The research conducted under this AFOSR award was mainly concerned with effects of coherence of light in various problems involving propagation and interaction of stochastic optical fields.
COHERENCE EFFECTS IN LIGHT PROPAGATION, IN SCATTERING AND IN SPECTROSCOPY

Final Report
Grant No: F4620-03-1-0138

Emil Wolf
The University of Rochester
Department of Physics and Astronomy
Rochester, New York

December 2005

Submitted to the
Air Force Office of Scientific Research

20060208 124

UNIVERSITY OF ROCHESTER
DEPARTMENT OF PHYSICS AND ASTRONOMY

DISTRIBUTION STATEMENT A
Approved for Public Release
Distribution Unlimited
I. ABSTRACT

In this report account is presented of results of research carried out during the period December 1, 2002 – November 30, 2005 under sponsorship of the Air Force Office of Scientific Research under grant F4620-03-1-0138. The research was mainly concerned with effects of coherence of light in various problems involving propagation and interaction of stochastic optical fields.

The results of our investigations were reported in 56 publications. They are listed on pages 4 - 7. Summaries of these publications are given on pages 8 - 18. Scientists who have participated in this research are listed on pages 19 and 20.
II. LIST OF PUBLICATIONS RESULTING FROM PREVIOUS AFOSR-SUPPORTED RESEARCH FROM DECEMBER 1, 2002 – NOVEMBER 30, 2005


III. SUMMARIES OF PUBLICATIONS RESULTING FROM PREVIOUS AFOSR-SUPPORTED RESEARCH

Spatial coherence properties are examined of the far field produced by partially coherent Gaussian Schell-model sources which generate the same distributions of radiant intensity as a fully coherent single-mode laser. The results are illustrated by numerical examples.

Some published computational work has suggested that partially coherent beams may be less susceptible to distortions caused by propagation through random media than fully coherent beams. In this paper this suggestion is studied quantitatively by examining the mean squared width of partially coherent conditions, and to what extent, partially coherent beams are less affected by the medium.

In a recent paper [Phys. Rev. Lett. 88, 013901 (2002)] it was shown that when a convergent spatially coherent polychromatic wave is diffracted at an aperture, remarkable spectral changes take place on axis in the neighborhood of certain points near the geometrical focus. In particular, it was shown that the spectrum is redshifted at some points, blueshifted at others, and split into two lines elsewhere. In the present paper we extend the analysis and show that similar changes take place in the focal plane, in the neighborhood of the dark rings of the Airy pattern.

We show that spectacular spectral changes take place in the vicinity of the dark rings of the Airy pattern formed with spatially coherent, polychromatic light diffracted at a circular aperture.

A modified form of diffraction tomography is presented in which measurements of the phase of the scattered field are replaced with measurements of the intensity on two planes beyond the scatterer. The new method is illustrated by an example.

We study the spreading of the rms spatial and angular characteristics of partially coherent beam sin turbulent media. The angular broadening of a beam is shown to be diffusion-like. The dynamics of the rms width of the beam is found to be determined by the interplay of free-space diffraction and turbulent diffusion. Our results indicate the conditions under which partially coherent beams are less sensitive to distortion caused by the atmospheric turbulence than are fully coherent beams.

We investigate the inverse scattering problem for statistically homogeneous, isotropic random media under conditions of strong fluctuations of optical wavefields. We present a method for determining the spectral density of the dielectric constant fluctuations in such media from scattering of partially coherent light. The method may find applications to a wide class of turbulent media such as the turbulent atmosphere and certain turbulent plasmas where backscattering and depolarization effects are negligible.


In usual measurements of the phase of an optical field it is generally assumed that the field is monochromatic. In reality this assumption is never justified. The distinction between monochromaticity and complete spatial coherence is first discussed, and it is then shown that with every spatially coherent field (e.g., a laser mode) one can associate a monochromatic wave that, in a well-defined sense, represents the average behavior of the field. Its phase can be measured by standard interferometric techniques and also by techniques developed in recent years for the measurement of the spectral degree of coherence of fields of arbitrary states of coherence.


