**NUMERICAL TREATMENT OF GEOPHYSICAL INTERACTIONS OF ELECTRODYNAMIC TETHERS**

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**ABSTRACT**

This Report summarizes the results achieved during three years under an AFOSR Grant for the development of numerical methods aimed at physical clarification and rational design of Electrodynamic Tethers with electron collection on the bare wire surface. The work was mainly carried out as part of one Doctoral Thesis (Tatsuo Onishi, MIT, Aug. 2002) and three MS Theses (Jean Benoit Ferry, MIT Aero/Astro, Dec. 2004; C.H. Zeineh, MIT Aero/Astro, underway). The original PIC code of Onishi was extended by Ferry, and their results have been documented in our two previous Yearly Reports. During the third year of the Grant, a new and more advanced code by Dr. Oleg Batischev, of our Space Propulsion Lab, was adapted and exploited by J. Marie Deux, whose Thesis is included as part of this Final Report. All the preceding work was aimed at the bare tether electron collection problem, for propulsion and power applications, in addition, work was also initiated by C. Zeineh on the application of high-voltage tethers to Radiation Belt Remediation; a preliminary account of this work (which is being extended under a new AFOSR Grant) is also included in this Report.
Abstract

This Report summarizes the results achieved during three years under an AFOSR Grant for the development of numerical methods aimed at physical clarification and rational design of Electrodynamic Tethers with electron collection on the bare wire surface. The work was mainly carried out as part of one Doctoral Thesis (Tatsuo Onishi, MIT, Aug. 2002) and three MS Theses (Jean Benoit Ferry, MIT Aero/Astro, Dec. 2004; C.H. Zeineh, MIT Aero/Astro, underway). The original PIC code of Onishi was extended by Ferry, and their results have been documented in our two previous Yearly Reports. During the third year of the Grant, a new and more advanced code by Dr. Oleg Batischev, of our Space Propulsion Lab, was adapted and exploited by J. Marie Deux, whose Thesis is included as part of this Final Report. All the preceding work was aimed at the bare tether electron collection problem, for propulsion and power applications. In addition, work was also initiated by C. Zeineh on the application of high-voltage tethers to Radiation Belt Remediation; a preliminary account of this work (which is being extended under a new AFOSR Grant) is also included in this Report.
1. Outline of the Problem

Many previous studies have identified the possibility of using conductive orbiting tethers for propulsion and/or power generation purposes. The propulsive effect is based on the appearance of substantial electrodynamic forces $F = IBL$ on the tether, when current $I$ is made to circulate along a tether of length $L$ transverse to the geomagnetic field $B$. The current may be driven by the motional emf $V_{emf} = vBL$ due to the orbital velocity $\vec{v}$ of the tether (assumed perpendicular to both $\vec{B}$ and $\vec{L}$), and in this case power is resistively dissipated in the wire and possibly in a series load, and mechanical energy is taken from the orbital motion through $\vec{F}$ being directed opposite to $\vec{v}$. Alternatively, power may be injected by an on-board power source and a voltage opposite and greater than $V_{emf}$ can be used to reverse $I$, creating a forward force and adding energy to the orbital motion. Fig. 1 represents these two configurations.

![Fig. 1: EDT can be used in two configurations: active (orbist-boost), where the Lorentz force created by the interaction of the flowing current with the Earth’s magnetic field is a thrust, and passive (de-orbit), where it is a drag.](image)

The main practical problem faced by any such application is the closure of the electrical circuit outside the tether itself. Earlier studies [11, 12] have made it clear that the dominant voltage drops in this outer path are the anodic and cathodic contacts between the ionospheric plasma and the tether or its terminating structures, while the drop along the current path though the ionosphere is usually negligible. Of the two, the cathodic contact (electron ejection) is regarded as less difficult, since several types of electron source are already available and can be adapted (hollow cathodes, electron guns, cold cathodes). In contrast, the difficulty at the anodic contact is of a fundamental nature, since the plasma density in the Ionosphere is rather low, and thousands of square meters are typically
required to collect Ampere-level currents. The most promising approach is to expose a fraction (several km) of the tether directly, with a positive bias, and to rely on the concentrating effect of this bias to achieve the same goal with a much reduced physical surface. The concept was advanced by Sanmartin, Martinez-Sanchez and Ahedo \[13\], and was initially analyzed using the Orbital Motion Limit (OML) theory of attracting plasma probes. This was motivated by (a) Order-of-magnitude estimates showing hat this simple limit would apply despite the complicating effects of orbital motion and magnetic field, and (b) The fact that collection is maximum for given area and bias under OML conditions.

