Abstract. It has been shown previously that the number of very-large-fluence solar proton events inferred for the period since 1561 were more frequent at times of low solar activity (e.g., following the recovery from the Maunder minimum), than in the present epoch of high solar activity. An inverse dependence is demonstrated between the probability of observation of the very large-fluence solar proton events and the strength of the interplanetary magnetic field derived from empirical predictions. Using the observed dependence, it is predicted and demonstrated that large-fluence solar proton events have been observed at Earth more frequently near the recurrent minima of the solar activity cycle in the past than during the present epoch. We show that these results are explicable in terms of the linear dependence of the Alfvén velocity upon the strength of the interplanetary magnetic field, leading to higher shock compression ratios in the past. These results indicate that this aspect of "solar weather" will be significantly influenced by the prevailing strength of the interplanetary magnetic field, and that recurrence of solar conditions similar to those of the solar activity minimum of solar cycles 12-14 (1878.9-1913.6) would be accompanied by a factor of ~4 increase in the occurrence of large-fluence solar proton events.
A STUDY OF THE FREQUENCY OF OCCURRENCE OF LARGE-FLUENCE SOLAR PROTON EVENTS AND THE STRENGTH OF THE INTERPLANETARY MAGNETIC FIELD

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Abstract. It has been shown previously that the number of very-large-fluence solar proton events inferred for the period since 1561 were more frequent at times of low solar activity (e.g., following the recovery from the Maunder minimum), than in the present epoch of high solar activity. An inverse dependence is demonstrated between the probability of observation of the very large-fluence solar proton events and the strength of the interplanetary magnetic field derived from empirical predictions. Using the observed dependence, it is predicted and demonstrated that large-fluence solar proton events have been observed at Earth more frequently near the recurrent minima of the solar activity cycle in the past than during the present epoch. We show that these results are explicable in terms of the linear dependence of the Alfvén velocity upon the strength of the interplanetary magnetic field, leading to higher shock compression ratios in the past. These results indicate that this aspect of "solar weather" will be significantly influenced by the prevailing strength of the interplanetary magnetic field, and that recurrence of solar conditions similar to those of the solar activity minimum of solar cycles 12–14 (1878.9–1913.6) would be accompanied by a factor of ~4 increase in the occurrence of large-fluence solar proton events.

1. Introduction

In an earlier paper (McCracken et al., 2001a) we have demonstrated that large-fluence solar proton events (SPEs), those with a \( > 30 \text{ MeV} \) omni-directional fluence of \( \geq 10^9 \text{ protons cm}^{-2} \), leave a recognizable impulsive signal in the nitrates sequestered in Arctic and Antarctic ice. The large-fluence SPE-produced nitrates are detectable as thin (<2 month duration) layers superimposed on a large annual variation due to nitrates produced at low latitudes by photoelectric and other processes, and transported to the polar caps via tropospheric circulation. As a consequence, these SPEs can only be recognized through high-resolution measurements of the nitrate concentration in ice cores using ~18 samples per year. McCracken et al. (2001a) have determined the conversion factor that allows the observed nitrate signal to be converted to solar proton fluence, and using this technique identified 125 large-fluence solar proton events that occurred during the period 1561–1950 AD. The threshold for these 125 large-fluence solar proton events was \( 1.0 \times 10^9 \text{ cm}^{-2} \) above 30 MeV (McCracken et al., 2001b).
In the 60 years since the initial recognition that charged particles are accelerated to cosmic ray energies in the vicinity of the Sun (Forbush, 1946), ground-based detectors, ionospheric techniques, and later satellite observations have shown that solar proton events are a relatively common phenomenon (Reames, 1999). However, all of these recent observations have been made during a period of relatively high solar activity. Over the past 500 years, the Sun has reverted to three extended periods of lower solar activity, as evidenced by low sunspot numbers, infrequent geomagnetic storms, and low auroral activity (e.g., Křivský and Pejml, 1988). Further, the analysis of geomagnetic records (Lockwood, Stamper, and Wild, 1999), the inversion of cosmogenic $^{10}\text{Be}$ records (Caballero-Lopez $\textit{et al.}$, 2004), and mathematical models of the evolution of the solar magnetic field (Solanki, Schüssler, and Fligge, 2000; Schrijver, DeRosa, and Title, 2002; Wang, Lean and Sheeley, 2000) have indicated that the interplanetary and coronal magnetic fields may have increased by a factor of two to three since 1700 AD. The acceleration of charged particles is invariably associated with magnetic fields, and it therefore appears possible that the occurrence and character of SPEs may have changed with time as a consequence of this long-term change.

