CHAMBER CARBON DIOXIDE AND VENTILATION

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Title: Chamber Carbon Dioxide and Ventilation

Abstract:
Maintenance of atmospheric carbon dioxide (CO₂) concentrations within physiologically acceptable limits is critical to the safe operation of hyperbaric chambers and manned diving systems. Quantitative understanding of CO₂ accumulation in such systems is required to specify minimum ventilation rates and CO₂ scrubbing capacities. Quantitative expressions are derived for computing the maximum carbon dioxide partial pressure (P₀₂) attained in a well-stirred chamber at constant temperature and pressure during continuous and intermittent ventilation, and for estimating the time course of CO₂ accumulation in such a chamber where an internal scrubber continuously removes CO₂.
GLOSSARY

\( \Delta_{CO_2} \) chamber \( CO_2 \) partial pressure difference between values at the beginning and end of a no-ventilation – ventilation cycle

\( \Delta_{CO_2,i} \) chamber \( CO_2 \) partial pressure difference between value at the beginning of a series of no-ventilation – ventilation cycles and the value at the end of cycle \( i \)

\( \Delta P_{CO_2}(t,v)_i \) change in chamber \( CO_2 \) partial pressure during ventilation in no-ventilation – ventilation cycle \( i \)

\( F \) \( CO_2 \) scrubber efficiency factor

\( i \) cycle number in a series of no-ventilation – ventilation cycles

\( k \) ventilation time constant with chamber (or scrubber, depending on context) volumetric ventilation rate at standard temperature and pressure (STP)

\( k_A \) ventilation time constant with chamber (or scrubber, depending on context) volumetric ventilation rate at chamber temperature and pressure

\( n_{CO_2} \) chamber \( CO_2 \) content, moles

\( n_g \) number of moles of gas species \( g \)

\( n_T \) chamber gas content, moles

\( \dot{n}_{CO_2} \) molar rate of change of chamber \( CO_2 \) content

\( \dot{n}_{CO_2, in} \) molar rate of \( CO_2 \) input to chamber via input ventilation

\( \dot{n}_{CO_2, met} \) molar rate of \( CO_2 \) input to chamber via metabolism

\( \dot{n}_{CO_2, scrub} \) molar rate of \( CO_2 \) removal from chamber via scrubbing

\( \dot{n}_{CO_2, out} \) molar rate of \( CO_2 \) removal from chamber via exhaust ventilation

\( \dot{n}_{g, met} \) molar production or consumption rate of gas species \( g \) by metabolism
\(\dot{n}_{g,\text{scrub}}\) molar consumption rate of gas species \(g\) by scrubbing

\(\dot{n}_{O_2,\text{met}}\) molar rate of metabolic \(O_2\) consumption

\(\dot{n}_{T,\text{in}}\) total molar rate of chamber input ventilation

\(\dot{n}_{T,\text{out}}\) total molar rate of chamber exhaust ventilation

\(P_{ch}\) chamber pressure

\(P_{CO_2}\) chamber \(CO_2\) partial pressure

\(P_{CO_2}^o\) initial chamber \(CO_2\) partial pressure

\(P_{CO_2,\text{vin}}\) \(CO_2\) partial pressure in input ventilation gas

\(P_{CO_2}(t_{\text{mv}})_i\) chamber \(CO_2\) partial pressure at end of no-ventilation period in chamber no-ventilation – ventilation cycle \(i\)

\(P_{CO_2}(t_{\text{v}})_i\) chamber \(CO_2\) partial pressure at end of ventilation period in chamber no-ventilation – ventilation cycle \(i\)

\(P_{CO_2}(\infty)\) steady-state \(CO_2\) partial pressure in a continuously ventilated chamber

\(P_{ex}\) pressure of exhaust ventilation gas

\(P_{in}\) pressure of input ventilation gas

\(P_S\) standard pressure

\(Q\) \((1-q)\); remaining fraction of total response

\(q\) fraction of total response

\(R\) gas constant

\(RQ\) respiratory quotient; ratio of \(CO_2\) produced to \(O_2\) consumed

\(T_{ch}\) chamber temperature

\(T_S\) standard temperature
\( t \)  
  time

\( t_\infty \)  
  infinite time

\( t_{v2} \)  
  half-time

\( t_{nv} \)  
  chamber no-ventilation time

\( (t_{nv})_i \)  
  no-ventilation time in chamber no-ventilation – ventilation cycle \( i \)

\( t_q \)  
  time for \( q \) response

\( t_v \)  
  chamber ventilation time

\( t_{v,1/2} \)  
  chamber ventilation half-time

\( (t_v)_i \)  
  ventilation time in chamber no-ventilation – ventilation cycle \( i \)

\( V_{ch} \)  
  chamber volume

\( \dot{V}_{CO_2, \text{in}} \)  
  rate of \( CO_2 \) input via ventilation at chamber temperature and pressure

\( \dot{V}_{CO_2, \text{in,STP}} \)  
  rate of \( CO_2 \) input via ventilation at standard temperature and pressure

\( \dot{V}_{CO_2, \text{scrub,STP}} \)  
  rate of \( CO_2 \) uptake via scrubbing at standard temperature and pressure

\( \dot{V}_{\text{vin}} \)  
  overall input ventilation rate at chamber temperature and pressure

\( \dot{V}_{\text{vin,STP}} \)  
  overall input ventilation rate at standard temperature and pressure

\( \dot{V}_{\text{vin,scr}} \)  
  overall \( CO_2 \) scrubber input ventilation rate at chamber temperature and pressure

\( \dot{V}_{CO_2, \text{met}} \)  
  metabolic \( CO_2 \) production rate at chamber temperature and pressure

\( \dot{V}_{CO_2, \text{met,STP}} \)  
  metabolic \( CO_2 \) production rate at standard temperature and pressure

\( \dot{V}_{CO_2, \text{out}} \)  
  \( CO_2 \) exhaust rate via ventilation at chamber temperature and pressure

\( \dot{V}_{CO_2, \text{out,STP}} \)  
  \( CO_2 \) exhaust rate via ventilation at standard temperature and pressure
<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$\dot{V}_{vex}$</td>
<td>overall exhaust rate at chamber temperature and pressure</td>
</tr>
<tr>
<td>$\dot{V}_{vex,STP}$</td>
<td>overall exhaust rate at standard temperature and pressure</td>
</tr>
<tr>
<td>$\dot{V}_{O_2,met}$</td>
<td>metabolic O$_2$ consumption rate at chamber temperature and pressure</td>
</tr>
<tr>
<td>$\dot{V}_{O_2,met,STP}$</td>
<td>metabolic O$_2$ consumption rate at standard temperature and pressure</td>
</tr>
<tr>
<td>$X_{CO_2}$</td>
<td>CO$_2$ mole fraction</td>
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1. INTRODUCTION

Maintenance of atmospheric carbon dioxide (CO₂) concentrations within physiologically acceptable limits is critical to the safe operation of hyperbaric chambers and manned diving systems. Quantitative understanding of CO₂ accumulation in such systems is required to specify minimum ventilation rates and CO₂ scrubbing capacities, but existing theoretical treatments that we could find in the literature are few and incomplete. For example, Nuckols et al.¹ present an equation giving the time course of CO₂ partial pressure \( P_{\text{CO}_2} \) change in a continuously ventilated chamber, but the equation is valid only if the net respiratory quotient (RQ) of all chamber occupants is unity and the initial \( P_{\text{CO}_2} \) is zero. The latter constraint precludes consideration of the maximum chamber \( P_{\text{CO}_2} \) attained during realistic intermittent ventilation scenarios. In this report we present a more general solution to the problem with expressions for computing the maximum chamber \( P_{\text{CO}_2} \) attained during intermittent ventilation, and for estimating the time course of CO₂ accumulation in chambers with an internal scrubber that continuously removes CO₂.

