**REPORT DOCUMENTATION PAGE**

The principal goals of this project were the further analysis and the experimental demonstration of the phenomenon of nonlinear energy pumping (targeted energy transfer), in which the vibration of a structure following a transient disturbance is irreversibly localized to a preferred subsystem, termed a nonlinear energy sink, and dissipated there. The occurrence of energy pumping depends on the essential nonlinearity of the sink stiffness; that is, this stiffness must be nonlinearizable, with no linear term in its force-displacement relation.

Two types of nonlinear energy sink (NES) have been investigated: Configuration I, in which the sink mass is coupled weakly and linearly to the primary structure and essentially nonlinearly to ground; and Configuration II, in which the coupling to the primary structure is essentially nonlinear but there is no connection of the sink to ground.

**14. ABSTRACT**

The principal goals of this project were the further analysis and the experimental demonstration of the phenomenon of nonlinear energy pumping (targeted energy transfer), in which the vibration of a structure following a transient disturbance is irreversibly localized to a preferred subsystem, termed a nonlinear energy sink, and dissipated there. The occurrence of energy pumping depends on the essential nonlinearity of the sink stiffness; that is, this stiffness must be nonlinearizable, with no linear term in its force-displacement relation.

Two types of nonlinear energy sink (NES) have been investigated: Configuration I, in which the sink mass is coupled weakly and linearly to the primary structure and essentially nonlinearly to ground; and Configuration II, in which the coupling to the primary structure is essentially nonlinear but there is no connection of the sink to ground.
Principal Investigators

Prof. Lawrence A. Bergman
Department of Aerospace Engineering
306 Talbot Lab, MC–236
104 South Wright Street
Phone 217–333–4970, Fax 217–244–0720
Email lbergman@uiuc.edu

Prof. Alexander F. Vakakis
Department of Mechanical and Industrial Engineering
Mechanical Engineering Building, MC–244
1206 West Green Street
Email avakakis@uiuc.edu

University of Illinois
Urbana, IL 61801

NOVEL PASSIVE VIBRATION CONTROL METHODS FOR AEROSTRUCTURES

Grant Number: F49620–01–1–0208
Grant Period: 28 Feb 2001 — 28 Feb 2004 (with no-cost extension through 28 Feb 2005)
Grant Monitor: Dr. Clark Allred
Abstract

The principal goals of this project were the further analysis and the experimental demonstration of the phenomenon of nonlinear energy pumping (targeted energy transfer), in which the vibration of a structure following a transient disturbance is irreversibly localized to a preferred subsystem, termed a nonlinear energy sink, and dissipated there. The occurrence of energy pumping depends on the essential nonlinearity of the sink stiffness; that is, this stiffness must be nonlinearizable, with no linear term in its force-displacement relation.

Two types of nonlinear energy sink (NES) have been investigated: Configuration I, in which the sink mass is coupled weakly and linearly to the primary structure and essentially nonlinearly to ground; and Configuration II, in which the coupling to the primary structure is essentially nonlinear but there is no connection of the sink to ground. The latter is obviously more practical for aerospace applications, and furthermore has been found to have greater potential for rapid energy transfer to the sink.

Much of the work done in this research program has focused on the dynamics of the NES interacting with a multi-degree-of-freedom linear structure. The topology of the underlying undamped nonlinear normal modes has been summarized in a frequency-energy plot, a schematic diagram which not only depicts the physics of transient resonant capture in these systems but allows the establishment of conditions under which energy pumping can occur (e.g., the initial deformation of the primary structure and the minimum initial energy). The potential for one sink to draw energy from several different modes of the linear structure to which it is attached, by resonating with each mode in turn, has been demonstrated by numerical simulation and through experiments.

We have advanced the analysis of these systems by refining the frequency-energy plots which summarize the nonlinear normal modes of one- and two-degree-of-freedom primary structures coupled to grounded and ungrounded energy sinks. This has led to improved understanding of $m: n$ resonance capture in such systems, and to a much more detailed picture of the feasible periodic solutions. The role of nonlinear beating as a bridge from arbitrary initial conditions to the onset of energy pumping has been explored, and the influence of the parameters of the physical system has been explained analytically and verified experimentally.

Two- and three-degree-of-freedom systems comprising linear, one- or two-degree-of-freedom primary structures coupled to a nonlinear energy sink have been designed, numerically simulated, and fabricated. The primary and sink masses, each of the order of 1 kg, are supported and guided by a linear air track. Essential nonlinearity is achieved in the test rig by exploiting the geometrical nonlinearity of a transversely deflected wire, initially untensioned, which produces a nonlinearizable, cubic hardening restoring force. These springs have been calibrated for several wire spans and diameters. The remaining, linear portions of the test structures were characterized by modal analysis, following which the energy flow within the system was quantitatively measured.

