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14. ABSTRACT

The focus of this work was on the synthesis of control and optimization theory, numerical methods, and fluid mechanics. There are three primary thrusts in our study of model-based flow control: Optimization, Feedback, and Characterization of Fundamental Limitations. Work on this contract focused primarily on some delicate unsolved problems related to regularization in the first of these three areas, optimization, and how, by addressing these problems, some practical applications involving the adjoint analysis of complex turbulent flows (specifically, the mixing of cross-flow dilution jets in a jet engine combustor) could be tackled that could not formerly be considered using these approaches. Work on this problem led directly to significant advancements in the other two areas as well (that is, feedback and characterization of fundamental limitations in fluid systems).

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Final Technical Report, AFOSR Grant F49620-01-1-0048

PI: Thomas R. Bewley, UC San Deigo

This document summarizes the research acccomplishments performed under AFOSR Grant F49620-01-1-0048, entitled

Adjoint-based optimization and control of complex dynamics in fluid systems,

as well as discussing some of the new research directions that this grant helped to open up. I truly hope this document serves to further stimulate interest at the AFOSR in these research directions, as I honestly believe this work is both highly relevant to the Air Force mission and represents several fundamental new advancements on the integration of control theory and fluid mechanics. I very much look forward to pursuing further some of these new research avenues on future contracts with the AFOSR.

The focus of this work is on the synthesis of control and optimization theory, numerical methods, and fluid mechanics. Individually, these fields are fairly mature. However, systematic attempts at their practical integration are still fairly new, and involve a myriad of subtle challenges. Addressing these peculiar challenges is the primary aim of my lab. Indeed, a one-sentence summary of our research objectives, many aspects of which were supported directly by this contract, follows:

> To synthesize model-based control theory with computational fluid dynamics to develop effective strategies for the control, estimation, optimization, and forecasting of unsteady fluid-mechanical systems, focusing specifically on the drag, heat transfer, mixing, and noise caused by transition and turbulence.

There are three primary thrusts in our study of model-based flow control: *Optimization*, *Feedback*, and *Characterization of Fundamental Limitations.* Work on this contract focused primarily on some delicate unsolved problems in the first of these three areas, optimization, and how, by addressing these problems, some practical applications involving the adjoint analysis of complex turbulent flows could be tackled that could not formerly be considered. Work on this problem led directly to advancements in the other two areas as well (that is, feedback and characterization of fundamental limitations in fluid systems), so our advancements in all three areas will be discussed in this document.

Further technical introduction of this research area is given in the recent review article, **B01**, which I was invited to submit to *Progress in Aerospace Sciences*, and the short review paper, **b02**, which I wrote to accompany my recent plenary lecture at the European Turbulence Conference, and will be reviewed further my forthcoming article in the *Annual Reviews of Fluid Mechanics.* In addition, during the period of this contract, I was invited twice to prepare and deliver full-day minicourses on *Tools for model-based control of transitional and turbulent flow systems*, sponsored by the NATO Research and Technology Organization (RTO)—once at the School of Aeronautics in Madrid, Spain (April 2002, hosted by Prof. Javier Jimenez), and once at Ecole Polytechnique in Palaiseau, France (May 2003, hosted by Prof. Patrick Huerre). These minicourses covered many of the advancements made under the present contract; the slides from these minicourses can be downloaded **here**.

Note that this document uses the convention that references with the authors' initials in uppercase denote refereed journal articles, references with initials in lowercase denote conference proceedings, and all such references (in blue) are linked directly to the corresponding articles in the pdf version of this document. For those reading a printed version of this document, the pdf version may be downloaded directly from:

http://turbulence.ucsd.edu/afosrwriteup.pdf

and all references may be obtained directly from:

http://turbulence.ucsd.edu/references.html

1. Optimization

1.1 Introduction

Within the field of fluid mechanics, the capability is emerging to simulate accurately a wide variety of unsteady fluid-mechanical systems. At the same time, within the field of computer science, the capability is emerging to assemble increasingly powerful parallel computers from inexpensive components. The time has come to ask ourselves what we can actually accomplish with the remarkable flow simulation capability that results from these two developments. In many cases, the natural answer is clear: we may use this simulation capability to *optimize* the flow properties of interest in engineering devices.

