satellite or in other words explains what the ground coverage of the satellite depends on. He states that the satellites ground coverage depends on the user and satellite altitudes, the minimal elevation angle and the distance from the user to the satellite (3: 611). He concludes his discussion of a single satellite coverage by saying that once the size of the satellite's ground coverage is known then "it will be easy to deduce a satellite altitude form a specific minimal elevation angle [or vice versa]" (3: 611). He then discusses global coverage and how Walker's satellite triplets "technique alone cannot achieve a coverage assessment of a constellation pattern with non-global coverage" (3: 612). He continues by discussing his zonal coverage method and states that it only deals with "coverage objectives defined as latitude bands" (3: 613). He also adds that for zonal coverage analysis, "areas of interest are delimited by zonal boundaries and must comply with a specific coverage level" (3: 613). This method expands on Walkers triplets method so that it can provide information when the "center

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point falls outside the target area even though the associated circumcircle overlaps the target area", he calls this the worst-seen point (3: 613). He provides examples for zones defined by latitude bands and states that longitude boundaries, while difficult to incorporate, are possible with his methods.

2.2.5 Emery et al's Constellations in matched inclination orbits

The thesis research done by Emery et al dealt with regional coverage of small-satellite constellations in with matched inclination LEO orbits (4: 1). Their work dealt with a specific region of interest where a satellite needed to take images of target areas within that region. They were given specific specifications that they used to develop a Matlab program that simulated orbits for satellites that would be used. This researched only dealt with satellite constellations that were on circular orbits and that had an inclination equal to the target region's latitude. The Matlab program they developed outputted orbital information for a single satellite in orbit for mission duration of one month. Within this month duration the program calculated the number of passes the satellite had over the targeted region for the thirty day duration. The given specifications for the imaging device on board the satellite constrained the total number of satellite passes to only the passes the satellite had over the target area in daylight. Their program had to deal with determining the satellite's position with respect to an Earth Centered Inertial frame which enabled them to determine the Sun's relative position every time the satellite passed over the target area. This let them know if the pass was during daylight or not. The orbit parameters that were used for the simulations were determine and used to provide the optimal amount of passes for a single satellite. The parameters that were left constant for the simulation were the semi-major axis, eccentricity, and inclination.

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These constant parameters were use to output different combinations of right ascension of the ascending nodes and argument of perigee and provide the total number of daylight passes the satellite passed over that area.

With the advancement of satellites and the higher cost of satellite launches, the optimum configuration of satellite constellations is necessary in order to be able to get the maximum use of the system. There have been many configurations that have been looked at, some concentrate on the orbits of the constellation, either being polar or non-polar. Yet other researchers focus on the effect that the shape of the constellation can have on the amount of coverage it can achieve. There are many different criteria that need to be taken into account in order to choose the optimum configuration, but the most important is the amount of coverage that the satellite constellation can achieve.

Most of the research that has been done during the years that deal with satellite constellations has dealt with continuous global coverage with a satellite constellation size of over twenty satellites. These papers don't necessarily consider optimizing coverage over one target area up until recently.

3 Methodology

3.1 Overview

Satellite constellations can be used to obtain imagery data from a region of interest. As mentioned earlier, the motivation behind this research was to determine the optimal configuration of a satellite constellation to maximize the number of daylight passes over a target area. The initial analysis was done using a single satellite optimizing the coverage for the target area. Once coverage of a single satellite was optimized, then the implementation of two or more satellites was considered. The optimal configuration for a single satellite was determined with a numerical search method varying inclination and eccentricity for the satellite's orbit. Since the satellite uses an optical imaging device, only daylight passes were of interest. Varying the orbital parameters yielded different values for satellite daylight passes over the target area. These passes were collected for a period of 30 days.

The distance between the satellite and the target, slant range, was also a parameter with which the optimal configuration was determined. The configuration that had the maximum number of daylight passes while remaining within an acceptable slant range was deemed the optimal orbit. After obtaining the optimal orbit configuration for a single satellite the same process was done for two or more satellites in the constellation.

3.2 Parameters for an Eccentric Orbit

After using the inclination to affect the total number of daylight passes, the next step was to observe the effects of a non-zero eccentricity had on the total number of passes.

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For non-zero eccentricity simulations, the minimum and maximum altitudes of the satellite needed to be known. The satellite specifications stated that the imaging device on board the satellite provides the greatest resolution at an altitude of 350 km, but it still provides acceptable resolution up to an altitude of 800 km. From these specifications, the minimum and maximum altitudes of the satellite were set. The perigee is defined as the minimum altitude of the satellite plus the radius of the Earth, 6378 km. On the other hand, the apogee is defined as the maximum altitude of the satellite plus the radius of the Earth. Since the minimum altitude was 350 km, the perigee of the satellite was 6728 km. The maximum altitude that the satellite can be at was 800 km, so the apogee was 7178 km.

$$R_p = 6728 \ \text{km}$$

$$R_a = 7178 \ \text{km}$$

where R_p and R_a is the radius of the perigee and apogee, respectively.

After perigee and apogee of the orbit were determined, the next step was to find the eccentricity. Using the following equations for the perigee and apogee of an orbit,

$$R_p = a(1-e) \tag{1}$$

$$R_a = a(1+e) \tag{2}$$

the eccentricity e was determined. Equation (1) was modified to solve for the semi-major axis in the following equation.

$$a = \frac{R_p}{(1-e)} \tag{3}$$

Taking Equation (3) and substituting it into Equation (2) yielded,

$$R_a = \frac{R_p}{(1-e)}(1+e) \tag{4}$$

Equation (4) was modified to solve for the eccentricity and the following equation was obtained.

$$e = \frac{(R_a - R_p)}{(R_a + R_p)} \tag{5}$$

Using the values of 6728 km and 7178 km for R_p and R_a respectively, Equation (5) generated the eccentricity value of $e = 0.032$.

Once the eccentricity was obtained, the next step was to determine the corresponding semi-major axis. Using Equation (1) along with the values for the perigee and the eccentricity, the value for the semi-major axis was determined to be $a = 6950.41$ km.

The perigee, apogee, semi-major axis, and the corresponding eccentricity were needed to determine if configuration was acceptable. The value for eccentricity was used in the algorithm to determine if it produced a greater total number of daylight passes than a zero eccentricity (circular orbit) configuration.

3.3 Two-Body Orbit

The orbital motion of a spacecraft around the Earth is frequently described by the dynamic associated with what is known as the "two-body problem." The name stems from the assumption that the motion can be modeled as "two point masses orbiting under their mutual gravitational attraction (8: 45)." The two body problem helps determine the position and velocity of an object in orbit. Any particular orbit is completely determined by six orbit elements: semi-major axis, eccentricity, inclination, right ascension of the

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ascending node (RAAN), the argument of perigee, and mean anomaly. Under the

assumptions of two-body motion, five orbital elements stay constant as shown below,

$$a(t) = a(t_o) \tag{6}$$

$$e(t) = e(t_o) \tag{7}$$

$$i(t) = i(t_o) \tag{8}$$

$$\Omega(t) = \Omega(t_o) \tag{9}$$

$$\omega(t) = \omega(t_o) \tag{10}$$

The semi-major axis, a, determines the orbit's size while the eccentricity, e, determines

the orbit's shape (8: 57). The inclination, i, is the angle that the orbit makes with respect

to the Earth's equator. The right ascension of the ascending node, Ω, and the argument

of perigee, ω, completes the definition of the orbit's orientation with respect to inertial

space. The mean anomaly changes due to the mean motion because the mean anomaly

gives the position of the satellite within the orbit (6: 1). The mean anomaly, on the other

hand does change. Equation (11) shows the relationship between the mean anomaly and

time

$$M(t) = M_o + n(t_o) \tag{11}$$

where M is the mean anomaly, M_o is the mean anomaly at epoch, and n is the mean

motion or the orbit's angular frequency (4: 56).

3.4 Approach

A numerical approach was used to ascertain the total number of passes a satellite

made over a specific target. In order to determine the optimum orbital configuration, the

Matlab program developed by Emery et al. was modified to account for the variance in

inclination and eccentricity. Emery et al. concluded that improved data transmission between the satellite and receiving ground station occurred when the receiving station was placed at a latitude lower than the satellite's inclination (4: 230). From their conclusions, an inclination that is higher than the target latitude would yield an increased number of daylight passes. The eccentricity was also varied to determine if similar effects occurred. When a satellite constellation is in an elliptical orbit, the altitude of the satellites with respect to the target location will vary; this is different from a circular orbit where the altitude remains relatively constant. The minimum and maximum altitudes are designated as the perigee and apogee, respectively. The eccentricity of the orbit is defined by the perigee and apogee. Eccentricity for elliptical orbits range between zero and one, zero being circular and one being parabolic. When the eccentricity is greater than zero, the satellite's orbit will be affected so as to pass over the target area at different elevations. Passes will occur at different altitudes ranging between the perigee and apogee. It is important to know the orbit's apogee because the resolution of the images is dependent on the satellite's altitude. Lower altitude passes provide better image resolution than higher altitudes due to reduced distances between the satellite and target.

3.5 Inclination

The code developed by Emery et al. used an inclination of 33 degrees. This inclination was chosen to match the target latitude, and is often referred to as a "matched inclination orbit." The inclination was varied starting at the target latitude of 33 degrees and initial altitude of 100 km. This inclination variance (Δi) was defined as the difference between inclination and target latitude. Next the same inclination variation process was completed for different altitudes: 350, 600, and 800 km. The Δi that yielded

the maximum number of daylight passes was plotted against its respective altitude. This was conducted to determine if an increasing trend for Δi versus satellite altitude occurred, which was important because the total daylight passes is expected to increase with increasing Δi and altitude. This entire procedure, from varying Δi at different altitudes to plotting the Δi with maximum number of daylight passes at their respective altitudes, was repeated for target latitudes of 10 and 50 degrees.

3.6 Eccentricity

Since a non-zero eccentricity was used, the satellite orbit was elliptical. The semi-major axis is defined as the average of the perigee and apogee. Only one eccentricity value was focused on in this research. Because of the specifications and design parameters of the satellite and imaging device, the value of the eccentricity used was constrained. The maximum altitude used for the satellite was 800 km. This altitude was chosen because it is the maximum distance at which the imaging device payload can provide acceptable imagery data. The minimum altitude was set to be 350 km for the satellite's orbit. This value was specified as the minimum orbit altitude by the sponsors for previous studies. From these minimum and maximum altitudes, the eccentricity value used was $e = 0.032$.

The reason why this eccentricity value was used is because of the corresponding semi-major axis, perigee, and apogee values. For an eccentricity value of $e = 0.032$, the perigee and apogee are 6728 km and 7178 km respectively.

Both the argument of perigee and the RAAN play an important part on a satellite's orbit. The argument of perigee "defines where the low point, perigee, of the orbit is with respect to the Earth's surface" (6: 1). The RAAN "defines the location of the ascending

and descending orbit locations with respect to the Earth's equatorial plane (6: 1)." Both

of these orbital elements are involved in determining the position of the satellite with

respect to the Earth.

3.7 Orbital Perturbation (J_2)

There are many forces acting on a satellite in addition to the point mass gravity

force described above. These include air drag, third body effects, and gravitational effects

caused by a non-spherical Earth. Each of these "perturbing" forces acts to slightly

change the two body solution. In this study, only the greatest of these effects was

considered. This effect is due to the non-spherical, ellipsoid nature of the Earth; this is

often termed the J_2 effect (9: 20). This affects all the equations discussed above; J_2

affects the semi-major axis, the eccentricity, and the inclination in similar ways. This

effect produces a periodic change to these three orbital parameters. Even though they are

changing, the average value for each parameter over a period of thirty days is nearly

constant, as if it were a simple two body orbit. On the other hand, the mean anomaly,

RAAN, and the argument of perigee all experience the J_2 effect in different ways. When

J_2 is taken into consideration, the equations for these three parameters must be modified.

In a regular two-body problem the mean anomaly was the only orbital element that varied

with time, but if the J_2 effect is taken into account, the equation for the mean anomaly

changes as follows

$$M(t) = M(t_o) + \dot{M}_o t + nt \qquad (12)$$

Here, the mean anomaly depends on the position of the satellite at time equals to zero as

well as the mean motion at any given time. The RAAN and the argument of perigee are

similarly affected when the J_2 perturbation is taken into consideration. Because of the perturbation, Equations (9) and (10) are changed as follows

$$\Omega(t) = \Omega(t_o) + \dot{\Omega}t \tag{13}$$

$$\omega(t) = \omega(t_o) + \dot{\omega}t \tag{14}$$

They each depend on the initial value for the RAAN, $\Omega(t_o)$, and argument of perigee, $\omega(t_o)$, but an additional value needs to be added for the J_2 perturbation effect, which are $\dot{\Omega}t$ and $\dot{\omega}t$.

Even though the J_2 perturbation effects on the orbital elements are minuscule, they were taken into consideration because of the mission length. If a satellite's orbit needed to be calculated for a period of one or two days, the J_2 perturbation effect could be neglected. This J_2 value is small enough that using the orbital equations for a two body problem would be sufficient. However, for the 30-day period investigated in this study, the effects were non-negligible. In this case, neglecting the J_2 perturbation effect on the mean anomaly, RAAN, and the argument of perigee would produce inaccurate results.