It is shown that the recently discovered phenomenon of so-called spectral switches has a natural interpretation in the framework of singular optics with polychromatic light and that it should be regarded as being primarily a manifestation of diffraction-induced spectral changes rather than correlation-induced spectral changes as was suggested in the original papers [the first one appearing in *Opt. Commun.* 162, 57 (1999)] reporting this effect.


A modified form of diffraction tomography is presented in which measurements of the phase of the scattered field are replaced with measurements of the intensity on two planes beyond the scatterer. The new method is illustrated by example.


It is shown that the scattering amplitude for any direction of incidence and any direction of scattering and, consequently, also the extinction cross-section, of a scattering object may be determined from measurements of the scattered field over a plane at an arbitrary distance from it.


The effect of the state of coherence on the three-dimensional spectral intensity distribution in the focal region of a system with a high Fresnel number is investigated. The structure of a Gaussian Schell-model field is studied in detail.
We introduce a hybrid tomographic method, based on recent investigations concerning the connection between computed tomography and diffraction tomography, that allows direct reconstruction of scattering objects from intensity measurements. This technique is noniterative and its intuitively easier to understand and easier to implement than some other methods described in the literature. The manner in which the new method reduces to computed tomography at short wavelengths is discussed. Numerical examples of reconstruction are presented.

The spectrum of a field generated by a planar, secondary, quasi-homogeneous source of any state of coherence that obeys the scaling law is investigated. It is shown that throughout the half-space into which such a source radiates, the normalized spectrum of the radiated field is, to a good approximation, equal to the normalized source spectrum, except perhaps at points at distances of the order of a wavelength or less from the source plane.

Expressions are derived for the spectrum of the field generated by a planar, homogeneous, secondary source of any spectral distribution and of any state of spatial coherence. It is shown that the state of coherence affects the contributions of the homogeneous as well as the evanescent waves of the emitted field. The near-field spectra are studied in detail. The analysis is illustrated by examples.

We analyze the coherence properties of a partially coherent optical field emerging from the two pinholes in a plane opaque screen. We show that at certain pairs of points in the region of superposition the light is fully coherent, regardless of the state of coherence of the light at the pinholes. In particular, this result also holds if each pinhole is illuminated by a different laser.

It is known that some partially coherent Gaussian Schell-model beams may generate, in free space, the same angular distribution of radiant intensity as a fully coherent laser beam. We show that this result also holds even if the beams propagate in atmospheric turbulence, irrespective of the particular model of turbulence used. The result is illustrated by example.

Two simple theorems are established which indicate some advantages gained from the use of partially coherent sources rather than fully coherent ones for generating beams that propagate over a long distance through a turbulent atmosphere.

The spreading of partially coherent beams propagating through atmospheric turbulence is studied by use of the coherent-mode representation of the beams. Specifically, we consider partially coherent Gaussian Schell-model beams entering the atmosphere, and we examine the spreading of each coherent mode, represented by a Hermite-Gaussian function, on propagation. We find that in atmospheric turbulence the relative spreading of higher-order modes is smaller than that of lower-order modes, whereas the relative spreading of all order modes is the same as in free space. This model behavior successfully explains why under certain circumstances partially coherent beams are less affected by atmospheric turbulence than are fully spatially coherent laser beams.


We analyze the coherence properties of a partially coherent field emerging from two pinholes in an opaque screen and show that the spectral degree of coherence possesses phase singularities on certain surfaces in the region of superposition. To our knowledge, this is the first illustration of the singular behavior of the spectral degree of coherence, and the results extend the field of singular optics to the study of phase singularities of correlation functions.


We propose and experimentally implement a method for the generation of a wide class of partially spatially coherent vortex beams whose cross-spectral density has a separable functional form in polar coordinates. We study phase singularities of the spectral degree of coherence of the new beams.


It was recently shown that the structure of the weakly scattering object may be reconstructed from intensity measurements made in a half-space beyond the scatterer. We review these results and show how they may be extended to more complicated situations, and illustrate them by examples.