We first obtained detailed, quantitative experimental data on the interaction of a nonlinear energy sink with a linear, single-degree-of-freedom primary structure. Excellent agreement was found between these experimental results and the output of a calibrated numerical simulation model in both the high- and low-energy regimes, where energy pumping does and does not occur, respectively. The primary structure was then extended to two DOF and the NES was shown to be capable of pumping energy from either structural mode following transient forcing of one primary mass. Other experiments have illustrated the resonance capture cascade, in which a nonlinear energy sink couples successively to multiple modes of the primary structure, and demonstrated the application of the NES to a continuous structure (a beam).
1 Objectives

The principal goals of this project were the analysis and experimental demonstration of the phenomenon of \textit{nonlinear energy pumping}, in which the vibration of a structure following a transient disturbance is irreversibly localized to a preferred subsystem and dissipated there. These objectives are unchanged from those originally proposed.

2 Status of Effort

The understanding of multi-degree-of-freedom discrete systems (including modal series representations of continua) coupled to smoothly nonlinear sinks has been advanced through analysis and numerical simulation. Some damping is desirable in a practical device, but the dynamics of a system which may exhibit energy pumping can be explained and analyzed largely in terms of the nonlinear normal modes of the corresponding undamped, unforced system. The topology of the underlying undamped nonlinear normal modes has been summarized in a schematic diagram. Using a 2-DOF laboratory model, energy pumping has been shown experimentally to be robust and repeatable. This apparatus has been extended to examine MDOF structures in the laboratory.

A second configuration of the NES has been devised which eliminates the earlier requirement of a connection between the mass of the NES and ground. Not only is this a more practical design for installation on an aircraft or space structure, but this new NES configuration has been shown to capable of pumping energy through additional, more efficacious mechanisms not found in the simpler NES investigated previously.

The goals of understanding and demonstrating energy pumping using relatively simple models and apparatus have been met. The potential of this technology is clear, and our focus is now shifting towards larger scale and more realistic applications.

3 Accomplishments and New Findings

3.1 Research Highlights

Advances made in each year of this research program are summarized briefly here. Much more detail may be found in the publications listed in a later section or in the previously submitted annual and interim progress reports.

3.1.1 Year 1

- SDOF linear structure plus NES
  
  - Extensive numerical simulations were carried out in the course of designing an experimental rig consisting of a Configuration I (grounded) nonlinear energy sink coupled to a single-degree-of-freedom linear primary structure.
  
  - This apparatus was constructed and its various mass, damping, and linear and nonlinear stiffness parameters were identified.
  
  - Experiments demonstrated transient resonant captures between the two subsystems following loading of the primary structure.
  
  - Analysis of possible interactions between the sink and primary structure led to improved understanding of energy transfer mechanisms and the influence of system parameters and initial conditions on energy pumping performance.

3.1.2 Year 2

- SDOF linear structure plus NES
The topology of the nonlinear normal modes of the Configuration I system was fully described and summarized in frequency-energy plots depicting amplitude vs. energy or resonant frequency vs. energy.

Simulations and experiments were begun to examine the effectiveness of the NES as a broad-band vibration absorber by driving this configuration with a random load.

- **MDOF linear structure plus NES**
  - The formulation of the equations of motion of a general linear system coupled to an essentially nonlinear energy sink, and their analysis by complexification and averaging, was completed.
  - The topology of the NNMs of this system was synthesized as was previously done for the SDOF structure and sink.
  - The possibility of resonance capture cascades, in which the sink resonates with two or more modes of the primary structure in turn, was demonstrated by simulation and the physics of this phenomenon were described in terms of the NNMs of the underlying undamped, unforced system.
  - The laboratory air track previously used to demonstrate energy pumping in a 2-DOF system was lengthened and otherwise improved. The new track is shown in Fig. 1.

- **Use of non-smooth nonlinearities**
  - Shock isolation through energy pumping in systems employing clearance (gap or vibro-impact) nonlinearities was investigated through extensive numerical simulations.

- **Steady-state energy pumping**
  - Localized NNMs were investigated for application in isolating substructures subject to periodic disturbances.
1. Schematic of the masses, connections and transducers in the 2-DOF system.

Figure 2. Experimental apparatus.

2. Two cars, representing the primary and sink masses, mounted on the air track.

Figure 2. Experimental apparatus.

3. Directly excited linear subsystem.

Figure 3. Example response when energy pumping occurs: experimental (solid line) and simulated (dashed line) acceleration time series.

- Analytical and numerical results showed these systems to be capable of a rich variety of steady-state responses.
- Good agreement was found between theory and experiment in a preliminary steady-state trial.

3.1.3 Year 3

- SDOF linear structure plus NES

- The existing experimental apparatus was refined (e.g., to further reduce unwanted friction between the moving masses and the air track during the initial portion of the transient response) and more completely characterized by dynamic tests and static measurements.

- Extensive experimental data were collected describing the transient input force and structural response of this system at both low and high energies. The experimental set-up is shown in Fig. 2.

- These experiments were compared to numerical simulations with excellent agreement, especially of the time evolution of the response signal envelopes. Examples of this agreement may be seen in Fig. 3.