2. Review of our previous work (years 1 + 2)
The theory of OML collection and its limitations has subsequently been the subject of a series of detailed analytical studies by Sanmartin and Estes \[4,5,6\]. Unfortunately, the impact of flow and magnetic field are such as to make analytical approaches impossible, and detailed data for attracting probes under such conditions were lacking as well. This motivated the start of our 2D numerical modeling efforts, based on a Particle-in-Cell (IC) code that was developed by T. Onishi \[7,8,9,10\]. Onishi’s code assumed weak variations along the tether, and solved a 2D –3V problem its vicinity (two spatial, three velocity components). Both electrons and ions were kinetically tracked between solutions for an updated potential. The magnetic field was assumed to be perpendicular to both, velocity and tether directions (some variations from this were explored in Ref. [10]), and no neutrals were modeled. These assumptions have by and large been also adopted by Ferry and Deux, with the exceptions to be noted later. Onishi’s initial grid was polar, but this was later modified to a hybrid mesh that was near-polar next to the tether, and transitioned to rectangular in the far field, so as to avoid difficulties satisfying the $\Delta x, \Delta y < d_{\text{Debye}}$ conditions. Because of numerical limitations, the sheath (especially at high potentials) tended to push against the computational boundaries, so that “unperturbed” conditions could not be trusted on these boundaries, and efforts were made to formulate boundary conditions (based on imposing neutrality) that allowed the local potential to float. Conversely, near the tether electrons reach high velocities, and simultaneously the grid size is small, and it is therefore difficult to satisfy Courant-Friedrichs’ conditions ($u \Delta t < \Delta x$). Onishi implemented two schemes to deal with this difficulty: (a) Numerically advancing electrons at a faster clock rate in this region, and (b) Defining an “Analytical Domain” inside of which the trajectories were calculated analytically by assuming the potential to be nearly axisymmetric. This worked reasonably well, although Ferry found later a need to tighten the accuracy of these procedures, especially when dealing with potentials well above the $V = 25 \, k T_e/e$ treated originally by Laframboise \[11\] for the symmetric case.

The results of these calculations were described in our previous Yearly Reports \[12,13\] and the accompanying Theses. The most striking and persisting result has been a current collection well in excess of the OML level. Strictly speaking, OML provides an upper limit only for stationary, steady, non-magnetized conditions, but qualitative arguments have in the past been used to justify the expectation that it would in fact provide a more general limitation. Because of this unexpected outcome, we have devoted extra attention to verification of the code for those few conditions where independent solutions exist,
which means in practice the symmetric case results of Laframboise\textsuperscript{[11]} and the extensions and detailed profiles by Sanmartin and Estes\textsuperscript{[4,5,6]}. Unfortunately, these are precisely the cases for which the OML limitation is indeed obeyed, and although our checks (which were continued and refined by J.M. Deux) have been quite successful, they have not explained or refuted the violation of OML for the non-symmetric cases.

Several candidate effects have been studied in an attempt to understand the excess current for cases with flow (without or with magnetic field). The magnitude of the excess is between 50% and 250%. Among these effects have been:

- **Imperfections in the boundary conditions** at the edge of the numerical domain. Ferry\textsuperscript{[15]} did identify a numerical feedback from these boundary conditions as the cause of some potential oscillations, but after comparing several types of conditions and several domain sizes, we have concluded that this is not the excess current mechanism. It is, however, important to make the numerical domain much larger than the sheath size, which became an issue at the highest potentials attempted by Onishi\textsuperscript{[10]} and Ferry\textsuperscript{[14,15]}.