2. THEORY

2.1. Basic Equations

If we assume that all gas in the problem is ideal, the total chamber gas content in moles is

\[
n_T = \frac{P_{\text{ch}} V_{\text{ch}}}{RT_{\text{ch}}} ,
\]

where \( P_{\text{ch}} \) is chamber pressure, \( V_{\text{ch}} \) is chamber volume, \( T_{\text{ch}} \) is chamber temperature, and \( R \) is the gas constant. The chamber CO₂ content is

\[
n_{\text{CO}_2} = X_{\text{CO}_2} n_T = \frac{X_{\text{CO}_2} P_{\text{ch}} V_{\text{ch}}}{RT_{\text{ch}}} ,
\]

where \( X_{\text{CO}_2} \) is the CO₂ mole fraction in the chamber. The CO₂ partial volume \( V_{\text{CO}_2} \) and partial pressure \( P_{\text{CO}_2} \) in the chamber are then obtained from Dalton’s Law —

\[
V_{\text{CO}_2} = X_{\text{CO}_2} V_{\text{ch}}
\]

(3.a)

and

\[
P_{\text{CO}_2} = X_{\text{CO}_2} P_{\text{ch}}
\]

(3.b)

— and are substituted into Eq. (2) to yield, respectively:
\[ n_{CO_2} = \frac{P_{ch}V_{CO_2}}{RT_{ch}} \]  
(4.a)

and

\[ n_{CO_2} = \frac{V_{ch}P_{CO_2}}{RT_{ch}} \]  
(4.b)

We express \( V_{CO_2} \) at standard temperature and pressure (STP), \( T_s \) and \( P_s \), respectively:

\[ V_{CO_2,STP} = \frac{P_{ch}T_s}{P_sT_{ch}}V_{CO_2}. \]  
(5)

We then substitute Eq. (5) into Eq. (4.a) to yield

\[ n_{CO_2} = \frac{P_sV_{CO_2,STP}}{RT_s}. \]  
(4.c)

The rate of change of chamber \( CO_2 \) content is obtained by differentiating Eq. (4.b) at constant chamber temperature and volume to obtain

\[ \left( \frac{\partial n_{CO_2}}{\partial t} \right)_{RT_{ch}} = \frac{V_{ch}V_{CO_2}}{RT_{ch}} = \frac{V_{ch}\dot{V}_{CO_2}}{RT_{ch}}, \]  
(6.a)

or by differentiating Eq. (4.c) to obtain

\[ \left( \frac{dn_{CO_2}}{dt} \right) = \frac{P_s}{RT_s} \left( \frac{dV_{CO_2,STP}}{dt} \right) = \frac{P_s\dot{V}_{CO_2,STP}}{RT_s}. \]  
(6.b)

Finally, mass balance requires that the change in chamber \( CO_2 \) content be equal to the sum of changes from the chamber input and exhaust ventilation, the amount of metabolic \( CO_2 \) produced by the occupants, and the amount of \( CO_2 \) scrubbed by chemical means in the chamber:

\[ \dot{n}_{CO_2} = \dot{n}_{CO_2,in} + \dot{n}_{CO_2,met} - \dot{n}_{CO_2,scrub} - \dot{n}_{CO_2,out}, \]  
(7)

where \( \dot{n}_{CO_2} = \left( \frac{dn_{CO_2}}{dt} \right) \). Using Eq. (6.b), Eq. (7) becomes

\[ \dot{V}_{CO_2,STP} = \dot{V}_{CO_2,in,STP} + \dot{V}_{CO_2,met,STP} - \dot{V}_{CO_2,scrub,STP} - \dot{V}_{CO_2,out,STP}, \]  
(8)

where \( \dot{V}_{CO_2,met,STP} \) is the standard rate of metabolic \( CO_2 \) production by the chamber occupants. All quantities on the right of Eq. (8), except \( \dot{V}_{CO_2,scrub,STP} \) when \( \dot{V}_{CO_2,scrub,STP} > 0 \), are assumed to be
constant under each set of conditions considered in this analysis. The assumption of constant $\dot{V}_{CO_2,met,STP}$ requires a sufficiently high chamber $P_O_2$ to sustain the assumed $\dot{V}_{CO_2,met,STP}$.

2.2. CO$_2$ Accumulation in a Nonventilated Constant Temperature Chamber

With no CO$_2$ flow into the chamber, no gas exhausted from the chamber, and no CO$_2$ scrubbing, the standard partial CO$_2$ volume in the chamber changes at a rate equal to $\dot{V}_{CO_2,met,STP}$:

\[ \dot{V}_{CO_2,STP} = \dot{V}_{CO_2,met,STP} . \]  \hspace{1cm} (9)

Equation (9) is substituted into Eq. (6.b) to obtain

\[ \left( \frac{d n_{CO_2}}{dt} \right) = \frac{P_S \dot{V}_{CO_2,met,STP}}{RT} . \]  \hspace{1cm} (10)

Equations (10) and (6.a) are then combined to obtain

\[ \frac{V_{ch}}{RT_{ch}} \left( \frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} = \frac{P_S \dot{V}_{CO_2,met,STP}}{RT} , \]  \hspace{1cm} (11)

which is rearranged to yield

\[ \left( \frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} = \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} . \]  \hspace{1cm} (12)

The $P_{CO_2}$ at time $t$ in a no-ventilation period between $t = 0$ and $t = t_n$ is given by integrating Eq. (12):

\[ P_{CO_2}(t) = \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} \cdot t + P_{CO_2}^0 , \]  \hspace{1cm} (13)

where $P_{CO_2}(t)$ is the $P_{CO_2}$ at $0 \leq t \leq t_n$, and $P_{CO_2}^0$ is the initial $P_{CO_2}$ at $t = 0$.

2.3. CO$_2$ Accumulation/Washout in a Continuously Ventilated Constant Temperature Chamber

The rate of change in CO$_2$ content of a continuously ventilated constant temperature chamber with no scrubbing is obtained in terms of the CO$_2$ partial pressure from Eqs. (6.a) and (6.b), and the mass balance equation, Eq. (8), with $\dot{V}_{CO_2,scrub,STP} = 0$:
\[
\left( \frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} = \frac{T_{ch} P_S}{T_S V_{ch}} \left( \dot{V}_{CO_2, \text{met,STP}} + \dot{V}_{CO_2, \text{in,STP}} - \dot{V}_{CO_2, \text{out,STP}} \right). \tag{14}
\]