A study of the large-fluence solar proton events (McCracken $\textit{et al.}$, 2001b) derived from the nitrate records indicated that the frequency of occurrence of large-fluence SPEs has varied substantially with time. In particular, it was shown that these large-fluence events have occurred more frequently during the sunspot cycles associated with the extended periods of low solar activity (e.g., the recovery period at the end of the Maunder minimum, 1700–1711) and less frequently at times of high solar activity. This result initially appears to be counter-intuitive. In this paper, we first validate this result, and then demonstrate that there is an inverse dependence between the observed probability of occurrence of SPEs, and the strength of the interplanetary magnetic field as modeled by Solanki, Schüssler and Fligge (2000) and others. From this relationship, we predict and then demonstrate that large-fluence SPEs have occurred near sunspot minimum more frequently in the past than in the modern epoch. We then show that a reduction in the magnitude of the interplanetary magnetic field (IMF) can cause a substantial increase in the Alfvén Mach number of the shocks associated with coronal mass ejections (CMEs), and that this is consistent with the solar proton event observations.

Reames (1999) has reviewed the two mechanisms proposed to explain “gradual” and “impulsive” SPEs. Shea and Smart (1996) and Smart and Shea (2003) have shown that the majority of large-fluence solar proton events at the Earth are associated with solar activity near the center of the solar disk and have concluded that the largest fluence events at Earth are due to “gradual events” in which there is continuous particle acceleration by the shocks associated with fast coronal mass ejections as they transit from the Sun to the Earth. We, therefore, take the view that the majority of the large-fluence solar proton events identified by the impulsive nitrate increases are associated with fast CMEs. Kahler (2001) has shown that the proton intensities in gradual events vary approximately as the fourth power of
the CME velocity, and noted that other factors influence the intensity as well. He has identified the role of a “seed” population and the variation of the proton energy spectra from one event to the next, as having a significant effect. In this paper, we consider another factor that Kahler did not consider, the magnitude of the magnetic field in the shock driven by the CME.

Webb and Howard (1994) have shown that there is a good correlation between sunspot number, and the occurrence rates of CMEs. For a single sunspot cycle, we define the frequency of occurrence of solar proton events, $F_{\text{spe}}$, as the number of SPEs occurring within that cycle. For that sunspot cycle, the Webb and Howard (1994) result indicates that the number of coronal mass ejections ($N_{\text{cme}}$) can be approximated by $K_e \int R \, dt$, where $K_e$ is a constant and $R$ is the sunspot number. We now extrapolate to the probability that a CME will generate a detectable large-fluence solar proton event; this probability is then $P_{\text{spe}} = F_{\text{spe}}/N_{\text{cme}}$. To a first approximation, we estimate that $\int R \, dt$ is proportional to the peak sunspot number $R_{\text{max}}$. From this we have $P_{\text{spe}} = K_e F_{\text{spe}}/R_{\text{max}}$ with $K_e$ a new constant. Using this estimate, we now examine the behavior of $F_{\text{spe}}$ and $P_{\text{spe}}$ throughout the period 1700–1950, for which both the derived large-fluence solar proton event data and sunspot numbers are available.

2. SPE Occurrence, 1700–1985

We have determined the number of large-fluence solar proton events, for each solar cycle in the interval 1700–1985 (i.e., solar sunspot cycles $-4$ through 21). The events prior to 1950 were obtained from the nitrate record, and after that date from ground-level and satellite data. The analysis reported in the first part of this paper is restricted to those solar proton events having a $>30$ MeV fluence $\geq 2 \times 10^9$ cm$^{-2}$ which corresponds to an impulsive nitrate signal five standard deviations above the background. We have excluded events with fluence $<2 \times 10^9$ cm$^{-2}$ since there is a small probability that a few of these events prior to the consolidation of the ice may be of meteorological origin and thereby distort the long term variation in event occurrence.