- **Turbulent plasma heating**. The results here are inconclusive. J.M. Deux\textsuperscript{[16]} did calculate very large electron temperatures inside the sheath, but this is easily accounted for as a result of the non-directed energy of the electrons in missing trajectories, which can go in multiple directions though a given point. Most of the recorded oscillations are fairly coherent, near $\omega_{pe}$ and $2\omega_{pe}$, and appear to be driven by the limited number of macroparticles per cell (about 16 typically).

- **Magnetic breaking of two-dimensionality**. Onishi argued that, to some extent at least, the relevant OML limit for magnetized conditions should be that for 3D probes, despite the two-dimensional geometry. This is because energy can be transferred by the electron gyrations between that of motion perpendicular to the tether and that along the tether. For typical conditions, the 3D OML current can be 10-14 times larger than the 2D limit, and this could explain the computed excess. The argument has never been made rigorous, however, and would not explain the excess also found in non-magnetized, but flowing cases (Onishi\textsuperscript{[10]}, Ferry\textsuperscript{[14,15]}).

- **Particle trapping**. During the initial voltage turn-on, most of the electrons residing in the neighborhood of the tether become trapped by the rising local potential (adiabatic trapping) and continue to orbit the tether until captured or scattered away by fluctuations. This had been observed numerically by Ferry\textsuperscript{[14]}, who found that these trapped electrons contributed substantially to the current early in the computation, but more or less disappeared after a few tens of plasma times. The issue of trapping was also investigated by Onishi and Sanmartin\textsuperscript{[9]}, who extended the classical treatment to two-dimensions, and in more detail by J. Marie Deux\textsuperscript{[16]}, who, among other things, confirmed that the excess over OML current cannot be explained by the small contribution of the trapped (negative energy) electrons, except in the initial transient.
3. Outline of the Year 3 work
Because of the residual concern that some structural feature of the Onishi-Ferry code might be responsible for the excess current, we decided to adapt for this application a different PIC code that had been developed by Dr. O. Batishchev. This code used a 2D rectangular grid, which allows the use of Fast Fourier Transform (FFT) methods for the repeated solution of Poisson’s equation. Because of the large gain in computational speed this affords, we are now able to extend the size of the computational domain to, in some cases, several hundred Debye lengths in each direction, well clear of the sheath and wake limits. The code also enforces a number of macro-particles per cell (16-36), and introduces upstream and downstream buffer zones to smooth the transition from a Maxwellian plasma to the tether’s pre-sheath. The side boundaries are given periodic boundary conditions. Figure 2 illustrates these boundary conditions and the domain layout. A variety of consistency checks were performed, as detailed in J.M. Deux’ Thesis [16].

Taking advantage of the added resolution, the new code also features the possibility of a non-geomagnetic field, and in particular, the effects of the additional $\tilde{B}$ field generated by the tether current itself can be reproduced. Some analytical studies exist on these effects [17,18], but the new computational capabilities greatly add to the understanding of the complex electrodynamics in the combined geomagnetic self-induced field. Figs. 3a, 3b, from Ref. [16], show the very different type of trajectory depending whether the electron

![Diagram of periodic boundary conditions](image)

Fig. 2
Fig. 3a

Fig. 3b
does not or does penetrate the closed-lines domain bound by the magnetic separatrix around the tether. Figure 4 shows the lack of symmetry of the wake due to the combined effect of both fields.

In general, it was confirmed that the effect of the self-field on current collection is small as long as the separatrix remains inside of the electrostatic sheath, and that is expected to be the case for envisioned parameters. It was also found that the presence of the closed magnetic lines can have the effect of multiplying the trapped electron population, due to the difficulty in de-trapping these magnetically confined low-energy electrons.

