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Gains, losses, and thresholds of influence within a social network: A modeling approach

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The views expressed in this briefing are those of the authors and do not reflect the official policy or position of the United States Air Force, the Department of Defense, or the United States government.

Approved for public release.
Overview

• Problem Statement
• Social Sciences and Network Flows
  – Initial mappings
  – Gains, losses, and thresholds
  – Relationships to network flow formulations
• Notional Example
  – Generalized Network Flow Problem (GFP)
  – Post-optimality analyses
Problem Statement

• Extend previous methodologies to generate and analyze courses of action applied to networks of individuals

• Overall goal - ‘shaping intentions’ through influence
  – … in the context of military psychological operations that strive to influence an adversary’s “… emotions, motives, reasoning, and ultimately, their behavior…” in order to achieve a given political goal. (JP 3-13, 1998:II-4)

• The means - extend previous SNA and OR mappings
# Current Mappings

<table>
<thead>
<tr>
<th>Social Closeness Terms</th>
<th>Flow Model Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>People or groups</td>
<td>Nodes (sinks, sources, or transshipment)</td>
</tr>
<tr>
<td>Connectivity or affinity</td>
<td>Capacitated arcs (or edges) between nodes</td>
</tr>
<tr>
<td>Social Closeness</td>
<td>Capacity</td>
</tr>
<tr>
<td>Influence</td>
<td>Commodity</td>
</tr>
<tr>
<td>Potential Influence</td>
<td>Magnitude of flow</td>
</tr>
<tr>
<td>People or groups initiating influence in the network</td>
<td>Source(s)</td>
</tr>
<tr>
<td>Target people or groups to be influenced</td>
<td>Sink(s)</td>
</tr>
<tr>
<td>People or groups involved in influencing</td>
<td>Transshipment node(s)</td>
</tr>
<tr>
<td>Multi-Criteria within a shared context</td>
<td>Multi-Commodity, contexts share capacity</td>
</tr>
<tr>
<td>Multi-Context or Multi-Criteria in different contexts</td>
<td>Multiple independent single-commodity models for each context or criteria</td>
</tr>
</tbody>
</table>

(Renfro, 2001:95)
Underlying Assumptions

• Renfro mappings are appropriate
• Accurate and complete network data
• Amount of influence generated by COA is measurable
  – Interpretation of influence amount is inviolate among individuals and their interactions
• Directed network mimics the anticipated operational channels of communication
  – No discussion or interaction, as seen in traditional SNA approaches, is modeled
Research Focus

• “Gains and losses represent predispositions, communication problems, and other similar factors based on the specific scenario under consideration.” (Renfro, 2001:67)

• “Thresholds can also be set for cases where individuals or groups require a minimum level of influence before they take a specific course of action.” (Renfro, 2001:67)

• Requires Generalized Network Flow
  – Arcs may consume or generate flow
  – Seen in power networks, canals, transportation of perishable commodities, and cash management (Ahuja, et al, 1993:8)
  – Develop maximum flow and minimum cost, maximum flow approaches
Influence

• As a commodity...
  – Transfers between individuals to enable group opinion formation (Friedkin and Cook, 1990)
  – Contagion of behavior (Leenders, 2002)
  – Diffusion of innovations (Valente, 1996)
  – Propagation of extremist opinions (Amblard and Deffuant, 2004)
  – Basis for interpersonal power of one individual over another (French, 1956)
“Outflow minus inflow must equal supply (or demand)”

Amount of flow from node $i$ to node $j$ on arc $(i, j)$ is $x_{ij}$

Mass Balance Constraints (Three cases)
Supply node: outflow $>$ inflow $\Rightarrow$ outflow $= \text{inflow} + b_j$
\[ x_{jk} - x_{ij} = b_j \]

Demand node: outflow $<$ inflow $\Rightarrow$ outflow $= \text{inflow} - b_j$
\[ x_{jk} - x_{ij} = -b_j \]

Transshipment node: outflow $= \text{inflow}$
\[ x_{jk} - x_{ij} = 0 \]

(Ahuja, Magnanti, and Orlin, 1993:5)
Network Flow

Target Flow

• Given the following…
  – A network structure
  – Social closeness measures for all arcs \((i, j)\)

• The objectives…
  – Identify key actors that serve as ultimate targets of influence
  – Identify actors that are accessible and likely to propagate influence through the network
  – Identify the minimum amount of influence required
Notional Network