Figure 1 is a plot of the number of large-fluence ($\geq 2 \times 10^9$ cm$^{-2}$ above 30 MeV) solar proton events against the peak group sunspot number (Hoyt and Schatten, 1998) for each solar cycle. (See later note regarding the solar cycle 1700–1711). Considering the entire population of points in Figure 1, it is clear that there is no significant correlation between these two parameters (correlation coefficient $= -0.120$). The number of large-fluence SPEs observed at Earth for each solar cycle appears to be poorly correlated with the sunspot number.

Examination of Figure 1 shows a subset of events that lie in the middle to upper left-hand corner of the scatter diagram (highlighted in bold symbols) that occurred during three periods of relatively low solar activity – the end of the Maunder minimum (1700–1715), the Dalton minimum (1800–1830), and the period $\sim$78
Figure 1. The number of large-fluence solar proton events derived from the nitrate content of polar ice cores, and ground-level and satellite data, plotted against the peak annual sunspot number acting as a proxy for solar activity. The triangles correspond to the Maunder minimum; the squares the Dalton minimum; and the diamonds the Gleissberg minimum. These indicated periods are those exhibiting the lowest sunspot numbers. The proton events all have a >30 MeV omni-directional solar proton fluence of $\geq 2 \times 10^9$ cm$^{-2}$.

years later (1879–1914) which we will call the Gleissberg minimum. This strongly suggests that even when the solar activity as indicated by the sunspot number is not excessively large, there are times when a CME has a high probability of producing a large-fluence SPE at Earth.

Lockwood, Stamper and Wild (1999) have presented observations that suggest that the magnitude of the interplanetary magnetic field (IMF) has increased by a factor of 2.3 since 1900 AD. The Solanki, Schüssler and Fligge (2002) and Schrijver, DeRosa and Title (2002) models of the open field, and the Caballero-Lopez et al. (2004) inversions of the cosmogenic $^{10}$Be data are in agreement that the interplanetary magnetic field near Earth has increased substantially over the past three centuries. We now investigate the correlation between the magnitude of the IMF and the observation of large-fluence solar proton events at Earth.

We have computed $P_{spe}$ as defined in Section 1 for each sunspot cycle in the interval 1700–1985. The group sunspot number $R_g$ is used herein. The yearly mean values of the Zürich sunspot number, $R_z$, given by McKinnon (1987) usually agree to within a factor of 1.5 (Hoyt and Schatten, 1998); these differences have no significant influence on the conclusions of this paper. There is one exception in the period considered here; the sunspot cycle 1700–1711, for which the peak sunspot numbers differ by a factor of ten ($R_g = 5.5$; $R_z = 58$). In view of this large discrepancy, we have computed $P_{spe}$ for that cycle alone using the geometric mean
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1.0 V

Maunder

0.9 N

Dalton

S08*Gleissberg

0.8

S0.7

0.6

Z 0.5

0.4

0.3

0.2 ,

0.1

0.0

L

I

Lp

Li

10

0 1 2 3 4 5 6 7 8 9 10

IMF Magnitude (nT)

Figure 2. The 22-year running means of the relative probability that a CME will produce an observable large-fluence solar proton event at Earth ($P_{rel}$), plotted against the estimated concurrent 22-year average strength of the interplanetary magnetic field. The triangles correspond to the Maunder minimum; the squares the Dalton minimum; and the diamonds the Gleissberg minimum. These Maunder, Dalton and Gleissberg intervals are those for which the IMF was predicted to be the lowest. The calculation of $P_{rel}$ is detailed in the text.

of $R_x$ and $R_z$, $(R_x \times R_z)^{0.5} \approx 18$, and that value is also used in Figure 1. The general conclusions of this paper are not affected by this choice, as discussed below.