The usual treatment of polarization of randomly varying electromagnetic waves is based on the well-known Stokes parameter or, equivalently, on $2 \times 2$ coherence matrices (also called polarization matrices). Neither of them is adequate to elucidate the recently discovered changes of the state of polarization which a light beam may undergo as the beam propagates. We present a unified theory of coherence and polarization of random electromagnetic beams which brings out clearly the intimate relationship which exists between these two phenomena and which makes it possible to predict the changes in the state of polarization of a partially coherent electromagnetic beam on propagation. We illustrate the analysis by showing that a completely unpolarized beam may be spatially fully coherent.

A method is described, based on a recently formulated unified theory of coherence and polarization, for determining the changes that the degree of polarization, the degree of coherence, and the spectrum of a random electromagnetic beam may undergo as the beam propagates. Propagation in free space as well as in linear media, both deterministic and random, is discussed.


The influence of the state of spatial coherence on the spectrum of light in the vicinity of phase singularities of focused wavefields is investigated. It is found that a decrease of the degree of coherence of the field reduces the spectral changes of the field in the focal region.


We introduce a new definition of the energy spectrum of a nonstationary ensemble of pulses that reduces to the usual ones in the limit of statistically stationary ensembles of signals and of fully temporarily coherent ensembles.


It has been shown in a recent publication that in order to calculate the changes which the degree of polarization, the degree of coherence and the spectrum of a random electromagnetic beam undergo as the beam propagates, it is necessary to know the $2 \times 2$ cross-spectral density matrix of the fluctuating electric field. In this paper, a procedure is described for measuring the four elements of this matrix, with the help of two polarizers and a rotator. The procedure makes use of the Young interference experiment and the recently derived spectral interference law for such beams.


We determine a general form of the cross-spectral density matrix and the spectral degree of coherence of the electric field of a fully spatially coherent electromagnetic beam. Our result implies that complete spatial coherence does not, in general, impose any conditions on the state of polarization of the beam.


Using a recently developed unified theory of coherence and polarization [E. Wolf, *Phys. Lett.* A312, 263-267 (2003)] it is shown that a spatially completely coherent random electromagnetic beam can have a degree of polarization $\mathcal{P}$ which may take on any value ($0 \leq \mathcal{P} \leq 1$) at points in the beam. In particular the beam can be fully polarized at one point and completely unpolarized or partially polarized at another point. This result also holds for electromagnetic beams which are not completely spatially coherent.
Using a recently developed unified theory of coherence and polarization, we investigate the changes in the degree of polarization of an electromagnetic Gaussian Schell-model beam, as the beam propagates through the turbulent atmosphere.

Expressions are derived for the cross-spectral density and for the spectral degree of coherence of the near field generated by a planar, secondary, homogeneous source of any spectral distribution and of any state of spatial coherence. The behavior of the spectral degree of coherence and the spectral intensity in the immediate vicinity of the source are illustrated by computed examples.

The coherence properties of sunlight were first studied by Verdet around 1869 and were later examined by other scientists. However, all the previous calculations assumed that the Earth is in the far zone of the Sun, an assumption that is incorrect. An investigation of why Verdet’s result is nevertheless correct reveals a surprising property of radiation from incoherent sources.

It is well known that the light fields which are partially coherent and/or polychromatic do not typically possess regions of zero intensity and hence do not possess any obvious phase singularities. It is of interest to ask whether or not such fields possess singularities in some ‘hidden’ form, and in this paper we discuss the singular optics of partially coherent fields and the nature of the singularities in such fields.

The recently developed theory that unifies the treatment of polarization and coherence of random electromagnetic beams is applied to study field correlations in Young’s interference experiment. It is found that at certain pairs of points the transmitted field is spatially fully coherent, irrespective of the state of coherence and polarization of the field is incident on the two pinholes.

