ENVIRONMENTAL ACOUSTIC TRANSFER FUNCTIONS AND FILTERING ACOUSTIC SIGNALS

THESIS

Brandon P. Dias, Second Lieutenant, USAF

AFIT/GAM/ENC/05-2

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GAM/ENC/05-2

ENVIRONMENTAL ACOUSTIC TRANSFER FUNCTIONS AND THE FILTERING OF ACOUSTIC SIGNALS

THESIS

Brandon P. Dias
Second Lieutenant, USAF

AFIT/GAM/ENC/05-2

AFIT/GAM/ENC/05-2

ENVIRONMENTAL ACOUSTIC TRANSFER FUNCTIONS AND THE FILTERING OF ACOUSTIC SIGNALS

THESIS

Presented to the Faculty of the Graduate School of Engineering and Management

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science

Brandon P. Dias, B.S.

Second Lieutenant, USAF

March 2005

AFIT/GAM/ENC/05-2

ENVIRONMENTAL ACOUSTIC TRANSFER FUNCTIONS AND THE FILTERING OF ACOUSTIC SIGNALS

Brandon P. Dias, B.S.

Second Lieutenant, USAF

Approved:

Dr. Mark Oxley Thesis Advisor	Date
Dr. William Baker Committee Member	Date

Table of Contents

Page

List of Figures

AFIT/GAM/ENC/05-2

Abstract

When dealing with acoustic signals, the environment in which the signal propagates affects the signal in measurable ways. These effects lead to echoes, changes in amplitude, and sometimes a confluence of other signals which may render the signal useless in terms of information retrieval. When considering acoustics in terms of pressure changes due to the driving signal, we are able to model and measure the effects of the environment on a given signal. Once this model is known, we are able to completely invert the environmental transfer enacted on the signal and filter out other signals, as long as some assumptions are held in the implementation of this procedure.

ENVIRONMENTAL ACOUSTIC TRANSFER FUNCTIONS AND THE FILTERING OF ACOUSTIC SIGNALS

I. Introduction

1.1 Background

Signal processing is the method of taking a given signal and extracting useful information, usually by a means of a transformation of some kind. Acoustic signals are functions of time in which the output is a pressure or a velocity potential response. An acoustic signal is affected by the environment in which it propagates, so one can attempt to remove the environmental effects to extract the useful information, in this case the original signal. The classic example of this problem and how the human brain deals with it is called the Cocktail Party Problem (3). Imagine yourself in a room full of people, where different conversations are happening all around you, and you attempt to either participate in a conversation around you or listen to a conversation near you. Somehow, your brain processes this multitude of signals by localization techniques and filters the information in such a way as you are able to ignore these extraneous conversations and concentrate on a particular conversation. This thesis will derive, in a mathematical framework, the process of filtering extraneous signals in a way that yields the original signal, and will then apply this process to the Cocktail Party Problem in an attempt to describe how useful this ability can be.

This ability has many applications in both the Department of Defense and commercial industries. Considering an acoustic signal to be a form of information, the basic purpose of filtering the received signal is to retrieve the original information, such as in intelligence gathering. One use for the Department of Defense is the use in "bugging" a room with listening devices that record the information being disseminated in, for example, a terrorist meeting. If a number of people conversing in a meeting, the received signal will be the sum of these signals at their respective locations. Taking into consideration their different locations, the recording can begin to sound quite garbled and unintelligible.

Since this process is being treated as information retrieval, this garbled recording would simply be useless information. By finding a method for separating the signals and removing environmental effects, the original information would then be recovered, potentially containing useful information.