Target Flow

Legend

\[
\begin{align*}
&i \quad \rightarrow \quad j \\
&[u_{ij}] \\
&u_{ij}: \text{upper bound of flow on arc } (i, j)
\end{align*}
\]
Notional Network

Target Flow

Legend

- $i \rightarrow j$: upper bound of flow on arc $(i, j)$

- $u_{ij}$: upper bound of flow on arc $(i, j)$

- $b_a = v$

- $b_{tgt} = -v$

"Influence Action"

"Targets"
Notional Network

Target Flow

Solution ($v^* = 2$)

Legend

$u_{ij}$: upper bound of flow on arc $(i, j)$
Gains

• Influence is not necessarily equitable between two actors (Renfro, 2001:103)
• Predispositions of individuals favoring influence (Renfro, 2001, 88)
• “… person’s opinions may be tugged in various directions by the influences of their significant others and that individuals deal with these cross-pressures by shifting their opinions into positions where pressures are balanced.” (Friedkin and Cook, 1990:130)
• Interpersonal power – “maximum force which A can induce on B minus the maximum resisting force which B can mobilize in the opposite direction” (French, 1956:183-4)
  – Five Bases: Attraction, Expert, Reward, Coercive, Legitimate
  – Must be measured from A’s and/or B’s perspective
Gains

For node $j$, “out – in” is represented by $x_{jk} - g_{ij}x_{ij} = 0$

Given $x_{ij} = 1$ and $g_{ij} = 2 \Rightarrow x_{jk} = 2$

$g_{ij} > 1$
Losses

- Theory presented by (French, 1956) also applies
- “… communication problems such as misunderstanding the message.” (Renfro, 2001:88)
- (Lopez, et al, 2002) link organizational structure to efficiency of information flow
- (Friedkin and Johnsen, 2002) analyze impact of organizational structure and span of control
  - Mitigation via “Fayol’s gangplanks”
  - Traces back to book by Williamson (1971)
Losses

For node $j$, “out – in” is represented by $x_{jk} - g_{ij}x_{ij} = 0$

Given $x_{ij} = 2$ and $g_{ij} = 0.5$ $\Rightarrow x_{jk} = 1$

"loss"

$g_{ij} < 1$
Thresholds

• “Models of collective behavior are developed for situations where actors have two alternative and the costs and/or benefits of each depend on how many other actors choose which alternative.” (Granovetter, 1978:1420)
  - Threshold – number or proportion required at point where benefits exceed costs for that actor
  - Innovations, rumors and diseases, strikes, voting, educational attainment, leaving social occasions, migration, and experimental social psychology (1423-4)
• (Valente, 1996) developed a (social) network threshold model for diffusion of innovations
• Two modeling options are presented
Thresholds

“m out of n”

Assuming one unit of flow from any \( i \) to \( j \), at least two of the three individuals must “influence” \( j \) before \( j \) will “influence” \( k \)

Given only one \( x_{ij} = 1 \) ⇒ \( x_{jk} = 0 \)
Given any two \( x_{ij} = 1 \) ⇒ \( x_{jk} = 1 \)
Given all three \( x_{ij} = 1 \) ⇒ \( x_{jk} = 2 \)
Notional Network

Maximum Flow

Legend

\[
\begin{align*}
\text{Legend} & : \quad g_{ij} \\
\text{uij: upper bound of flow on arc (i, j)}
\end{align*}
\]
Notional Network

Maximum Flow

Legend

- $g_{ij}$
- $u_{ij}$: upper bound of flow on arc $(i, j)$

- $b_a = v$
- $b_{tgt} = -v$

- $b_4 = -1$

- “Influence Action”
- “Targets”
Notional Network

Maximum Flow

Legend

- **Arc flow = 1**
- **Arc flow = 0**

Solution ($v^* = 3$)

$\triangledown_4 = \frac{4}{3}$

$b_4 = -1$
Minimum cost, maximum flow

- External Costs – Course of Action
  - Represent risk friendly forces are subjected to when implementing the COA
    - Node “a” to all initial target nodes - execution
    - Target nodes to “tgt” node - observation

- Internal Costs
  - Represent risks perceived by individuals within the network
  - Operational – Fear of compromise
  - Personal – Fear of retribution
  - May also apply to individuals external to the network of interest
Notional Network

Minimum cost, maximum flow

Legend

- $u_{ij}$: upper bound of flow on arc $(i, j)$
- $c_{ij}$: cost per unit flow on arc $(i, j)$
- $g_{ij}$: gain/loss factor for arc $(i, j)$
Notional Network