The spectral degree of coherence and of polarization of some model electromagnetic beams modulated by a polarization-dependent phase-modulating device, such as a liquid-crystal spatial light modulator, acting as a random phase screen are examined on the basis of the recent theory formulated in terms of the $2 \times 2$ cross-spectral density matrix of the beam. The phase-modulating device is assumed to have strong polarization dependence that modulates only one of the orthogonal components of the electric vector, and the phase of the phase-
A modulating device is assumed to be a random function of position imitating a random phase screen and is assumed to obey Gaussian statistics with zero mean. The propagation of the modulated beam is also examined to show how the spectral degree of coherence and the polarization of the beam change on propagation, even in free space. The results are illustrated by numerical examples.


It is shown that, for a $3 \times 3$ correlation matrix $W_y(r, r, \omega)$, $(i, j = x, y, z)$ of the electric vector of a random, stationary electromagnetic field to represent a field that is completely polarized at a point $r$ and frequency $\omega$, each element of the matrix must factorize. More precisely, a necessary and sufficient condition for the correlation matrix to represent a fully polarized field at a point $r$ is that the matrix has the form $W_y(r, r, \omega) = \mathcal{E}_i^*(r, \omega)\mathcal{E}_j(r, \omega)$, where $\mathcal{E}_i(r, \omega)$ $(i = x, y, z)$ are deterministic functions, i.e., that all pairs of the Cartesian components of the electric field at a point $r$ and frequency $\omega$ are completely correlated.


The analysis presented in this paper resolves an outstanding controversial issue of statistical optics, concerning the existence of the degree of polarization of any random, statistically stationary electromagnetic field. We show that the second-order electric spectral correlation matrix at any point in such a field may be uniquely expressed as the sum of three matrices, the first of which represents a completely polarized contribution. The ratio of the average intensity of the polarized part to the total average intensity provides a unique and unambiguous definition of the spectral degree of polarization of the electric field. It may be expressed by a simple formula in terms of the eigenvalues of the correlation matrix of the electric field and it reduces, for the two-dimensional case, to the usual well-known expression for the degree of polarization of beam-like fields. The results of this paper are of special interest for near-field optics.


It is shown analytically that the degree of polarization of a beam generated by an electromagnetic Gaussian Schell-model source which propagates through atmospheric turbulence tends to its value at the source plane with increasing distance of propagation. This result is independent of the spectral degrees of correlation of the source and of the strength of atmospheric turbulence. These conclusions are illustrated by a numerical example.


It was shown recently that the basic properties of a fluctuating electromagnetic beam can be derived from knowledge of a $2 \times 2$ cross-spectral density matrix of the electric field in the source plane. However, not every such matrix represents a source that will generate a beamlike field. We derive conditions that the matrix must satisfy for the source to generate an electromagnetic Gaussian Schell-model beam.
A recent paper entitled “Radiation from arbitrarily polarized incoherent source” contains an invalid demonstration that a degree of coherence of a statistically stationary electromagnetic field employed in several publications and rigorously derived in a recent paper cannot correctly represent the state of coherence of the field.

We present a derivation of the optical theorem that makes it possible to obtain expressions for the extinguished power in a very general class of problems not previously treated. The results are applied to the analysis of the extinction of power by a scatterer in the presence of a lossless half space. Applications to microscopy and tomography are discussed.

We show that after propagating through a turbulent medium the normalized spectrum of a beam generated by a class of quasi-homogeneous sources is, to a good approximation, equal to the normalized source spectrum. This result is a generalization of a previously derived result for propagation in free space.

It is pointed out that a recently introduced definition of the degree of coherence of a random electromagnetic field is not a true quantitative measure of coherence.

In this paper we study the effects of turbulent atmosphere on the degree of polarization of a partially coherent electromagnetic beam, which propagates through it. The beam is described by a $2 \times 2$ cross-spectral density matrix and is assumed to be generated by a planar, secondary, electromagnetic Gaussian Schell-model source. The analysis is based on a recently formulated unified theory of coherence and polarization and on the extended Huygens-Fresnel principle. We study the behavior of the degree of polarization in the intermediate zone, i.e. in the region of space where coherence properties of the beam and the atmospheric turbulence are competing. We illustrate the analysis by numerical examples.