The private sector also has many uses for this method, and while none of them may not have as direct an impact on our daily lives as the defense application, they can be helpful at a relatively low cost from a computing standpoint. Consider a boardroom meeting transpiring in a room. While at most times, there will only be one person talking, there may be at other times many people talking at once, arguing with each other. This is when the application of this signal processing will become most useful. The entire meeting could be recorded with every word clearly spoken and usable for future reference. Also, consider the explosive proliferation of cellular phones in the past decade. While in most cases, cellular phones are easy to use, there are some noisy environments in which they are practically useless. The user of the phone can eliminate this problem by using an earpiece, sending the signal directly to his or her ear, but the person on the other end of the line is still stuck with the multitude of signals entering the mouthpiece of the phone. One way to eliminate these sounds would be to filter the other voices out of the incoming signal, by somehow correlating the user's voice with an amplitude function or some other way and to then filter the sounds by software included in the phone. One product currently on the market that performs this task is a mobile phone headset called Jawbone. Jawbone uses technology developed by DARPA to correlate the motion of the user's jawbone with the signal being received. When the user's jawbone is not moving, the headset effectively turns off and uses the ambient sound being received during this time to create a noise filter for use when the user is talking. This is a perfect example of this technology being marketed in the private sector (1).

1.2 Definitions

This section presents and defines certain terms which are used throughout this thesis.

Environment refers to the finite region of influence acting on a signal. In the mathematical depiction of the problem, this will be replaced by the domain with boundary conditions.

Attenuation is the effective reduction in amplitude of a signal caused by environmental factors. For the purposes of this thesis, attenuation will be described as either atmospheric or boundary.

Isothermic describes the property of a medium having a uniform temperature, i.e. the temperature gradient with respect to the three spatial variables and time is equal to zero.

Isobaric describes the property of a medium having a uniform pressure density, i.e. the dispersion of the medium's constituent gases is constant throughout the environment.

The *coefficient of reflection* is the ratio of the energy of a wave after encountering a barrier to its original energy. While the coefficient of reflection can take complex values, the examples in this thesis deal only with real-valued coefficients. This implies that there is no phase shift, or modulation, on the signal when encountering a boundary, only attenuation.

Ultrasonic describes a frequency above the human threshold of hearing.

A *Source* is the origin of a signal, which will be a person speaking, unless stated otherwise.

A *Receiver* is the instrument with which a recording of an environmentally altered signal will be made. For the purposes of this thesis, a receiver will refer to an omnidirectional microphone unless stated otherwise.

Transfer Function refers to the rule by which an environment affects a signal. It is our goal to find the transfer function of a given environment.

1.3 The Problem

An acoustic signal can be distorted in a number of ways by an environment, thereby making information contained within the signal less accessible. The effect of the environment must be determined in order to retrieve the original information. This thesis pursues those goals by finding solutions to the following three problems:

1. Problem 1 is defined as that of finding and inverting the room transfer function.
2. Problem 2 is defined as the problem of filtering different signals being transmitted at once.
3. Problem 3 is defined as retrieving the signal of a mobile source.

1.4 Research Objectives

This thesis develops a rule for environmental effects on a signal, as well as a rule for removing these effects. This removing, or filtering, enables us, to a certain degree, to separate out multiple time-dependent signals into separate, whole signals containing the original information contained in each signal.

1.5 Scope

The results of this thesis enable us to determine under which conditions a filtering operation can successfully be performed on a set of received signals. While Problem 2 is primarily concerned with the two-receiver/two-source case, this can be applied to numerous other cases, such as multiple (more than two) signals and signals with a variable location (from here on, referred to as mobile sources for the purpose of this thesis).

The results of Problem 2 also lead to a development of a rule for judging how "good" a filter is, based on the condition number of the matrix of transfer functions.

While the scope of the thesis seems somewhat narrow at the beginning, the findings can be broadened to apply to a general case (the most general of which is the Cocktail Party Problem with no limitations on source locations). The benefit of directional microphones in this setting is discussed, but not implemented in the mathematics.

1.6 Approach

The approach to the very basic problem of finding the room transfer function is that of solving the three-dimensional wave equation modified to incorporate atmospheric attenuation. The boundary conditions and initial conditions are developed in the latter part of Chapter II.

