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6. AUTHOR(S)
Adrian Wheelock
David L. Cooke
Nikolaos A. Gatsonis*

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Force Reseearch Laboratory/VSBXR
29 Randolph Road
Hanscom AFB, MA 01731-3010

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*Worcester Polytechnic Institute, Mechanical engineering Department, Worcester, MA

14. ABSTRACT
Neutralization of ion beams in electric propulsion applications is well-known phenomenon. The physics behind the robust matching of ion and electron currents and densities, are not. As electric propulsion devices move into micro and macro regimes with colloids, FEEPs, and thruster arrays, thruster-neutralizer interactions are under increasing scrutiny. A series of 2D simulations using PIC codes are presented, detailing starting and steady state interactions. It is shown that the expectations of full current coupling of electrons to the beam is not seen, but there is similar coupling behavior exhibited. Numerical mechanisms for coupling the current to the beam are examined for effect on coupling behavior.

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COMPUTATIONAL ANALYSIS OF CURRENT COUPLING OF ION BEAM-NEUTRALIZER INTERACTIONS

Adrian Wheelock* and David L. Cooke.†
Air Force Research Laboratory, Space Vehicles Directorate, Hanscom AFB, MA, 01731

Nikolaos A. Gatsonis‡
Mechanical Engineering Department, Worcester Polytechnic Institute, Worcester, MA 01609

Neutralization of ion beams in electric propulsion applications is a well-known phenomenon. The physics behind the robust matching of ion and electron currents and densities, are not. With electric propulsion devices moving into micro and macro regimes with colloids, FEEPs, and thruster arrays, thruster-neutralizer interactions are under increasing scrutiny. A series of 2D simulations using PIC codes are presented, detailing starting and steady state interactions between an ion and electron beam. A parametric investigation of ion beam and electron beam interactions is given in 2-D. It is shown that starting conditions require careful matching of currents to propagate without space charge effects while steady state conditions are robust regardless of ion or electron currents. Investigation of exiting electron currents shows no preference for electron-ion current coupling.

I. Introduction

Ion beam neutralization during operation of electric propulsion devices requires both current and charge density matching with an emitted electron beam. This current coupling is easily accomplished in reality, yet the exact process remains unknown. Currently an "effective collision frequency" that binds electrons to the ion beam describes the neutralization process. As electric propulsion becomes more prevalent and new regimes of electric propulsion are explored in space missions, this matter garners significant importance. Proper modeling of current coupling and neutralization will enable development of low-current neutralizers and optimization of neutralizers for micropropulsion devices and clusters of engines. Explanation of the effective collision frequency also has bearing on space instrument calibration, electrodynamic tethers, and ionospheric research.

A dense ion beam requires space charge neutralization to avoid a potential barrier that can divert or reflect the beam. The vehicle on which the thruster operates needs current neutrality to avoid unwanted charging. In the context of collisionless plasma theory, achieving both current and charge neutrality with the same source of electrons appears to be nearly impossible owing mostly to the large difference in mass between electrons and the ions. For example, define the ion flux, $F_i = N_i v_i$, and the net electron flux, $F_e = \frac{1}{4} N_e v_{eth}$, where N is density, v is velocity, i and e are ion and electron subscripts and eth designates the electron thermal velocity for an idealized electron source. Equal density and flux requires $v_{eth} = 4v_i$. A 1 keV Xenon beam has v_i=38,000 m/s so a matching electron velocity requires a source temperature of about 0.05 eV. A challenging, but not impossible number, but a collisionless analysis suggests careful balancing is required, whereas real systems quite easily achieve 'beam coupling.' Of course a higher temperature, lower density electron source will lead to a positive potential well that does trap electrons, but then the theory must explain by what process the trapped electrons shed energy so as to actually fill the well. Another observation, presented in more detail below, is that when ion beams and neutralizers

* Electronics Engineer, Battlespace Environment Division, Space Weather Center of Excellence, Ph.D. Candidate, WPI, Student Member AIAA.
† Tech Advisor, Battlespace Environment Division, Space Weather Center of Excellence, Senior Member AIAA.
‡ Associate Professor, Mechanical Engineering Department, Senior Member AIAA.

are operated in conducting vacuum tanks, the currents are closely coupled even though the grounded tank eliminates the charge accumulation that could provide feedback for current balance so it appears that one or more plasma mechanisms must be responsible for this collective phenomena -- charge and current neutrality – which we hereafter call current coupling.

Our immediate goal is to determine if what might be considered standard Particle-In-Cell (PIC) techniques are adequate to understand and capture the beam coupling process, or if additional treatment is needed. In this paper we present first a brief review of neutralization studies. Next we present some experimental data to illustrate our ideas, followed by a series of parameter studies looking at the neutralization of beams in 2D presented with analysis.