The original Matlab code, written by Smuck, was based on similar equations in which $\dot{M}_0, \dot{\Omega},$ and $\dot{\omega}$ were empirically determined by comparing results to similar predictions by Satellite Tool Kit. These constants have since been compared to the theoretical values, and provided reasonable approximations.

The number of daylight passes over a target, for a given orbit, was determined using a Matlab algorithm as part of a previous study that examined the problem with an orbit inclination matched to the target's latitude. This algorithm "identifies pseudo-optimal initial conditions for a single satellite that maximized the number of daylight

passes…for a given time span (4: 51)." This algorithm took into consideration the six

orbital elements. Inclination, eccentricity, and semi-major axis, were set as constants.

RAAN and argument of perigee were varied from zero degrees to 360 degrees with a step

size of 36 degrees and 30 degrees respectively. The mean anomaly was set to zero

degrees. The RAAN and argument of perigee combinations yielded a total number of

daylight passes over the specified target at each RAAN step (4: 52).

Since the satellite was required to have visibility of the target during daylight, in

order for the pass to be counted, the algorithm required the capability to distinguish such

passes.

3.8 Daylight Determination (4: 54-67)

In order to determine if the satellite's pass over the target site was during daylight,

the positions of the target site, the satellite, and the position of the sun, with respect to an

Earth Centered Inertial (ECI) reference frame, were tracked throughout the simulation.

The code started by determining the position of the target vector. From the latitude,

longitude, and altitude of the target, the target's position vector in an Earth Centered

Fixed (ECF) frame was determined. This was done by using the following equations

$$R_{SiteECF} = \begin{pmatrix} (N+h)\cos(\phi)\cos(\lambda) \\ (N+h)\cos(\phi)\sin(\lambda) \\ [N(1-a^2)+h]\sin(\phi) \end{pmatrix} \qquad (15)$$

where

$$N = \frac{R_\oplus}{\sqrt{(1-\alpha^2\sin^2(\phi))}} \qquad (16)$$

$$\alpha^2 = 2f - f^2 \qquad (17)$$

$f = $ *Earth's flattening factor*

$R_\oplus = $ *Earth's radius*

$\phi = $ *geodetic latitude*

$\lambda = $ *east longitude*

$h = $ *geodetic altitude*

After determining the position of the target, the next step was to compare it to the position of the TACSAT. The satellite's position was calculated with respect to an ECI coordinate system at a given simulation epoch by transforming the classical orbital elements equations, which were discussed in the previous section, at any point in time. The satellite's ECI position vector was calculated for a period of 30 days at one minute increments.

The next step was to convert all the position vectors calculated to a single coordinate frame; this was done to evaluate the interaction of the system. In order to do this, the epoch time was determined in Julian Centuries using a reference Julian date of 1 January 2000 at 1200Z. Next, the Greenwich Mean Sidereal Time (GMST) at the simulated epoch was calculated by using the Earth's rotation rate and the time past epoch

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at each time step. The resulting GMST was then transferred from ECF to ECI coordinates using the following equation.

$$M_{ECF-ECI} = \begin{pmatrix} \cos_{GMST} & \sin_{GMST} & 0 \\ -\sin_{GMST} & \cos_{GMST} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (18)$$

Next, the target to satellite elevation was calculated in order to determine "if the satellite is within view of the [target]."
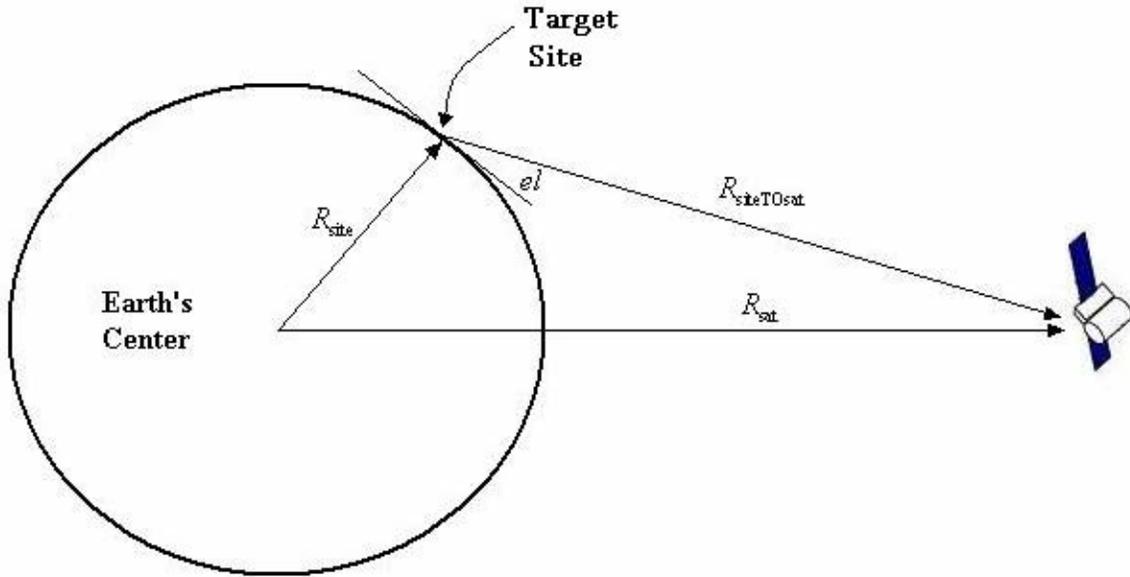

Figure 1: Target to Satellite elevation geometry

Figure 1 shows how the satellite's elevation with respect to the target was calculated.

$$el = \frac{\pi}{2} - \arccos\left(\frac{R_{siteECF} \bullet R_{siteTOsat}}{|R_{siteECF}||R_{siteTOsat}|}\right) \qquad (19)$$

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Using Equation (19), the elevation angle was determined; if a positive elevation angle is calculated, then the satellite is in view of the target.

Once the target and satellite positions were known, the next task was to calculate the Sun's position vector so as to determine if the satellite's pass over the target was in daylight. The time in Julian centuries referencing 1 January 2000 was used for this calculation. Emery et al. began by calculating "any time past the simulation epoch"

$$T = T_0 + \frac{t/86400}{36525} \tag{20}$$

where

$T = time$ (Julian Centuries)

$T_0 = epoch\ time$ (Julian Centuries)

and the same $t = time\ (sec)$ that was used to determine the satellite's position vector. Next, Emery et al. calculated the mean longitude and the mean anomaly of the Sun, as well as the distance from the Earth to the Sun ($\lambda_\odot^m$, $M_\odot$ and $r_\odot$ respectively) to acquire the following:

longitude of the ecliptic

$$\lambda_\odot = \lambda_\odot^m + 1.914666471\sin(M_\odot) + 0.019994643\sin(2M\odot) \tag{21}$$

obliquity of the ecliptic

$$\varepsilon = 23.439291 - 0.0130042T \tag{22}$$

Once both the longitude and the obliquity of the ecliptic were known, the ECI position vector of the Sun was calculated using

$$R_\odot = r_\odot \begin{pmatrix} \cos(\lambda_\odot) \\ \cos(\varepsilon)\sin(\lambda_\odot) \\ \sin(\varepsilon)\sin(\lambda_\odot) \end{pmatrix} \tag{23}$$

Once the ECI position vector of the Sun was known, then the angle between the Sun position vector and the target position vector could be calculated. If this angle was less than 90 degrees, the target would be in daylight at the time of the pass.
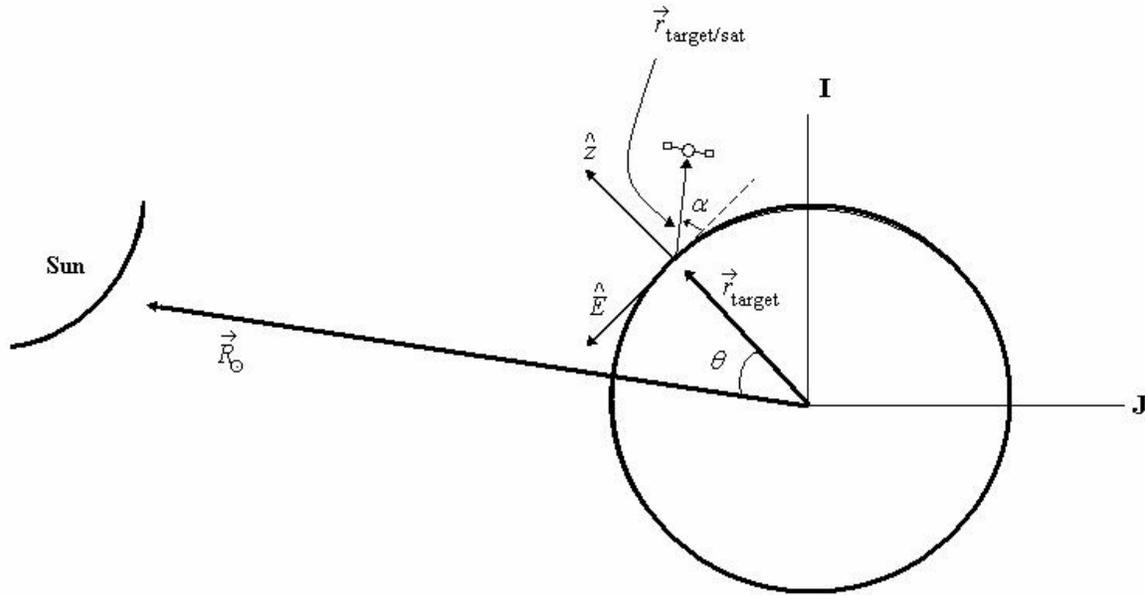
Figure 2: Satellite angle determination relative to the Sun's position to determine if pass is in daylight.

Figure 2 shows how the daylight determination of the satellite over the target was calculated. The value for θ was the determining factor whether the satellite's pass over the target was in daylight or not. The value for θ was calculated by using the following equation:

$$\cos\theta = \frac{\vec{R}_{\odot} \bullet \vec{r}_{\text{target}}}{\| \vec{R}_{\odot} \| \bullet \| \vec{r}_{\text{target}} \|} \qquad (24)$$

The angle θ depends on the position vector, $\vec{R}_{\odot}$, of the Sun and the position vector of

the target, $\vec{r}_{\text{target}}$. The angle θ needed to be less than 90 degrees in order for the pass to

occur during daylight.

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3.9 Matlab Algorithm

A block diagram of the algorithm adapted from the Emery et al. thesis is shown below in Figure 3 (4: 58).
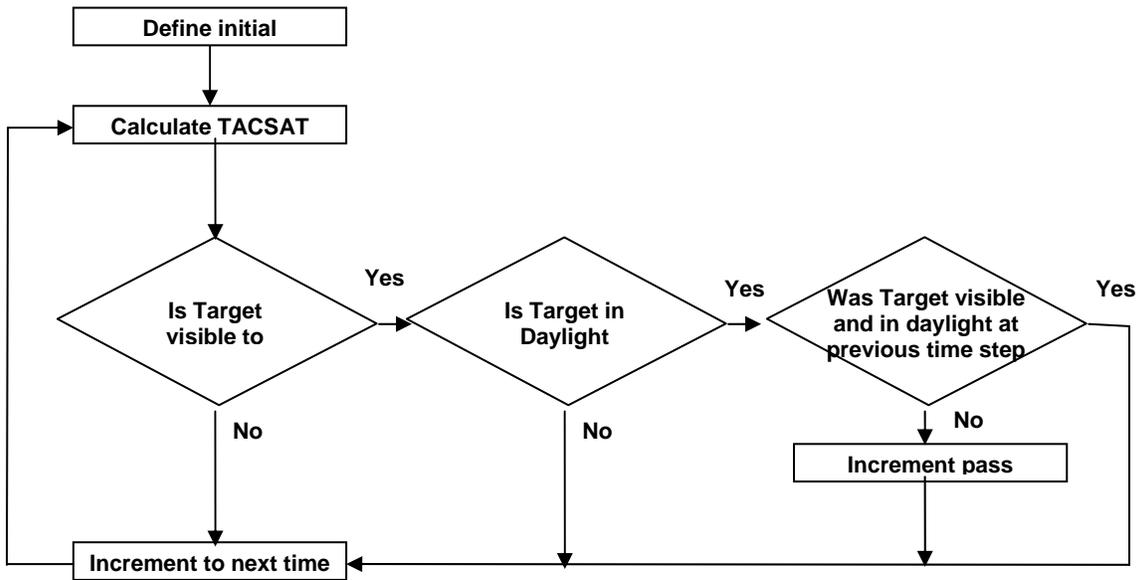
Figure 3: High-Level TACSAT Orbit Optimization Algorithm Flow

The first step in the algorithm was to define the initial conditions for the orbital elements. Next, the position of the satellite was calculated. The algorithm then determined if the target was visible to the satellite. If so, it proceeded to the next step; if not, the algorithm incremented to the next time step and calculated the new satellite position until the target was view of the satellite. Once the target was visible to the satellite, the algorithm checked if the target was in daylight. If true, it proceeded to the next step; if false, then it incremented to the next time step and calculated the new position of the satellite. If the target was visible during daylight, a pass counter was incremented and the process repeated, by then calculating the next satellite position. This process was repeated for each combination of RAAN and argument of perigee, summing all daylight passes, and

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finally, displaying the results on a graph. The output displayed the daylight pass totals versus the corresponding RAAN value.