The number of large-fluence solar proton events in a solar cycle (i.e., $F_{spe}$) is a small integer in the range 0–6, and the values of $P_{spe}$, therefore, exhibit substantial fluctuations due to binomial statistics. To reduce this variability, and to emphasize the effects of the longer-term changes in solar activity, we have computed the 22-year running averages of $P_{spe}$. The highest value of $P_{spe}$ corresponds to the end of the Maunder minimum, and we denote it by $P_{MM}$. We then computed $P_{rel} = P_{spe}/P_{MM}$, the 22-year running mean probability expressed as a fraction of $P_{MM}$. In Figure 2, $P_{rel}$ is plotted against the magnitude of the concurrent 22-year running mean of the interplanetary magnetic field derived from the flux estimates of Solanki, Schüssler and Fligge (2002). (The conversion from flux is outlined in the caption of Figure 4). In Figure 2 the events during the episodes of low solar activity are again highlighted with bold symbols.

A clear trend is evident; the highest relative probabilities of occurrence of large-fluence solar proton events are usually associated with the lowest values of the IMF at Earth. High values of the IMF (>6.5 nT) are invariably associated with low values of $P_{rel}$. The data in Figure 2 show that a fast CME that occurs when the IMF is weak, in the range of ~2 nT ($P_{rel} \approx 1$), is approximately 8–10 times more likely to result in a detectable large-fluence SPE at Earth than when the IMF strength is strong, in the range of ~6 nT ($P_{rel} \approx 0.1$). To explore this relationship further, an
empirical fit to the data is given in Figure 2; and shows that \( P_{\text{rel}} \approx 4.8/B^2 \). We will refer to this as \( P_{\text{rel}}(B) \). From the definition of \( P_{\text{rel}} \), then,

\[
P_{\text{spe}} = (4.8) P_{\text{MM}} / B^2.
\]  

(1)

As noted above, the geometric mean of \( R_g \) and \( R_z \) was used to calculate \( P_{\text{spe}} \) for the solar cycle 1700–1711. Using either \( R_g \) or \( R_z \) alone affects the value of \( P_{\text{MM}} \) and the absolute values of \( P_{\text{rel}} \) in Figure 2. Nevertheless \( P_{\text{rel}} \) declines with increasing IMF strength when either \( R_g \) or \( R_z \) are used alone. The most significant consequence of using \( R_g \) and \( R_z \) instead of the geometric mean is in the functional form of \( P_{\text{rel}}(B) \); using \( R_g \) yields a steeper dependence \((-B^{-2.8})\); using \( R_z \) a weaker dependence \((-B^{-1.2})\). The qualitative features of the results obtained in the following sections, and the overall conclusions are unchanged independent of whether \( R_g \), \( R_z \), or the geometric mean is used.

3. SPE Occurrence during a Solar Cycle: a Prediction

Satellite measurements of the lower energy portion of the solar cosmic ray spectrum commenced in the 1960s, as did direct measurements of the interplanetary magnetic field (IMF) near Earth. Figure 3 displays the 6-month mean strength of the IMF since 1965. From this figure it is clear that the values of the IMF at sunspot minimum \( (B_{\text{min}}) \) have been consistently \(~5.3 \text{ nT}\) while the IMF values at sunspot maxima \( (B_{\text{max}}) \) have ranged from 6.5 to 10.2 nT with an average value of \(~8.5 \text{ nT}\). The ratio of \( B_{\text{max}}/B_{\text{min}} \approx 1.6 \). Reference to Figure 2 shows that \( P_{\text{rel}}(B) \) varies between 0.15 and 0.05 over that range. Figure 4 shows the predicted IMF magnitude at 1 AU since

![Figure 3](image-url)  

*Figure 3.* Six-month average values of the magnitude of the interplanetary field near Earth, from 1963 to 2003. Solar cycle 20 commenced in 1964.9, solar cycle 21 commenced in 1976.5, solar cycle 22 commenced in 1986.8, and solar cycle 23 commenced in 1996.8.
1700 AD as derived from the data given by Solanki, Schüssler and Fligge (2002). For the period 1856–1933, $B_{	ext{min}}$ is $\approx 2.5$ nT and $B_{\text{max}}$ is $\approx 7.5$ nT, yielding a ratio of $B_{\text{max}}/B_{\text{min}} = 3.0$. That is, the predicted interplanetary magnetic field exhibited larger variations throughout the solar activity cycle during this earlier period which includes the years of the Gleissberg minimum in solar activity as defined earlier. Reference to Figure 2 shows that $P_{\text{rel}}(B)$ would, therefore, vary between $\sim 1.0$ near sunspot minimum to $\sim 0.10$ near sunspot maximum. Thus, the dependence of $P_{\text{rel}}(B)$ upon $B$ inferred from Figure 2 implies that large-fluence solar proton events may have been more frequent during low solar activity as indicated by the sunspot number than in the present epoch. We now explore the quantitative consequences of this prediction upon the frequency of occurrence of large-fluence SPEs throughout an 11-year solar cycle.