II. History of EP Neutralization

Electric propulsion plumes need to be properly mixed with electrons or else severe space-charge effects would result. Before the first space tests, serious doubts lingered as to the stability of any neutralization approach to the ion beam. The general idea was to neutralize the beam shortly after emission, preventing beam slowdown or return. A lack of understanding of electron trapping mechanisms and stability of these mechanisms brought about significant research activity. The Ramo-Wooldridge staff performed the first comprehensive review of the entire electrostatic thruster problem in 1960[1]. Their one-dimensional investigation into neutralization was admittedly unrealistic enough to provide a satisfactory indication as to the stability and practicality of neutralization.

Over the next few years, many theorists who looked at 1- and 2-D models predicted growing instabilities that could turn the beam back to the spacecraft. Some research pointed towards the possibility of neutralization, such as French[2] and Mirels[3]. Other work pointed towards potential problems, such as Seitz et al.[4] Some of the earliest computational studies were brought to bear on the problem, and Buneman and Kooyers[5], using a one-dimensional code in 1963, were able to provide a neutralized beam when electrons were injected at energies lower than the directed ion energy and velocities on the same order. Fluctuations in the space charge field provided mixing of the beam. Two years later Wadhwa et al.[6] performed a two-dimensional PIC simulation showing that electrons would oscillate within the beam to allow for neutralization, but theorized the oscillations were not the only mechanism at work. One method suggested was that fluctuations in the space-charge field allowed for entropy increase to mix the electrons, but these fluctuations were not found far downstream of the neutralizer, suggesting a collective cooling mechanism. Work thus far looked only at space charge neutralization, not current neutralization.

The 1964 Space Electric Rocket Test I (SERT I) and its successor SERT II in the 1970s found that it was quite easy to neutralize ion beams in space using straightforward neutralizer geometry. In a series of tests it was shown that the ion thruster developed thrust at a level indicating complete beam neutralization. This indicated that the ion and electron velocities were matching so that current coupling was happening without impact on vehicle potential or thrust produced. After SERT I, proof of concept was achieved and the theoretical discussion of beam neutralization dwindled in favor of engineering new thrusters.

Interest in the topic was kept alive by Parks, Katz, and Mandell[7,8], which led to the present method of using an effective collision frequency to trap electrons in potential wells. A small resurgence of interest has occurred recently, with a few numerical simulations performed recently, including Othmer et al.[9,10,11] using a relativistic 3D PIC simulation and Tajmar and Wang[12] investigating FEEP neutralizer placement. Othmer suggested that electrons reflected from the ends of the beam therefore eventually matching velocities, but this does not explain why current coupling can be observed in a vacuum chamber, where the beam is nominally stationary and bounded. Tajmar was not investigating the coupling effect directly. Work in the nuclear fusion community has recently investigated pulsed plasma beams being neutralized by background plasma[13], but the high powers and densities involved make a direct connection difficult.

Despite decades of research and the broad acceptance of electric propulsion devices today, full understanding of the process by which an ionized beam is neutralized in space is still unknown. Assorted methods to fit data with theory have been found, but the actual process has yet to be studied in sufficient detail to fully understand the subject. Additionally, electric micropropulsion devices such as FEEP or colloidal thrusters as well as large arrays of ion and Hall thrusters are still not guaranteed to function as expected. We also desire a means to predict and optimize neutralizer operations, especially for the extremes of micropropulsion and clusters of thrusters. Thus, a simulation technique exhibiting beam coupling is needed. Results from ion beam neutralization modeling will also be applicable to ion beams for instrument calibration, electrodynamic tethers, ionospheric research, and fundamental plasma physics.

III. Current Coupling Observations in a Vacuum Tank

In order to observe this phenomenon directly, we utilized the JUMBO large vacuum facility (2 meter diameter) at AFRL/Hanscom. A 3-cm Veeco ion source, functionally identical to an electrostatic ion engine, was installed. This source used a hot tungsten wire placed across the beam to provide neutralization, although in a vacuum tank it is not required for operation due to neutralization at the conducting wall. The ion source was able to run on a wide variety of gases; for these tests we used nitrogen.

The controller enabled accurate control of beam (extractor) voltage to 1V, neutralizer filament current to 10mA and measurement of beam (extracted) current and neutralizer emission current to an accuracy of 1mA. With other settings shown in Table 1, three tests were performed, one with the beam voltage set to zero, one at 450 V and one at 800 V. In each test, the heater current was increased from zero to 3.50 A, a level sufficient for emitted current saturation, then brought back to zero. The results can be seen in Fig. 1, 2, and 3.