Minimum cost, maximum flow

Legend

- $u_{ij}$: upper bound of flow on arc $(i, j)$
- $c_{ij}$: cost per unit flow on arc $(i, j)$
- $g_{ij}$: gain/loss factor for arc $(i, j)$
Notional Network

Minimum cost, maximum flow

Solution (z* = 93.32)

Legend

$i$ to $j$: upper bound of flow on arc $(i, j)$
$x_{ij}$: amount of flow on arc $(i, j)$
Post-optimality Analysis

\[ b_a = a \]
\[ b_4 = -1 \]
\[ b_{tgt} = - t \]

Legend:
- \( g_{ij} \): gain/loss factor for arc \((i, j)\)
- \( u_{ij} \): upper bound of flow on arc \((i, j)\)
- \( c_{ij} \): cost per unit flow on arc \((i, j)\)
Post-optimality Analysis

- Objective Function Coefficients
- Right-hand side
  - Thresholds, $b_a, b_{tgt}$
  - Upper bounds (if included as a constraint)
- Technological coefficients
  - Gains and losses
- Parametric and multiple changes
## Post-optimality Analysis

### Objective Function Coefficients

<table>
<thead>
<tr>
<th>Variable</th>
<th>Current</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1_4</td>
<td>8</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>X1_6</td>
<td>20</td>
<td>3</td>
<td>0.339</td>
</tr>
<tr>
<td>X2_4</td>
<td>6</td>
<td>∞</td>
<td>0</td>
</tr>
<tr>
<td>X3_4</td>
<td>2</td>
<td>5</td>
<td>31.662</td>
</tr>
<tr>
<td>X4_5</td>
<td>9</td>
<td>0.339</td>
<td>12.662</td>
</tr>
<tr>
<td>X4_6</td>
<td>15</td>
<td>∞</td>
<td>3</td>
</tr>
<tr>
<td>X4_7</td>
<td>24</td>
<td>∞</td>
<td>13.339</td>
</tr>
<tr>
<td>X5_6</td>
<td>10</td>
<td>∞</td>
<td>29.823</td>
</tr>
<tr>
<td>X5_7</td>
<td>6</td>
<td>0.254</td>
<td>9.498</td>
</tr>
<tr>
<td>X6_10</td>
<td>6</td>
<td>∞</td>
<td>0.339</td>
</tr>
<tr>
<td>X6_11</td>
<td>5</td>
<td>12.662</td>
<td>3.339</td>
</tr>
<tr>
<td>X7_8</td>
<td>4</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>X7_9</td>
<td>9</td>
<td>3</td>
<td>1.015</td>
</tr>
<tr>
<td>X7_10</td>
<td>7</td>
<td>0.339</td>
<td>∞</td>
</tr>
<tr>
<td>X8_9</td>
<td>8</td>
<td>∞</td>
<td>3</td>
</tr>
<tr>
<td>X9_tgt</td>
<td>10</td>
<td>∞</td>
<td>1.015</td>
</tr>
<tr>
<td>X10_tgt</td>
<td>9</td>
<td>3</td>
<td>∞</td>
</tr>
<tr>
<td>X11_10</td>
<td>4</td>
<td>∞</td>
<td>3.339</td>
</tr>
<tr>
<td>Xa_1</td>
<td>6</td>
<td>0</td>
<td>∞</td>
</tr>
<tr>
<td>Xa_2</td>
<td>8</td>
<td>∞</td>
<td>0</td>
</tr>
<tr>
<td>Xa_3</td>
<td>7</td>
<td>5</td>
<td>∞</td>
</tr>
</tbody>
</table>

Currently in basis

Potentially of interest
Notional Network

Cost Coefficients

Legend

- $g_{ij}$: upper bound of flow on arc $(i, j)$
- $u_{ij}$: amount of flow on arc $(i, j)$
- $x_{ij}$: amount of flow on arc $(i, j)$

Solution ($z^* = 92.66$)

change in $c_{1,6}$
From 20 to 19

$b_{gt} = -2$

$b_{a} = 3$

$4/3$

$2/3$

$5/3$
### Post-optimality Analysis

**Right-hand Side**

<table>
<thead>
<tr>
<th>Row (Node)</th>
<th>Current Value</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3</td>
<td>0</td>
<td>0.499</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.499</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0.499</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>1</td>
<td>0.499</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1.33</td>
<td>0.67</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0.67</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.67</td>
<td>0.33</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0.33</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.67</td>
<td>0.33</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0.67</td>
<td>0.33</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>tgt</td>
<td>-2</td>
<td>0.67</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