However, in the unsteady setting, there are some challenges in achieving this goal. The primary challenge is associated with the fact that fluid systems are both nonlinear and "infinite dimensional". That is, such systems possess a huge number of degrees of freedom, and it is often unclear (unless one does the proper analysis) in what manner one should "tweak" a system in order to improve overall system performance. In industry, such tweaking is largely performed by hand, based on the engineer's intuition. However, in complex unsteady flow systems, such intuition often fails.

My lab has pioneered the development of automated techniques to optimize unsteady flow systems. With such techniques, the "best" direction to tweak the system in order to reduce a metric quantifying the system behavior is calculated via a procedure called adjoint analysis. The iterative tweaking process used to get the best performance possible is called gradient-based optimization. When used properly, such tools open up a flood of exciting new opportunities for leveraging large-scale numerical simulation capability to optimize the flows in engineering devices.

1.2 Technical accomplishments

As mentioned above, our lab has pioneered the use of adjoint analysis and gradient-based optimization strategies to perform iterative model-based optimization of high-dimensional control distributions for nonlinear fluid systems. Our first major result in this area was accomplished during my PhD thesis, in which we solved the benchmark problem of relaminarizing fully-developed channel-flow turbulence via a distribution of blowing/suction on the wall for the first time, using adjoint-based optimization of high-dimensional unsteady control distributions according to a control paradigm commonly known as Model Predictive Control (MPC), as discussed in **BMT01**.

The adjoint-based optimization technique has since proven to be extensible to the optimization of other turbulent flow systems; we are now refining and extending this technique to more complex flow systems of greater engineering relevance. Specifically, the practical applications we are now focusing on with extensions of this technique are:

i. Optimization of open-loop forcing to excite the break-up of jets in crossflow in order to improve mixing and reduce pattern factor in jet engines, thereby extending engine life, improving overall system efficiency, and reducing pollutant formation, as discussed further in **bnb01**. This problem was in fact the primary application area on which the work funded by this contract was focused.

ii. Optimization of open-loop forcing near the exit of a round jet for the reduction of jet noise, as discussed in **cbfl02** and **cb03b**.

iii. Optimization of the structural parameters of a compliant surface (using a novel type of surface substructure which we refer to as a tensegrity fabric) in order to achieve turbulent drag reduction, as discussed in **lb03b**.[1]

We are also beginning to look at the application of estimation and forecasting of chaotic multiscale uncertain flow systems. Meteorologists have started to use similar adjoint-based techniques (they call it 4D-VAR) to address the problem of weather forecasting. As my group is coming at this problem from a flow control background, we have some very different perspectives on how best to address the problem of forecasting in the face of both multiscale complexity and model uncertainty. Due to the close relationship between the problems of flow control and weather forecasting, this is a community we plan to collaborate with closely in the coming years.

In the process of applying adjoint-based optimization to turbulent flow systems at Reynolds numbers high enough to

[1] Note that the problem of accurately computing a 3D turbulent flow over a compliant surface presents some difficult computational challenges. Through our CFD training, we are prepared to attack such computational challenges head on in order to accurately represent the fluid system of interest, which is often nontrivial. However, the ultimate goal of our research program is not the development of computational models *per se*, but rather is the integration of such accurate computational models of unsteady fluid systems with open-loop control optimization strategies and closed-loop feedback control strategies in order to alter effectively the unsteady dynamics expressed by the fluid system.

exhibit dynamics over a broad range of length scales and time scales, as encountered in the above-mentioned applications, we have found the standard formulation of adjoint analysis sometimes to be numerically ill-behaved. Thus, our fundamental work in the field of adjoint-based optimization is largely related to appropriately "regularizing" the adjoint analysis approach; that is, tuning the spectra of the various fields that are computed during an adjoint analysis and preconditioning the gradient in order to capture adequately the flow sensitivities of interest with marginally-resolved numerical grids, and to accelerate convergence of the associated optimization algorithm, as discussed in **PBH04**. The key to making this work is to recognize the fact that, in the framing of an adjoint analysis, there are in fact three inner products involved, each of which one has flexibility in defining. These three inner products are essentially: the norm used in the cost function to measure the state, the duality pairing in the identity used to define the adjoint operator, and the inner product used to extract the gradient from the expression for the perturbation of the cost function. There is a subtle relationship between the effects of modifying these inner products in order to adjust the regularity of the resulting adjoint problem and the speed of convergence of the associated gradient-based optimization problem. *In terms of fundamental contributions, this work on regularization of the adjoint field probably reflects the most significant direct research result of the present contract.*

Noncooperative analysis also plays a potentially important role for "robustifying" adjoint analyses, which are often quite "fragile" (that is, prone to "over-optimization"), in order to make them less sensitive to both unmodeled external disturbances and modeling errors; we have performed an extensive mathematical study of adjoint analysis in this noncooperative setting, as discussed in **BTZ00a**.