Table 1: Ion Source Settings

Cathode Current	3.00 A
Discharge Voltage	55.0 V
Accelerator/Beam Current Limit	20 %
Background Pressure	5E-4 Torr

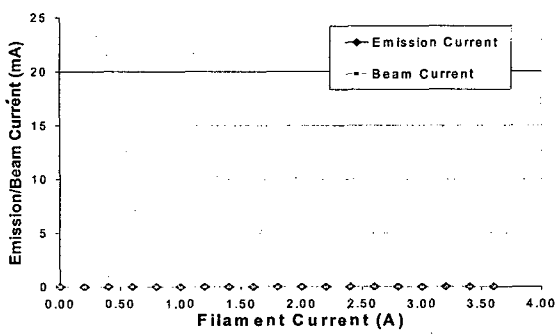

Figure 1: Currents with no ion beam

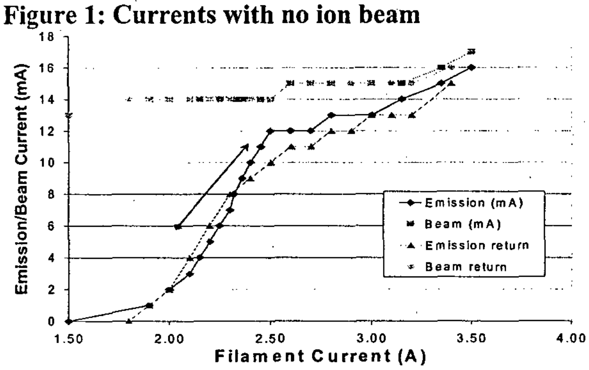

Figure 2: Currents with 450V ion beam

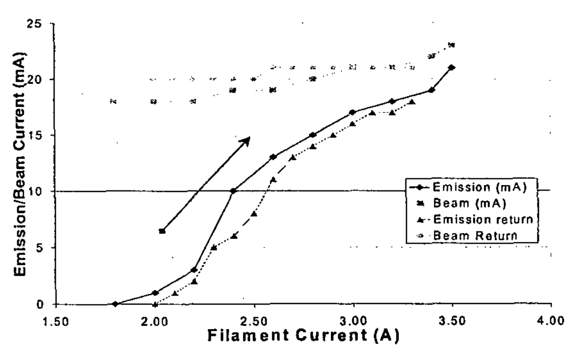

Figure 3: Currents with 800V ion beam

As expected, without an ion beam present, even though the filament current was at over 3 A, the filament emission current saturated at less than 0.1mA. Once a beam was provided, however, the emission current quickly rose with increasing filament current to near the beam current, though never quite reached it. The increase in beam current with increasing filament emission current we theorize is due to backstreaming electrons from the filament. The gap between neutralizer and beam current may be due to charge-exchange ions due to the high background pressure at which we operated as well as electrons released from chamber walls partially neutralizing the beam.

IV. 2-D Results

Simulations in 2-D have provided a quick method to enable parameter studies without the complexities and hassle of a 3-D code, at the cost of giving up realistic geometries. For our studies, we have used XOOPIC[14], an object-oriented code that is easily available off the web from the Berkeley Plasma Theory and Simulation Group.

Previous work by the authors has established there are two cases that XOOPIC is capable of performing. The "filling" case is that of a beam expanding into a vacuum, while the "chamber" case is a beam propagating across a bounded domain that is grounded. Since current coupling can be observed in a chamber, as demonstrated above, we focus on the "chamber" case. Discussion of the filling case can be found in Wheelock et al. [15,16]

In the chamber case, if current coupling is modeled by standard PIC, then there should be a preference for electron flow in the direction of ion flow. Utilizing the ability of computer code to manipulate the ions, we can determine if there is an effect of ion motion or numerical parameters on the electrons. The easiest way to measure this is through the particle flux through either side. If a bias exists that is created by ion motion, it would be evident by comparing the number of electrons leaving each side. Electrons are injected from each end to avoid any bias in the collected current created by ballistic electrons.

A manipulation available only in simulation is the "freezing" and "unfreezing" of ions through use of ion subcycling. By doing this we can isolate ion motion effects. A quick comparison of two simulations using otherwise identical 1keV Xenon ion beams, one frozen, one with mobile ions, creates a slight difference in the flux of electrons through each side, on the order of 5%. A beam with frozen ions has matching fluxes out either end. To further examine this, we took a beam with frozen ions and allowed it to subcycle once during a simulation. The ions and electrons were loaded with a cold quiet start, so before the ions move, there is zero motion in the simulation as seen in Fig. 5. After the ions cycle, the electrons are set in motion with the same 5% bias to the downstream side by induced fluctuations in the electric field. This shows some coupling is observed, but the question becomes, "Is this what is expected in PIC?"