The complex spectral degree of coherence of a general random, statistically stationary electromagnetic field is introduced in a manner similar to the way it is defined for a beamlike field, namely, by means of Young’s interference experiment. Both its modulus and its phase are measurable. We illustrate the definition by applying it to blackbody radiation emerging from a cavity. The results are of particular interest for near-field optics.
A three-point source model is used to study the interference of wavefields which are mutually partially coherent. It is shown that complete destructive interference of the fields is possible in such a “three-hole interferometer” even if the sources are not fully coherent with respect to each other. An explanation of this surprising effect is given, and conditions necessary for complete destructive interference are started.

A new general interference law is derived for the superposition of two random electromagnetic beams of any state of coherence and of any state of polarization when transmitted through polarizers and rotators. It includes, as special cases, a variety of interference laws which apply to particular situations. Some of them have a close bearing on the classic interference experiments of Fresnel and Arago which have played a basic role in elucidating the concept of polarization of light.

We show that complete coherence of light fluctuations at two points in a statistically stationary optical field implies that the fluctuations are either identical or are proportional to each other, a property which may be called statistical similarity. In particular for light to be completely coherent it need not be monochromatic nor do the fluctuations need to be deterministic.

In this paper a generalization of the Stokes parameters of a random electromagnetic beam is introduced. Unlike the usual Stokes parameters which depend on one spatial variable, the generalized Stokes parameters depend on two spatial variables. They obey precise laws of propagation, both in free space and in any linear medium, whether deterministic or random. With the help of the generalized Stokes parameters, the changes of the ordinary Stokes parameters on propagation can be determined. Numerical examples of such changes are presented. The generalized Stokes parameters contain information not only about the polarization properties of the beam but also about its coherence properties. We illustrate this fact by expressing the degree of coherence of the electromagnetic beam in terms of one of the generalized Stokes parameters.

We show that the state of polarization of the polarized portion of a random, statistically stationary, electromagnetic beam may change on propagation, even in free space. We derive analytic formulas for the orientation angle and for the magnitudes of the major and of the minor axes of the polarization ellipse in terms of elements of the $2 \times 2$ cross-spectral density matrix of the electric field. We also obtain conditions for the invariance of the state of polarization on propagation. We illustrate the results by a numerical example relating to an electromagnetic Gaussian Schell-model beam.
A general expression is derived for the electric cross-spectral density matrix of a random electromagnetic beam in an observation plane parallel to the plane of the pinholes in a Young interferometer when the pinholes are illuminated by light of any state of coherence and of polarization. It is shown, that the degree of polarization $P$ in the observation plane may take on values different from those of the degree of polarization $P$ at each pinhole, depending on the value of the degree of coherence of the light incident on the pinholes.

In studies of radiation from partially coherent model sources the so-called quasi-homogeneous (QH) sources have been very useful, for instance in elucidating the behavior of fields produced by thermal sources. The analysis of the fields generated by such sources has, however, been primarily carried out in the framework of scalar wave theory. In this paper we generalize the concept of the QH source to electronic theory and derive expressions for the elements of the cross-spectral density matrix, the spectral density, the spectral degree of coherence, the degree of polarization and the Stokes parameters of the far-field generated by a QH source with a uniform state of polarization. We then derive reciprocity relations analogous to those familiar in connection with the generalized van Cittert-Zernike theorem for radiation from scalar sources. We illustrate the results by calculating the properties of the far field produced by transmission of an electromagnetic beam through a system of spatial light modulators.

A method of generating electromagnetic Gaussian Schell-model source from two coherent linearly polarized plane waves is described. This method involves two mutually correlated phase-only liquid-crystal spatial light modulators placed in the arms of a Mach-Zehnder interferometer. The sources produced by this method can be used to generated a wide class of electromagnetic beams with prescribed coherence and polarization properties.

In the last few years changes in the state of polarization of a model random electromagnetic beam (so-called electromagnetic Gaussian Schell-model beam) propagating in free space have been investigated. In the present paper we extend the analysis to propagation of such beams in homogeneous, isotropic, non-absorbing atmospheric turbulence. We find that the effects of turbulence on the state of polarization are most significant when the atmospheric fluctuations are weak or moderate, while in strong regime of atmospheric fluctuations the state of polarization of the beam returns to its original state. Our results might find useful applications for sensing, imaging and communication through the atmosphere.