Using the definition $P_{\text{spe}}(B) = P_{\text{spe}}/N_{\text{cme}}$, the frequency of occurrence of solar proton events is given by $F_{\text{spe}} = P_{\text{spe}}(B)N_{\text{cme}}$. Using the Webb and Howard (1994) result that $N_{\text{cme}} \approx kZ$, where $Z$ is the sunspot number, and the definition $P_{\text{rel}} = P_{\text{spe}}/P_{\text{MM}}$, then

$$F_{\text{spe}} = (\text{constant})P_{\text{spe}}(B) \times Z = \text{const.} P_{\text{rel}}(B) \times Z. \quad (2)$$

We consider two epochs:

(1) The period of nine sunspot cycles, 1844–1944 (cycles 9–17), centered on the period of relatively low solar activity as indicated by the sunspot numbers for the period 1879–1913. From Figure 4, and the above discussion, the interplanetary magnetic field at sunspot minimum was as low as $B_{\text{min}} \approx 2.0$ nT, and at sunspot maximum, $B_{\text{max}} \approx 7.5$ nT. Using the empirical fit from Figure 2, this indicates that $P_{\text{rel}}(B)$ varied between $P_{\text{rel}} \approx 1$ at sunspot minimum, and $P_{\text{rel}} \approx 0.10$ at sunspot minimum.
maximum. From Equation (2), we write the predicted frequency of occurrence of large-fluence solar proton events for year $t$ of the sunspot cycle as

$$F_{spe}(t) = \text{const. } P_{\text{rel}}(B) \times Z(t).$$

(3)

For the variation of the sunspot number throughout the solar cycle, $Z(t)$, we have averaged the data from the nine solar cycles, for each year $t$, after solar minimum. For this initial investigation, we have approximated $P_{\text{rel}}(B)$ over the sunspot cycle by $P_{\text{max}} - (P_{\text{max}} - P_{\text{min}}) \sin(\pi t / T)$, where $T$ is the period of the solar cycle, $P_{\text{max}} = 1$, and $P_{\text{min}} = 0.10$ as discussed above. Using Equation (3), we have computed $F_{spe}(t)$; this is plotted as the dotted line in Figure 5(A) after normalization relative to the highest value.

For the nitrate data used here, it is occasionally difficult to determine the year in which a solar proton event occurred. We estimate that there is a probability of 0.2 that the assigned year is in error by ±1 year and a probability of 0.6 that the assigned year is correct. We have applied a 1, 3, 1 running average to the predictions in Figure 5(A) to allow for this uncertainty, the result displayed as the solid curve.

**Figure 5.** The predicted frequency of occurrence of solar proton events derived from Equation (3) as detailed in the text. (A): for the interval 1844–1944, corresponding to relatively low IMF strengths at sunspot minimum (to 2.5 nT). The dotted line is as computed using Equation (3); the solid line is after allowance is made for dating uncertainties discussed in Section 3. (B): for the modern interval, 1954–1996, corresponding to an IMF of ~5 nT at sunspot minimum.
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Comparing the solid and dotted curves, it is clear that both curves exhibit similar general features, as discussed below.

The second epoch is the period of high solar sunspot activity, 1954–1996 (sunspot cycles 19–22). Figure 3 shows that the observed IMF at 1 AU varied between $B_{\text{min}} \approx 5.3$ nT, and $B_{\text{max}} \approx 8.5$ nT between the start of solar cycle 20 in 1964.9 and the end of cycle 22 in 1996.8. The predicted IMF in Figure 4 indicates similar IMF values for sunspot cycle 19 (1954–1965). From Figure 2 we note that these field strengths correspond to $P_{\text{rel}} = 0.15$ at solar minimum and $P_{\text{rel}} = 0.05$ at solar maximum. Using Equation (3), and the procedure outlined above, $F_{\text{spe}}(t)$ has been calculated, and shown in Figure 5(B). Since the solar proton events for this period were all obtained from satellite or ground-based instrumentation, there is no timing ambiguity, and a running average has not been applied to Figure 5(B).