Under the assumption that the chamber is well stirred, we then have that
\[
\dot{V}_{CO_2, \text{out,STP}} = X_{CO_2} \dot{V}_{\text{ex,STP}} = \frac{P_{CO_2}}{P_{ch}} \dot{V}_{\text{ex,STP}}, \tag{15}
\]
where \( \dot{V}_{\text{ex,STP}} \) is the ventilation exhaust rate at STP, and Eq. (3.b) has been used to express the mole fraction of chamber \( CO_2 \) in terms of the chamber \( P_{CO_2} \) and total pressure. Substitution of Eq. (15) into Eq. (14) yields
\[
\left( \frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} = \frac{T_{ch} P_S}{T_S V_{ch}} \left( \dot{V}_{CO_2, \text{met,STP}} + \dot{V}_{CO_2, \text{in,STP}} - \frac{P_{CO_2}}{P_{A}} \dot{V}_{\text{ex,STP}} \right), \tag{16}
\]
which is rearranged to obtain
\[
\left( \frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} + \left( \frac{T_{ch} P_S \dot{V}_{\text{ex,STP}}}{T_S P_{ch} V_{ch}} \right) P_{CO_2} = \frac{T_{ch} P_S}{T_S V_{ch}} \left( \dot{V}_{CO_2, \text{met,STP}} + \dot{V}_{CO_2, \text{in,STP}} \right) \tag{17}
\]
Note that Eq. (17) reduces to Eq. (12) when \( \dot{V}_{\text{ex,STP}} = \dot{V}_{CO_2, \text{in,STP}} = 0 \); the conditions assumed to derive Eq. (12) in Section 2.2. However, with constant \( P_{ch} (> 0), \dot{V}_{\text{ex,STP}} (> 0), \) and \( \dot{V}_{CO_2, \text{in,STP}} (\geq 0) \), Eq. (17) is a linear first-order differential equation of form
\[
y' + ky = c \tag{18}
\]
with solution (see Section 3.1)
\[
y = \frac{c(1 - e^{-kt})}{k} + C \cdot e^{-kt}, \tag{19}
\]
where
\[
y = P_{CO_2}(t), \tag{20.a}
\]
\[
k = \frac{T_{ch} P_S \dot{V}_{\text{ex,STP}}}{T_S P_{ch} V_{ch}}, \tag{20.b}
\]
\[
c = \frac{T_{ch} P_S}{T_S V_{ch}} \left( \dot{V}_{CO_2, \text{met,STP}} + \dot{V}_{CO_2, \text{in,STP}} \right), \tag{20.c}
\]
and $C$ is a constant determined from initial conditions for the problem. Cancellation of terms after appropriate substitutions into Eq. (19) yields:

$$P_{CO_2}(t) = \frac{P_{ch} \left( \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \right)}{V_{vex,STP}} \left[ 1 - e^{-kt} \right] + Ce^{-kt}. \quad (21)$$

Solution of Eq. (21) at $t = 0$ shows that $C = P_{CO_2}^o$, the partial pressure of CO$_2$ in the chamber at time 0, so that our final result is

$$P_{CO_2}(t) = \frac{P_{ch} \left( \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \right)}{V_{vex,STP}} \left[ 1 - e^{-kt} \right] + P_{CO_2}^o e^{-kt}, \quad \text{with } k = \frac{T_{ch} P_{vex,STP}}{T_S P_{ch} V_{ch}}; \quad (22.a)$$

and

$$P_{CO_2}(t) = \frac{P_{ch} T_{ch} \left( \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \right)}{T_S V_{vex}} \left[ 1 - e^{-k_d t} \right] + P_{CO_2}^o e^{-k_d t}, \quad \text{with } k_d = \frac{\dot{V}_{vex}}{V_{ch}}; \quad (22.b)$$

where Eq. (22.b) applies when the ventilation rate is expressed as the volume flux at actual chamber temperature and pressure, $T_{ch}$ and $P_{ch}$.

Note that $V_{ch}$ is a constant with units of volume but without reference to any temperature or pressure, while the dependence of $P_{CO_2}(t)$ on $P_{ch}$ evident in Eq. (22.a) is obscured in the units ratio, $\frac{V_{STP}}{V} = \frac{P_{ch} T_{ch}}{P_{vex,STP}}$, in Eq. (22.b). In addition, maintenance of the constant $P_{ch}$ required by either Eq. (22.a) or Eq. (22.b) requires a minimum constant gas bleed of $\dot{V}_{vin} = \dot{V}_{O_2,met}(1 - RQ)$ into the chamber, where $RQ$ is the respiratory quotient (see Section 3.3). This gas bleed is insufficient to sustain any specified $\dot{V}_{CO_2,met}$ indefinitely. The minimum input ventilation of 100% O$_2$ required for this purpose, $\dot{V}_{vin,STP} = \dot{V}_{O_2,met,STP} = \dot{V}_{CO_2,met,STP} / RQ$, then requires $\dot{V}_{vex,STP} > 0$ to keep $P_{ch}$ constant (see Section 3.3). The requirement for $\dot{V}_{vex,STP} > 0$ also becomes clear when we next consider the steady state $P_{CO_2}$ approached at the other boundary condition on Eqs. (22.a) and (22.b); namely, that at $t = \infty$. Finally, if $RQ = 1$, maintenance of constant chamber pressure requires $\dot{V}_{vex,STP} = \dot{V}_{vin,STP}$ (see Section 3.3), from which it follows that $\dot{V}_{vex} = \dot{V}_{vin}$. If we also set $P_{CO_2}^o = 0$ and express the metabolic CO$_2$ production rate at chamber temperature and pressure, Eq. (22.b) becomes identical to that given by Nuckols, et al.: \(^1\)

$$P_{CO_2}(t) = \left[ \frac{P_{ch} \dot{V}_{CO_2,met}}{\dot{V}_{vin}} + P_{CO_2,in} \right] \left[ 1 - e^{-k_d t} \right], \quad \text{with } RQ = 1 \text{ and } P_{CO_2}^o = 0; \quad (23)$$

where $P_{CO_2,in}$ is the CO$_2$ partial pressure in the input ventilation gas.
Steady-State $P_{CO_2}$

As $t \to \infty$, the $P_{CO_2}$ in a continuously ventilated chamber approaches a steady-state value given by the solution of Eq. (22.a) or (22.b) at $t = \infty$:

$$P_{CO_2}(\infty) = \frac{P_{ch} \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}}{\dot{V}_{vex,STP}} \quad (24.a)$$

$$= \frac{P_{ch} T_{ch} \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}}{T_S \dot{V}_{vex}} \quad (24.b)$$

Because the chamber $P_{CO_2}$ can never exceed the prevailing chamber pressure, the ratio $rac{\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}}{\dot{V}_{vex,STP}}$ can never exceed unity, and $\dot{V}_{vex,STP} \geq \frac{\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}}{\dot{V}_{vex}}$. At the smallest value of $\dot{V}_{vex,STP}$ for which Eq. (24.a) remains applicable, or the smallest value of $\dot{V}_{vex}$ for which Eq. (24.b) remains applicable, the steady-state chamber $P_{CO_2}$ is simply the chamber pressure, $P_{ch}$. The latter result is a consequence of the well-stirred assumption, which applies to the metabolic processes in the chamber as well as to the chamber gases. At minimum $\dot{V}_{vex}$, the accompanying minimum O$_2$ influx, $\dot{V}_{vex}(100\% \text{O}_2) = \dot{V}_{O_2,met}$ (see Section 3.3) is instantaneously mixed and metabolized to CO$_2$ so that no O$_2$ remains.