The wavefront spacing in the focal region of an aplanatic focusing system is investigated, using the vector theory of electromagnetic diffraction, for monochromatic, linearly polarized
incident light. It is shown that in systems of high numerical aperture, the wavefront spacing near the focus is significantly larger than the wavelength of the incident light, and that the wavefront spacing changes significantly within a few wavelengths of the focus and can be less than a wavelength.


The electromagnetic Gaussian Schell-model (EGSM) beam is the simplest analytical model of a random electromagnetic beam. The model can be successfully used in studies of spectral, polarization and coherence properties of random electromagnetic beams on propagation in free space or in any linear medium, deterministic or random. The most general type of source which produces electromagnetic Gaussian Schell-model beams is characterized by 10 parameters. Using properties of the cross-spectral density matrix of such a beam, we derive necessary and sufficient conditions which the parameters of the source must satisfy in order to generate a physically realizable beam of this type. The conditions provide certain constraints for synthesis, simulations and any applications of such beams.
IV. SCIENTIFIC COLLABORATORS

In addition to Professor Emil Wolf, the Principal Investigator for this grant, the following scientists have taken part in this research:

AGARWAL, G. S. Professor, Department of Physics, Oklahoma State University, Stillwater, OK.

BOGATYRYOVA, G. Professor, Cherovsti National University, Chernovtsi, Ukraine.

CARNEY, P. S. Assistant Professor of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL.

DOGARIU, A. Associate Professor, College of Optics, University of Central Florida, Orlando, FL.

ELLIS, J. Graduate Student, College of Optics, University of Central Florida, Orlando, FL.

FEL’DE, V. Professor, Cherovsti National University, Chernovtsi, Ukraine.

FOLEY, J. T. Professor of Physics, Dept. of Physics and Astronomy, Mississippi State Univ., Mississippi State, MS.

GAMIZ, V. Program Manager, Senior Research Scientist, Air Force Research Laboratory, Directed Energy Doctrate, Surveillance Technologies Branch – AFRL/DEBS, Kirtland AFB, NM.

GBUR, G. Assistant Professor, North Carolina Charlotte, Dept. of Physics and Optical Science, Charlotte, NC.

GREFFET, J.-J. Professor, Laboratoire EM2C, Ecole Centrale Paris, Cedex, France.

HOOVER, B. Optical Scientist, Advanced Optical Technologies, Albuquerque, NM.

KOROTKOVA, O. Research Associate, University of Rochester, Rochester, NY.

MUJAT, M. Research Fellow, Harvard Medical School and Wellman Center for Photo-medicine Massachusetts General Hospital, Boston, MA.

POLYANSKII, P. Professor, Cherovsti National University, Chernovtsi, Ukraine.

PONOMARENKO,S. Research Fellow, Los Alamos National Laboratory, Los Alamos, NM.

ROWCHOWDURY, H. Graduate Student, University of Rochester, Rochester, NY.

SALEM, M. Graduate Student, College of Optics, University of Central Florida, Orlando, FL.

SCHOTLAND, J. Professor, Dept. of Bioengineering, University of Pennsylvania, Philadelphia, PA.
<table>
<thead>
<tr>
<th>Name</th>
<th>Institution and Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCHOUTEN, H.</td>
<td>Graduate Student, Vrije Universiteit, Amsterdam, The Netherlands.</td>
</tr>
<tr>
<td>SHIRAI, T.</td>
<td>Research Scientist, Mechanical Engineering Laboratory, MITI, Tsukuba, Japan.</td>
</tr>
<tr>
<td>SOSKIN, M. S.</td>
<td>Professor, Institute of Physics, National Academy of Sciences, Kiev, Ukraine.</td>
</tr>
<tr>
<td>VISSER, T.</td>
<td>Visiting Associate Professor at the University of Rochester, Rochester, NY and Associate Professor, Vrije Universiteit, Amsterdam, The Netherlands.</td>
</tr>
</tbody>
</table>