In order to verify the accuracy of the gradient information obtained with the adjoint-based approach, the gradient may be projected onto a particular direction and compared with a direct calculation of the directional derivative. The most straightforward way of doing this direct calculation while leveraging an existing nonlinear simulation code, using a finite difference method, is itself highly prone to accuracy problems. Under the present funding, our lab has pioneered the extension of the Complex Step Derivative method, which is not prone to such problems, to pseudospectral simulation codes such as those commonly used to study transition and turbulence. Certain care must be exercised to make this extension successfully, as described in **CB03**.

Finally, even when gradient information from the adjoint analysis is leveraged, the iterative adjoint-based optimization of controls for unsteady flow systems requires a huge number of flow calculations with candidate small perturbations to the control forcing distribution. This has motivated us to fundamentally rethink our approach to the flow simulation problem, and in particular (for reasons of computational efficiency) to consider optimizing the worst-case time-periodic orbits embedded within the attractor of the turbulent flow system, as discussed in **bt02**. Our current work in this area is focusing on making this approach practical by identifying maximally efficient computational techniques to determine such periodic orbits.

2. Feedback

2.1 Introduction

It is often advantageous to attempt to coordinate the unsteady forcing of a system based on measurements of the system dynamics itself. Such coordinated forcing is called feedback control, and is sometimes necessary to stabilize a system which, when left on its own, tends to "trip" and exhibit unsteady behavior, which is often undesireable. The dynamics of the system when operating under the influence of such coordinated control forcing (the so-called "closed-loop" system), is usually completely different than the dynamics expressed by the unforced system or by the system forced by "open-loop" control inputs that are not coordinated with system measurements.

The problem of coordinating effective control feedback for a given objective based on measurements of the system and approximate knowledge of the system's governing equation is a fundamentally harder problem than the problem of optimizing open-loop control forcing. To illustrate this point, taking N as the size of the model of the system in the computer, the solution of the "Riccati equation" at the heart of the former problem has N^2 elements, whereas the solution of the "adjoint equation" at the heart of the latter problem has N elements. For small systems described by, say, $N = 10$ states, both problems are easily solved. However, for accurate approximations of fluid systems, which typically require, say, $N = 10^6$ states, the prosaic approach to the former problem renders it, literally, a million times larger than the latter problem, essentially making it computationally intractable.

My lab has pioneered the development of techniques to finesse oneself out of this dimensionality predicament, thereby rendering the calculation of model-based feedback as a tractable problem in many flow systems of interest. This extension of the established body of feedback control theory to fluid-mechanical systems holds the potential to open the door to new levels of system efficiency not previously imagined in many flows of engineering significance.

2.2 Technical accomplishments

As mentioned above, we typically use Riccati-based tools to address the problem of closing the loop around a fluid-dynamical system using model-based feedback. Our first major result in this area was also accomplished during my PhD thesis, in which we completely decoupled the problem of controlling small (linear) 3D perturbations to a laminar channel flow at each Fourier mode, discretized the resulting PDEs in the remaining (wall-normal) spatial coordinate, and solved the resulting control problem at representative streamwise and spanwise wavenumbers using $\mathcal{H}_2/\mathcal{H}_\infty$ control theory, as reported in **BL98**. This set the stage for our later work in the feedback control of shear-driven instabilities, establishing the link between transient energy growth in the controlled fluid system, non-normality of the closed-loop eigenvectors, and the transfer function norms characterizing the excitation of the state by external disturbances in both the benign (Gaussian) setting and the malevolent worst case; this work demonstrated how all of these effects can be mitigated via linear optimal/robust control theory. This work also alerted us to the dangers of classical (eigenvalue-based) control approaches and model reduction strategies which fail to account for the significant effects of eigenvector non-normality.