**Ventilation Half-Time**

The ventilation half-time $t_{v,1/2}$ for approach to the steady state given by Eq. (24.a) or Eq. (24.b) is (see Section 3.2)

$$t_{v,1/2} = \frac{\ln(2)}{k} = \frac{0.693 \cdot T_s P_{ch} V_{ch}}{T_{ch} P_{vex,STP}} \quad (25.a)$$

$$= \frac{\ln(2)}{k_A} = \frac{0.693 \cdot V_{ch}}{\dot{V}_{vex}} \quad (25.b)$$

The ventilation half-time increases with chamber volume and decreases with increasing ventilation rate at the prevailing chamber pressure and temperature.
2.4. CO₂ Accumulation/Washout in an Intermittently Ventilated Constant Temperature Chamber

The $P_{CO₂}$ at the end of the no-ventilation period between $t = 0$ and $t = t_n$ in a no-ventilation – ventilation cycle is given by Eq. (13), with $P_{CO₂}^o$ equal to the chamber $P_{CO₂}$ at the beginning of the cycle:

$$P_{CO₂}(t_n) = \frac{T_{ch}P_S\dot{V}_{CO₂,met,STP}}{T_SV_{ch}}t_n + P_{CO₂}^o.$$  (26)

The $P_{CO₂}$ at the end of the subsequent ventilation period of duration $t_v$ is given by Eq. (22.a), with $P_{CO₂}$ at the beginning of the ventilation period given by Eq. (26):

$$P_{CO₂}(t_v) = \frac{P_{ch}\left[\dot{V}_{CO₂,met,STP} + \dot{V}_{CO₂,in,STP}\right]}{\dot{V}_{ex,STP}}\left[1 - e^{-kt_v}\right] + \left(\frac{T_{ch}P_S\dot{V}_{CO₂,met,STP}}{T_SV_{ch}}t_n + P_{CO₂}^o\right)e^{-kt_v}.$$  (27)

The difference between the end-cycle $P_{CO₂}$ at $t = t_n + t_v$ and the initial $P_{CO₂}$ at $t = 0$ is

$$\Delta CO₂ = P_{CO₂}(t_v) - P_{CO₂}^o.$$  (28)

Equation (27) is substituted into Eq. (28) to yield

$$\Delta CO₂ = \frac{P_{ch}\left[\dot{V}_{CO₂,met,STP} + \dot{V}_{CO₂,in,STP}\right]}{\dot{V}_{ex,STP}}\left[1 - e^{-kt_v}\right] + \left(\frac{T_{ch}P_S\dot{V}_{CO₂,met,STP}}{T_SV_{ch}}t_n + P_{CO₂}^o\right)e^{-kt_v} - P_{CO₂}^o,$$  (29)

which is rearranged to obtain

$$\Delta CO₂ = \left[\frac{P_{ch}\left[\dot{V}_{CO₂,met,STP} + \dot{V}_{CO₂,in,STP}\right]}{\dot{V}_{ex,STP}} - P_{CO₂}^o\right]\left[-e^{-kt_v}\right] + \left(\frac{T_{ch}P_S\dot{V}_{CO₂,met,STP}}{T_SV_{ch}}t_n\right)e^{-kt_v}.$$  (30)

2.4.1. "Large $kt_v$" No-Ventilation – Ventilation Steady State

If $kt_v$ is sufficiently large to assume attainment of end-ventilation steady state, then $e^{-kt_v} \to 0$ and Eq. (30) is reduced to

$$\Delta CO₂ = \frac{P_{ch}\left[\dot{V}_{CO₂,met,STP} + \dot{V}_{CO₂,in,STP}\right]}{\dot{V}_{ex,STP}} - P_{CO₂}^o.$$  (31.a)

With substitution of Eq. (24.a), this becomes
\[ \Delta_{CO_2} = P_{CO_2}^{(\infty)} - P_{CO_2}^o. \]  

(31.b)

Note that \( \Delta_{CO_2} \) is independent of \( P_{CO_2} \) at the end of the no-ventilation period. The latter \( P_{CO_2} \), given by Eq. (26), will be the maximum chamber \( P_{CO_2} \) in the no-ventilation – ventilation cycle only with sufficiently high \( \dot{V}_{ex} \) during the ventilation period. If ventilation is just sufficient to return chamber \( P_{CO_2} \) to its initial value, then \( \Delta_{CO_2} = 0 \) and successive no-ventilation – ventilation cycles are in a steady state, for which Eq. (31.a) and (31.b) are combined to obtain

\[ P_{CO_2}^o = P_{CO_2}^{(\infty)} = \frac{P_{ch} \left[ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \right]}{\dot{V}_{ex,STP}}. \]  

(32)

Moreover, if the chamber \( P_{CO_2} \) is not returned to its initial value at the end of the first cycle \( (\Delta_{CO_2} \neq 0) \), a steady state in which the right side of Eq. (32) holds is still achieved in subsequent cycles because they all begin and end with chamber \( P_{CO_2} \) equal to \( P_{CO_2}^{(\infty)} \). The large \( k_t \) steady state achieved in either case has minimum chamber \( P_{CO_2} \) equal to \( P_{CO_2}^{(\infty)} \) which, like \( \Delta_{CO_2} \), is independent of the \( P_{CO_2} \) at the end of the no-ventilation periods. Thus, \( t_v \) and \( t_{mv} \) are specified for such steady-state cycles at given chamber pressure after specification of desired \( P_{CO_2} \) maxima and minima in the cycles, and with given values of \( \dot{V}_{CO_2,met,STP} \) and ventilation gas composition.

Eqs. (25.a) and (32) are used to specify \( t_v \) in terms of a number of ventilation half-times sufficient to meet the end-ventilation steady-state assumption and the ventilation rate required to maintain the specified \( P_{CO_2}^o \). Equation (26.a) or Eq. (26.b) is then used to specify \( t_{mv} \) to limit the maximum \( P_{CO_2} \).

It follows from Eq. (22.b) that chamber volume must be replaced many times during ventilation in order to assume attainment of end-ventilation steady state. For example, attainment of \( e^{-k_t} \leq 0.001 \) requires \( \dot{V}_{ex,t_v} \geq 6.9 \cdot V_{ch} \). This requirement is met only with relatively high ventilation rates and times. Specifically, the actual per minute ventilation rate must exceed 70% of the chamber volume for the required ventilation time to be 10 minutes or less. If the actual per minute ventilation rate is only 10% of the chamber volume, the ventilation time must exceed 70 minutes to attain \( e^{-k_t} < 0.001 \).

2.4.2. “Small \( k_t \),” No-Ventilation – Ventilation Steady State

If ventilation during the ventilation periods is insufficient to approximate the end-ventilation steady-state assumption, each no-ventilation – ventilation cycle \( i \) in a series of \( n \) total cycles ends with a \( \Delta_{CO_2,i} > 0 \). However, the cycle-to-cycle accumulation of chamber CO2 is progressively attenuated, because the end-ventilation steady state approached at the end of each cycle is independent of chamber \( P_{CO_2} \) [see Eqs. (24.a) and (24.b)].

The change in chamber \( P_{CO_2} \) during the \( i^{th} \) ventilation period is
\[ \Delta P_{CO_2}(t_v)_i = q \cdot [P_{CO_2}(\infty) - P_{CO_2}(t_{mv})_i], \]  
(33)

where \( P_{CO_2}(\infty) \) is given by Eq. (24.a) or Eq. (24.b), and \( q \) is the fraction of maximum possible washout response, \( P_{CO_2}(t_{mv})_i - P_{CO_2}(\infty) \), actually achieved during the ventilation period given by

\[ q = 1 - 0.5^{(t_v/t_{v,1/2})}, \]  
(34)

where \( t_{v,1/2} \) is the ventilation half-time given by Eq. (25.a) or Eq. (25.b) [see Table 1]. Note that \( \Delta P_{CO_2}(t_v)_i < 0 \) when the ventilation rate and time are adequate to reduce chamber \( P_{CO_2} \) during the ventilation period. Also, \( q \) in this context has an absolute lower limit, because it depends on \( \dot{V}_{\text{ex}} \) [see Eqs. (24.a) and (24.b) and Eqs. (25.a) and (25.b) and \( \dot{V}_{\text{ex}} \) is constrained by the constant \( P_{ch} \) and \( \dot{V}_{CO_2,\text{met},STP} \) assumptions (see Section 3.3).