Examining Figure 5(A) and 5(B), note that for the modern epoch, 1954–1996, the predicted large-fluence solar proton event frequency varies by a factor of $\sim 7$ between the years adjacent to sunspot minimum, and sunspot maximum, with a steady monotonic decay of $F_{\text{spe}}(t)$ thereafter. For the epoch 1844–1944, however, the ratio is $\sim 5$, and there is a very pronounced slow decay of $F_{\text{spe}}(t)$ for the last half of the solar cycle. These features are evident in both the solid and dotted curves in Figure 5(A). Thus, the function $P_{\text{rel}}(B)$ derived from Figure 2 has resulted in the prediction that the distribution of large-fluence SPEs throughout the solar cycle will be flatter for the earlier interval centered on the Gleissberg minimum (1879–1913), than for the present-day epoch of high solar activity. Stated differently, Equation (3) and Figure 5(A) imply that there is a higher relative probability that a detectable large-fluence SPE will be observed in the vicinity of sunspot minima for the 1844–1944 epoch, than in the present epoch. This is a direct consequence of the strong variation of $P_{\text{rel}}(B)$ from 1.0 to $\approx 0.10$ between sunspot minimum and sunspot maximum for the earlier epoch.

4. SPE Occurrence during a Solar Cycle: Observations

McCracken et al. (2001a) have interpreted the thin, impulsive layers of nitrate observed in ice cores from the polar caps as the terrestrial record of large-fluence solar proton events that have occurred in the past. On that basis, they identified 125 large-fluence solar proton events in the interval 1561–1950 AD, and estimated the $>30$ MeV omnidirectional fluence of each event. Using those identifications, Figure 6(A) displays the observed frequency of impulsive nitrate events ($>30$ MeV fluence $\geq 10^9$ protons cm$^{-2}$) versus phase of the solar cycle for the interval 1844–1944 considered in the previous section. To improve our statistics, we have now included all events with $>30$ MeV fluences $\geq 10^9$ cm$^{-2}$.

Figure 6(B) provides the equivalent dependence based on satellite and ground-level observations for the period 1954–1996 (Shea and Smart, 1990; Smart and Shea, 2002). Comparing Figure 6(A) and 6(B), it is clear from the distribution for 1844–1944 that impulsive nitrate enhancements (i.e., large-fluence SPEs) occurred
relatively frequently during the 2 years on either side of solar minimum. By way of contrast, the distribution for the present epoch (1954–1996) is concentrated near sunspot maximum. This difference was predicted in the previous section, as a consequence of the inferred variation of $P_{spe}(B)$ from 1.0 to 0.1 in Equation (3).

The relatively low number of large-fluence solar proton events in Figure 6(A) and 6(B) means that statistical fluctuations are substantial, and therefore we now verify the above conclusion in another manner. We define the relative large-fluence solar proton event distribution index as the sum of the number of large-fluence solar proton events observed in the two tenths of the length of the solar cycle, before and after sunspot minimum in Figure 6 divided by the remainder of.

![Graph A](image)

![Graph B](image)

**Figure 6.** The observed frequency of occurrence of large-fluence solar proton events plotted against the phase of the solar activity cycle. (A): for the interval 1844–1944, using the large-fluence solar proton events identified in the nitrate content of the polar ice record. (B): for the modern interval, 1954–1996, using solar proton events identified by satellite and ground-level event data. In both cases, the $>30$ MeV omni-directional fluence for each event was $\geq10^9$ protons cm$^{-2}$.
the large-fluence SPEs in the time averaged plots in Figure 6. The relative distribution indices computed from the data in Figure 6(A) and 6(B) are 0.41 for the period 1844–1944, and 0.14 for the period 1954–1996. These values confirm that large-fluence solar proton events were more frequently observed during periods of low solar activity (as identified by the sunspot number) in the earlier period, than in the modern data.