Our first important extension of this work at UCSD involved the determination of effective linear feedback gains of the form $\mathbf{u} = K\mathbf{x}$ at a large array of wavenumber pairs; upon inverse transform to physical space, this led to well-resolved convolution kernels which relate the measurement at any particular sensor to forcing on the estimator model nearby that sensor, and further relate the control forcing at any actuator to the value of the state estimate nearby that actuator, as discussed in **HBH03a**. Again, the issue of regularity comes to play in order to solve this type of problem correctly; without enforcing regularity (via penalization of either one time derivative or two space derivatives of the control distribution on the wall), well resolved kernels which converge upon grid refinement and increase of computational box size can not be obtained for problems of this type. Thus, the primary result under the present contract related to the proper regularization of adjoint analyses, as summarized on page 3 of this report and discussed in depth in **PBH04**, played an essential role in our understanding of how to solve correctly the feedback control problem for fluid systems.

Our early numerical experiments on the linear feedback control of a low-order chaotic system (governed by the Lorenz equation) indicated potential pitfalls. In particular, applying linear control feedback to nonlinear systems on their chaotic attractor may destabilize the closed-loop system, as reported in **B99**. However, the power and flexibility of linear control theory combined with evidence of the importance of linear mechanisms for sustaining shear-driven turbulence motivated us to attempt (cautiously!) to use linear control theory to relaminarize fully-developed channel-flow turbulence using blowing/suction controls, as accomplished previously by our group using adjoint-based MPC. Via full state feedback and a simple gain scheduling algorithm which tunes the feedback gains to the instantaneous mean velocity profile, we showed that this may indeed be done, as reported in **HBH03b**. The two control approaches my lab has introduced to solve this problem (adjoint-based MPC and gain-scheduled Riccati-based $\mathcal{H}_2/\mathcal{H}_\infty$ control) have in fact been the *only* two approaches to solve successfully the benchmark problem of relaminarization of fully-developed channel-flow turbulence using blowing/suction actuation, despite the concerted attention given to this problem by many groups.

Given our recent successes in the full-state feedback control of both transition and turbulence in wall-bounded flows, we are currently focusing on the dual problem of estimation. We have several investigations ongoing in this area; many of the relevant issues are discussed in **BP04**. Our two most recent results in this area are discussed in **HCBH05** and **CHBH05**.

My lab has focused primarily on the "design then reduce" philosophy to feedback control design; that is, we focus first on solving the feedback control problem in the resolved setting, then attempt to reduce the complexity of the resulting feedback strategy. This is due to the tendency which we have observed for open-loop model reduction strategies (such as Proper Orthogonal Decomposition), which are key to the alternative "reduce then design" approach, to "lose the baby with the bathwater" in highly non-normal fluid systems. That is, it is difficult to know what in the system model may be thrown out before the control problem is solved; it is much easier to maintain closed-loop performance by reducing the complexity of a compensator designed in the fully-resolved setting, and there are existing strategies for such reductions that are well founded.

To make the Riccati-based "design then reduce" strategy numerically tractable when there are many inputs and/or outputs, the control problem must usually be decoupled or simplified down to a PDE to be discretized in at most one spatial direction. This was done in the work described above using Fourier transforms by assuming a parallel flow in the x and z directions. An alternative strategy we have developed is to assume a *parabolic* development of the perturbations in the streamwise coordinate, as is sometimes justified in a boundary layer. In order to solve this control problem correctly, standard optimal control theory for systems which are parabolic in time must be extended to account for the unique *noncausal* capability of control algorithms in this parabolic-in-space setting. That is, measurements at a particular streamwise location may be used to update both downstream and upstream controls to neutralize the effects of disturbances that enter the boundary layer both upstream and downstream of the actuator itself, as formulated in **CB04a** and demonstrated numerically in **CB04b**. Chandrasekhar's method is an alternative strategy we have used (see **HCBH05**) to make model-based feedback control calculations tractable in the case in which both the number of inputs and the number of outputs is small, but the number of states might be huge. With this method, the differential Riccati equation is split into low-rank factors and then these factors are computed directly. We are currently examining this approach for practical flow control problems which are "fundamentally 2D" even in the linear setting.

We are also exploring a certain class of "reduce then design" strategies which are based not on a black-box eigenvalue-based perspective to open-loop model reduction (such as balanced truncation) but rather on leveraging surrogate 1D PDE models such as the complex Ginzburg-Landau (CGL) model, which have been shown to accurately capture the dynamics of certain global instabilities such as jets and wakes even in the controlled setting. This work will eventually combine adjoint-based system identification, as discussed in **lb03a**, together with Riccati-based feedback applied to the CGL model with the identified parameters, as discussed in **LB02**.