This algorithm was used by Emery et al. to determine maximum daylight passes with a matched inclination to the target latitude for circular orbits. However, in this study, it was modified and used to determine if varying the inclination and eccentricity had a positive effect on the total number of daylight passes. The inclination was varied in one degree increments, starting from the initial match inclination. As mentioned in a previous section, it was hypothesized that if the inclination of the orbit was greater than the target latitude, the total number of daylight passes would also increase when compared to the number of daylight passes for a matched inclination orbit. The algorithm was iterated for each inclination value. If each greater inclination value yielded more daylight passes, the process continued to be implemented until the total number of passes peaked.

3.10 Satellite to Target Range

The satellite did not always take pictures of the target when it was passing directly above; most of the time the images were taken when the satellite was closest to the target for the given pass. Since some of the tests that were conducted involved varying the inclination and the eccentricity, the satellite's passes were sometimes further away from the target. Therefore, the distance between the satellite and the target needed to be known, especially in an eccentric orbit, where the satellite has a minimum and a maximum altitude. The distance between the satellite and the target was called the slant range. This distance was calculated whenever the target was in view of the satellite.

Figure 4 shows the slant range between the satellite and the target at three different times.
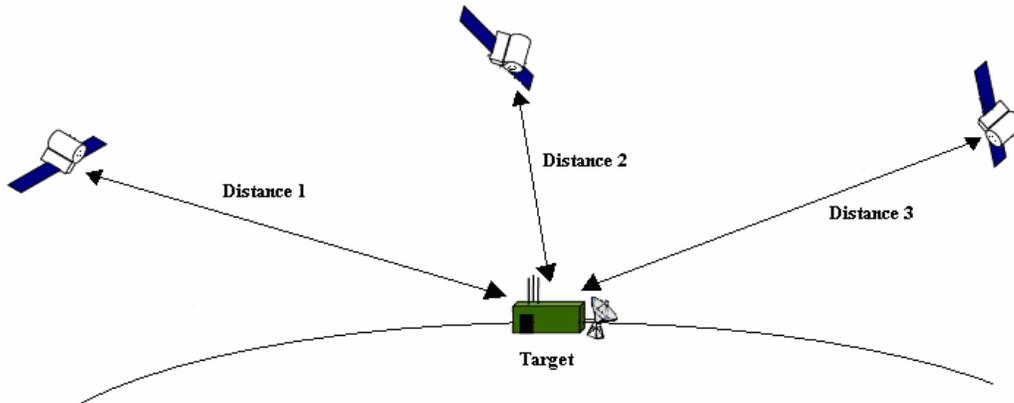
Figure 4: Slant range depiction at the time the target is in view of the satellite.

These distances are not the altitude of the satellite; the altitude is measured straight down with respect to the surface of the Earth. The distances from the satellite to the target were recorded whenever a satellite pass was counted. The average of these distances was determined for the different RAAN values. This average distance was called the average slant range.

4 Results

4.1 Single Satellite Coverage

As discussed in previous chapters, the purpose of this study was to identify the orbital elements which maximized the number of daylight passes over specific target latitudes. This was done by a numerical search method which varied the orbital parameters and recorded the effect each had on the number of daylight passes. Table 1 shows the range in which each element was varied to form the complete search space.

Table 1: Range of Orbital Elements

Altitude (Semi-major axis for eccentric orbits)	100 km, 350 km, 600 km, 800 km (6478 km), (6728 km), (6978 km), (7178 km)
Inclination	target latitude 33° + Δi = -2, -1, 0, 1, 2,…,18°
Eccentricity	0 and 0.032
Right Ascension of the Ascending Node	0, 36, 72,…, $(360-36)^o$
Argument of Perigee	0, 30, 60,…, 360°
True Anomaly (single sat)	0°
at epoch (two sat)	0, 60, 120, 180,…, 360°
Target latitude	10, 33, 50°

Note that for circular orbits, the argument of perigee and true anomaly are summed to give argument of latitude.

4.1.1 Inclination Effects

The inclination was varied with respect to the target site latitude in order to determine if a change in the inclination had any effect on the satellite's coverage. The inclination was varied for different altitudes of the satellite; these altitudes include 100

km, 350 km, 600 km, and 800 km. This process was conducted at target latitudes of 10, 33, 50 degrees.

4.1.1.1 Target Latitude of 10 Degrees

For an altitude of 100 km, the inclination was varied starting at 10 degrees. For this 10 degree inclination value, the maximum number of satellite daylight passes was approximately 72. This total number of satellite daylight passes was obtained by maximizing the RAAN and argument of perigee. When the inclination was changed to a higher degree i.e. 11, 12 degrees etc., the total number of satellite passes also changed as well. It was determined that the increase of total number of satellite passes peaked at an inclination of 12 degrees or a two degree difference from the target site latitude. The maximum number of satellite passes over the target area was 88 for an inclination of 12 degrees. Tests were also conducted for an inclination less than the target latitude; this was done to show the increasing trend of the daylight passes with respect to the inclination variance. It was found that if the inclination was less than the target latitude, the total number of daylight passes decreases. The values obtained for this simulation are shown on Figure 5.

Daylight passes vs i (at 100 kms Altitude, Target= 10° Lat)
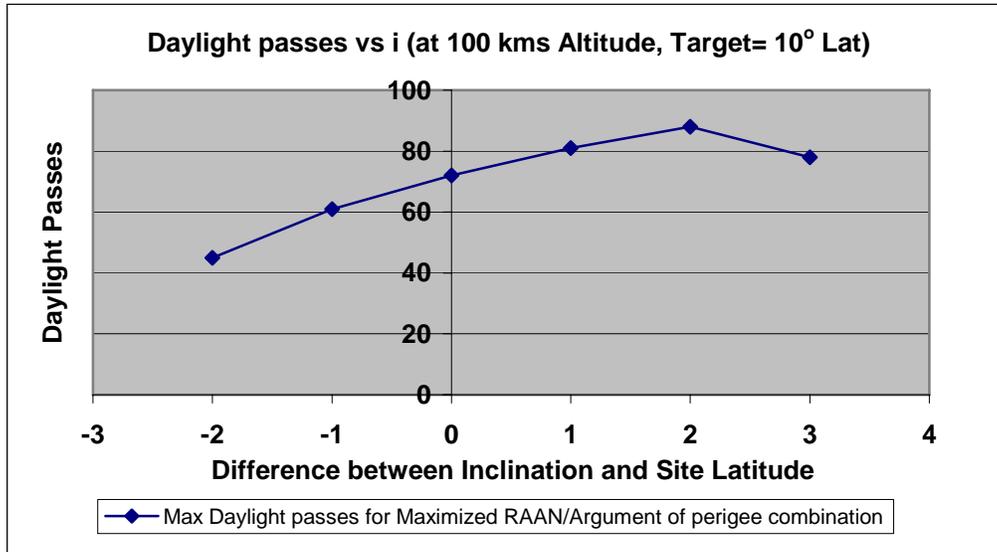
Figure 5: Daylight Passes for a satellite at 100km altitude as inclination changes with respect to target latitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

The same analysis was done for a satellite at an altitude of 350 km; the inclination started equal to the target latitude, 10 degrees. This inclination allowed the satellite to pass over the target site for a maximum number of daylight passes of 133. When the inclination was changed, the total number of passes over the target sight also increased. The total number of satellite passes increased when the inclination was changed to be greater than the target latitude. Even though there was not a steady increase in the total number of daylight passes, this simulation showed that varying the inclination had a positive effect on the total number of daylight passes a satellite had of the target. The number of daylight passes peaked at an inclination of 18 degrees or an eight degree difference from the target latitude. The maximum number of satellite passes for this inclination value was 143. The complete simulation output for this test is seen on Figure 6.
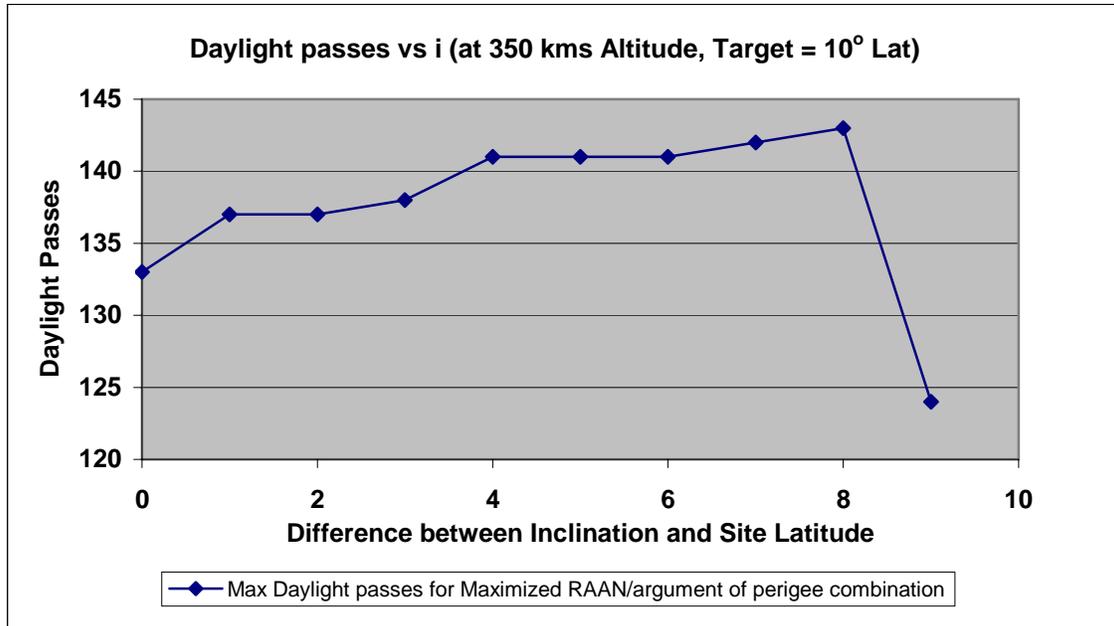
Figure 6: Daylight Passes for a satellite at 350km altitude as inclination changes with respect to target latitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

The next altitude that was used was 600 km, and again, the inclination was varied starting at 10 degrees. For this inclination value the total number of satellite passes was 166 passes. For this simulation the total number of daylight passes decreased if the inclination was greater than the target latitude. However, the number of daylight passes did increase when the inclination was set to be less than the latitude of the target. The change in inclination with respect to the latitude still provided a positive effect on the number of daylight passes when the absolute difference between inclination and latitude was increased (when the inclination was set lower than the latitude). The total number of satellite passes increased when the inclination was lowered, until the total number of daylight passes stabilized until the inclination was set to zero. At this point the number of daylight passes began to drop after an inclination of two degrees south of the Equator or equivalently a two degree inclination. From the results obtained in this simulation, the

total number of satellite passes peaked at an inclination of two degrees south of the Equator or an inclination to target difference of 12 degrees from the target latitude. The maximum number of satellite passes that was obtained for this inclination value was 223. The daylight pass values for this simulation are seen on Figure 7.
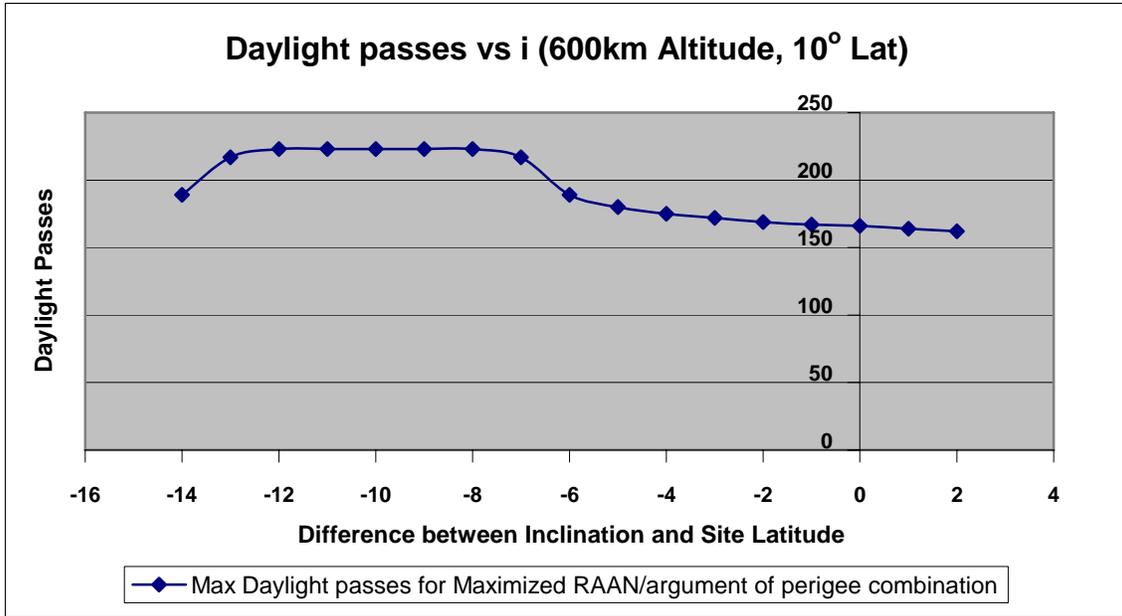
Figure 7: Daylight Passes for a satellite at 600km altitude as inclination changes with respect to target latitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

The final simulation conducted at a target latitude of 10 degrees was at an altitude of 800 km. Like the previous simulations, the inclination was varied starting at and inclination of 10 degrees. The total number of daylight passes for this inclination value, was 180. A similar pattern occurred at this altitude as in the simulation with a satellite altitude of 600 km. The total number of daylight passes increased when the inclination was less than the target latitude. The total number of daylight passes increased as the inclination to target latitude difference increased until the number of passes began to

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stabilize. The total number of daylight passes remained constant at 212 passes from an inclination of five degrees above the Equator until five degrees below the Equator; these two inclination values are basically the same. Therefore the number of daylight passes peaked at an inclination value of five degrees below the Equator, or an inclination to target latitude difference of 15 degrees. The results for this simulation are seen on Figure 8.