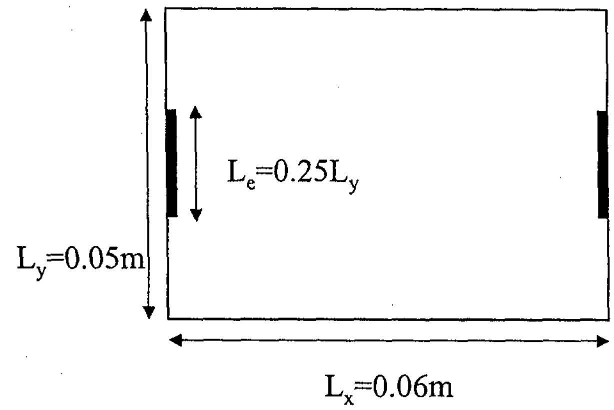

Figure 4: 2-D Simulation Domain

To answer this question, we begin by looking at the drifting 1-D Boltzmann-Maxwell distribution function. For now, we assume that the drift is caused by a complete coupling of the electrons to the beam, indicating $v_{drift} = v_{beam}$.

$$f_e(c) = \left(\frac{m_e}{2\pi kT}\right)^{\frac{1}{2}} \exp\left(-\frac{m_e(c-u)^2}{2kT}\right) \tag{1}$$

Multiplying by the thermal velocity and integrating over the halves of velocity space gives us the flux either in the direction of the drift (beam), or opposite it.

$$\int_0^\infty c f_e(c)\,dc = -\frac{1}{2}\left(\frac{a}{\pi}\right)^{1/2}\left(\sqrt{a} + u\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{a}u\right) a\exp\left(au^2\right) + u\sqrt{\pi}a\exp\left(au^2\right)\right)\Big/ a^{3/2}\exp\left(au^2\right). \tag{2}$$

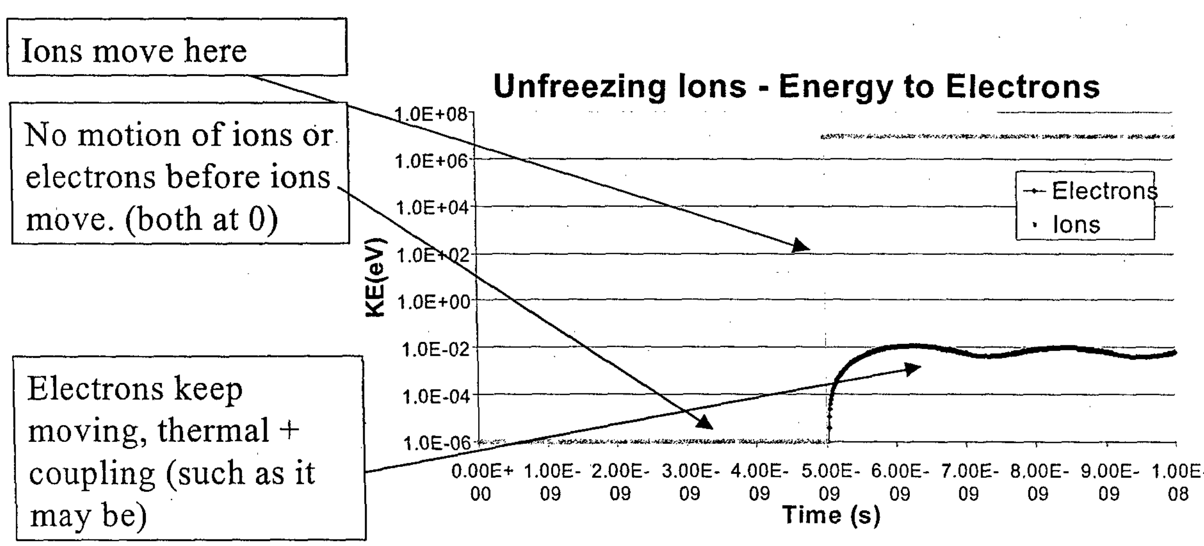

Figure 5: Ion/Electron energy before/after ion timestep.

where $a = m_e/2\pi kT$. This gives us a ratio of

$$\frac{R}{L} = -\frac{u\sqrt{\pi}a\exp\left(au^2\right)^4 + \sqrt{a} + u\sqrt{\pi}\operatorname{erf}\left(\sqrt{a}u\right)a\left[\exp\left(au^2\right)\right]^4}{-u\sqrt{\pi}a\exp\left(au^2\right)^4 + \sqrt{a} + u\sqrt{\pi}\operatorname{erf}\left(\sqrt{a}u\right)a\left[\exp\left(au^2\right)\right]^4} \tag{3}$$

Comparison of this result to the fluxes generated by XOOPIC can provide a basis for comparison of the observed coupling compared to expected coupling. This analysis provides expected electron motion based on the beam energy, which is easily tested in simulation. Fig. 6 shows the simulated and projected flux ratios and Fig. 7 shows the fraction of electrons exiting through the downstream side. As can be clearly seen, the simulations provide the same basic behavior, but at a drastically reduced rate. This may indicate incomplete coupling or a numerical effect based on a gridded electrostatic potential.