3. Characterization of Fundamental Limitations

3.1 Introduction

A new and valuable role for model-based control theory in fluid mechanics is the characterization of fundamental limitations present in fluid systems to which controls might be applied. Such fundamental limitations may be computed in advance of determining any particular candidate control strategies of a given class, and can be valuable in providing new physical insight into the flow control problem at hand as well as indicating whether or not attempting to determine an effective control strategy for a given purpose is even a worthwhile endeavor to pursue. My lab is the first (and, to date, only) group in the flow control community to consider the development of such rigorous bounds on control system effectiveness.

3.2 Technical accomplishments

There are two major types of results which we seek in this area: *Stabilization Limitations* and *Performance Limitations.*

The first *Fundamental Stabilization Limitation* we have characterized is related to the gradual loss of stabilizability as the Reynolds number is increased in the CGL model of spatially-developing flows, as discussed in **LB03**. We have related this gradual loss of stabilizability to an increase of non-normality of the eigenvectors of the closed-loop system as the Reynolds number is increased, and have established a metric based on adjoint eigenvector analysis which quantifies this loss of stabilizability of individual modes of the open-loop system which we are currently extending to 3D CFD codes via the implicitly-restarted Arnoldi method.

The first *Fundamental Performance Limitations* we have studied are related to determining the minimum heat transfer and minimum momentum transfer of a channel flow (with constant-temperature walls) that can be sustained with zero-net blowing/suction controls on the walls. We have proven mathematically that the minimum sustainable heat transfer is given by the laminar flow, as discussed in a manuscript we currently have under review. We have also investigated numerical evidence that the minimum sustainable momentum transfer (that is, the minimum sustainable drag) might also given by the laminar flow, as discussed in **BA04**.

PUBLICATIONS

As discussed above, we have over 2 dozen recent refereed journal publications on related topics, most of which have been impacted to some degree or another by work performed under this contract. A full listing of all of these publications, in addition to links to pdf versions of all of them, is available here: **http://turbulence.ucsd.edu/references.html**

RELATED SERVICE ACTIVITIES

In the area of service to the flow control community, in addition to organizing several flow control workshops and minisymposia, reviewing papers, etc., my lab has made two unique contributions worth mentioning here:

First, I am about 75% done with a new Masters-level textbook entitled *Efficient Numerical Methods for Simulation, Optimization, and Control*; a draft is available **here**. This textbook is the first book that I know of to attempt to fit together a unified perspective on this vast range of topics that forms the foundation for model-based flow control, starting from an in-depth treatment of numerical linear algebra and leading all the way to many of the advanced topics summarized in this document and the papers it references.

Second, we have just released version 1.0 of a GNU open-source project called **Diablo** (Dns In A Box, Laptop Optimized). This code is a streamlined, easy-to-understand, cartesian-coordinate, structured-grid direct numerical simulation (DNS) code. It uses pseudo-spectral and energy-conserving second-order finite-difference methods for spatial discretization and a low-storage CN/RKW3 method for temporal discretation, and is completely described from first principles in the above mentioned textbook. Our goal with this open-source project is to assist others to study flow control problems by providing them with an easy-to-use and easy-to-extend tool that can make turbulence simulations accessible to flow control researchers that haven't spent an entire PhD studying turbulence simulation methods.

TRANSITIONS

Under this contract, we spent a substantial effort collaborating with United Technologies Research Center (UTRC). In fact, one of the post-docs funded under this contract, Greg Hagen, now works at UTRC. He took with him our lab's DNS/adjoint-based optimization code for turbulent flow systems in simple geometries, and he and his collaborators at UTRC have used this code extensively to study the problem of cross-flow jet mixing.

PATENTS

None.

SUMMARY

My lab is pursuing a targeted series of focused investigations in the three primary areas which I believe will lay the conceptual and algorithmic foundations for many future investigations in model-based flow control: *Optimization*, *Feedback*, and *Characterization of Fundamental Limitations*. The funding under this particular AFOSR contract was an essential piece in setting the foundation for a large number of exciting activities in this field. The several articles we have written so far have established us on a good trajectory, but much remains to be done to finish the job of setting the field of flow control on a solid theoretical foundation, and bringing these techniques to bear on practical problems of Air Force interst. The field of model-based flow control is extremely fertile territory for future university research in collaboration with both the US aerospace industry and the USAF labs.

Please feel absolutely free to contact me if you have any further questions related to the advancements discussed in this document, how they are applicable to the Air Force mission, and how they may readily be extended to future problems of Air Force interest in a variety of potential new projects.

Best regards,

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