5
• MDOF linear structure plus NES
  – Experiments were conducted in which a second degree of freedom was added to the primary structure of the system above in such a way that only one of the two primary masses was elastically coupled directly to ground. While it was straightforward to demonstrate energy pumping from the first (lower-frequency) natural mode of this structure to the sink, it proved difficult to excite the second (higher-frequency) mode at a sufficiently high amplitude.
  – Simulations indicated that coupling both masses of the primary system to ground by introducing an additional linear spring might result in a system in which both modes could be excited at levels sufficient to produce energy pumping.
  – The experimental rig was modified along these lines in an effort to demonstrate an energy pumping cascade, in which transient forcing or prescribed initial conditions lead to pumping from both modes in succession.

• NES Configuration II
  – A new arrangement of the elements of the nonlinear energy sink was devised, offering the following advantages:
    * Elimination of any direct connection of the sink to ground, making possible the construction of a modular NES to be attached to a primary structure at a single point.
    * Additional and more efficacious mechanisms for energy pumping than were found to exist in the grounded sink configuration previously studied.
  – Analysis and simulation of systems incorporating a sink of this new configuration included the examination of the potential of this device to suppress limit cycle oscillation of aircraft structures.

3.1.4 Year 4
• The role of $m:n$ (i.e., non-1:1) resonance capture was analyzed and the results incorporated in the frequency-energy and nonlinear normal mode explanations of energy pumping.
• The importance of nonlinear beating in the response to arbitrary initial conditions was more completely explained, and the corresponding bridging orbits leading to one-way energy pumping were analyzed. Experiments clearly showed the transition from beating to pumping following a transient input.
• Experiments were performed with the new Configuration II sink.
• Additional experiments were performed in which the primary system was a 2-DOF linear structure or an Euler-Bernoulli beam. Good agreement with analysis and simulation was found.
• The existence of a resonance capture cascade, in which a single nonlinear energy sink interacted with multiple modes of the primary structure, was demonstrated experimentally.
4 Significance to the Field

The analysis and simulation of two configurations of the nonlinear energy sink, including one in which the sink requires no connection to ground, has been expanded and corroborated by experiments. Experimental results have been found to agree well with simulation data and with analysis. Experiments continue with more complex primary structures, namely MDOF discrete structures and beams, results of which verify the existence of resonance capture cascades. These are necessary steps toward the practical application of the phenomenon to the reduction of the transient vibratory response of realistic structures. Numerous applications outside the laboratory, for example to aerospace and land vehicles and civil structures, are expected to follow once a firm basis for the technology has been established.

4.1 Relationship to Original Goals

We found no cause to deviate from our original goals, defined in our proposal.

4.2 Relevance to the Air Force's Mission

While continuing to develop our understanding of the energy pumping mechanism for simple systems, as demonstrated in analyses, simulations and laboratory experiments, we have proceeded with applications to structures with more complex configurations; the ultimate aim of this research is the design of passive nonlinear energy sinks in practical (realistic) flexible structures. This research will pave the way for a more complete understanding of the nonlinear energy pumping phenomena and their application to practical engineering systems. This is highly relevant to the mission of the Air Force, as it promises to provide a new, effective means of passive vibration isolation and attenuation in complex flexible structures of immediate importance, such as aircraft and spacecraft.

5 Potential Applications to Air Force and Civilian Technologies

Immediate application of this technology lies in protection of flexible structures from localized transient forces and potentially unstable aeroelastic responses. The nonlinear energy sink (NES) is designed to quickly direct vibratory energy from the structure itself to the sink, where it can be optimally dissipated. The sinks themselves may even be designed to be sacrificial. Examples of applications include protection of large satellite-borne solar arrays from space debris, stabilization of aircraft- and spacecraft-borne mechanical systems, limit cycle oscillation (flutter) suppression, and shock isolation of sensitive avionics and airborne fire-control systems.

6 Personnel

6.1 Supported Personnel

- Prof. Lawrence A. Bergman, Principal Investigator
- Prof. Alexander F. Vakakis, Principal Investigator
- Dr. D. Michael McFarland, Senior Research Scientist
- Prof. Leonid Issakovich Manevitch, Co-Investigator
- Prof. Oleg Gendelman, Co-Investigator
- Prof. Yuri V. Mikhlin, Co-Investigator
- Mr. Christian John Hartwigsen, Graduate Assistant
- Mr. Young S. Lee, Graduate Assistant
• Mr. Jeffrey J. Kowtko, Graduate Assistant
• Mr. Darien L. Gipson, Undergraduate Assistant
• Ms. Amanda J. Kinney, Undergraduate Assistant
• Ms. Melonee M. Wise, Undergraduate Assistant

6.2 Other Personnel
• Dr. Gaetan Kerschen (Fulbright Fellow), University of Liege, Belgium
• Dr. Panagiotis Panagopoulos, National Technical University of Athens, Greece

7 Publications


8 Interactions and Transitions

8.1 Conference Presentations


8.2 Invited Lecture


8.3 Consultative and Advisory Functions


8.4 Workshop Participation


8.5 Transitions

None to date.

9 New Discoveries, Inventions, or Patent Disclosures

- Invention Disclosure TF02010, “Passive Energy Pumping from a Vibrating System to an Attached Nonlinear Sink,” filed 8 February 2002 with the Office of Technology Management of the University of Illinois at Urbana-Champaign.
