FDTD Formulations for Scattering from Three Dimensional Chiral Objects

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Abstract

Two finite difference time-domain (FDTD) scattered-field formulations for chiral objects are developed and presented in this paper. The first formulation provides single frequency results, while the second formulation provides multi frequency results from one FDTD simulation. Both formulations are developed for three dimensional electromagnetic applications. Numerical results from the single frequency formulation is presented for the scattering from a chiral sphere. In the multi-frequency formulation, the evaluation of both the electric and magnetic field values at every half-time step, unlike the conventional leap-frog algorithm, is needed to minimize the memory size required to store past values of the field components. Results of this formulation are presented for the co-polarization and cross-polarization of the reflected and transmitted waves from a chiral slab due to normal incidence of a plane wave and for the scattered field from a chiral sphere. Validation is performed by comparing the results with those based on the exact solution.

I. Introduction

The interaction of electromagnetic fields with chiral matters has been studied over the years. Chiral media were used in many applications involving antennas and arrays, antenna radomes, microstrip substrates and waveguides. A chiral object is, by definition, a body that lacks bilateral symmetry, which means that it cannot be superimposed on its mirror image neither by translation nor rotation. This can also be known as handedness. Objects that have the property of handedness are said to be either right-handed or left-handed. Chiral media are optically active – a property caused by asymmetrical molecular structure that enables a substance to rotate the plane of incident polarized light, where the amount of rotation in the plane of polarization is proportional to the thickness of the medium traversed as well as to the light wavelength [1-5]. Chiral medium are therefore has an effect on the attenuation rate of the right hand and left hand circularly polarized waves. Unlike dielectric or conducting cylinders, chiral scatterers produce both co-polarized and cross-polarized scattered fields. Coating with chiral material is therefore attempted for reducing radar cross-section of targets.

The electromagnetic wave propagation in chiral and bi-isotropic media has been modeled by the FDTD technique recently in various studies [6-12]. These studies are based on various assumptions of constitutive relations of bi-isotropic and chiral media. However, non of the published literature provides a complete three dimensional solution for composite scatterers containing chiral objects. In this paper, the scattering of electromagnetic plane wave from three-dimensional chiral scatterers as well as the reflection and transmission from one-dimensional slab are presented. In the first part of this paper a single frequency scattered field FDTD formulation is developed for the chiral media and calculated scattered fields from a chiral sphere are presented. In the second part of this paper a new formulation is introduced where the frequency term in the constitutive relations is replaced by complex differentiation and the new FDTD updating equations are discretized by the second order accurate finite difference schemes. An additional storage of the field values at a previous one and one half time steps are required in this FDTD formulation. Furthermore, unlike the conventional leap-frog FDTD techniques, both the electric and magnetic field values need to be calculated at every half time step. This FDTD formulation is used to

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calculate the co-polarization and cross-polarization of the reflected and transmitted waves from a chiral slab and a chiral sphere due to an incident plane wave. Very good agreements are observed while comparing the numerical results based on these new formulations and the corresponding values based on the exact solutions for these canonical problems.

II. Single Frequency FDTD Formulation for Chiral Media

Starting from the constitutive relations for a chiral media, one can write:

$$\overline{D} = \varepsilon\overline{E} - j\chi\sqrt{\mu_0\varepsilon_0}\,\overline{H} \quad , \quad \overline{B} = \mu\overline{H} + j\chi\sqrt{\mu_0\varepsilon_0}\,\overline{E} \tag{1}$$

where χ is the chirality parameter and e^{jwt} is the assumed time harmonic convention. Using (1) in the complex Maxwell curl equations

$$\nabla \times \overline{E} = -j\omega\overline{B} - \sigma^m\overline{H} \quad , \quad \nabla \times \overline{H} = +j\omega\overline{D} + \sigma^e\overline{E} \tag{2}$$

we get

$$\nabla \times \overline{E} = -j\omega\mu\overline{H} + \omega\chi\sqrt{\mu_0\varepsilon_0}\,\overline{E} - \sigma^m\overline{H} \tag{3}$$

$$\nabla \times \overline{H} = +j\omega\varepsilon\overline{E} + \omega\chi\sqrt{\mu_0\varepsilon_0}\,\overline{H} + \sigma^e\overline{E} \tag{4}$$