**Daylight passes vs i (800km Altitude, 10° Lat)**
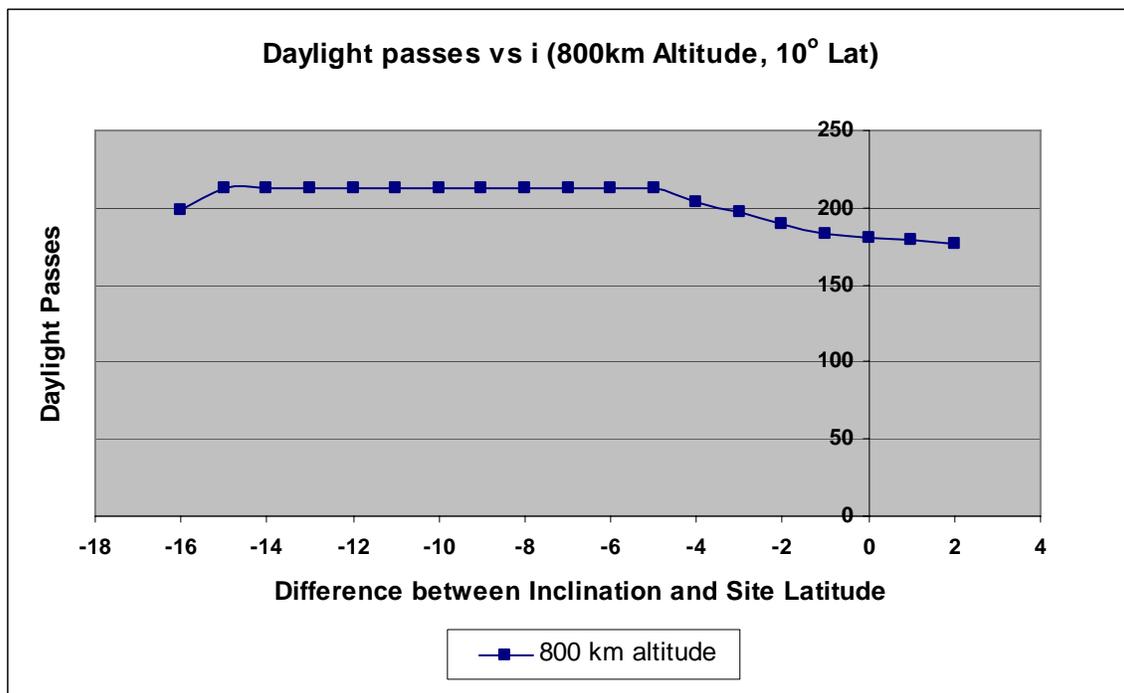
Figure 8: Daylight Passes for a satellite at 800km altitude as inclination changes with respect to target latitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

From these four cases, it was concluded that the maximum number of satellite passes over a target site was increased by using a greater inclination value than the target site latitude, as well as orbiting at a higher altitude. The inclination value that resulted in the maximum number of daylight passes changed whenever a different satellite altitude

was used. In other words, the higher the altitude, the greater the inclination value

corresponding to the maximum number of daylight passes. The reason for this is because

at higher altitudes, the satellite has a greater field of view of the Earth's surface. Figure

14 shows the optimal inclination (Δi) as a function of satellite altitude. Based on this

graph, the user could estimate that the optimum inclination is approximately equal to the

target latitude plus 2° for each 100 km increase in the altitude of the satellite. However,

the maximum altitude is constrained by the imagery equipment used on the satellite. For

the sensors envisioned for TACSAT altitudes greater than 800 km would not be

acceptable. The reason for this is because at altitudes greater than 800 km, the resolution

of the images taken by the satellite would not be clear enough to be used for planning

purposes.

Figure 9: The optimal inclination difference (vs. site latitude) increases with satellite altitude. This
is due to the increase in the sensor footprint at the higher altitude.

4.1.1.2 Target Latitude of 33 Degrees

The simulations that were conducted for a target latitude of 10 degrees were next done for a target latitude of 33 degrees. For an altitude of 100 km, the inclination was varied starting at 33 degrees, which is equal to the target site latitude. For this test, the maximum amount of satellite daylight passes was about 55. This amount for the total number of satellite daylight passes was obtained by maximizing the RAAN and argument of perigee. When the inclination was changed to a higher degree i.e. 34, 35 degrees etc. the total number of satellite passes also changed as well. It was determined that the increase of total number of satellite passes peaked at an inclination of 35 degrees or an inclination to latitude difference of two degrees. The maximum number of satellite passes over the target area was 72 passes for an inclination of 35 degrees. Tests were also conducted for an inclination less than the target latitude; this was done to show the increasing trend of the daylight passes with respect to the inclination variance. This increasing trend is similar to the trend observed for the same simulation scenario for a target latitude of 10 degrees. It was determined that if the inclination is less than the target latitude the total number of daylight passes decreases. The results of this test can be seen on Figure 10.

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Daylight Passes vs i (100 kms satellite altitude, 33° latitude)
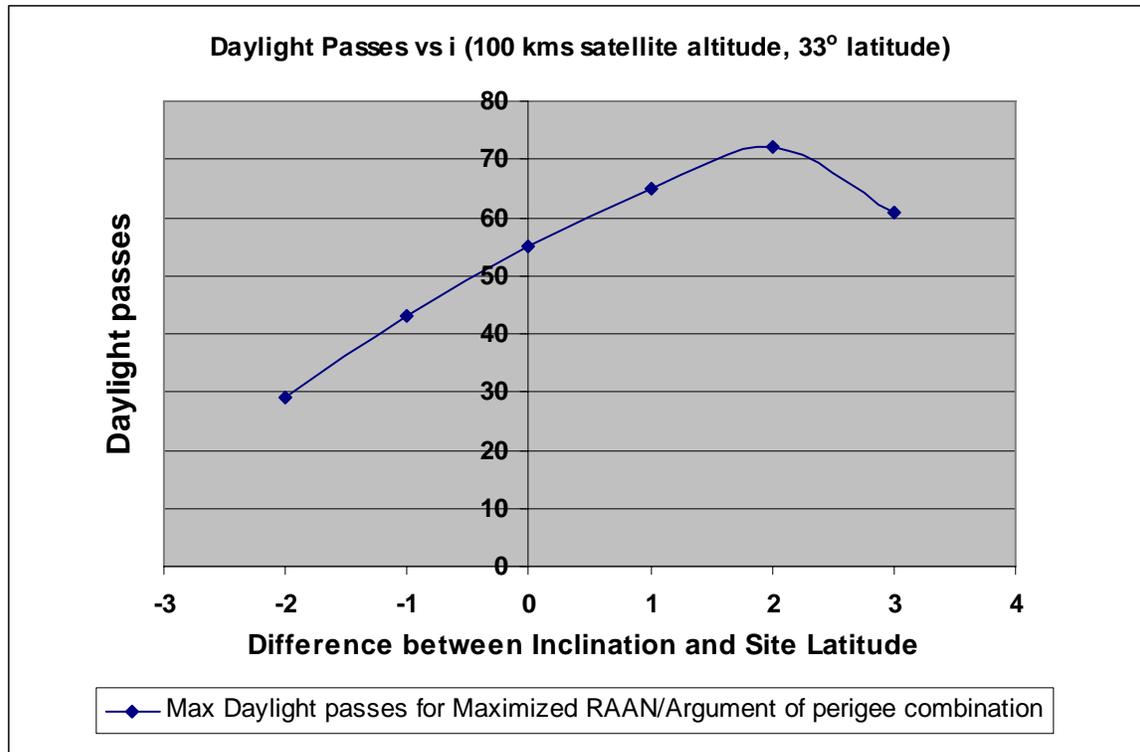
Figure 10: Daylight Passes for a satellite at 100km altitude as inclination changes with respect to target latitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

The next simulation was done at a satellite altitude of 350 km, the inclination started equal to the target site latitude. This inclination allowed the satellite to pass over the target site for maximum daylight passes of 98. When the inclination was changed, the total number of passes over the target sight also changed. The total number of satellite passes increased by five more passes for every degree added to the inclination. This test showed that the total number of satellite passes peaked at an inclination of 41 degrees or an inclination to target latitude difference of eight degrees. The maximum number of satellite passes for this inclination was 131 passes over the target location.

The results for the total number of daylight passes for a satellite at an altitude of 350 km are shown on Figure 11.

**Daylight Passes vs i (350 kms Altitude, 33° Latitude)**
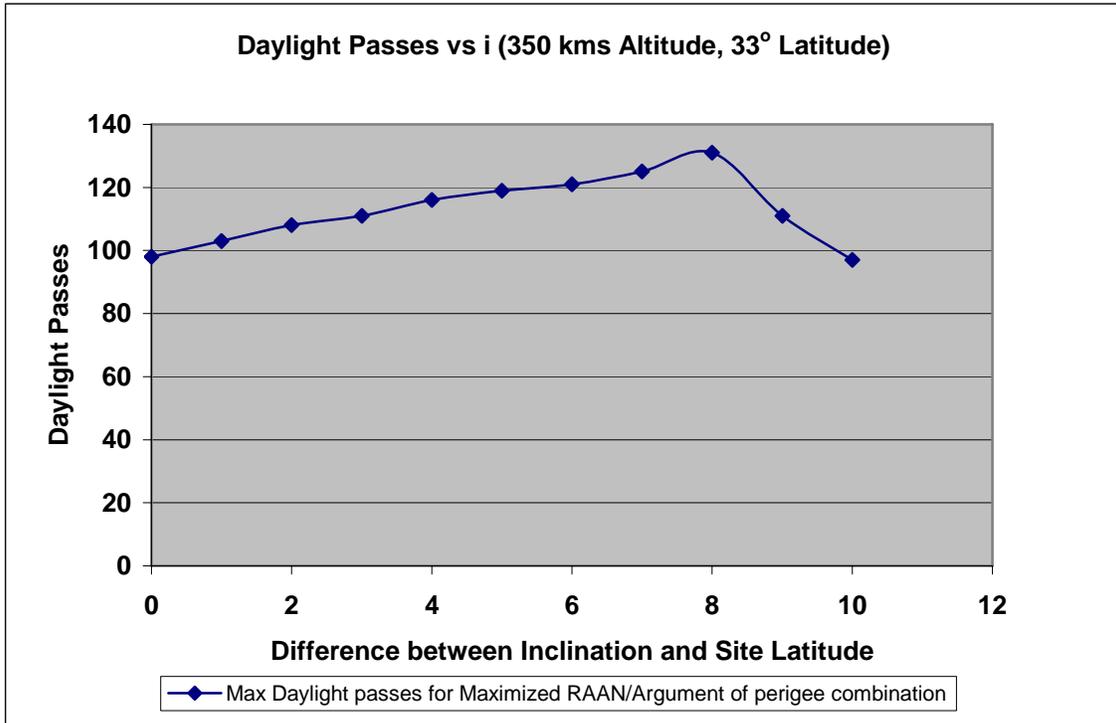
Figure 11: **Daylight Passes for a satellite at 350km altitude as inclination changes with respect to target latitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)**

The third altitude that was used was an altitude of 600 km and again the was inclination varied starting at 33 degrees. For this inclination value the total number of satellite passes was 119 passes. The same situation happen during this test as it did the previous two tests which were that the total number of satellite passes increased with respect to a greater inclination to target latitude difference. The total number of satellite passes increased by three to four extra passes with each additional degree added to the inclination. The total number of satellite passes peaked at an inclination of 45 degrees or a 12 degree satellite to altitude target difference. The maximum number of satellite

passes for a 45 degree inclination was 154 passes. The peak of the total number of passes can be seen on Figure 12.