1.7 Thesis Outline

This thesis takes a bottom-up approach. Chapter II deals with a literary review of most of the concepts and topics discussed in this introductory chapter. Chapter III begins with a statement of the foundational assumptions made in solving this problem, as well as detailing the solution to the initial problem. Finally, the results of the core problem of finding the room transfer function are extended in the same mathematical framework to the filtering problem (Problem 2) and the tracking problem (Problem 3). Chapter IV discusses the implementation of this process on a specific situation where numerical values are assigned to the parameters used in the derivation of the solution to the initial problem. Chapter V summarizes the findings of this thesis as well as presenting recommendations for future research in this area.

II. Literature Review

2.1 Introduction

This chapter consists of a literary review of the two basic aspects of the three problems. The first part consists of a review of the fundamental aspects of acoustics and how different environments affect a signal. This will deal primarily with part of Problem 1, that of finding the room transfer function. The second part will consist of a review of filtering techniques and processes, which generalizes the transformation used in Problem 1 to a more complicated system.

2.2 Fundamentals of Acoustics

2.2.1 A Definition of Sound. Before beginning the goal of determining a rule by which an environment affects an acoustic signal, a description of sound must be given. "The whole study of sound is the study of vibrations" (8:36). By this one sentence, written in one of the most prolific texts on acoustics, a great deal can be deduced. The key word in this sentence is "vibrations". When considering the mathematical description of vibrations, the likely tool for describing them will be the trigonometric functions because of their oscillating qualities. Morse begins his description of what sound really is by speaking of an oscillating system, using the example of a spring-mass oscillator. By introducing this example, he shows that in a real-world oscillating system, there is dampening caused by friction (8:37). In an acoustic environment, this dampening is the atmospheric attenuation.

2.2.2 Measuring Sounds. Now that sound has been defined the system in which it propagates has been described, a method to quantify these effects must be discussed. Thus far, the environment has simply been described as air and been called a medium of propagation. Since air is composed of gases, it follows the laws of fluid dynamics, and so these laws must be used to describe the effect of a signal on the air. A fluid has a number of characteristics that determine how it behaves. In this case, the fluid density, the pressure, and the temperature are the key factors that determine how a wave will propagate through this fluid. When discussing the propagation of a wave, there must be a way to quantify the magnitude of the wave's effects at a spatial point in the environment at a certain time. Since the direct measurement of temperature and density is rather difficult, it is more efficient to measure the amplitude by its pressure. Measurements of pressure are also more accurate than those of temperature and density (8:229). The familiar measurement of the intensity of sound, decibels, is a logarithmic measurement of pressure. It is in this framework of pressure changes caused by sound waves that a signal and the environment's effects are measured.

2.2.3 Deriving the Wave Equation. Speaking in the framework of pressure, Morse derives the wave equation from a succession of equations used to describe the law of conservation of mass and the law of conservation of momentum in a fluid. For a fluid, the law of conservation of mass in three spatial dimensions is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{\nu} = 0 \tag{2.1}$$

and the law of conservation of momentum is

$$\rho \left[\frac{\partial \vec{\nu}}{\partial t} + (\vec{\nu} \cdot \nabla) \vec{\nu} \right] + \nabla p = 0 \ , \tag{2.2}$$

where $\rho = \rho(\vec{x}, t)$ is the particle density of the fluid at location $\vec{x}$ at time t, $\vec{\nu}(\vec{x}, t)$ is the particle velocity in the fluid, and $p(\vec{x}, t)$ is the fluid pressure. Since the environment consists of a homogeneous fluid, then

$$\rho(\vec{x}, t) = \rho = c \tag{2.3}$$

for some constant $c > 0$. In a fluid, the pressure is a function of the particle density, so define f such that

$$p = f(\rho) \ . \tag{2.4}$$

This definition is called the equation of state. These parameters can be linearized about initial conditions such that

$$p(\vec{x},t) = p_0 + \hat{p}(\vec{x},t) \tag{2.5}$$

$$\rho = \rho_0 + \hat{\rho} \tag{2.6}$$

$$\vec{\nu}(\vec{x},t) = \nu_0 + \hat{\nu}(\vec{x},t) \tag{2.7}$$

where $\nu_0 = 0$ since the initial conditions are quiescent. Using these linearized functions, the equations of mass, momentum, and state can be restated as