Now assuming a constant frequency $\omega = \omega_o$, then we can write (3) and (4) in time domain as:

$$\nabla \times \overline{E} = -\mu\frac{\partial\overline{H}}{\partial t} + \omega_o\chi\sqrt{\mu_0\varepsilon_0}\,\overline{E} - \sigma^m\overline{H} \tag{5}$$

$$\nabla \times \overline{H} = \varepsilon\frac{\partial\overline{E}}{\partial t} + \omega_o\chi\sqrt{\mu_0\varepsilon_0}\,\overline{H} + \sigma^e\overline{E} \tag{6}$$

One should notice that the use of equations (5) and (6) in a FDTD updating equations will be valid at only $\omega = \omega_o$. After decomposing the total fields into the incident and scattered field components and discretizing (5) and (6) we can obtain the FDTD updating equations in the following form

$$E_{scat,x}^{n+1}(i,j,k)\left[\frac{1}{\Delta t} + \frac{\sigma_x^e}{2\varepsilon_x(m)}\right] = E_{scat,x}^n(i,j,k)\left[\frac{1}{\Delta t} - \frac{\sigma_x^e}{2\varepsilon_x}\right] + \frac{1}{\varepsilon_x}\left[\frac{H_{scat,z}^{n+1/2}(i,j,k) - H_{scat,z}^{n+1/2}(i,j-1,k)}{\Delta y}\right]$$

$$-\frac{1}{\varepsilon_x}\left[\frac{H_{scat,y}^{n+1/2}(i,j,k) - H_{scat,y}^{n+1/2}(i,j,k-1)}{\Delta z}\right]$$

$$-\frac{\sigma_x^e}{2\varepsilon_x}\left[E_{inc,x}^{n+1}(i,j,k) + E_{inc,x}^n(i,j,k)\right]$$

$$-\frac{(\varepsilon_x - \varepsilon_0)}{\varepsilon_x}\left[\frac{E_{inc,x}^{n+1}(i,j,k) - E_{inc,x}^n(i,j,k)}{\Delta t}\right]$$

$$-\frac{\omega_o\chi\sqrt{\mu_0\varepsilon_0}}{\varepsilon_x}\left[H_{scat,x}^{n+1/2}(i,j,k) + H_{inc,x}^{n+1/2}(i,j,k)\right]$$

$$\tag{7}$$

The updating equations for the other field components follow the same procedure. The problem shown in Fig. 1 is constructed for the validation of this new formulation. An x-polarized, z traveling plane wave of frequency 1GHz, is incident on a sphere of radius 7.2 cm and relative permittivity = 4. The center of the sphere is coinciding with the origin of the coordinates system. The far fields at 100 m are calculated and