With \( P_{CO_2}^o \) equal to the chamber \( P_{CO_2} \) at the beginning of the first no-ventilation period, the chamber \( P_{CO_2} \) at the end of the \( i^{th} \) no-ventilation period is

\[ P_{CO_2}(t_{mv})_i = P_{CO_2}^o + \Delta CO_2,i-1 + \frac{T_{ch}P_S\dot{V}_{CO_2,\text{met},STP}}{T_SV_{ch}}(t_{mv}), \]  
(35)

where, for \( i = 1 \), \( \Delta CO_2,0 = 0 \) and, for subsequent cycles,

\[ \Delta CO_2,i = P_{CO_2}(t_{mv})_i + \Delta P_{CO_2}(t_v)_i - P_{CO_2}^o. \]  
(36)

Equations (33) and (35) are substituted into Eq. (36) to obtain

\[ \Delta CO_2,i = \Delta CO_2,i-1 + \frac{T_{ch}P_S\dot{V}_{CO_2,\text{met},STP}}{T_SV_{ch}}(t_{mv}) + q \cdot [P_{CO_2}(\infty) - P_{CO_2}(t_{mv})_i]. \]  
(37)

If \( P_{CO_2}(t_{mv})_i > P_{CO_2}(\infty) \), the last term in Eq. (37) consists of a constant \( q \) times a quantity that becomes increasingly negative in successive cycles. As the number of cycles \( n \) approaches infinity under such conditions and the condition \( P_{CO_2}(t_{mv})_i < P_{ch} \) holds, this term asymptotically approaches a value equal to the amount of \( CO_2 \) added during each no-ventilation period. As a result, \( \Delta CO_2,i \rightarrow \Delta CO_2,i-1 \), and a different type of no-ventilation – ventilation steady state is approached, a state for which Eq. (37) reduces to

\[ q \cdot [P_{CO_2}(t_{mv})_i - P_{CO_2}(\infty)] = \frac{T_{ch}P_S\dot{V}_{CO_2,\text{met},STP}}{T_SV_{ch}}(t_{mv}). \]  
(38)
The maximum chamber $P_{CO_2}$ in this "small ktv" no-ventilation – ventilation steady state is the $P_{CO_2}$ at the end of each no-ventilation period. This $P_{CO_2}$ is obtained for the $i^{th}$ period by rearranging Eq. (38) to obtain

$$P_{CO_2}(t_{nv})_i = P_{CO_2}(\infty) + \left\{ \frac{T_{ch} P_{S} \dot{V}_{CO_2,met,STP}}{T_{S} \dot{V}_{ch}} \left( \frac{t_{nv}}{q} \right) \right\}. \quad (39)$$

The minimum chamber $P_{CO_2}$ in this steady state is that at the end of each ventilation period and that given by

$$P_{CO_2}(t_v)_i = P_{CO_2}^0 + \Delta_{CO_2,i}, \quad (40)$$

an expression that, after Eqs. (33) and (36) are substituted, becomes

$$P_{CO_2}(t_v)_i = P_{CO_2}(t_{nv})_i - q [P_{CO_2}(t_{nv})_i - P_{CO_2}(\infty)] \quad (41)$$

An alternative expression for the maximum chamber $P_{CO_2}$ in this steady state is obtained by noting that

$$q \cdot [P_{CO_2}(t_{nv})_i - P_{CO_2}(\infty)] = P_{CO_2}(t_{nv})_i - P_{CO_2}(t_v)_i. \quad (42)$$

Substituting Eq. (22.a) into Eq. (42) then yields

$$q \cdot [P_{CO_2}(t_{nv})_i - P_{CO_2}(\infty)] = P_{CO_2}(t_{nv})_i - \left\{ \frac{P_{ch} \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}}{\dot{V}_{vex,STP}} \left[ 1 - e^{-kt_v} \right] + P_{CO_2}(t_{nv})_i \cdot e^{-kt_v} \right\}, \quad (43)$$

which is substituted into Eq. (38) to obtain

$$\frac{T_{ch} P_{S} \dot{V}_{CO_2,met,STP}}{T_{S} \dot{V}_{ch}} (t_{nv})_i = P_{CO_2}(t_{nv})_i - \left\{ \frac{P_{ch} \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}}{\dot{V}_{vex,STP}} \left[ 1 - e^{-kt_v} \right] + P_{CO_2}(t_{nv})_i \cdot e^{-kt_v} \right\}. \quad (44)$$

Finally, Eq. (44) is solved for $P_{CO_2}(t_{nv})_i$ to obtain:
or, with the ventilation rate expressed as the volume flux at actual chamber temperature and pressure,

\[
P_{\text{CO}_2}(t_{\text{nv}}) = \frac{T_{ch} P_{sv} \dot{V}_{\text{CO}_2,met,STP}(t_{\text{nv}})}{T s \dot{V}_{ch} \left[ 1 - e^{-k_c t_v} \right]} + \left[ \frac{P_{ch} \left( \dot{V}_{\text{CO}_2,met,STP} + \dot{V}_{\text{CO}_2,\text{in},STP} \right)}{\dot{V}_{vex,STP}} \right], \quad (45.a)
\]

Alternatively, Eq. (44) is solved for \( t_{\text{nv}} \) to obtain the no-ventilation time required to sustain a given maximum chamber \( P_{\text{CO}_2} \) with given ventilation time and \( \dot{V}_{\text{CO}_2,met,STP} \):

\[
t_{\text{nv}} = \frac{T_s \dot{V}_{ch} \left[ 1 - e^{-k_c t_v} \right]}{T_{ch} P_{sv} \dot{V}_{\text{CO}_2,met,STP}} \left[ \frac{P_{\text{CO}_2}(t_{\text{nv}}) - P_{ch} \left( \dot{V}_{\text{CO}_2,met,STP} + \dot{V}_{\text{CO}_2,\text{in},STP} \right)}{\dot{V}_{vex,STP}} \right], \quad (46.a)
\]

or, with the ventilation rate expressed as the volume flux at actual chamber temperature and pressure,

\[
t_{\text{nv}} = \frac{\dot{V}_{ch} \left[ 1 - e^{-k_c t_v} \right]}{\dot{V}_{\text{CO}_2,met,STP}} \left[ \frac{T_s P_{\text{CO}_2}(t_{\text{nv}}) - P_{ch} \left( \dot{V}_{\text{CO}_2,met,STP} + \dot{V}_{\text{CO}_2,\text{in},STP} \right)}{T_{ch} P_{sv} \dot{V}_{vex,STP}} \right]. \quad (46.b)
\]

2.5. \( \text{CO}_2 \) Accumulation in a Continuously \( \text{CO}_2 \)-Scrubbed Constant Temperature Chamber