The new code also permits the analysis of multiple tethers in close proximity. An example is shown in Figs. 5 and 6, where a reduction of over 30% in the current per tether is observed due to the mutual interference.
The self-field and multiple tether options were verified, but not studied systematically, and much remains to be done here. Instead, the advanced code was put to use mainly to continue the refinement of our previous findings concerning details of the symmetric case and current excess in the flow cases. An example of the results in the symmetric case is the calculated sheath radius versus bias, shown in Fig. 7.
Our results compare well with the analytical results of Sanmartin (unpublished):

\[
\frac{e\phi_p}{kT_e} = 2.554 \left( \frac{R_s}{\lambda_D} \right)^{3/4} \ell nR
\]

where \( \phi_p \) is the potential bias, \( T_e \) the electron temperature, \( R_s \) and \( R \) the sheath and probe radii, and \( \lambda_D \) the Debye length. This also confirms independent numerical work (unpublished) by Choiniere and Gilchrist, who used a steady-state model similar to Laframboise’s.

The current collection is found to be very close to, but systematically slightly in excess of that from the classical Laframboise’s calculations, including OML cases. Fig. 8 shows results for \( R_s = \lambda_D \), close to the upper limit of OML.
Careful examination of the results showed that the 5-10% excess is attributable to the collection of negative energy electrons, a population that classical theory does not consider. This is exemplified in Fig. 9, where the $E < 0$ and $E > 0$ contributions to current are shown separately.
Turning now to non-symmetric cases, J.M. Deux considered first situations with no magnetic field, but with flow. This case had caused convergence problems for Onishi possibly due to grid limitations, but Onishi had fairly clearly established the collection of a current 50-100% above OML. The new code converges well in these situations, and confirms Onishi’s excess current results. Fig. 10 shows a close-up view of the potential near a tether at 29V, exposed to orbital velocity plasma.

![Electric potential, V=29 V, with 8 km/s orbital velocity](image)

**Fig. 10**

There is a well-developed negative wake, due to the inertial non-penetration of ions into the region directly behind the tether. A slight positive potential ridge is also seen near the sheath-wake junction, which strongly disturbs particle trajectories. The current collected versus bias is shown in Fig. 11, and, as noted, it is up to 100% greater than the OML limit.
Once again, the large trapped population might be suspected of being behind this extra current, but this is disproved by reckoning separately their contribution, which turns out to be small, as in the symmetric cases (Figure 12).

As noted above, J.M. Deux also did a careful study, part analytical and part numerical, of the trapped electron issue. Using an un-shielded Coulomb potential between the tether and the sheath edge, an effective 1-D potential was derived, including angular momentum effects, and regions identified for specified energy $E$ and angular momentum $J$ where electrons could be trapped. This turns out to be possible even for some $E > 0$ situations, so that some orbiting electrons are in a sort of metastable state bounded away from
infinity by an intermediate barrier in the effective potential. A time-averaged Vlasov equation was then solved for the energy distribution of the trapped population, with the matching condition of continuity at the trapping boundary. This further generalizes Sanmartin’s and Onishi’s 2D adiabatic trapping model, and yields a spatial distribution of trapped electrons that agrees qualitatively with what is observed at early computational times. Depending on bias voltage, the trapped density can be many times greater than the free (E > 0) density at infinity. Analytical results are shown in Fig. 13 for $e\varphi_p / kT_e = 450$; the corresponding computed early distribution is as shown in Fig. 14.

![Trapped Electron Density in the Sheath](image1)

**Fig. 13**

![Electron density in the sheath: 45V biased probe in OML regime](image2)

**Fig. 14**

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The overprediction is partly due to the effect of the negative space charge due to the trapped population itself, that was not accounted for in the calculations. It was at least verified that this space charge was not sufficient to create potential barriers that would affect the OML status of the case studied.

It is observed computationally that the relatively high level of potential fluctuations is constant removing electrons from this trapped populations at a fairly high rate, sometimes as much as 50% in one time step. However, it is also observed that a steady state trapped population is nevertheless maintained. Deux constructed a semi-quantitative model for the balance between de-trapping and replenishment. The latter was postulated to be a consequence of the small positive potential transient that occurs whenever an electron is scattered away or captured by the tether; under the proper conditions (an eigenvalue problem) each loss leads to one new trapping event, and a steady level is reached. The results are qualitatively similar to observed final distributions, but the model may be too crude for quantitative purposes.