the magnitude of $\overline{E}_\theta$ in the x-z plane is plotted in Fig. 2 for $\chi = 0$ (dielectric) and $\chi = 0.5$ (chiral). The calculated results are compared to the exact analytical solution of the scattering by a sphere [13,14].
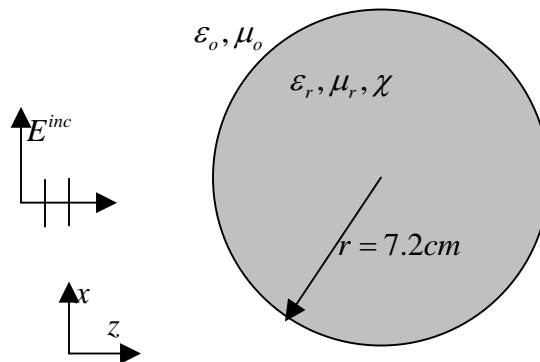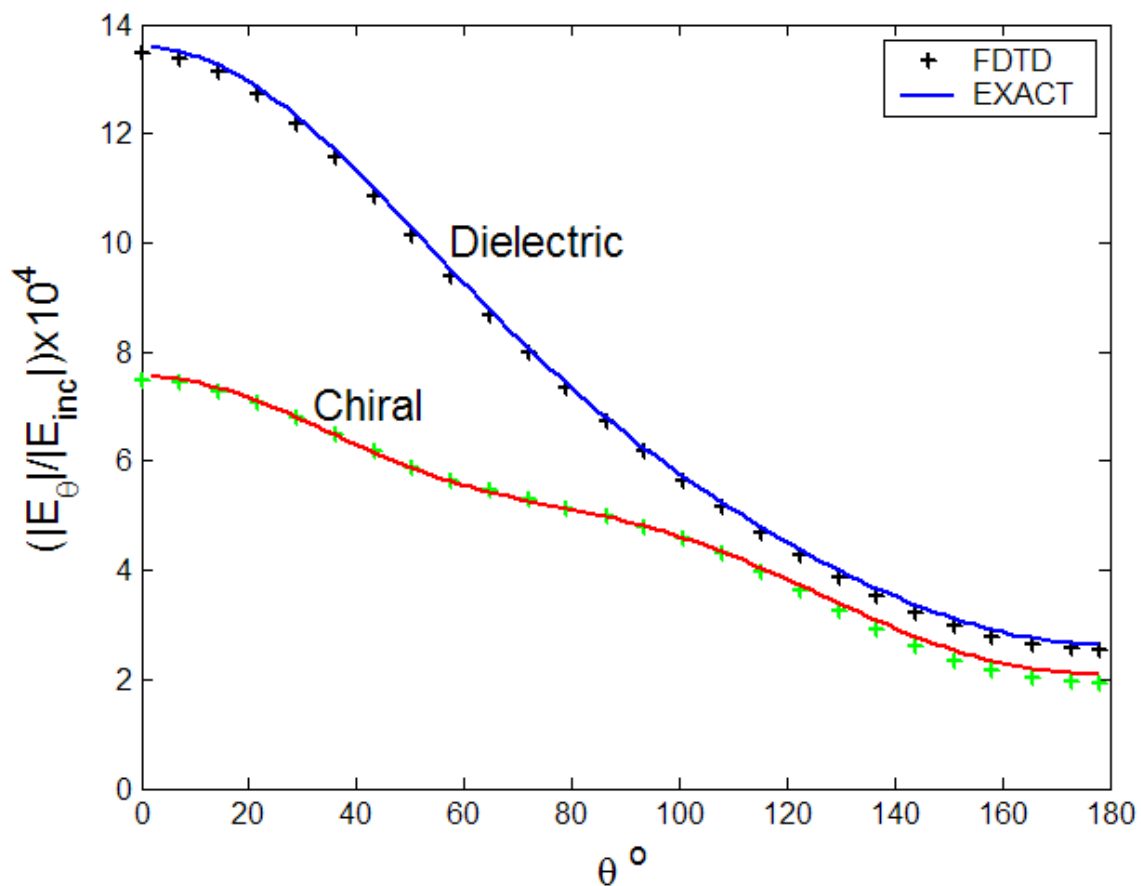

Fig. 1 Chiral sphere in free space.

Fig. 2 Scattering from a sphere, $\chi = 0$ (dielectric), $\chi = 0.5$ (chiral).

III. Multiple Frequency FDTD Formulation for Chiral Media

Starting from equations (3-4) and using the substitutions $j\omega = \dfrac{\partial}{\partial t}$ and $\omega = -j\dfrac{\partial}{\partial t}$ we obtain

$$\nabla \times \overline{E} = -\mu \frac{\partial \overline{H}}{\partial t} - j\chi\sqrt{\mu_0\varepsilon_0}\frac{\partial \overline{E}}{\partial t} - \sigma^m \overline{H} \tag{8}$$

$$\nabla \times \overline{H} = \varepsilon \frac{\partial \overline{E}}{\partial t} - j\chi\sqrt{\mu_0\varepsilon_0}\frac{\partial \overline{H}}{\partial t} + \sigma^e \overline{E} \tag{9}$$

Here the assumption of time harmonic fields is used and hence the values of the electric and magnetic fields are complex. After applying the second order central difference approximation to the derivatives, these equations reduce to the following form

$$\frac{\mu_x}{\Delta t}H_{scat,x}^{n+1}(i,j,k) + \frac{j\chi\sqrt{\mu_0\varepsilon_0}}{\Delta t}E_{scat,x}^{n+1}(i,j,k) = \frac{\mu_x}{\Delta t}H_{scat,x}^{n}(i,j,k) + \frac{j\chi\sqrt{\mu_0\varepsilon_0}}{\Delta t}E_{scat,x}^{n}(i,j,k)$$

$$\frac{1}{\Delta z}\left[E_{scat,y}^{n+0.5}(i,j,k+1) - E_{scat,y}^{n+0.5}(i,j,k)\right] - \frac{1}{\Delta y}\left[E_{scat,z}^{n+0.5}(i,j+1,k) - E_{scat,z}^{n+0.5}(i,j,k)\right]$$