**Daylight passes vs i (at 600 kms Altitude, 33° Latitude)**
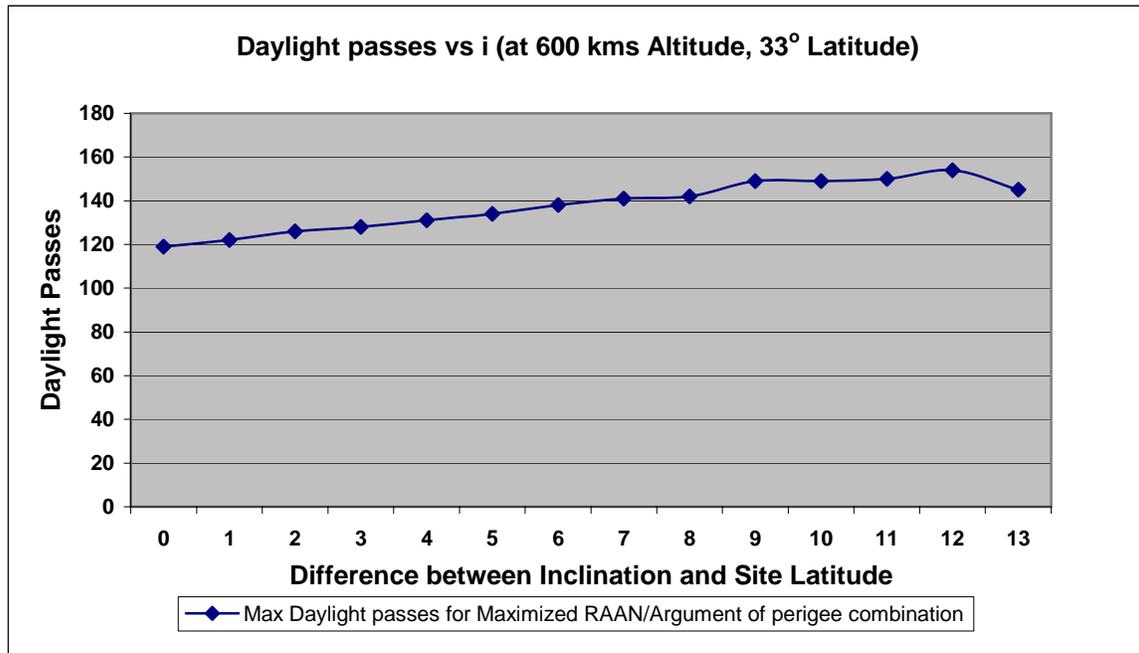
Figure 12: Daylight Passes for a satellite at 600km altitude as inclination changes with respect to target latitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

The final test done was simulated at an altitude of 800 km; like the previous tests, the inclination was varied starting at an inclination of 33 degrees. The total number of daylight passes for this inclination was 131 daylight passes. The total number of passes increased as expected; based on the three previous tests conducted. There was a constant increase in the number of daylight passes when the inclination to target latitude difference was increased. Though, this daylight pass increase was not as great when compared to the three previous simulation conducted for the same target latitude; the daylight passes improved by two, for each one degree difference between the inclination

and the target latitude. The increase in daylight passes peaked at an inclination of 48 degrees; inclination to target latitude difference of 15 degrees. Figure 13 shows the increase of the total number of daylight passes for the inclination variance at a satellite altitude of 800 km.
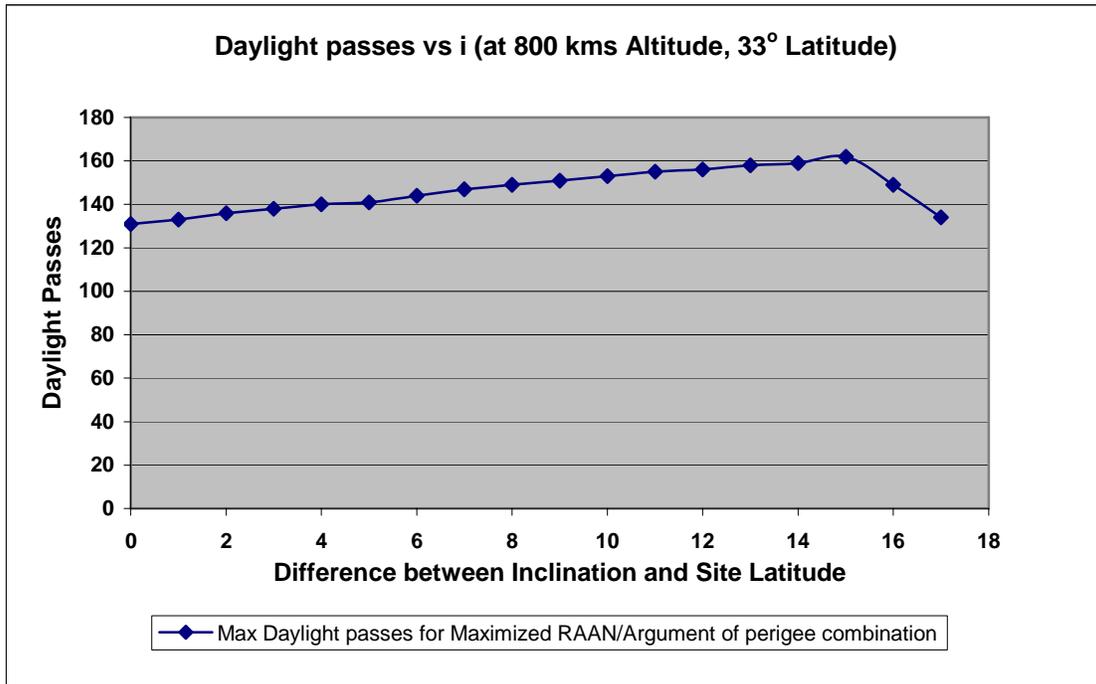
Figure 13: The optimal inclination difference (vs. site latitude) increases with satellite altitude. This is due to the increase in the sensor footprint at the higher altitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

From these four cases, it was concluded that the maximum number of satellite passes over a target site was increased by using a greater inclination value than the target site latitude, as well as orbiting at a higher altitude. The inclination value that resulted in the maximum number of daylight passes changed whenever a different satellite altitude was used. In other words, the higher the altitude, the greater the inclination value corresponding to the maximum number of daylight passes. As stated in the previous section, the reason for this trend is because at higher altitudes, the satellite has a greater

field of view of the Earth's surface. Figure 14 shows the optimal inclination (Δi) as a function of satellite altitude. Based on this graph, the user could estimate that the optimum inclination is approximately equal to the target latitude plus 2° for each 100 km increase in the altitude of the satellite. However, the maximum altitude is constrained by the imagery equipment used on the satellite. For the sensors envisioned for TACSAT, altitudes greater than 800 km would not be acceptable. The reason for this is because at altitudes greater than 800 km, the resolution of the images taken by the satellite would not be clear enough to be used for planning purposes.

Max Inclination vs Satellite Altitude (33° Latitude)

◆ Peak Inclination that resulted in the maximum number of satellite daylight passes

Figure 14: The optimal inclination difference (vs. site latitude) increases with satellite altitude. This is due to the increase in the sensor footprint at the higher altitude. (Ω, ω are chosen to maximize daylight passes)

4.1.1.3 Target Latitude of 50 Degrees

The last simulation was conducted for a target latitude of 50 degrees. As was done for the previous latitudes used, the inclination was varied for four different satellite altitudes. For the first altitude of 100 km, the inclination was varied starting from the target latitude (50 degrees). For this test, the maximum amount of satellite daylight passes was 63. When the inclination was changed to a higher degree i.e. 51, 52 degrees etc., the total number of satellite passes increased. It was determined that the daylight passes increase of the satellite peaked at an inclination of 35 degrees or a two degree inclination to target latitude difference. At this inclination value, the maximum number of satellite passes over the target area was 83. Tests were also done for an inclination less than the target latitude in order to determine what effect a smaller inclination value had on the total number of daylight satellite passes. It was determined that if the inclination is less than the target latitude the total number of daylight passes decreased. The results of this test can be seen on Figure 15.
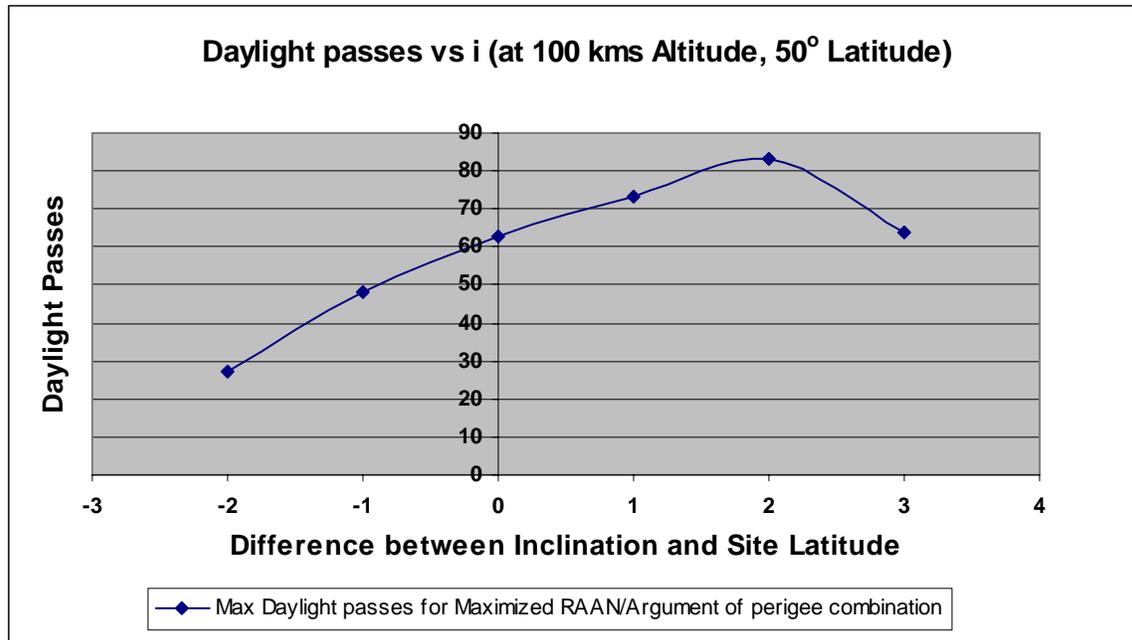
Figure 15: The optimal inclination difference (vs. site latitude) increases with satellite altitude. This is due to the increase in the sensor footprint at the higher altitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

The next satellite altitude was 350 km; again, the inclination variance began equal to the target site latitude (50 degrees). This inclination allowed the satellite to pass over the target site for maximum daylight passes of 112. When the inclination was increased, the total number of passes over the target sight also increased. This test showed that the total number of satellite passes peaked at an inclination of 58 degrees or at an inclination to target latitude difference of eight degrees. The maximum number of satellite passes for this inclination was 150 passes over the target. Figure 16 shows the results of this simulation.
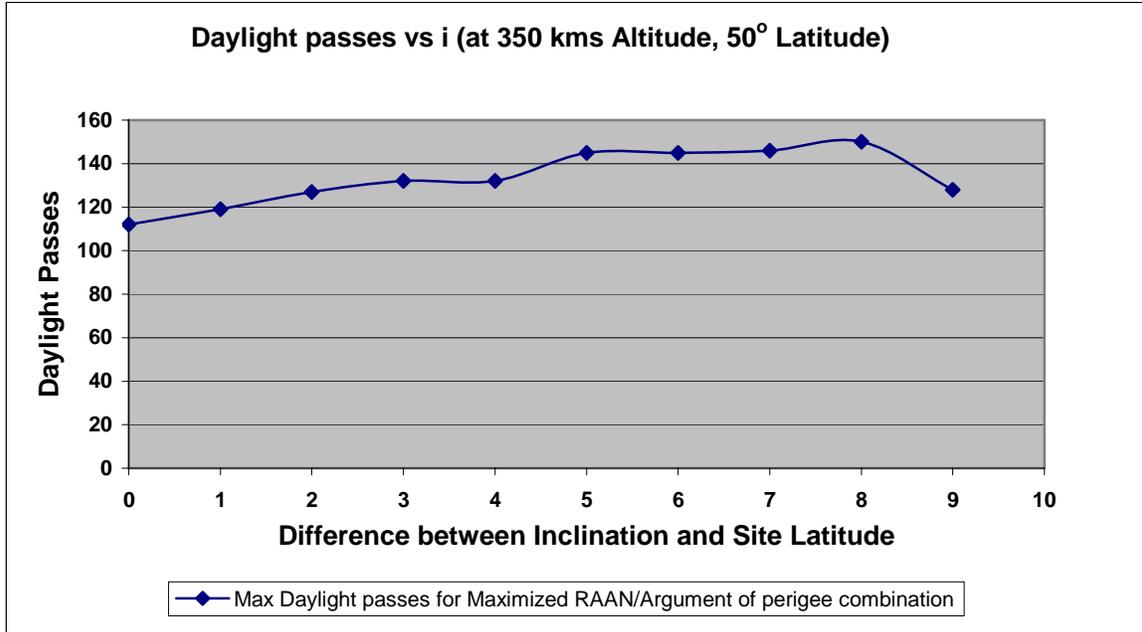
Figure 16: The optimal inclination difference (vs. site latitude) increases with satellite altitude. This is due to the increase in the sensor footprint at the higher altitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