A final observation on these calculations is that, as noted before, even the dense "accretion disk" of trapped electrons that builds up (possibly through non-physical pseudo-collisions) contributes in the end no more than 10% to the current collection. This would seem to provide strong validation to our results, but in the absence still of a clear mechanistic explanation for OML violation, one cannot rule out a spurious, undetected numerical artifact. Continued work on these questions is therefore recommended.

4. Initial work on electrostatic Radiation Belt Remediation (RBR)

The problem of Radiation Belt Remediation (RBR) arises from the possibility of man-made intensification of the already dangerous levels of high-energy particle populations trapped by the geomagnetic field in the altitude range of 1,000-15,000km. These particles, typically protons, have pitch angels with respect to the magnetic field high enough to induce repeated reflections above the dense atmosphere altitude. Any intervention that can reduce this pitch angle can lead to absorption of the particle as it dips farther into the polar atmosphere. Among the possible mechanisms that have been proposed for this purpose, two deserve mention: (a) reduction of the cross-field energy by interaction with left circularly polarized radiation at the ion cyclotron frequency, and (b) scattering by some high potential structure (a metallic tether or combination of tethers has been suggested) that will send a fraction of the protons into the loss cone for their magnetic line.

The study of RBR tether operation can be attempted at two levels of accuracy. For design studies, where multiple parametric trials need to be evaluated, a simple model can be developed that uses an approximate analytical potential distribution around the tether, evaluates the scattering cross-section for particle removal, and uses it to calculated total loss rate. On the other hand, for a clear understanding of the physical issues and for numerical precision, one needs to resort to a detailed kinetic simulation capable of including magnetic effects, relativistic corrections and complicated geometries.
A good example of the simplified approach is the original study of Danilov et al. Danilov pointed out that a pair of tethers biased in a double-probe arrangement could be used, with most of the potential deviation from the background being taken up by the negative tether, which then becomes the active scattering center. Since the trapped proton energy is up to 1 MeV, the bias must also be of this order. The electron temperature in the background magnetospheric plasma is of the order of 100 eV, and the plasma density at the high altitudes considered can be as low as $10^8$ m$^{-3}$, for a Debye length of nearly 25 m. The radius of the sheath formed around the negative tether can be estimated from Eq. (1) (section 3). For the conditions quoted above, with a 1 MV potential, the sheath radius ranges from 1,400 m when the tether radius is 0.01 mm to 1,900 m when the tether radius is 10 mm. The ratio of sheath to tether radii is then from $1.4 \times 10^8$ to $1.9 \times 10^5$. The energetic particle density is much smaller than the background density, which sets the Debye length and, together with the tether radius, determines the current capture rate by the two tethers, and hence the power required by the system. We have initiated analytical work towards a model of RBR through electrostatic scattering. The model assumes a purely Coulombic potential around a negative tether, extending to zero value at the radius $r_s$ of the sheath. The tether is assumed to be in a circular equatorial orbit, directed vertically. Magnetic effects are ignored during the scattering interaction, but a background magnetic field is assumed, with its dipole axis coincident with the Geographic NS axis. A classical scattering formulation furnishes a relationship between the incoming particle “miss distance” in the equatorial plane and its angular deflection by the electrostatic potential field. Analytical conditions are derived to test whether the leaving direction falls within the Loss Cone at the location of the interaction (defined as the angular range around the magnetic direction that will allow penetration of the particle below 110 km in either polar region), and an integration is performed over all velocities belonging to a “hollow cone” Maxwellian distribution and over all miss distances consistent with entering the Loss Cone. The calculations are only partially done analytically, the rest being performed as a series of nested numerical quadratures.

The program embodying the above formulation has been coded and is currently in its testing phase. Extensions are planned to calculate the current collected by either polarity tether, hence the DC power required to sustain the potential. The OML formulation will be used for this purpose. This tool will then be exercised to postulate and evaluate a number of system configurations (orbital parameters, number of tethers, tether dimensions, system mass, expected life, cost, etc.). These will be assessed against the system’s performance, namely, the time to reduce the initial RB energetic particle population by a desired percentage. Natural as well as enhanced initial conditions will be considered, and the competition of the natural capture rate of cosmic radiation will be an input to the calculations.
REFERENCES