$$+\frac{(\mu_o - \mu_x)}{\Delta t}\left[H_{inc,x}^{n+1}(i,j,k) - H_{inc,x}^{n}(i,j,k)\right] - \frac{j\chi\sqrt{\mu_0\varepsilon_0}}{\Delta t}\left[E_{inc,x}^{n+1}(i,j,k) - E_{inc,x}^{n}(i,j,k)\right]$$

$$-\sigma_x^m\left[H_{inc,x}^{n+0.5}(i,j,k) + H_{scat,x}^{n+0.5}(i,j,k)\right] \tag{10}$$

$$-\frac{\varepsilon_x}{\Delta t}E_{scat,x}^{n+1}(i,j,k) + \frac{j\chi\sqrt{\mu_0\varepsilon_0}}{\Delta t}H_{scat,x}^{n+1}(i,j,k) = -\frac{\varepsilon_x}{\Delta t}E_{scat,x}^{n}(i,j,k) + \frac{j\chi\sqrt{\mu_0\varepsilon_0}}{\Delta t}H_{scat,x}^{n}(i,j,k)$$

$$+\frac{1}{\Delta z}\left[H_{scat,y}^{n+0.5}(i,j,k) - H_{scat,y}^{n+0.5}(i,j,k-1)\right] - \frac{1}{\Delta y}\left[H_{scat,z}^{n+0.5}(i,j,k) - H_{scat,z}^{n+0.5}(i,j-1,k)\right]$$

$$+\frac{(\varepsilon - \varepsilon_o)}{\Delta t}\left[E_{inc,x}^{n+1}(i,j,k) - E_{inc,x}^{n}(i,j,k)\right] - \frac{j\chi\sqrt{\mu_0\varepsilon_0}}{\Delta t}\left[H_{inc,x}^{n+1}(i,j,k) - H_{inc,x}^{n}(i,j,k)\right]$$

$$+\sigma^e\left[E_{inc,x}^{n+0.5}(i,j,k) + E_{scat,x}^{n+0.5}(i,j,k)\right] \tag{11}$$

Denoting that the right hand side of the equations (10) and (11) as M_x and N_x, respectively, we can come up with two coupled equations for the new values of E_x and H_x field components. The solution of these two equations leads to the final updating equations for E_x and H_x as

$$E_{scat,x}^{n+1}(i,j,k) = \frac{\Delta t\mu_x}{\varepsilon_x\mu_x - \chi^2\mu_0\varepsilon_0}\left[\frac{j\chi\sqrt{\mu_0\varepsilon_0}}{\mu_x}M_x - N_x\right] \tag{12}$$

$$H_{scat,x}^{n+1}(i,j,k) = \frac{\Delta t\varepsilon_x}{\varepsilon_x\mu_x - \chi^2\mu_0\varepsilon_0}\left[M_x + \frac{j\chi\sqrt{\mu_0\varepsilon_0}}{\varepsilon_x}N_x\right] \tag{13}$$

One should notice that at every half time step new values of both the electric and magnetic field components should be calculated simultaneously. The storage of the field values of the previous one and half time steps are required in this formulation. Although time harmonic fields are

assumed in this formulation, the FDTD results from such formulation, due to band limited signals, are found to be accurate as will be shown in the following examples.

Reflection and Transmission from a One Dimensional Chiral Slab

Consider a Gaussian plane wave normally incident on a chiral slab as shown in Fig. 3. The co- and cross-polarization of the reflected waves are denoted by Γ_{co} and Γ_{cr}, while the co- and cross-polarization of the transmitted waves are denoted by T_{co} and T_{cr}. Figure 4 shows the Γ_{co} and T_{co} for a dielectric slab with $\varepsilon_r = 2$, and $\chi = 0$. The cross- polarization components are zero as expected. Figure 5 shows the Γ_{co} and T_{co} for $\varepsilon_r = 2$, and $\chi = 0.3$ while Fig. 6 shows the corresponding Γ_{cr} and T_{cr} for this chiral slab. Very good agreement, across a wide range of frequencies, can be observed compared to the exact results of this simple problem.
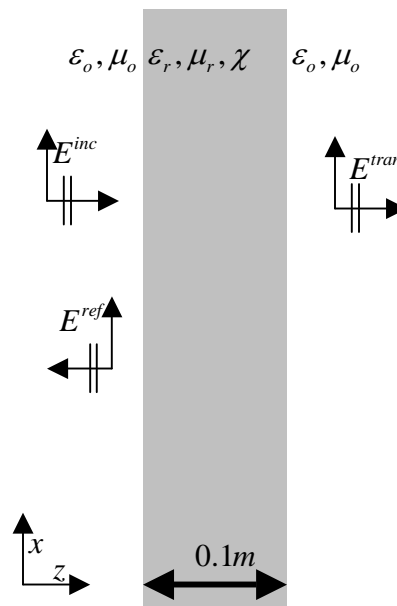