$$\frac{\partial \hat{\rho}}{\partial t} + \rho_0 \nabla \cdot \hat{\nu} = 0 \tag{2.8}$$

$$\nabla \hat{p} + \rho_0 \frac{\partial \hat{\nu}}{\partial t} = 0 \tag{2.9}$$

$$\hat{p} = c^2 \hat{\rho} \; . \tag{2.10}$$

Taking the derivative with respect to time of the equation of mass above, the following equations are found

$$\frac{\partial^2 \hat{\rho}}{\partial t^2} + \frac{\partial}{\partial t}(\rho_0 \nabla \cdot \hat{\nu}) = 0 \tag{2.11}$$

$$\frac{\partial^2 \hat{\rho}}{\partial t^2} - \frac{\partial}{\partial t}(\rho_0 \nabla \cdot \hat{\nu}) = \Delta \hat{p} \; , \tag{2.12}$$

and the equation relating the particle density and pressure emerges as

$$\frac{\partial^2 \hat{\rho}}{\partial t^2} = \Delta \hat{p} \; , \tag{2.13}$$

but the equation of state can be resubstituted back into this equation, leaving the familiar wave equation

$$\Delta \hat{p} - \frac{1}{c^2} \hat{p}_{tt} = 0 \; . \tag{2.14}$$

2.2.4 Attenuation–Atmospheric and Boundary. In Chapter I, attenuation is defined as the reduction in amplitude of an acoustic signal. Going back to Morse's example of an oscillating spring, he describes the friction in this system as a resistance to a change in inertia of the medium. In the acoustic setting, this friction is the resistance of air molecules to a displacement, thereby reducing the energy of the signal by a reduction in amplitude, which is the definition of attenuation. The amount of attenuation is given by a vector Γ, which defines the amount of attenuation in the directions of the domain. In this case, the vector has constant components, all of which are equal such that

$$\vec{\Gamma} = 2\Gamma \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{2.15}$$

This implies that there is no directional dependence in atmospheric attenuation. This assumption is justified in that the propagational environment is isothermal and isobaric, with no fluid flow. Since this attenuation takes places as the signal propagates through space, its effect on the pressure in the sound wave can be described using the gradient of the pressure. The pressure caused by the wave is decreased as the wave passes through space, and so the atmospheric attenuation can be given by the expression

$$\vec{\Gamma} \cdot \nabla p = 2\Gamma \left(p_x + p_y + p_z \right). \tag{2.16}$$

In the differential equation developed in later sections, this term is added to the wave equation, giving a modified form for the wave equation which takes into account atmospheric attenuation.

The atmospheric attenuation is not the only source of attenuation, however–in this setting there also exists boundary attenuation. In the case of the oscillating spring mass system, consider a second spring set at the lowest point at which the oscillating spring will reach in an oscillation. When the mass interacts with the boundary, it will encounter resistance to any further displacement by this second spring. This resistance will effectively reduce the energy of the oscillating spring, thereby reducing the amplitude of the next oscillation. Now, translating this idea to the acoustic setting, the signal will reach a boundary and will find a resistance to a change in inertia. This resistance will, analogous to the oscillating system, cause a reduction in amplitude, thereby attenuating the signal at the boundary.

The comparison to a spring on the boundary is fairly accurate. The spring constant translates to an impedance on the boundary, which characterizes how much sound the boundary absorbs and how much is reflected. If we assume that the boundary reflects a signal with constant attenuation, that is, no frequency dependence and no phase shift, then the coefficient of reflection of a boundary is a real, positive constant. A reflective coefficient of β leads to the Robin boundary condition

$$\frac{\partial p}{\partial n} = \beta p \tag{2.17}$$

where $\frac{\partial p}{\partial n}$ denotes the outward unit normal derivative of pressure with respect to the boundary. The left-hand side is the reflected signal and the right-hand side is the original signal.

