APPLICATION OF AN ITERATIVE METHOD FOR THE SOLUTION OF ELECTROMAGNETIC SCATTERING FROM WIRE ANTENNAS

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Abstract: In this paper we apply vector ε-algorithm to solve a system of linear equations arising in the method of moments solution of electromagnetic scattering from wire antennas. The method does not require the inversion of a matrix. Hence, it avoids the problems associated with matrix inversion of singular or ill-conditioned matrices.

Keywords: Method of moments, Singular matrix, system of linear equations

1.   Introduction

This paper presents an iterative technique for solving the system of equations obtained in a method of moments solution. The method is based on vector ε-algorithm (VEA) [1]. The application of the method is illustrated by applying it to find the current induced on a wire excited by a normally incident plane wave. Matrix inversion is not required as the method is iterative. The solution process can be stopped when a pre-defined convergence criterion is met. This allows the user to specify the degree of accuracy in the solution. Some of the iterative techniques utilized so far include the conjugate gradient method [2] and the method of steepest descent [3]. VEA presents an alternative to solving a system of equations, especially when other methods fail due to ill-conditioned or a singular matrix [4].

2.   Vector ε - Algorithm (VEA)

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Consider the solution of the following system of linear equations:

$$AX = b \tag{1}$$

where A is a known $N \times N$ moment method matrix, $A = [a_{ij}]$; X is the unknown $N \times 1$ column vector, $X = (x_1, x_2, \ldots x_N)^T$, and b is known $N \times 1$ forcing function vector, $b = (b_1, b_2 \ldots b_N)^T$. In VEA, estimates of X, designated $\tilde{X}$, are obtained without the use of matrix inversion.

The starting vectors $\{\varepsilon_{-1}^{(0)}\}$ and $\{\varepsilon_0^{(q)}\}$, each of length N, are constructed from the initial values:

$$\{\varepsilon_{-1}^{(q)}\} = \{0\}, \qquad q = 1, 2, \ldots \tag{2}$$

$$\{\varepsilon_0^{(0)}\} = \{S^{(0)}\} = \{0\}$$

$$\{\varepsilon_0^{(q)}\} = \{S^{(q)}\}, \qquad q = 1, 2, \ldots \tag{3}$$

The vectors, $S^{(q)} = (s_1^{(q)}, s_2^{(q)} \ldots s_N^{(q)})$, can be calculated using Gauss-Seidel relaxation [5]:

$$s_i^{(q)} = \left(b_i - \sum_{j=1}^{i-1} a_{ij} s_j^{(q)} - \sum_{j=i+1}^{N} a_{ij} s_j^{(q-1)}\right) \Big/ a_{ii}, \qquad \begin{array}{l} i = 1, 2, \ldots N \\ q = 1, 2, \ldots \end{array} \tag{4}$$

Once we have at least two initial vectors $S^{(0)}$ and $S^{(1)}$, then the VEA can be started to obtain $(r+1)^{th}$ order iterates, which are given by:

$$\varepsilon_{r+1,i}^{(q)} = \varepsilon_{r-1,i}^{(q+1)} + \frac{1}{\left(\varepsilon_{r,i}^{(q+1)} - \varepsilon_{r,i}^{(q)}\right)} \qquad \begin{array}{l} q = 0, 1, 2, \ldots \\ i = 1, 2, \ldots N \end{array} \tag{5}$$

The value of index r in the above equation takes on values of 0, 1, 2, …. In (5), the even order vectors $\{\varepsilon_2^{(q)}\}, \{\varepsilon_4^{(q)}\}, \ldots$ provide estimate of the solution vector X. The computations are stopped when the following condition is met:

$$\frac{\left\|\left(A\tilde{X} - b\right)\right\|_2}{\|b\|_2} \leq NC_f \tag{6}$$

where C_f is a pre-defined convergence factor.