To eliminate possible numerical effects, a series of parameter studies was conducted, examining the effects of duration of simulation, particle weighting, cell size, and length of the simulation domain. While these do show some variation, the effects mostly point to modes of the problem rather than distinct numerical effects.

Varying the particles per cell does not seem to have an appreciable effect at either 100eV or 1keV ion beam energies as seen in Fig. 8. Some potential modes are presented, but none that significantly affect the coupling observed. A similar effect is seen in Fig. 9, where modes are visible but the duration of the simulation only seems to enhance the low-energy ions. Fig. 10 shows the effect of domain size on the simulation, again with no significant variation in coupling rate.

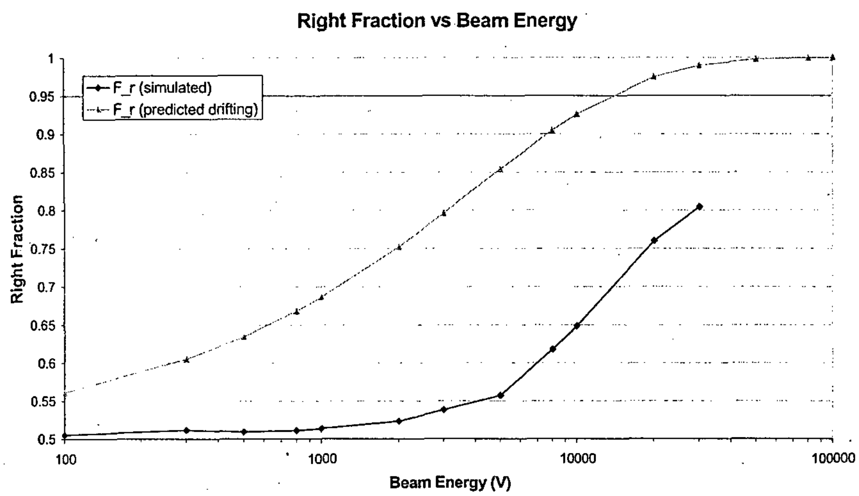

Figure 6: Right Fraction vs. Ion Beam Energy, simulated and calculated. Predictions assume ideal current coupling.

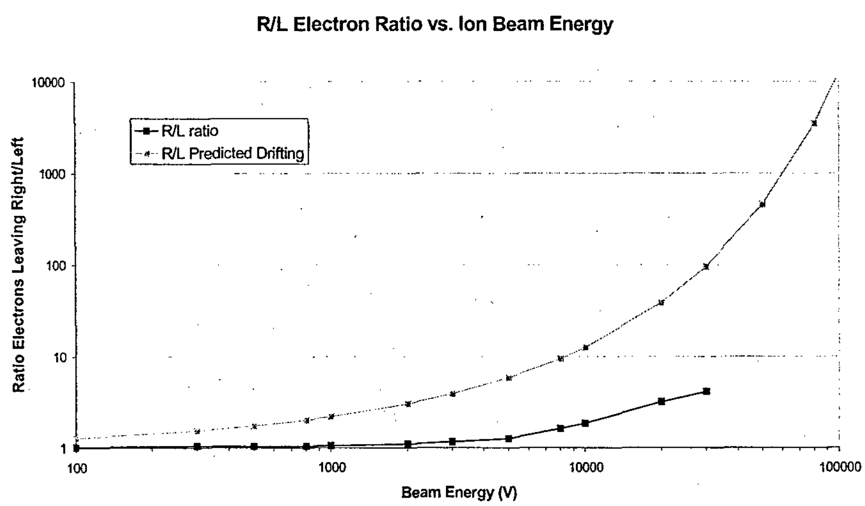

Figure 7: Electron Ratio vs. Ion Beam Energy, simulated and calculated. Predictions assume ideal current coupling.

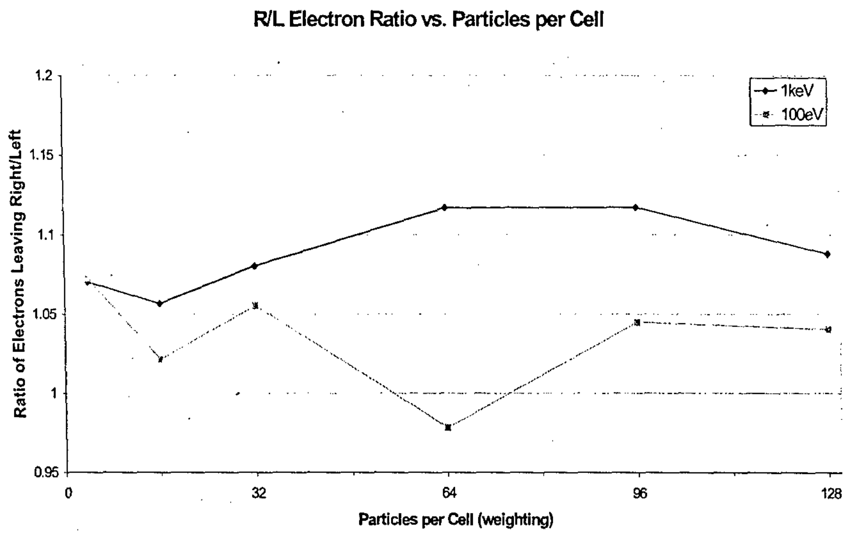

Figure 8: Weighting effects on electron ratio.