The analog of Eq. (14) to consider \( \text{CO}_2 \) content in a constant-pressure chamber with an internal scrubber that continuously removes \( \text{CO}_2 \) is

\[
\left( \frac{\partial P_{\text{CO}_2}}{\partial t} \right)_{T_A} = \frac{T_{ch} P_{sv}}{T_s \dot{V}_{ch}} \left( \dot{V}_{\text{CO}_2,\text{met,STP}} - \dot{V}_{\text{CO}_2,\text{in,STP}} - \dot{V}_{\text{CO}_2,\text{out,STP}} \right), \quad (47)
\]

where terms in the curly brackets denote the action of the scrubber — i.e., the change in partial volume of \( \text{CO}_2 \) in the overall ventilation of the scrubber, \( \dot{V}_{\text{vent,STP}} \). Note that the sense of the “in” and “out” subscripts is different from that used when chamber ventilation is considered. Here, \( \dot{V}_{\text{CO}_2,\text{in,STP}} \) is the standard \( \text{CO}_2 \) partial volume flow into the scrubber, and \( \dot{V}_{\text{CO}_2,\text{out,STP}} \) is the standard \( \text{CO}_2 \) partial volume flow in the scrubber exhaust. The analog of Eq. (15) for these conditions is consequently

\[
\dot{V}_{\text{CO}_2,\text{in,STP}} = \frac{P_{\text{CO}_2}}{P_{ch}} \dot{V}_{\text{vin,STP}}, \quad (48)
\]
where \( \dot{V}_{\text{vin,STP}} \) is the standard scrubber input ventilation rate. We then assume that the standard CO\(_2\) partial volume in the scrubber exhaust is a constant fraction \( f \) (0 \( \leq \)) of the standard CO\(_2\) partial volume entering the scrubber:

\[
\dot{V}_{\text{CO}_2,\text{out,STP}} = f \cdot \dot{V}_{\text{CO}_2,\text{in,STP}} = \frac{P_{\text{CO}_2}}{P_{\text{ch}}} \dot{V}_{\text{vin,STP}} \cdot f.
\] (49)

Rearrangement after Eqs. (48) and (49) are substituted into Eq. (47) yields:

\[
\left( \frac{\partial P_{\text{CO}_2}}{\partial t} \right)_{T_{\text{ch}}} + \frac{T_{\text{ch}} P_{\text{ch}} F \cdot \dot{V}_{\text{vin,STP}}}{T_{\text{ch}} P_{\text{ch}} V_{\text{ch}}} P_{\text{CO}_2} = \frac{T_{\text{ch}} P_{\text{ch}} F \cdot \dot{V}_{\text{vin,STP}}^{\text{out,STP}}}{T_{\text{ch}} V_{\text{ch}}},
\] (50)

where \( F = (1 - f) \) is the scrubber efficiency factor. Equation (50) is identical to Eq. (17), with \( \dot{V}_{\text{CO}_2,\text{in,STP}} = 0 \) and \( \dot{V}_{\text{vin,STP}} \) replaced by \( F \cdot \dot{V}_{\text{vin,STP}} \). With constant \( F \cdot \dot{V}_{\text{vin,STP}} \) and \( P_{\text{ch}} \), the solution of this equation is consequently obtained as before:

\[
P_{\text{CO}_2}(t) = \frac{P_{\text{ch}} \dot{V}_{\text{CO}_2,\text{met,STP}}}{F \cdot \dot{V}_{\text{vin,STP}}} \left[ -e^{-k t} + P_{\text{CO}_2}^0 e^{-k t} \right], \quad \text{with } k = \frac{T_{\text{ch}} P_{\text{ch}} F \cdot \dot{V}_{\text{vin,STP}}}{T_{\text{ch}} V_{\text{ch}}};
\] (51.a)

and

\[
P_{\text{CO}_2}(t) = \frac{P_{\text{ch}} T_{\text{ch}} \dot{V}_{\text{CO}_2,\text{met,STP}}}{T_{\text{ch}} F \cdot \dot{V}_{\text{vin}}} \left[ -e^{-k_A t} + P_{\text{CO}_2}^0 e^{-k_A t} \right], \quad \text{with } k_A = \frac{F \cdot \dot{V}_{\text{vin}}}{V_{\text{ch}}};
\] (51.b)

where Eq. (51.a) is for use when the scrubber ventilation is expressed at STP, and Eq. (51.b) is for use when the scrubber ventilation is expressed at actual chamber temperature and pressure.

Note that maintenance of the constant \( P_{\text{ch}} \) required by this equation requires a constant O\(_2\) bleed into the chamber at standard rate equal to \( \dot{V}_{\text{O}_2,\text{met,STP}} \), while with constant \( F \cdot \dot{V}_{\text{vin,STP}} \), \( \dot{V}_{\text{CO}_2,\text{scrub,STP}} \) varies with the chamber CO\(_2\) mole fraction.

Similar to the situation in a continuously ventilated chamber, the \( P_{\text{CO}_2} \) in a continuously scrubbed chamber approaches a steady-state value given by the respective solutions of Eq. (51.a) and Eq. (51.b) at \( t = \infty \):

\[
P_{\text{CO}_2}(\infty) = \frac{P_{\text{ch}} \dot{V}_{\text{CO}_2,\text{met,STP}}}{F \cdot \dot{V}_{\text{vin,STP}}};
\] (52.a)

\[
= \frac{P_{\text{ch}} T_{\text{ch}} \dot{V}_{\text{CO}_2,\text{met,STP}}}{T_{\text{ch}} F \cdot \dot{V}_{\text{vin}}},
\] (52.b)
where $F \cdot \dot{V}_{\text{vin}, \text{STP}} \geq \dot{V}_{\text{CO}_2, \text{met}, \text{STP}}$. The minimum scrubber ventilation $\times$ efficiency factor product stipulated by Eqs. (52.a) and (52.b) satisfies both physiological and constant $P_{ch}$ requirements at the required chamber input bleed of 100% $O_2$ (see Section 3.3).

The half-time $t_{v,1/2}$ for approach to the steady state given by Eqs. (52.a) and (52.b) is (see Section 3.2):

\[
\begin{align*}
   t_{v,1/2} &= \frac{\ln(2)}{k} = \frac{0.693 \cdot T_{ch} P_{ch} V_{ch}}{T_{ch} P_{ch} F \cdot \dot{V}_{\text{vin}, \text{STP}}} \quad (53.a) \\
   &= \frac{\ln(2)}{k_A} = \frac{0.693 \cdot V_{ch}}{F \cdot \dot{V}_{\text{vin}}} \quad (53.b)
\end{align*}
\]

3. **SUPPLEMENTAL MATERIAL**

3.1. Derivation of Equation (19), Solution of the Differential Equation $y' + ky = c$

The present theory of $CO_2$ accumulation in ventilated or $CO_2$-scrubbed systems is based on the solution of a simple first-order differential equation $y' + ky = c$, where $y$ is a function of $t$ ($y = y(t)$), $y' = dy/dt$, and $k$ and $c$ are constants. This solution is a standard problem in first courses in differential equations, but is presented here for completeness.

We start by multiplying both sides of the equation by the integrating factor $e^{kt}$ to obtain

\[ e^{kt} (y' + ky) = ce^{kt}, \]

which simplifies to

\[ (ye^{kt})' = ce^{kt}. \]  

(3.2)

Equation (3.2) is integrated to obtain

\[ ye^{kt} = \int ce^{kx} dx + C, \]

(3.3)

where $C$ is an integration constant, the value of which is governed by the initial conditions for the problem. Eq. (3.3) is rearranged, and the integral is evaluated to obtain
\[ y = ce^{-kt} \int e^{kx} dx + Ce^{-kt} = \frac{ce^{-kt}(e^{kt}x|_0)}{k} + Ce^{-kt} \] (3.4)

and

\[ y = \frac{ce^{-kt}(e^{kt} - 1)}{k} + Ce^{-kt}. \] (3.5)

Eq. (3.5) is rearranged to obtain our final result:

\[ y = c\left(1 - e^{-kt}\right) \frac{k}{k} + Ce^{-kt}. \] (3.6)