The third altitude used was at 600 km and again the inclination varied starting at 50 degrees. For this inclination value the total number of satellite passes was 131. The same situation happen during this test as it did on the previous two tests which was a satellite pass increase with respect to the increase between the inclination and target latitude. The total number of satellite passes peaked at an inclination of 62 degrees or at a 12 degree inclination to target latitude difference. The maximum number of satellite passes for this inclination value was 176. The simulation results for this simulation can be seen on Figure 17.
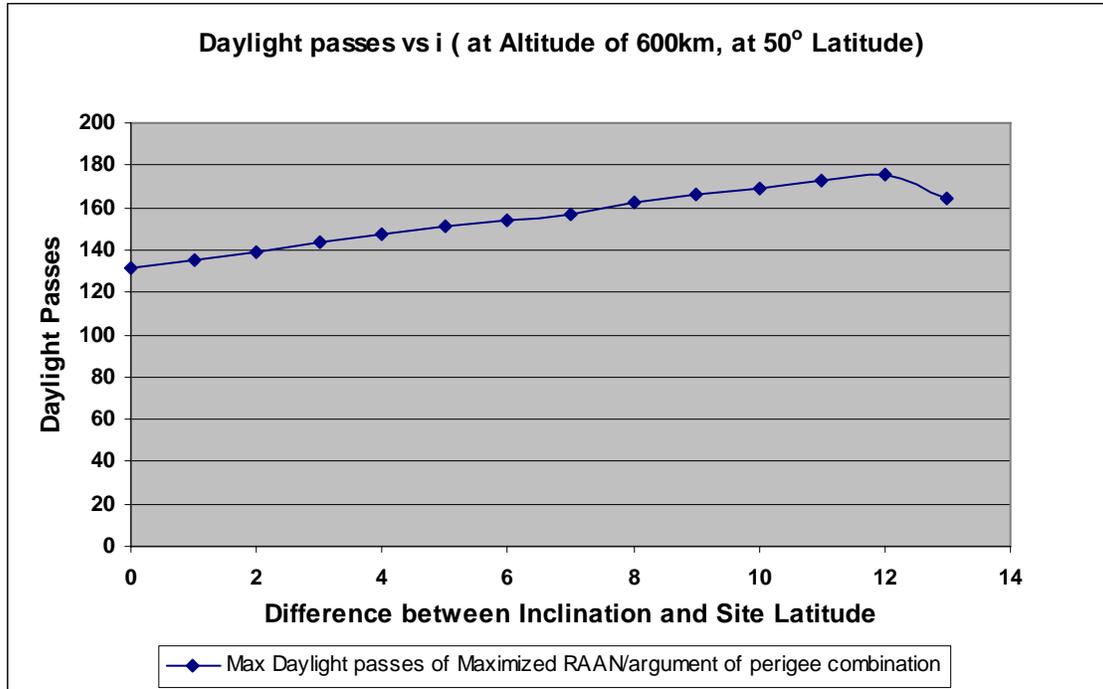
Figure 17: The optimal inclination difference (vs. site latitude) increases with satellite altitude. This is due to the increase in the sensor footprint at the higher altitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

The final test done was at an altitude of 800 km; like the previous tests, the inclination was varied starting at an inclination of 50 degrees. The total number of daylight passes for this inclination was 139. The total number of passes increased as expected, based on the prior simulation results. The average increase of total daylight passes for the 800 km test was 4 passes for every degree increment in the inclination. Figure 18 shows the increase of the total number of daylight passes for the inclination variance. The daylight passes peaked at an inclination of 65 degrees, or an inclination to target latitude difference of 15 degrees.
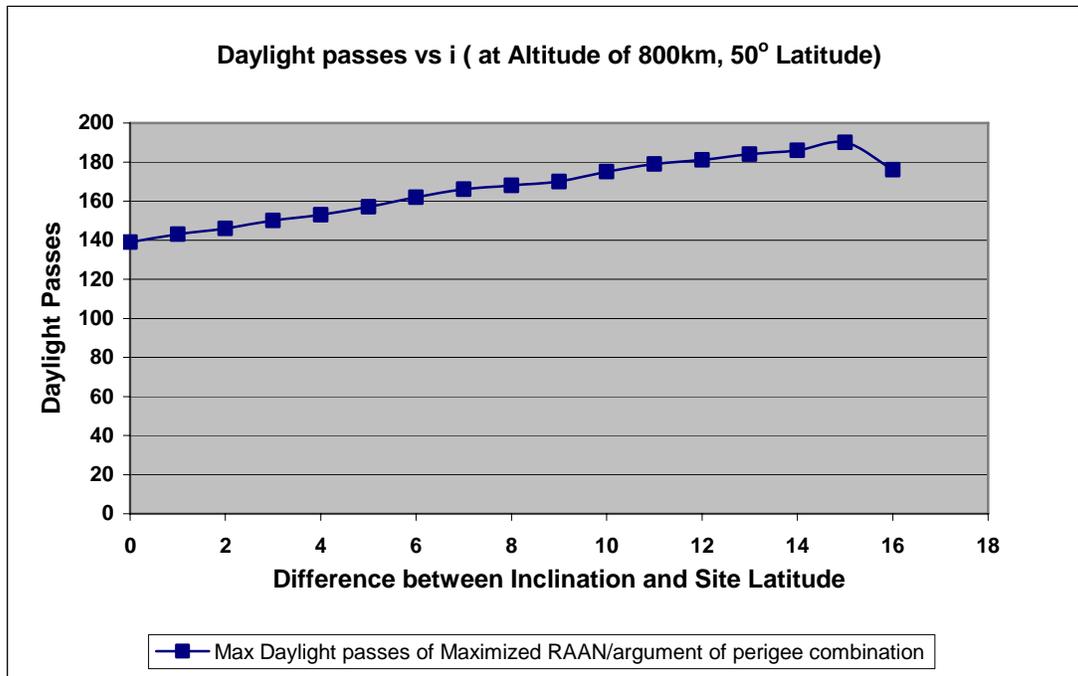
Figure 18: The optimal inclination difference (vs. site latitude) increases with satellite altitude. This is due to the increase in the sensor footprint at the higher altitude. Data collected over a 30-day period for each delta i. (Ω, ω are chosen to maximize daylight passes)

From the four cases conducted at a latitude of 50 degrees, it was concluded that the maximum number of satellite passes over a target site was increased by using a greater inclination value than the target site latitude, as well as orbiting at a higher altitude. The inclination value that resulted in the maximum number of daylight passes changed whenever a different satellite altitude was used. In other words, the higher the altitude, the greater the inclination value corresponding to the maximum number of daylight passes. As stated in the previous section, the reason for this trend is because at higher altitudes, the satellite has a greater field of view of the Earth's surface. Figure 19 shows the optimal inclination (Δi) as a function of satellite altitude. Based on this graph, the user could estimate that the optimum inclination is approximately equal to the target latitude plus 2° for each 100 km increase in the altitude of the satellite. However, the

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maximum altitude is constrained by the imagery equipment used on the satellite. For the

sensors envisioned for TACSAT, altitudes greater than 800 km would not be acceptable.

The reason for this is because at altitudes greater than 800 km, the resolution of the

images taken by the satellite would not be clear enough to be used for planning purposes.

Max Inclination vs Satellite Altitude (50 deg latitude)
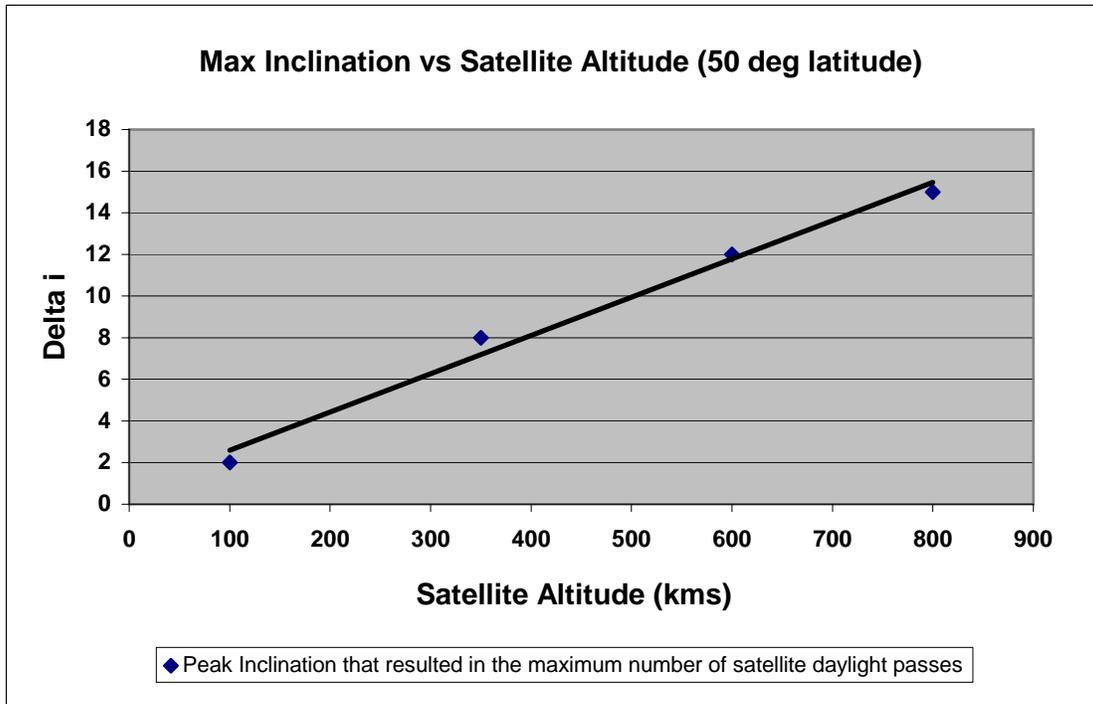
Figure 19: **The optimal inclination difference (vs. site latitude) increases with satellite altitude. This is due to the increase in the sensor footprint at the higher altitude.**

The results obtained from the simulations conducted at the three different target

latitudes indicate that a similar pattern is shared by these latitude bands. The same

optimum inclination with respect to the target latitude occurred for each of the three

latitude values. The maximum inclination which resulted in the maximum number of

daylight passes increased whenever a higher altitude was used. So from these tests; the

target latitude plus 2° per 100 km altitude increase for the optimum inclination holds true for latitude bands between 10 and 50 degrees.

4.1.2 Eccentricity Effects

Eccentricity was the other orbital element of primary interest in this study. In the cases shown in this section, the inclination was left equal to the target site latitude, $i = 33$ degrees in order to observe the effects an eccentric orbit had on the total number of satellite passes over the target site. To compute a reasonable value for eccentricity, first the minimum and maximum heights of the satellite orbit were determine, or equivalently, what perigee and apogee were going to be used. From the satellite specifications, for allowable altitudes, the apogee was set to have an altitude of 800 km. A perigee altitude of 350 km was chosen, matching the altitude used by Emery et al. The value for the semi-major axis was then calculated to correspond with the apogee and perigee. This resulted in a semi-major axis of $a = 6,950.41$ km. The satellite's mission duration of one month was not significantly affected be the values mentioned above. Since the satellite's lowest altitude was 350 km, the drag experienced at this altitude would not degrade its orbit so as to affect its mission. With these apogee and perigee, the maximum value of for the eccentricity was computed to be e $= 0.032$.

When simulations were run using an inclination of 33 degrees, at an altitude of 350 km, the Matlab code outputted maximum daylight passes for different RAAN and Argument of perigee combinations which are shown on Figure 20.
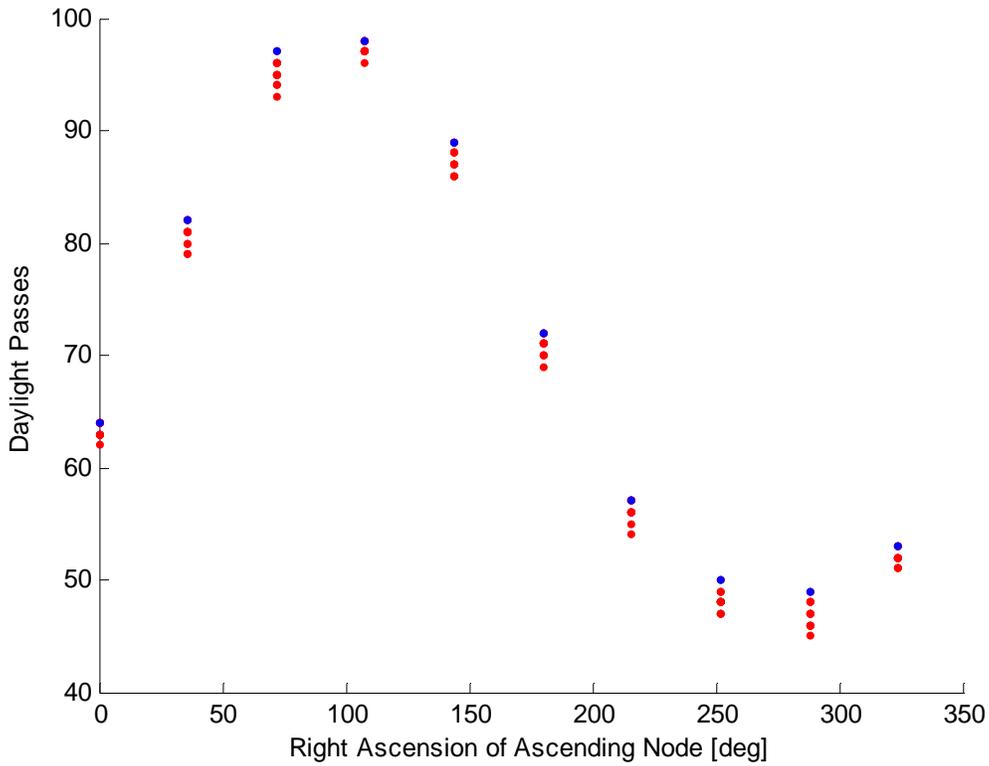
Figure 20: Total number of daylight passes for different RAAN and argument of perigee combinations for a circular orbit with an inclination of 33 degrees and zero eccentricity.