In the previous section we used Equation 3, and the empirical function $P_{\text{rel}}(B)$ derived from Figure 2, to predict that solar proton events would be observed more frequently near sunspot minimum in the interval 1844–1944, than in the present epoch since 1954. Examination of Figure 6(A) and 6(B), and of the large-fluence SPE distribution indices, confirms this prediction.

In summary, the observed data indicate that the probability of observing large-fluence SPEs was higher (a) during the solar cycles associated with extended periods of low solar activity (e.g., at the end of the Maunder minimum); and (b) around the sunspot minima in the interval 1879–1914, compared to the present epoch. Both observations are consistent with the hypothesis that the probability of occurrence of a large-fluence SPE varies approximately as the inverse square of the strength of the interplanetary magnetic field. In particular, this relationship, together with the observed and predicted interplanetary magnetic field strengths, successfully predicts that the time dependence of the occurrence of large-fluence SPEs throughout the solar cycle was significantly different in the past from that observed in the present epoch. We now discuss the theoretical implications of these results.

5. Discussion

In the present epoch, the strong anisotropies evident in solar cosmic rays, and their time profiles, indicate that they are propagating freely through an interplanetary magnetic field as originally predicted by Parker (1965). Their parallel mean free paths are frequently in the range 0.5–1.0 AU (Dröge, 2000), indicating that irregularities in the Parker spiral field are not impeding their propagation to Earth to any significant degree. We, therefore, conclude that the observed dependence of the probability of occurrence of large-fluence SPEs on the strength of the IMF is unlikely to be due to propagation processes from Sun to Earth.

The majority of the energetic protons in the “gradual events” associated with coronal mass ejections (CMEs) are understood to be accelerated in the shock wave that travels ahead of the CME (Reames, 1999). It has been shown that the spectral index of the energetic particles is a function of the compression ratio in the shock (Jones and Ellison, 1991, and references therein), which is, in turn, determined by the Alfvén Mach number. For a CME speed of $V$, and an interplanetary magnetic induction $B$, the Alfvén Mach number is given by $M_A = [V/B] \{4\pi n_o M\}^{-0.5}$, where $n_o$ is the plasma number density, and $M$ is the mean mass number of the plasma particles. Thus, equal increases in Alfvén Mach number, $\Delta M$, and identical changes
in the particle spectrum will be yielded by either an increase in CME speed, ΔV, or a decrease in magnetic induction, ΔB, if ΔV/V = ΔB/B.

Kahler (2001) and Reames (1999) have shown that the intensity of protons in the energy range 2 < E < 60 MeV in “gradual” solar energetic particle events varies approximately as the fourth power of the speed of the associated CME; \( I_1 \approx I_0 (V_1/V_0)^4 \). For constant B, this is equivalent to \( I_1 \approx I_0 (M_{A1}/M_{A0})^4 \). In the case where both B and V may change, and the plasma number density \( n_0 \) is assumed to remain constant, then \( M_{A1}/M_{A0} = V_1 B_0/V_0 B_1 \), and therefore \( I_1 \approx I_0 (V_1 B_0/V_0 B_1)^4 \). That is, for a given population of CME speeds, and varying B, the equivalence in the preceding paragraph indicates that the proton intensity would vary as the inverse fourth power of B. Lockwood (2001) and Cabalerro-Lopez et al. (2004) have suggested that the IMF at the end of the Maunder minimum was a factor of 2 to 3 times weaker than that of the present epoch, indicating that (all other factors being unchanged), the number of large-fluence solar proton events at the end of the Maunder minimum would have been substantially greater than in the present epoch for an equivalent CME speed.

The time periods having higher relative probabilities of the occurrence of large-fluence solar proton events, as shown in Figure 2, correspond to periods of relatively low solar activity. It is plausible that the CME velocities at such times were systematically lower than during the present epoch of high solar activity. Consequently the solar proton event fluence would be determined by the competing effects of an increase due to the lower IMF strength and a decrease due to the lower CME velocity.

The data shown in Figure 2 indicate there was a greater probability of large-fluence solar proton events (≥2 × 10^9 cm\(^{-2}\)) during the recovery from the Maunder minimum and during the Dalton and Gleissberg minima than during periods when the IMF was considerably higher (which includes the spacecraft era). This indicates that the effect of reduction in the strength of the IMF was the dominant factor for these periods of minimum solar activity; however, the total fluence may have been partially compensated by a reduction in the mean CME velocity during those time periods. The relationship between the observation of large solar proton fluence events for these two time periods will depend also upon the plasma number density, \( n_0 \), and the nature of the population of CME speeds. The mathematical treatment of these effects is beyond the scope of this paper.