*Potentially of interest*
Notional Network

Right-hand Side

change in \( b_a \)
From 3 to 2

\[ b_a = 2 \]

\[ b_{tgt} = -2 \]

Solution (\( z^* = 50 \))

Legend

\[ u_{ij} \): upper bound of flow on arc \((i, j)\)

\[ x_{ij} \): amount of flow on arc \((i, j)\)
Notional Network

Right-hand Side

change in $b_a$
From 3 to 1

$b_a = 1$

Solution ($z^* = 9$)

Legend

$I$ $j$

$g_{ij}$ $u_{ij}$: upper bound of flow on arc $(i, j)$

$x_{ij}$: amount of flow on arc $(i, j)$
## Post-optimality Analysis

### Arc Capacities

<table>
<thead>
<tr>
<th>Arcs with non-zero flow</th>
<th>Arcs with zero flow</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variable</strong></td>
<td><strong>Current (Range)</strong></td>
</tr>
<tr>
<td>X1_4</td>
<td>2 (1, ∞)</td>
</tr>
<tr>
<td>X2_4</td>
<td>3 (2, ∞)</td>
</tr>
<tr>
<td>X3_4</td>
<td>1 (0, ∞)</td>
</tr>
<tr>
<td>X4_5</td>
<td>7 (5, ∞)</td>
</tr>
<tr>
<td>X5_7</td>
<td>5 (2⅓, ∞)</td>
</tr>
<tr>
<td>X7_9</td>
<td>6 (5⅔, ∞)</td>
</tr>
<tr>
<td>X7_10</td>
<td>6 (5, ∞)</td>
</tr>
<tr>
<td>X9_tgt</td>
<td>1 (⅓, ∞)</td>
</tr>
<tr>
<td>X10_tgt</td>
<td>1 (⅓, ⅓)</td>
</tr>
<tr>
<td>Xa_1</td>
<td>1 (0, ∞)</td>
</tr>
<tr>
<td>Xa_2</td>
<td>1 (0, 1)</td>
</tr>
<tr>
<td>Xa_3</td>
<td>1 (0, 0)</td>
</tr>
</tbody>
</table>

* The (Range) indicates allowable decrease, $d$, and allowable increase, $i$, denoted by $(d, i)$. 
## Post-optimality Analysis

### Technological Coefficients

<table>
<thead>
<tr>
<th>Constraint $i$</th>
<th>Variable $j$</th>
<th>Acceptable Change</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td></td>
<td>$0 \leq \Delta a_{i,j} \leq 0$</td>
<td>--</td>
</tr>
<tr>
<td>Binding</td>
<td>Non-Basic</td>
<td>$\frac{c_j - c_B B^{-1} a_j}{w_i} \leq \Delta a_{i,j} \leq \infty$</td>
<td>For all $\leq$ constraints</td>
</tr>
<tr>
<td>Non-binding</td>
<td>Non-Basic</td>
<td>$-\infty \leq \Delta a_{i,j} \leq \frac{x_{n+i}}{x_j}$</td>
<td>For $\leq$ constraint ($x_{n+i}$ = slack)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-\frac{x_{n+i}}{x_j} \leq \Delta a_{i,j} \leq \infty$</td>
<td>For $\geq$ constraint ($x_{n+i}$ = surplus)</td>
</tr>
</tbody>
</table>

Although many optimization software packages do not provide this capability, similar analyses on the technological coefficients (i.e. the gains and losses) may be performed

Conclusions

• Social Sciences and Network Flows
  – Initial mappings
  – Gains, losses, and thresholds
  – Relationships to network flow formulations

• GFP and Notional Examples
  – Advantages - Post-optimality analyses
  – Disadvantages – Data, Deterministic, …

• Attractive option to analyze, better understand, and predict behavior of non-cooperative networks in response to external influence
Backups/Old Slides
Flow bound constraints are the upper and lower limits of $x_{ij}$

Social Closeness ($S_{ij}$), measured by a value-focused thinking model, is defined as "the maximum potential influence one person or group ($i$) has upon another person or group ($j$)..." in a given social network and under a given scenario. (Renfro, 2001:89)
Thresholds

“Absorbing node”

\[ b_j = - U_j \]

\[ U_j \geq \sum_{\{j:(i,j)\in A\}} s_{ij} \quad \text{Influence will not pass} \]

However, varying this input can have some interesting properties...