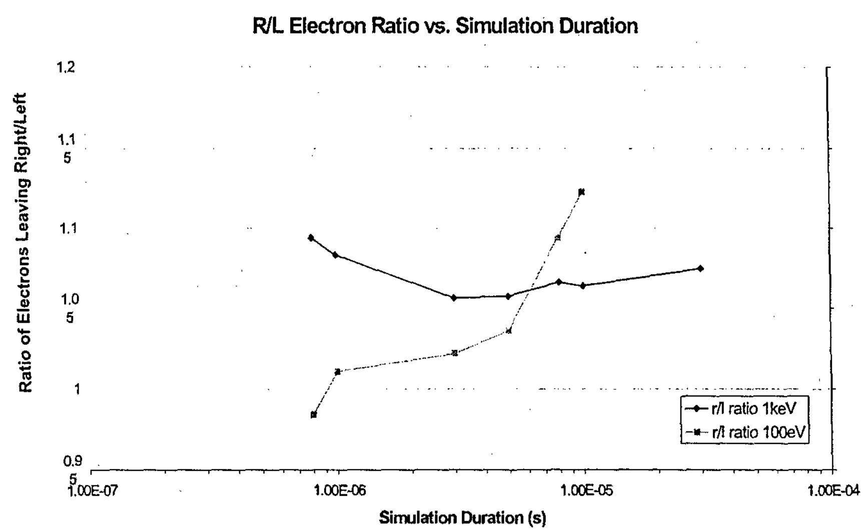

Figure 9: Electron Ratio vs Simulation Duration

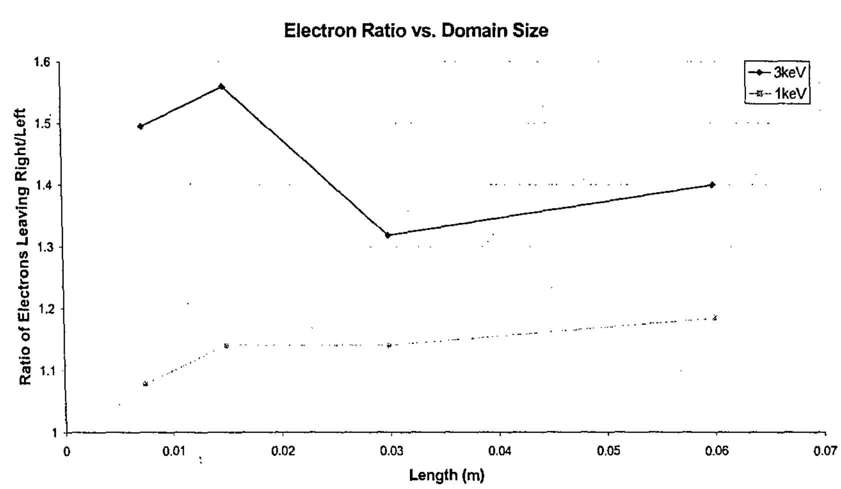

Figure 10: Electron Ratio vs. Domain Size

Fig. 11 shows the effects of cell size on the simulation while keeping the number of particles per cell constant. Since particle weighting was shown to have a negligible effect, the strong coupling of the beam to smaller cell size indicates that there is a coupling factor that is affected by the gridded electric field. Whether this is due to enhanced fluctuations by creating more cell boundary crossings and therefore disrupting the phase-space population in each cell or through reweighting of ion charge to the grid, the ion motion is still minimal, with at most a handful of crossings each timestep. An alternate explanation could center on better resolution of coulomb collisions and increased particle interaction. A curious note is the sudden drop in the low-energy coupling at 1024 cells, potentially indicating a coupling mode or instability.

V. Discussion of 2-D Coupling

While it is clear that there is some coupling observed in the simulations, it does not correspond to the calculated effects of full coupling as described above. By examining the simulated ratios, we can come to the conclusion that if some fraction of the beam is coupling, $f_c = n_c/n_0$, the ratio of the simulated curve to that predicted by full coupling would scale as

$$\frac{n_c}{n_0} = f_c = \frac{f_{r,simulated} - 1/2}{f_{r,predicted} - 1/2} \quad (4)$$

This examination leads us to the rough estimate that we are seeing 5-15% of the electrons coupling with the ion beam across most of the energy range investigated, with greater coupling as the ions increase in energy.