### 3.2. Ventilation Half-Time Derivation

Continuous chamber or CO₂ scrubber ventilation started at \( t = 0 \) with initial chamber \( P_{CO_2} \) equal to \( P_{CO_2}(t_o) \) causes a maximum chamber \( P_{CO_2} \) change equal to \( P_{CO_2}(t_o) - P_{CO_2}(t) \) at \( t = t_o \). The fraction \( q \) of this total response attained in period \( t_q \) beginning at \( t_o \) is

\[
q = \frac{P_{CO_2}(t_o + t_q) - P_{CO_2}(t_o)}{P_{CO_2}(t_o) - P_{CO_2}(t_o)} = \frac{Z\left[1 - e^{-k(t_o+t_q)}\right] + P_{CO_2} e^{-k(t_o+t_q)} - Z[1 - e^{-k_t_o}] - P_{CO_2} e^{-k_t_o}}{Z[1 - e^{-k_t_o}] + P_{CO_2} e^{-k_t_o} - Z[1 - e^{-k_t_o}] - P_{CO_2} e^{-k_t_o}},
\] (3.7)

where \( Z = \frac{P_{ch} [\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2, in, STP}]}{\dot{V}_{vex, STP}} \) for the ventilated chamber, and

\[
Z = \frac{P_{ch} \dot{V}_{CO_2, met, STP}}{F \cdot \dot{V}_{vin, STP}}
\]

for the scrubbed chamber.

In general, \( 0 \leq q \leq 1 \), but \( q \) has lower bounds \( > 0 \) with the present definitions of \( Z \).

Equation (3.7) is solved for \( t_q \) to obtain the time required to attain fraction \( q \) of the total remaining response at \( t = t_o \):

\[
q = \frac{Z[1 - e^{-k_t_o} e^{-k_t_q}] + P_{CO_2} e^{-k_t_o} e^{-k_t_q} - Z[1 - e^{-k_t_0}] - P_{CO_2} e^{-k_t_0}}{Z - Z[1 - e^{-k_t_0}] - P_{CO_2} e^{-k_t_0}}
\]
\[ q = \frac{Ze^{-kt_0}e^{-kt_q} + p_0^{\alpha}e^{-kt_0}e^{-kt_q} + Ze^{-kt_0} - p_0^{\alpha} e^{-kt_0}}{Ze^{-kt_0} - p_0^{\alpha} e^{-kt_0}} = \frac{\left(p_0^{\alpha} - Z\right)e^{-kt_0}e^{-kt_q} + \left(Z - p_0^{\alpha}\right)e^{-kt_0}}{\left(Z - p_0^{\alpha}\right)e^{-kt_0}} \]

\[ q = \left(\frac{Z - p_0^{\alpha}}{Z - p_0^{\alpha}}\right)e^{-kt_0} - \left(Z - p_0^{\alpha}\right)e^{-kt_0}e^{-kt_q} = 1 - e^{-kt_q} \]

\[ 1 - q = e^{-kt_q} \]

\[ -kt_q = \ln(1 - q) \]

\[ t_q = -\frac{\ln(1 - q)}{k} = -\frac{\ln Q}{k}, \quad (3.8) \]

where \( Q = 1 - q \) is the fraction of the \( t_o \rightarrow t_\infty \) response remaining at the end of the \( t_q \) period, \( t_0 + t_q \).

For the half-time, the time required to attain one-half of the remaining response at \( t = t_0, q = \frac{1}{2} \) and

\[ t_{v/2} = -\frac{\ln(0.5)}{k} = \frac{\ln 2}{k}, \quad (3.9) \]

Values of \( q, Q, \) and \(-\ln(Q)\) for various multiples of \( t_{v/2} \) are given in Table 1.

<table>
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<tr>
<th># half-times ((t/v_{1/2}))</th>
<th>response fraction at ( t_{v/2} )</th>
<th>remaining response fraction at ( t_{v/2} )</th>
<th>( Q, % )</th>
<th>(-\ln(Q))</th>
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<td>6.931</td>
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</table>
3.3. Consequences of Constant $P_{ch}$ and $\dot{V}_{CO_2,met}$ Assumptions

Present analytic solutions to the equations for $P_{CO_2}(t)$ in ventilated and CO$_2$-scrubbed chambers [Eqs. (22.a) and (22.b) and Eqs. (51.a) and (51.b), respectively] and all dependent results require constant chamber pressure, $P_{ch}$, and a constant rate of metabolic CO$_2$ production by the chamber occupants, $\dot{V}_{CO_2,met}$. We here consider minimum chamber input, chamber exhaust, and CO$_2$ scrubber ventilation rates required to meet these assumptions, regardless of whether the chamber is considered "ventilated," because these minimum rates preclude consideration of cases that otherwise yield undefined or nonsensical results [e.g., $P_{CO_2}(t) > P_{ch}$ in Eqs. (22.a) and (22.b) and in Eqs. (51.a) and (51.b)].

Equation (1) is solved for $P_{ch}$, and then differentiated with respect to time at constant chamber temperature, to obtain

$$\left( \frac{\partial P_{ch}}{\partial t} \right)_{T_{ch}} = \frac{RT_{ch}}{V_{ch}} \left( \frac{\partial n_T}{\partial t} \right)_{T_{ch}}.$$  \hspace{1cm} (3.10)

With $\left( \frac{\partial P_{ch}}{\partial t} \right)_{T_{ch}} = 0$ and $\left( \frac{\partial n_T}{\partial t} \right)_{T_{ch},P_{ch}} = \sum \left( \frac{\partial n_g}{\partial t} \right)_{T_{ch},P_{ch}} = \dot{n}_T = \dot{n}_T$, where each summation is over the all gas species $g$ in the mix comprising $n_T$, mass balance requires that

$$\frac{RT_{ch}}{V_{ch}} \left\{ \dot{n}_{T,in} + \sum \dot{n}_{g,met} + \sum \dot{n}_{g,scrub} \right\} = \frac{RT_{ch}}{V_{ch}} \dot{n}_{T,out},$$ \hspace{1cm} (3.11)

where $\sum \dot{n}_{g,met}$ is the sum of changes in the chamber gas content due to the metabolic activity of the chamber occupants, and $\sum \dot{n}_{g,scrub}$ is the sum of changes in the chamber gas content due to the action of a CO$_2$ scrubber. Equation (1) for the molar amounts of chamber input and exhaust gases is differentiated at constant temperature and pressure to obtain, respectively:

$$\dot{n}_{T,in} = \frac{P_{in}}{RT_{ch}} \left( \frac{\partial V_{T,in}}{\partial t} \right)_{T_{ch},P_{in}} = \frac{P_{in}}{RT_{ch}} \dot{V}_{vin},$$ \hspace{1cm} (3.12)

and

$$\dot{n}_{T,out} = \frac{P_{ex}}{RT_{ch}} \left( \frac{\partial V_{T,ex}}{\partial t} \right)_{T_{ch},P_{ex}} = \frac{P_{ex}}{RT_{ch}} \dot{V}_{ved},$$ \hspace{1cm} (3.13)
where \( P_{in} \) and \( P_{ex} \) are the respective pressures of the input and exhaust ventilation flows. If the input and exhaust ventilation rates are expressed at chamber pressure, then \( P_{in} = P_{ex} = P_{ch} \) and Eqs. (3.12) and (3.13) are substituted into Eq. (3.11) to obtain

\[
\dot{V}_{vin} P_{ch} + RT_{ch} \left[ \sum \dot{n}_{g,met} + \sum \dot{n}_{g,scrub} \right] = \dot{V}_{vex} P_{ch}.
\]  

(3.14)

If the chamber humidity is assumed to be constant, \( \dot{n}_{g,met} = 0 \) for all gases except \( O_2 \) and \( CO_2 \). For \( O_2 \), we have

\[
\dot{n}_{O_2,met} = -\frac{P_{ch} \dot{V}_{O_2,met}}{RT_{ch}},
\]  

(3.15)

and for \( CO_2 \),

\[
\dot{n}_{CO_2,met} = \frac{P_{ch} \dot{V}_{CO_2,met} RQ}{RT_{ch}},
\]  

(3.16)

where we have invoked the definition of the respiratory quotient, \( RQ = \dot{V}_{CO_2,met} / \dot{V}_{O_2,met} \). Equations (3.15) and (3.16) are substituted into Eq. (3.14) to obtain

\[
\dot{V}_{vin} + \dot{V}_{O_2,met} (RQ-1) + \frac{RT_{ch}}{P_{ch}} \sum \dot{n}_{g,scrub} = \dot{V}_{vex}.
\]  

(3.17)

Input and exhaust ventilation rates, as well as any \( CO_2 \) scrubbing rate, must satisfy Eq. (3.17) to maintain constant \( P_{ch} \).