3. Numerical Results

We show the application of VEA in solving the current induced on a thin wire scatterer. The integral equation for the induced current, given by Pocklington's equation, is solved using pulse expansion and triangular testing functions [6].

Fig.1 shows the current induced by a normally incident plane wave ($E_0 = 1\,\text{V/m}$) on a wire scatterer of length $\ell = \lambda/2$, radius $a = \lambda/1000$, frequency $f = 300\,\text{MHz}$ and convergence factor $C_f = 0.01$. The number of pulse expansion functions, N, is 31. We have run the code for higher values of N and obtained results in agreement with those obtained from direct methods. Presently work is in progress to apply the algorithm to solve 2-dimensional (flat plate) and 3-dimensional (conducting sphere) scattering problems using triangular patches in the method of moments..

4.   Conclusion

In this work, we have shown that vector ε-algorithm can be utilized to solve a system of linear equations obtained in a moment method solution of scattering problems. The results for the computation of current distribution on a thin wire scatterer show very good agreement with direct method (matrix inversion). Since this method is iterative, it does not require inversion of a matrix. This can be advantageous if one encounters a singular or ill-conditioned moment method matrix.

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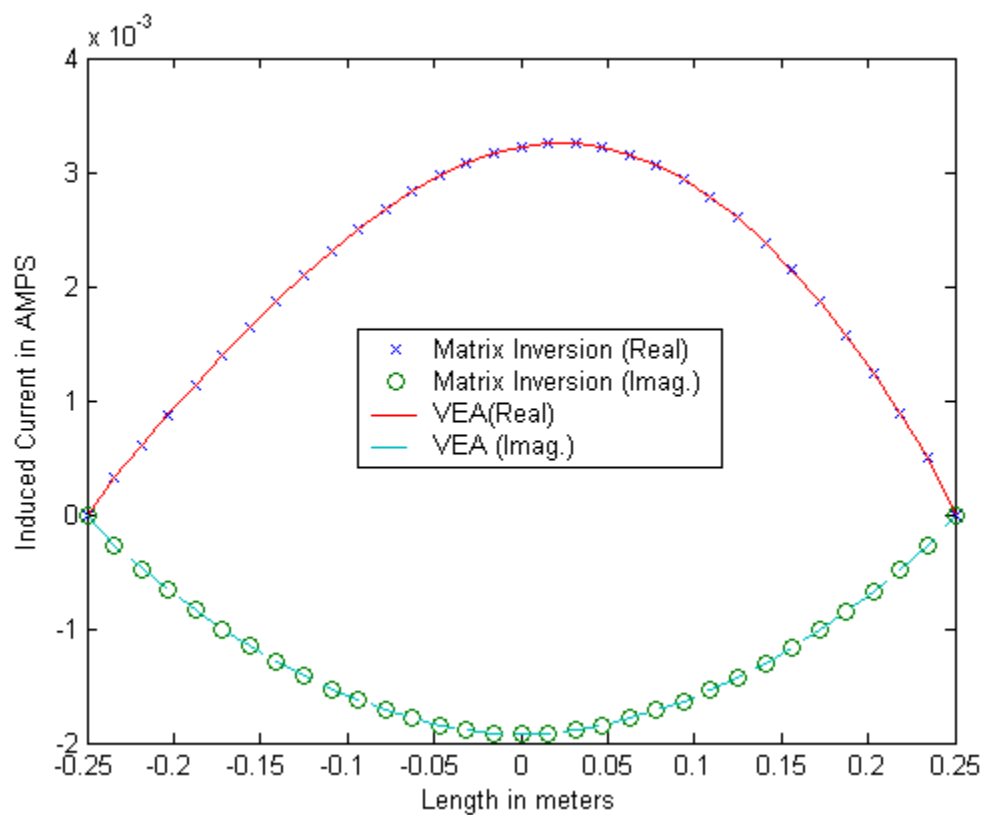
Figure 1:   The current induced on a thin wire of length = $\lambda/2$, radius = $\lambda/1000$ and N = 31.