As shown in Figure 6, a striking feature of the results presented in this paper is the apparent conflict between the relatively frequent occurrence of large-fluence solar proton events during the solar cycles associated with extended periods of low solar activity (e.g., 1844–1944), and the fact that large-fluence SPEs were infrequent near sunspot minimum in the modern epoch. We have proposed that this is explicable in terms of the 11-year average IMF being significantly weaker during the sunspot cycles associated with the Maunder, and other “Grand Minima”, compared to the present epoch. As an example, consider the sunspot cycle 1700–1711. The mean sunspot number was approximately a tenth of the sunspot numbers
in the present epoch. In fact the mean sunspot number between 1700–1711 is similar

to the sunspot numbers at solar minimum in the present epoch, the average of the

annual group sunspot numbers for the minima of 1965, 1976, and 1986 being

11.9. The frequency of occurrence of CMEs near the peak of the sunspot cycle

1700–1711 may therefore have been similar to that during modern sunspot minima

(extrapolating from the results of Webb and Howard, 1994).

The results in this paper suggest that the strength of coronal and interplanetary

magnetic fields in the region traversed by a CME driven shock wave will play an

important role in determining the intensity of solar protons that are subsequently

observed at Earth. The satellite observations that the IMF varies by a factor of

two throughout the solar cycle also suggest that the SPE intensities may exhibit
detectable correlations with the IMF throughout the solar cycle.

6. Conclusions

Using the large-fluence solar proton events identified in ice cores corresponding

to the interval 1561-1950 (McCracken et al., 2001a), it has been shown that the

probability that solar activity will result in a large-fluence solar proton event at Earth

exhibits a significant association with the strength of the interplanetary magnetic

field (IMF). Comparing the observed occurrence of SPEs with the strength of the

IMF, the probability exhibits an approximately \( B^{-2} \) dependence on \( B \) over the


Using that empirical relationship, we thus predicted that the occurrence of large-

fluence SPEs throughout the solar cycle would be significantly different for the 9

solar cycles (1844–1944) centered on the Gleissberg minimum, compared to the

distribution observed in the “instrumental era” since 1954. This empirical relation-

ship and Equation (3) predict that large-fluence SPEs will have occurred more

frequently during periods of relatively low solar activity (as defined by the sunspot

number) in the interval 1844–1944, compared to the modern era. This predicted dif-

ference is a direct consequence of the lower values of the IMF (~2.5 nT) predicted

for a number of the sunspot minima in the interval 1844–1944, compared to the higher

values at the sunspot minima in the modern era (~5nT). The large-fluence SPE data, based upon both the ice core data and modern satellite and ground-level

observations, confirm this prediction.

It has been shown that the observed dependence of the occurrence of large-

fluence SPEs upon the strength of the IMF is explicable in terms of the accepted

model for the acceleration of charged particles in the solar corona and interplan-

etary space. Thus, the Alfvén Mach number, which varies as the reciprocal of

the strength of the interplanetary magnetic field, is higher for weaker values of

the IMF. As a consequence, the compression ratios will be higher in shocks asso-

ciated with a weak IMF leading to the probability of occurrence of a large-

fluence SPE varying as \( \sim B^{-4} \) for no change in the speed of the interplanetary

shock. The observed \( B^{-2} \) dependence suggests that the mean shock speed near


the end of the Maunder minimum (1700–1715) was less than that in the present epoch.

In summary, we conclude that the characteristics of solar proton events observed at Earth depend quite strongly upon the prevailing strength of the IMF. This has significant implications regarding "space weather", as will be discussed elsewhere. Briefly, the results herein indicate that the occurrence of solar activity similar to that during the Gleissberg minimum between 1879 and 1913 (peak sunspot numbers ~75; minimum values of the IMF ~2.5–3.5 nT) would result in a factor of ≥4 in the probability of occurrence of large-fluence solar proton events detectable near Earth.

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References