When the simulation with a non-zero eccentricity value was run, the Matlab code was modified to account for eccentricity value of e = 0.032. The semi-major axis was changed to $a = 6,950.41 \, \text{km}$ as discussed above. The total number of daylight passes changed for the corresponding RAAN and argument of perigee combinations when this eccentricity values was used. The results from this test can be seen in Figure 21.
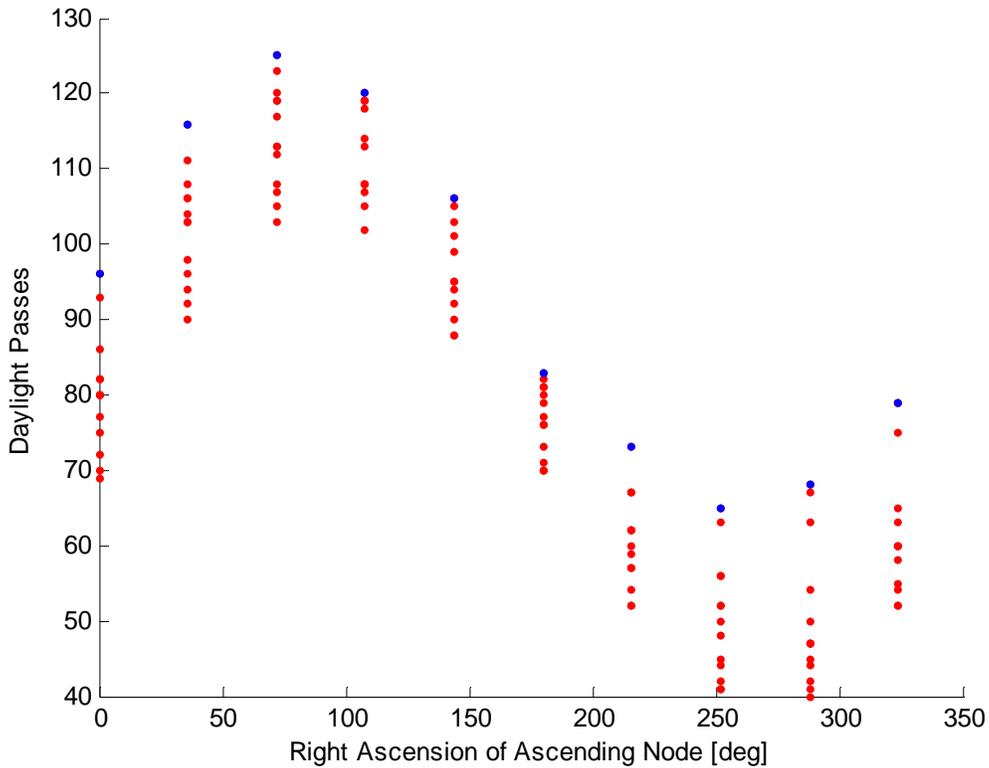
Figure 21: Total number of daylight passes for different RAAN and argument of perigee combinations for a circular orbit with an inclination of 33 degrees and an eccentricity e = 0.032.

Compared to the circular orbit, the eccentric orbit (e = 0.032) had a significant increase in daylight passes at all values of ascending node and argument of perigee. The total number of satellite passes of the target area increased by an average of 22 more passes number of tests. The maximum values for the eccentric case were compared to those from the circular case in Figure 22. It was clear that using an elliptical orbit yielded more daylight passes than a circular orbit. However, it will be shown in a subsequent section that there is a range penalty in this approach.
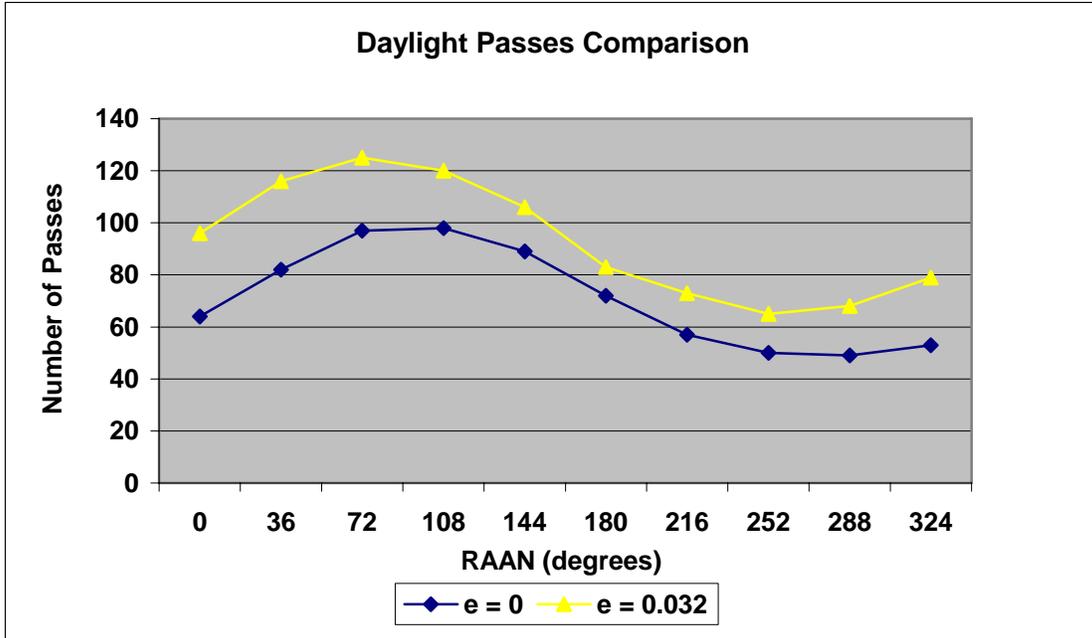
Figure 22: Daylight Passes comparison for a satellite with different eccentricities with equal inclination for a 30-day period.

4.1.3 Varying Inclination and Eccentricity

It was shown that total number of daylight passes was positively affected when the inclination was changed and when the eccentricity was changed. Another test was conducted to see what effect changing both the inclination and eccentricity had on the total number of satellite passes. This test was conducted with the following values of inclination and eccentricity, $i = 48$ degrees and $e = 0.032$. Figure 23 shows this comparison. Again the total number of daylight passes increased significantly. The biggest increase occurred on the second RAAN value, with an increase of 77 more daylight passes; RAAN = 36 degrees. The lowest increase occurred at the seventh RAAN value with an increase of only 10 daylight passes; RAAN = 216 degrees. The overall increase when comparing these two graphs was an average of more than 43 daylight passes.
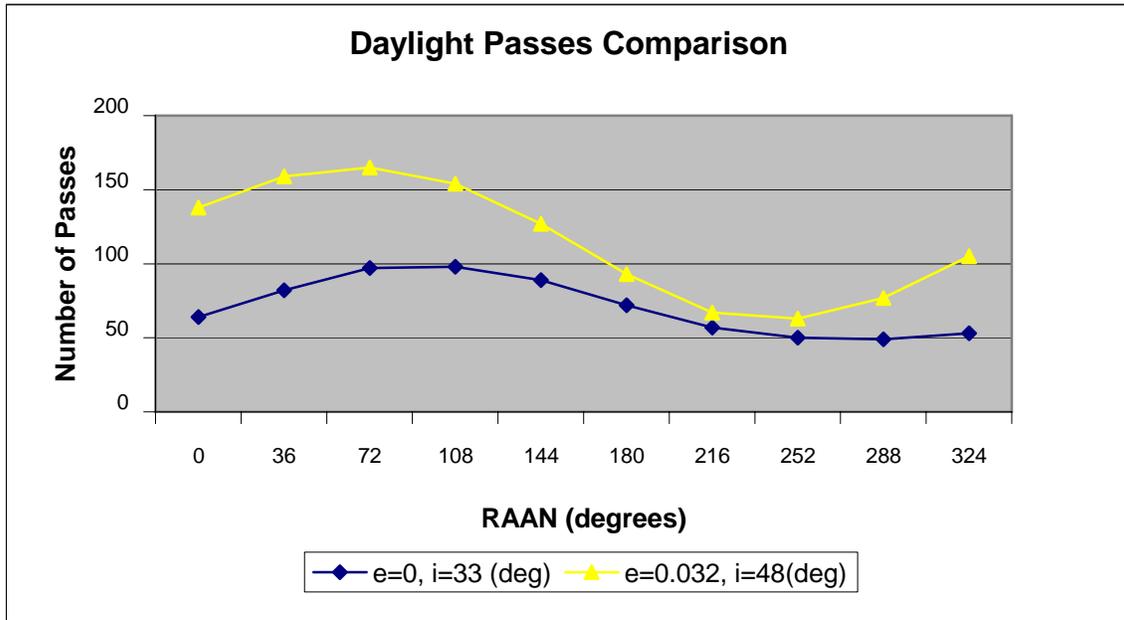
Figure 23: Daylight Passes comparison for a satellite with different eccentricity and inclination for a 30-day perion.

This resulted in an increase of the total number of daylight passes when compared to results for a test that used an inclination of 41 degrees and a zero eccentricity. When comparing the maximum number of satellite passes from these two tests it was concluded that the test conducted with values of 41 degrees and 0.037 for the inclination and eccentricity respectively had an average of 7.4 more passes than the test using an inclination of 41 degrees and a zero eccentricity.

4.2 Satellite to Target Slant Range

The slant range between the satellite and the target was calculated for two different tests. The first test dealt with showing the effect that varying the inclination has on the average slant range. The generic matched inclination circular orbit was compared to a

circular orbit that has an inclination of 41 degrees. The inclination difference between these two orbits is 8 degrees. The second test involved the effect of varying the eccentricity had on the average slant range. The elliptical orbit with an eccentricity of 0.032 and apogee and perigee of 7178 km and 6728 km respectively was used. This orbit was compared with the generic matched inclination circular orbit. The imaging device on the satellite can provide images with adequate resolution up to an altitude of 800 km. The optimal resolution occurs at an altitude of 350 km, so the resolution will degrade between this altitude and 800 km.

4.2.1 Inclination Comparison

It was previously determined that having an inclination value that is greater than the latitude of the target will provide a greater number of satellite daylight passes. Finding the average slant range of the passes at the greater inclination will show if this different configuration can be used. Table 2 shows the daylight pass, average slant range comparison of a satellite at a matched inclination orbit with a satellite that has an inclination of 41 degrees. Both are in a 350 km circular orbit.

Table 2: Slant range comparison between circular orbits at 33 and 41 degree inclination.

RAAN	circular, a = 350 km, i = 33 deg			circular, a = 350 km, i = 41 deg		
	daylight passes	avg height (km)	avg slant range (km)	daylight passes	avg height (km)	avg slant range (km)
0	64	350	696.13	92	350	778.71
36	82	350	698.63	117	350	787.05
72	97	350	698.42	131	350	785.38
108	98	350	691.08	128	350	772.98
144	89	350	704.11	113	350	776.93
180	72	350	703.17	90	350	787.98
216	57	350	710.19	70	350	777.51
252	50	350	704.26	59	350	774.54
288	49	350	704.38	59	350	774.94
324	53	350	709.48	73	350	767.54

The table shows that the change in inclination does affect the average slant range. The overall average slant range of the 33 degree inclination orbit is 702 km while the 41 degree inclination orbit had an overall slant range of about 779 km, a significant increase. But the important slant range change is for a RAAN value of 72 degrees, where the satellite provides the maximum number of daylight passes. For this RAAN value, the average slant range of the satellite with a 41 degree inclination is 785.38 km which is below the maximum allowable distance for the imaging device to produce images with adequate resolution. Therefore, the orbit with the greater inclination value is a better choice considering that it provides a greater number of daylight passes while not violating the maximum distance constraint of 800 km for the imaging device.