Still unanswered is the question of the method by which the electrons are being accelerated along the beam. Several possibilities come quickly to mind, including velocity drag and the two-stream instability.

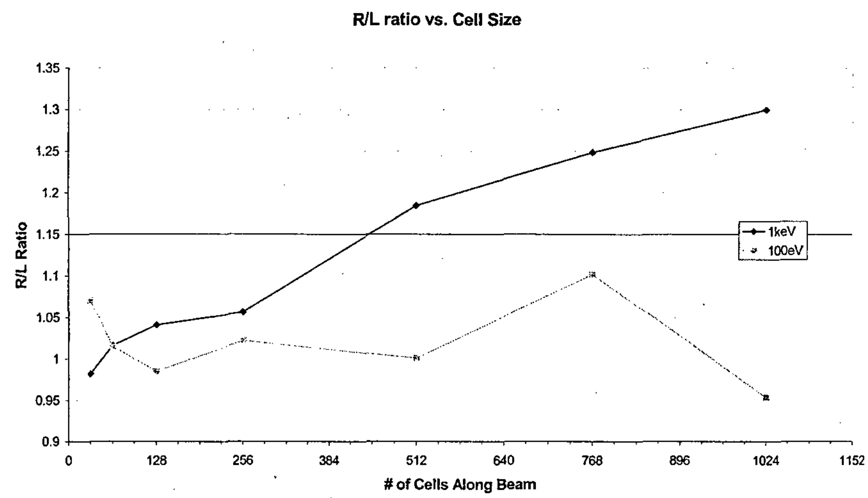

Figure 11: Electron Ratio vs. Cell Size

A. Velocity Drag

Looking at the velocity drag relations for a 1D problem as described in Birdsall[17], we can easily extrapolate to a 2D or 3D situation. As given by Birdsall, the acceleration felt by a particle in 1D is

$$\frac{dv}{dt} = -\frac{\omega_p v_t}{2N_D} = -\frac{q^2}{2m} \quad (5)$$

This can easily be shown in 2D to be

$$\frac{dv_x}{dt} = -\frac{q^2\theta}{\pi m} \quad (6)$$

Where $\theta = \arctan v_t / v_{x,i}$. This is due to the shock cone behind the particle moving at superthermal velocities through the background plasma with information of its passing spreading perpendicular to its direction of travel only at the thermal velocity v_t. In the limit of infinite v_t, this reduces to the 1D acceleration. This acceleration is constant, so we can quickly examine its effects. With realistic q, m, v_t, and 1kV ions, it is quickly shown that the acceleration time for electrons to match ion speeds in this case is on the order of years for v_t comparable to that observed (~0.3eV). Indeed, the 1D case shows a heavy bias towards higher thermal temperatures.

$$\text{1D:}\Delta v = 38337\text{m/s} = e^2 t / 2m \Rightarrow t = 2m\Delta v / e^2 \approx 2.72\text{e}12 \text{ sec}$$
$$\text{2D:}\Delta v = -e^2\theta t / \pi m \Rightarrow t = \pi m \Delta v / e^2\theta \approx 2.72276\text{e}12 \text{ sec}$$

B. Two-Stream Instability

Examination of the two-stream instability shows that any counterstreaming beams are unstable at certain wavelengths. The growth rate of the ion/electron two-stream instability is known to be

$$\gamma = \omega_p \left(\frac{m}{M}\right)^{\frac{1}{3}}. \quad (7)$$

At first glance, this growth rate appears to fail in providing significant coupling as the time for the oscillation to grow is strongly limited by the electron-ion mass ratio.

$$\mathbf{E} = E_0 \exp\left(i\left[k\vec{\mathbf{x}} - \alpha t\right]\right)\exp(\gamma t), \; \omega = \alpha + i\gamma \quad (8)$$

For a n_e=1e15 Xenon plasma, γ=2.87e7. But simulation duration is of order 1e-6 seconds, indicating instability growth is small until the simulation has progressed significantly. But the sheer number of plasma electron cycles and perturbation modes can create much more rapid growth.

The effects of the two-stream instability match several observations made during simulations. Oscillations in potential have been observed that flow downstream at electron timescales. The electric field energy grows until it saturates, then oscillates about a certain value, as in Fig. 12. Also, the electrons "slow down" in the ion frame of reference. This is matched by a loss of energy of the ions in the laboratory frame of reference, as shown in Fig. 13.

