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Inducing Stochastic Behavior in a Deterministic Model

Pete Bulanow
Steven P. Wilcox, Ph.D.

Northrop Grumman
Information Technology Sector
June 2005
Overview

- **Background**
  - Understanding the Value Added Analysis Process

- **Challenges**
  - Demand for quick-turnaround analysis
  - Non-monotonic relationships between parameters and VIC outcomes
  - Statistical analysis to support accurate decision making

- **Our Answer**
  - Induce stochastic variation in VIC runs through perturbation
  - Utilize statistical tests for comparisons of options

- **Benefit**
  - More accurate decisions about equipment trades using VIC as part of the Value-Added Analysis process
The Value Added Analysis Process

- Supporting the Center for Army Analysis (CAA)
- Uses the Vector-in-Commander (VIC) Corps-level combat simulation model
- Objective:
  - Estimate the incremental contribution of system trades to combat effectiveness
  - Perform a cost-benefit analysis to determine the actual ‘value-added’ of the systems of interest.
- Previous methodology was a typical DOE approach
- Now a perturbation methodology induces stochastic behavior in VIC

Figure 1. The VAA Process
The Force Exchange Ratio

- Primary Measure of Effectiveness (MOE) in the VAA process
- Force Exchange Ratio (FER)
  - Ratio of relative losses
  - Used as a proxy for the win probability
  - Only high-value equipment losses are counted in our version

\[
FER = \frac{\text{Losses}_R}{\text{Strength}_R} \div \frac{\text{Losses}_B}{\text{Strength}_B}
\]
Legacy Methodology

- Comparing FERs using a Design of Experiments
- A typical DOE is to run a number of combinations of experimental settings
  - And then analyze the MOEs using analysis of variance
  - Differences in the means between treatments indicate possible differences in effectiveness
- Statistical efficiency is achieved at the cost of elaborate run setups.

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DOEs aid in making statistical “decisions”:

FER(X₁ = 1) - FER(X₁ = -1) > 0 ?

How big is the difference?
What is the confidence interval?
Two Paradigm Shifts

- VIC and complexity
  - Battle is a complex dynamical system
  - The results of battle are somewhat uncertain
    - Especially when the foes are close to evenly matched
  - VIC battles are a realization of a complex dynamical system
    - Sensitivity to parameters and initial conditions should be expected

- Embrace complexity
  - In support of quick turnaround analysis
  - Using the statistical perspective
Inducing the Expected Variability

- Statistical methods require variability and replication.
- Key Requirements for inducing stochastic behavior consistent with accurate analysis:
  - Must not alter any performance data (Bailey, 2001)
  - Must affect many battlefield operating systems (Bailey, 2001)
  - Must continuously perturb the run – not just the initial conditions (Bailey, 2001)
  - Retains the original scenario setup within the precision of combat operations
A Perturbation Methodology

- Our method perturbs several things a “small” amount
  - Unit locations and waypoints
  - Helo path points
  - Airborne sensor orbit points
- See Bulanow et al. (2004) for validation with respect to using the outputs in statistical models
The Difficulty with the Two-Run Comparison

- Non-Monotonic effects have been observed in Deterministic Combat Models
  - Better settings do not necessarily mean a better FER
- Sensitivity to initial conditions and parameter values
  - Extensively noted in toy models of combat
    - The RAND model (Dewar, et al, 1991)
  - Also noted in VIC
    - Saeger & Hinch (2001)
- The DOE is a legacy solution to this problem, but a more responsive approach is required.

Kills of Selected Equipment by a Blue System of Interest

A three-way comparison of VIC results
Parametric Sensitivity in VIC

- A direct fire system (DF Sys) fraction of time firing (FTF) is multiplied by a number randomly selected from the interval (0.95, 1.05)
- Blue kills vary non-monotonically and significantly
- Any two runs selected from these might show a difference in the MOE
  - But is the difference statistically "significant"?
Inducing Variability Through Unit Locations

- Perturbing ground unit locations and waypoints by ±10 meters produces very different pictures of the loss exchange ratio.
- Each color line represents the plot of Blue versus Red kills over the run for the original and 64 replications
  - X and Y scales include zero but are not the same

Kills by Blue and Red of High-Value Equipment
More on VIC’s Stochastic Personality

- Perturbing ground unit locations small amounts (a non-performance parameter) reveals a world of stochastic variability
  - Like what might happen in combat
- Statistical methods can characterize this variability for decision-making purposes

Kills of Selected Equipment by the Blue System of Interest

Base Case, Perturbed
Analysis Without DOE Matrices

- Paired Comparisons can be performed without an elaborate DOE

<table>
<thead>
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<th>Perturbation Set</th>
<th>Base</th>
<th>Alternative A</th>
<th>Delta</th>
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- We also perform multiple comparisons between numerous options
- More efficient for the analyst due to fewer run setups than with a DOE
- Has been employed in a variety of trade comparisons
Effect of Replications on the Confidence Interval of Estimates

- Confidence intervals decrease as the inverse square root of sample size.
- In actual applications, the standard deviation would be estimated.

<table>
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<th>Confidence Interval (assuming a notional standard deviation, known in advance)</th>
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Conclusions and Way Forward

- Our perturbation analysis for VIC analysis aids in quick-turn analysis by:
  - Reducing run setups,
  - Simplifying design and analysis of experiments, and
  - Enabling statistical analysis with simple designs
- VIC run perturbation gives visibility to the complex system feature of combat
  - Even though VIC is deterministic
  - Thus providing an added window into the issue of outcome variability
References

- Randomization of VIC

- Sensitivity of VIC Settings

- Non-Monotonicity in General

- Design of Experiments