Fig. 3 Chiral slab and directions of the incident, reflected, and transmitted fields.
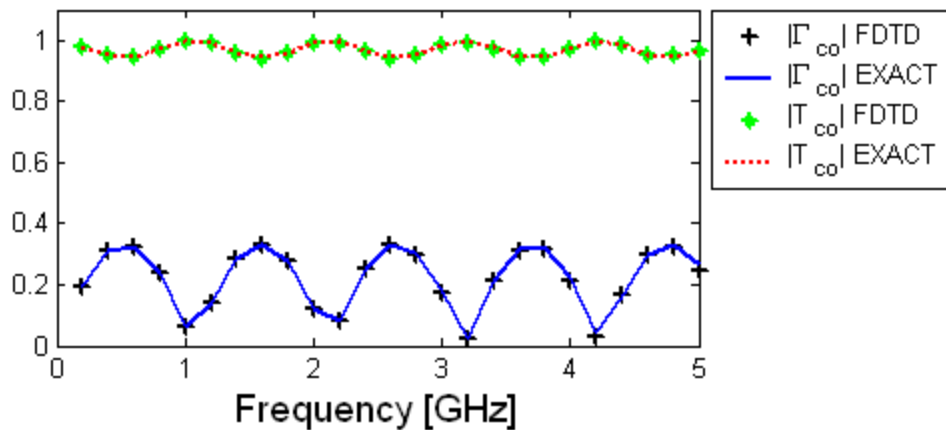

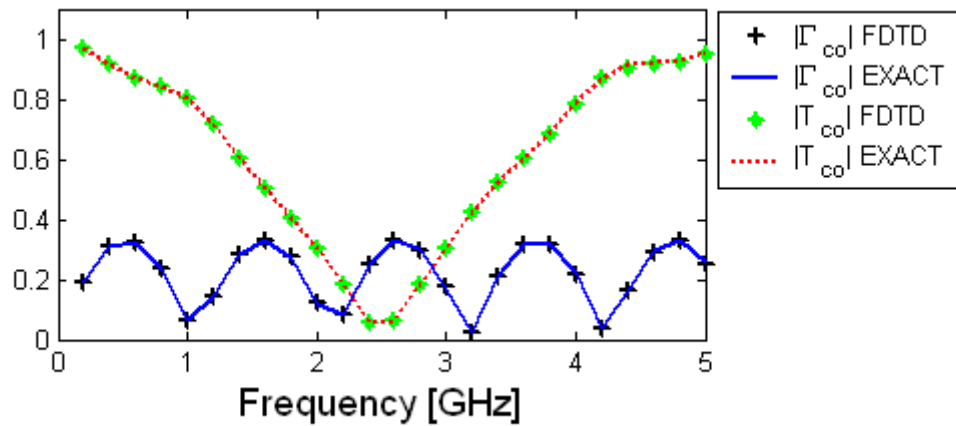Fig. 4 Co polarization of reflected and transmitted waves from a dielectric slab, $\varepsilon_r = 2, \chi = 0$.

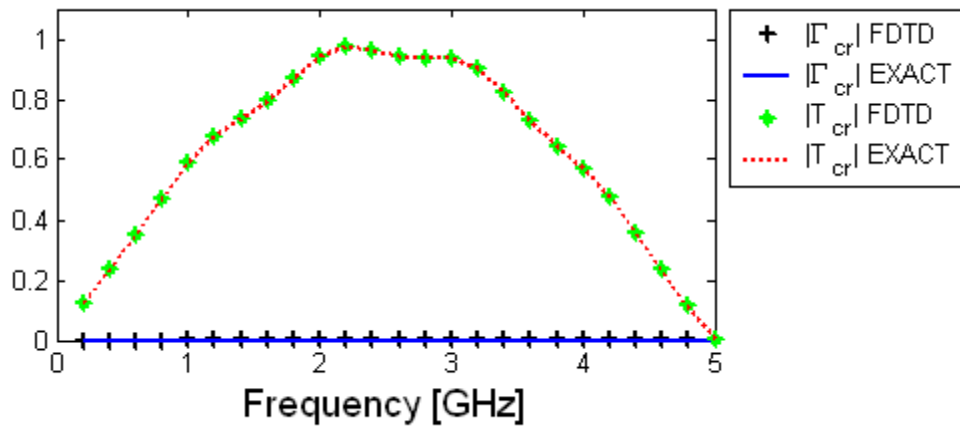Fig. 5 Co-polarization of reflected and transmitted waves from a chiral slab, $\varepsilon_r = 2, \chi = 0.3$.