The match is not perfect, however. The electric field only grows to ~0.01% of the system energy while the ions lose ~0.1% of their total energy. This indicates that the excess energy is being used to heat the electrons. Nevertheless, it allows us to examine if we are observing a fully developed two-stream instability. Because of the differences in the masses of the particles, we can use the weak-beam criteria as established by Birdsall[17], substituting the ion plasma frequency for the background electron plasma frequency:

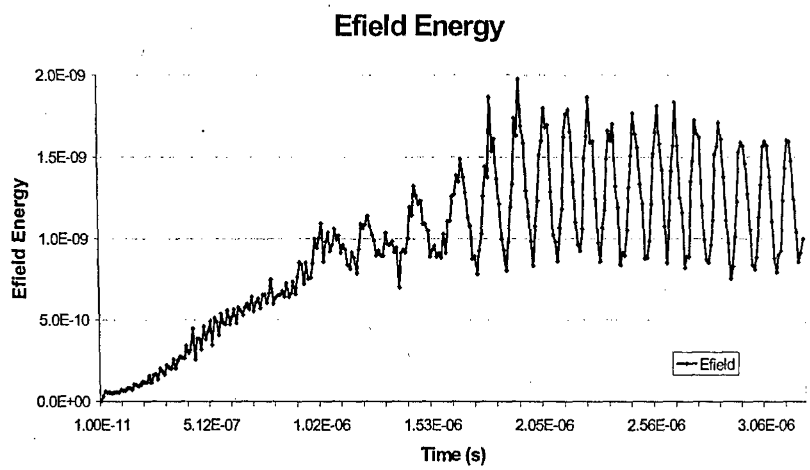

Figure 12: Electric Field Energy in Joules

$$R = \omega_{pb}/\omega_{pp} \Rightarrow R = \omega_{pb}/\omega_{pi} \ll 1 \tag{9}$$

Under the weak-beam criteria, the Strongest weak beam allowed is R=0.032. Using Xenon R=4.18e-6. This suggests that we are well within the weak-beam regime. In this case, it is estimated that the beam energy goes to the field energy at a rate of

$$\left(R/2\right)^{\frac{1}{3}} = 0.013 \tag{10}$$

This number is significantly higher than what we have observed, giving us an inconclusive result. While the two-stream instability seems to be the best candidate for providing the coupling mechanism, it still remains small, and we are not observing even the energy transfer, and hence the coupling, predicted. This indicates that we are not fully simulating the instability or the coupling.

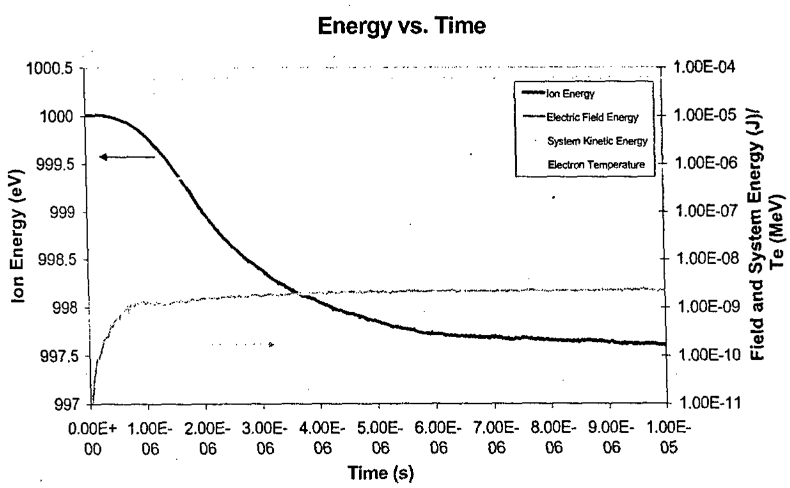

Figure 13: Comparison of ion energy, electric field, total system energy, and electron temperature. Note Te units on right scale.

VI. Conclusion

We have shown PIC simulations model behavior similar to the current coupling observed in electric propulsion devices. We have developed a theory for the expected behavior, but the discrepancy in simulation and theory is significant, although the general behavior is correct. Using the 2-D PIC code XOOPIC, we have determined that the degree of coupling depends primarily on the energy of the ion beam. Numerical effects such as particle weighting, simulation duration, and domain size were found to be of no significant effect on the observed coupling. Cell size did play a noticeable role, indicating some dependence on the granularity of the simulation.

Physical effects that could cause a mixing of the beams or an acceleration of the electrons were examined and found insufficient to produce the simulated effect. Velocity drag is an effect that requires a high temperature plasma, which we are not seeing, and the two-stream instability, while a strong contender for at least part of the observed phenomenon, also looks like it is producing insufficient coupling. Until it can be adequately explained what processes allow current coupling, neutralization will remain a feature that cannot be engineered for. Previous research has not pointed out an adequate explanation for this effect.

A series of laboratory experiments capable of reproducing the simulations is necessary to understand the actual dynamics of the system. With experimental data, theory can be refined to a point where calculations of engineering use are possible.

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