4.2.2 Eccentricity Comparison

Again since the satellite will use an imaging device that has altitude constraints, knowing the slant range of the satellite when using an elliptical orbit is important to know. When using an elliptical orbit, the altitude of the satellite will vary every time it

passes over the target. This altitude change depends on how elliptical the orbit is or in other words, how high the eccentricity is. If the eccentricity value is small, the altitude variance will also be small. But when the eccentricity starts to increase, the altitude variation over the target will start to increase as well. This altitude variation will have an effect on the slant range between satellite and target. The slant range is calculated at every time interval the satellite is in view of the target in daylight as shown on Figure 1. Table 2 shows a comparison of the average height and slant range for a circular orbit and an elliptical orbit with an eccentricity of e = 0.032. The height of a circular orbit is uniform and is indicated on Table 3, but the slant range between the satellite and the target varies from 696 km to 709 km. In the elliptical orbit case the height of the orbit changes as expected as well as the slant range. The average height of the satellite is 730 km which is close to the 800 km constraint of the imaging device. The average slant range for the satellite in an elliptical orbit is about 1300 km.

Table 3: Average Height and Average Slant Range Comparison of a satellite in a circular orbit with a semi-major axis of 350 km and a satellite in an elliptical orbit that has and eccentricity of *e* = 0.0323 with a semi-major axis of 575 km.

RAAN	circular orbit, a = 350 km, e = 0			elliptical orbit, a = 575 km, e = 0.032		
	daylight passes	avg height (km)	avg slant range (km)	daylight passes	avg height (km)	avg slant range (km)
0	64	350	696.13	96	768.71	1354.22
36	82	350	698.63	116	754.96	1313.12
72	97	350	698.42	125	707.51	1220.44
108	98	350	691.08	120	765.10	1320.97
144	89	350	704.11	106	728.50	1269.23
180	72	350	703.17	83	732.67	1285.12
216	57	350	710.19	73	633.61	1224.45
252	50	350	704.26	65	718.91	1341.85
288	49	350	704.38	68	725.88	1340.74
324	53	350	709.48	79	763.74	1367.05

This slant range is a much greater value than the imaging device altitude constraint. Even though this orbit configuration provides a greater number of daylight passes when compared to the circular orbit, it is not feasible option because the average slant range is greater than the maximum distance of 800 km for the imaging device.

4.3 Coverage of Two Satellites

Since this thesis deals with optimizing the coverage of a specific region by a satellite constellation; one satellite might not be sufficient to provide coverage for the specified one month mission duration. The reason for this is because of the shift in daylight passes through out the 30 day mission duration. This shift can be seen in Figure 24; the coverage or the number of daylight passes over the target is shifted depending on the RAAN value of the orbit.
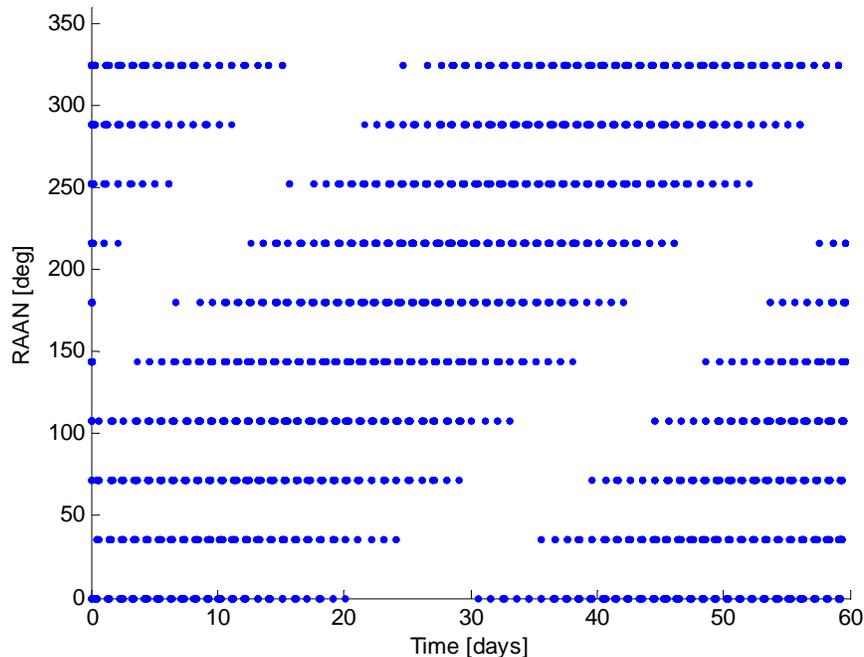
Figure 24: Daylight Passes Shift for different RAAN values

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Because of this the shift there will be a period of time during the mission that the satellite will not have daylight passes over the target. This is shown on Figure 24 by the absence of blue dots at the different RAAN values. Using two satellites instead of one will provide the necessary coverage of the target for the required mission duration.

From what is shown on Figure 21, which shows the total number of daylight passes for different RAAN/argument of perigee values, for a circular orbit with an inclination of 33 degrees, the first thought would be to choose the RAAN/argument of perigee combination that result in the second best total of daylight passes.

The RAAN values for the two satellites that will provide continuous coverage for the required mission duration can be obtain observing Figure 24. These RAAN values are 72 and 108 degrees. The daylight pass values for these two RAAN values correspond to the top two daylight passes values. This means that not only do using two satellites provide continuous coverage for the specified mission duration; it also provides more satellite daylight passes over the target. The daylight passes for a single satellite for circular orbits with an altitude of 350 km and an inclination of 33 degrees is shown on Figure 20. The maximum daylight passes for this test was 98 passes. Now if two satellites are used, the number of daylight passes for this case is the sum of the daylight pass values pertaining to the RAAN values of 72 and 108 degrees. So the total number of daylight passes using two satellites is 195 passes; this is almost double the daylight passes when compared to the single satellite case. On average, the two satellites would provide six and a half daylight passes per day over the target.

This can also be done for the optimal case when a single satellite was used; that is using an elliptical orbit with an eccentricity of $e = 0.032$, an inclination of 48 degrees, and with apogee and perigee being 800 km and 350 km respectively. For the single satellite case the maximum daylight passes was 165 passes at a RAAN value of 72 degrees. Now when using two satellites as was seen on the previous case the satellites will provide coverage for the mission duration of a month. But instead of having the maximum daylight passes occurring at RAAN values of 72 and 108 degrees; the maximum daylight passes occur at RAAN values of 36 and 72 degrees. The corresponding daylight passes for these RAAN values are 159 passes at 36 degrees and 165 passes at 72 degrees. This means that if two satellites are used to provide coverage of the desired target; these two satellites will provide 324 daylight passes for the month duration. So on average, this two satellite coverage will provide eleven daylight passes over the target per day.

Though putting two satellites on similar orbits at different RAAN/argument of perigee combinations yield better result compared to single satellite coverage; a more efficient way is to have the two satellites on the same orbit. Using the same RAAN value for both satellites and having them spaced out, in other words having different values for the true anomaly will provide optimal coverage. Table 4 shows the daylight passes and max Argument of perigee values for a satellite at true anomaly values of 0 and 180 degrees.

Table 4: A comparison of Daylight and Max Argument of Perigee for a satellite in a 350 km circular orbit at true anomaly of 0 and 180 degrees.

RAAN (deg)	a = 350 km/true= 0		a = 350 km/true= 180	
	day passes	max Arg (deg)	day passes	max Arg (deg)
0	64	0	64	180
36	82	60	82	150
72	97	120	97	300
108	98	60	98	60
144	89	0	89	180
180	72	60	72	60
216	57	30	57	120
252	50	300	50	120
288	49	150	49	330
324	53	60	53	240

For a RAAN value of 108 degrees, the daylight passes remained the same and so did the argument of perigee value, 60 degrees. This shows that it is feasible to have two satellites with the same RAAN and argument of perigee value that will provide twice the amount of coverage compared to a single satellite and the coverage of the two satellites will not overlap because they would be space 180 degrees apart. So using two satellites situated in the same orbit at different true anomaly values will produce 196 daylight passes over the target for the mission duration of a month.

A further presumption to this would be to combine the two ways discussed above to use three or more satellites. The idea for this would be to have two satellites on the same orbit but at spaced 180 degrees apart and then have a third satellite in an orbit with RAAN/argument of perigee values that yields the second most number of daylight passes. The data given in table one is not sufficient to support this presumption; further modification and analysis needs to be done in order to provide information that would support such a claim.

5 Conclusion

The simulation results demonstrate that the circular, matched inclination orbit does not optimize the total number daylight passes over a target. By varying the inclination and the eccentricity, it is possible to increase the daylight passes over a 30-day period by up to 33%.

For a circular orbit, the number of daylight passes increased as the inclination was raised from the target latitude. The optimal inclination was typically a few degrees above the target latitude, but the exact optimum was dependant on the altitude of the orbit. This is due to an increased sensor footprint as the satellite altitude is raised. For the cases studied, it appears that the best inclination is equal to the target latitude plus about 2° per 100 km of altitude.

The number of daylight passes also can be increased by increasing the eccentricity of the orbit. Only a small change to eccentricity was considered, but at an eccentricity of 0.032, the daylight passes were increased by 27.5%. This increase comes with a cost, however. By increasing eccentricity, the average slant range to the target during the daylight passes also increased by 86%.

One case varying both inclination and eccentricity was done. This case showed a greater increase than changing either inclination or eccentricity alone. For an eccentricity of 0.032 and inclination of 48° (15° above the target latitude), 164 daylight passes can be obtained over a 30 day period using the 350 km x 800 km orbit. This is compared to only 98 daylight passes for the circular, ($e = 0$), matched inclination ($i = 33^o$) with a semi-major axis (350 km altitude).

All these tests pertained to a single satellite. When a second satellite was added, it was seen that the total number of daylight passes could be nearly doubled by an appropriate change to the true anomaly of the second satellite. Because of the need to not "overlap" passes, where one or more satellites would provide coverage simultaneously, it became clear that additional satellites would not increase the daylight passes in a linear fashion. That is, it is not expected that three satellites would give three times the passes of one satellite. More study on how to use additional satellites is needed.

A final conclusion is that the specifications for what constitutes a successful pass should include consideration of elevation (and/or slant range) in addition to satellite altitude. The code used in this thesis assumed that any pass above 0° elevation during daylight was useful, and altitude was limited to between 350 km and 800 km, as suggested by the TACSAT specifications. However, the slant ranges varied significantly, and in the worst case (800 km altitude and 0° elevation) could have been as high as 2000 km. Whether this would meet the intended goals of the TACSAT is unclear.

5.1 Recommendations for Future work

Future research for this problem might include attempting an analytical solution instead of a numerical search to optimize the daylight passes of the LEO satellite. Research can also look into improving the orbit propagation model by including the drag effect on the satellite. Another area for future research would be using a finer resolution for the search parameters (inclination, eccentricity). Changing increment size for the inclination from 1° to 0.5^o degrees might yield a sharper trend for the relationship between increasing inclination and obtaining greater daylight pass totals. Similarly,

simulate satellite coverage of low eccentric, $e < 0.032$ orbits that would not violate the

imaging device's maximum altitude restriction. Varying the altitude values, for example

using altitude between 350 km and 800km at increments of 50 km, might provide

altitudes that would be more appropriate than a 350 km altitude. Finally, looking into the

effects of the inclination and eccentricity variation will have on the daylight passes at

different latitudes. This last recommendation would provide insight into a general idea of

satellite coverage of a more broad area.

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Vita

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REPORT DOCUMENTATION PAGE

1. REPORT DATE (DD-MM-YYYY) 24 Mar 06	2. REPORT TYPE **Master's Thesis**	3. DATES COVERED (From – To) Aug 04 – Mar 06

4. TITLE AND SUBTITLE Optimal Coverage of Theater Targets with Small Satellite Constellations	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) Rendon, Axel, 2d Lt, USAF	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(S) Air Force Institute of Technology Graduate School of Engineering and Management (AFIT/EN) 2950 Hobson Way WPAFB OH 45433-7765	8. PERFORMING ORGANIZATION REPORT NUMBER AFIT/GSS/ENY/06-M12
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFIT/ENY Attn: Lt Col Nathan Titus 2950 Hobson Way WPAFB OH 45433-7765 Comm: 937-427-3636 ext7469	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

The daylight passes of a low-Earth orbit satellite over a targeted latitude and longitude are optimized by varying the inclination and eccentricity of an orbit at different altitudes. This investigation extends the work by Emery et al, in which the optimal Right Ascension of the Ascending Node was determined for a circular, matched inclination orbit. The optimal values were determined by a numerical research method based on Emery et al.'s Matlab program. Results indicate that small increases in inclination raise the number of daylight passes up to 33%. These optimal inclinations depend on the satellite semi-major axis. Eccentricity increases also improve daylight pass numbers, but at a cost of increased range to the target.

15. SUBJECT TERMS
Satellite Constellations, Area Coverage, Inclination, Eccentricity

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON **Lt Col Nathan Titus**
REPORT **U**	ABSTRACT **U**	c. THIS PAGE **U**	**UU**	77	19b. TELEPHONE NUMBER (Include area code) (937) 427-3636, ext 7469; e-mail: @afit.edu