**No \( CO_2 \) Scrubbing**

With no scrubbing, the absolute minimum ventilation rates required to keep \( P_{ch} \) constant are obtained by rearranging Eq. (3.17) with \( \sum \dot{n}_{g,scrub} = 0 \):

\[
\dot{V}_{vin} = \dot{V}_{O_2,met} (1 - RQ), \text{ with } RQ \leq 1 \text{ and } \dot{V}_{vex} = 0,
\]  

(3.18.a)

or

\[
\dot{V}_{vex} = \dot{V}_{O_2,met} (RQ - 1), \text{ with } RQ \geq 1 \text{ and } \dot{V}_{vin} = 0.
\]  

(3.18.b)

Because \( \dot{V}_{vin} < \dot{V}_{O_2,met} \) in either case, the absolute minimum ventilations cannot indefinitely sustain any specified \( \dot{V}_{O_2,met} \), and the constant \( \dot{V}_{CO_2,met} \) assumption must ultimately be violated. This
violation must be avoided to consider the solutions of Eqs. (22.a) and (22.b) and Eqs. (51.a) and (51.b) at \( t = \infty \) [see Eqs. (24.a) and (24.b) and Eqs. (52.a) and (52.b)]. Thus, indefinite maintenance of any specified \( \dot{V}_{O_2,\text{met}} \) requires a minimum input ventilation of 100% \( O_2 \) at a rate equal to \( \dot{V}_{O_2,\text{met}} \).

This minimum input, however, exceeds the minimum \( \dot{V}_{\text{vin}} \) in either Eq. (3.18.a) or Eq. (3.18.b), so we then have the requirement that \( \dot{V}_{\text{vex}} > 0 \) to keep \( P_{ch} \) constant. Under these circumstances, Eq. (3.17) becomes

\[
\dot{V}_{\text{vex}} = \dot{V}_{O_2,\text{met}} + \dot{V}_{O_2,\text{met}}(RQ-1),
\]

which reduces to

\[
\dot{V}_{\text{vex}} = \dot{V}_{O_2,\text{met}} \cdot RQ = \dot{V}_{CO_2,\text{met}}, \text{ with } \dot{V}_{\text{vin}}(100\% O_2) = \dot{V}_{O_2,\text{met}}. \tag{3.20}
\]

\[\text{CO}_2\] Scrubbing

The absolute minimum scrubber ventilation rate required to keep \( P_{ch} \) constant in a scrubbed chamber is obtained from Eq. (3.17) with \( \dot{V}_{\text{vex}} = 0 \) and \( \dot{V}_{\text{vin}}(100\% O_2) = \dot{V}_{O_2,\text{met}} \) to indefinitely sustain any assumed \( \dot{V}_{O_2,\text{met}} \). Under these conditions, Eq. (3.17) reduces to

\[
\dot{V}_{O_2,\text{met}} RQ = F \cdot \dot{V}_{\text{vin,scr}} \left( \frac{P_{CO_2}}{P_{ch}} \right), \tag{3.21}
\]

where the definition of \( \frac{RT_{ch}}{P_{ch}} \sum \bar{n}_{g,\text{scrub}} \) from Eqs. (48) and (49) has been substituted. Equation (3.21) is solved for \( \dot{V}_{\text{vin,scr}} \) to obtain

\[
\dot{V}_{\text{vin,scr}} = \frac{P_{ch} \dot{V}_{O_2,\text{met}} RQ}{F \cdot P_{CO_2}} = \frac{P_{ch} \dot{V}_{CO_2,\text{met}}}{F \cdot P_{CO_2}}, \text{ with } \dot{V}_{\text{vin}}(100\% O_2) = \dot{V}_{O_2,\text{met}}. \tag{3.22}
\]

The absolute minimum scrubber ventilation rate, \( \dot{V}_{\text{vin,scr}} = \dot{V}_{CO_2,\text{met}} / F \), occurs in the worst case, where \( P_{CO_2} = P_{ch} \). Greater scrubber ventilation rates drive the chamber \( P_{CO_2} \) downward toward the steady-state value given by Eq. (52.a) or Eq. (52.b). Note that chamber input ventilation rates greater than \( \dot{V}_{O_2,\text{met}} \) require that \( \dot{V}_{\text{vex}} > 0 \). The chamber is then both scrubbed and ventilated, and Eqs. (51.a) and (51.b) no longer hold. Consideration of these combined conditions is beyond our present scope.
4. SUMMARY

Analytic expressions have been derived to compute the value of the atmospheric $P_{CO_2}$ in well-mixed, constant-volume manned diving systems during various operational conditions at a constant temperature $T_{ch}$.

Equation (13) gives the system $P_{CO_2}$ at any time $t$ in a no-ventilation period between $t = 0$ and $t = t_n$, with initial $P_{CO_2} = P_{CO_2}^0$.

If the diving system remains at constant pressure $P_{ch}$, Eq. (22.a) or Eq. (22.b) gives the system $P_{CO_2}$ at any time $t$ in a continuous ventilation period between $t = 0$ and $t = t_v$ with initial $P_{CO_2} = P_{CO_2}^0$. During such a ventilation period, the system asymptotically approaches a steady-state $P_{CO_2}$ given by Eq. (24.a) or Eq. (24.b), with half-time given by Eq. (25.a) or Eq. (25.b).

Intermittent ventilation with constant no-ventilation and ventilation times at constant system pressure causes system $P_{CO_2}$ to oscillate between maximum and minimum values in successive no-ventilation – ventilation cycles. Over time, these maxima and minima asymptotically approach steady-state values, with maximum system $P_{CO_2}$ given by Eq. (45.a) or Eq. (45.b) and minimum system $P_{CO_2}$ given by Eq. (41). The no-ventilation time required to sustain a given maximum system $P_{CO_2}$ at given ventilation time and $\dot{V}_{CO_2,met,STP}$ is given by Eq. (46.a) or Eq. (46.b).

If the diving system remains at a constant pressure $P_{ch}$ while $CO_2$ is continuously scrubbed from the system atmosphere, Eq. (51.a) or Eq. (51.b) gives the system $P_{CO_2}$ at any time $t$ in a continuous ventilation period between $t = 0$ and $t = t_v$ with an initial $P_{CO_2} = P_{CO_2}^0$. During such a scrubbing period, the system $P_{CO_2}$ asymptotically approaches a steady state value given by Eq. (52.a) or Eq. (52.b), with half-time given by Eq. (53.a) or Eq. (53.b).
5. REFERENCES