Fig. 6 Cross-polarization of reflected and transmitted waves from a chiral slab, $\varepsilon_r = 2, \chi = 0.3$.

Scattering from a Chiral Sphere

The multi frequency formulation is applied to the scattering problem shown in Fig. 1. A plane wave with a Gaussian waveform traveling in the +z direction and incident on a chiral sphere of radius = 7.2 cm, relative permittivity = 4, and $\chi = 0.5$. The scattered fields at 100 m from the center of the sphere in the x-z plane are calculated. The computed numerical results from one FDTD simulation are compared to those based on the exact solution calculated at different frequencies. Figure 7 shows this comparison at four different frequencies, 0.2 GHz, 0.5 GHz, 0.8 GHz, and 1 GHz.
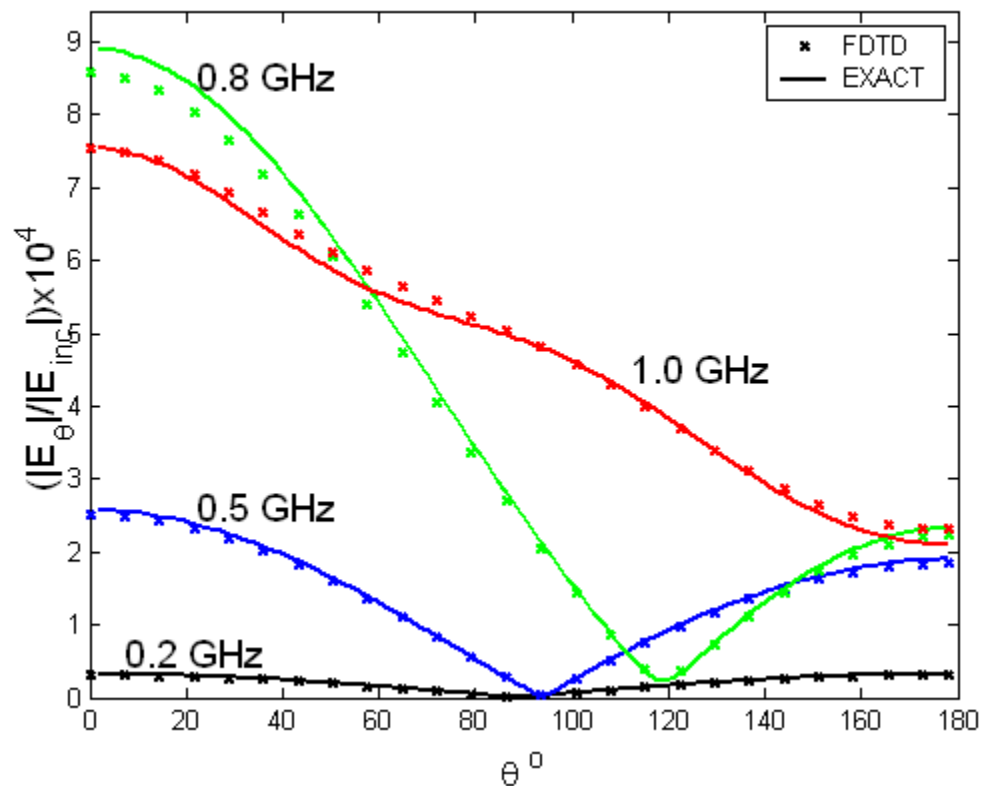
Fig. 7 Scattering from a chiral sphere, $\varepsilon_r = 4$, $\chi = 0.5$.

Conclusions

FDTD formulations for electromagnetic scattering from chiral objects in one and three dimensional domains are presented. The developed formulations are shown to provide accurate results for single and multiple frequencies for both dielectric and chiral objects.

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