A coefficient of reflection approaching infinity implies that the boundary absorbs all of the incident wave energy and none is reflected. This complete attenuation of the incident signal is described by

$$p = 0. \tag{2.18}$$

The most realistic case is that of a boundary that absorbs some, but not all of the signal, and so the reflected wave undergoes a reduction in amplitude. This boundary condition is given by

$$\frac{\partial p}{\partial n} = \beta p \tag{2.19}$$

where β is the reflective coefficient of the boundary. The reflection coefficient can be a function of frequency, attenuating different frequencies by different amounts, but for the purposes of this thesis, β is a positive, real constant.

2.2.5 Forcing Functions. Consider an atmospheric environment with isothermal and isobaric properties. Since the temperature and pressure gradients are both equal to zero, then the system will be at rest until another force acts on the environment if the initial conditions are quiescent. In this case, there is no sound being propagated. However, if the environment is acted upon by another force, an acoustic signal, then the environment will be disturbed from its quiescent state and a signal will be propagated through the medium. This force can be considered mathematically as a driving function to the system.

The shape of the source partially determines the nature of the sound wave being propagated. In the case of a point source, the sound wave propagated is spherical in geometry since there is no apparatus to act as a focusing antenna, as the human mouth does, but for the sake of simplicity, this thesis will deal with point sources that will radiate spherically from the source location. This point source can be described as a time dependent signal being propagated at a location $\vec{x}_0$, and so the use of the Dirac delta function is appropriate in the use of a forcing function. A time-dependent signal propagated from a location $\vec{x}_0$ therefore acts as a forcing function in the form

$$s(t)\,\delta(\vec{x}-\vec{x}_0)\,. \tag{2.20}$$

2.2.6 The Modified Wave Equation. Now that the different effects of the environment and the nature of the source have been discussed, it is possible to formulate a differential equation that combines these effects. It has been mentioned that the effect of atmospheric attenuation must be included in the differential operator, and so the left-hand side of the differential equation takes on the form

$$\Delta p + \vec{\Gamma}\cdot\nabla p - \frac{1}{c^2}p_{tt}. \tag{2.21}$$

The forcing function makes up the right-hand side of the equation, and so the differential equation is

$$\Delta p + \vec{\Gamma}\cdot\nabla p - \frac{1}{c^2}p_{tt} = s(t)\,\delta(\vec{x}-\vec{x}_0)\,, \quad \vec{x}\in\Omega,\ t>0 \tag{2.22}$$

The effects on the signal at the boundary must satisfy the Robin boundary condition

$$\frac{\partial p}{\partial n} = \beta p, \quad \vec{x}\in\partial\Omega,\ t>0. \tag{2.23}$$

and must have initial conditions

$$\begin{aligned} p(\vec{x},t) &= 0, \quad \vec{x} \in \Omega,\ t = 0 \\ p_t(\vec{x},t) &= 0, \quad \vec{x} \in \Omega,\ t = 0. \end{aligned} \tag{2.24}$$

These quiescent initial conditions are justified by the fact that the microphones can be calibrated such that the pre-existing pressure in the room is ignored, and so only the changes in pressure are measured.

2.3 Inversion and Filtering

2.3.1 Localization Cues and Qualitative Elements of a Signal. When a signal is acted upon by the environment, there are many changes that occur in the original signal, all of which are described in the differential operator and boundary conditions. These effects reflect the nature of the propagational environment and are instrumental in the location of the source's origin. Humans have auditory receivers in the form of ears. The size and shape of these ears help to directionalize the signal in that they are not open all around, but have a bowl shape to them. The cartilage-composed outer part of the ear is called pinna, and they help humans to determine where a signal is coming from–its azimuth and elevation. Since this model assumes that the microphones being used are omnidirectional, the azimuth and elevation of the source signal is not taken into account, and so these effects on the signal are not measured. Only the source and receiver locations with respect to boundaries and the distance between source and receiver affect the signal.

The nature of the environmental boundaries play a large role in the effect of the environment. If these boundaries are highly reflective, the signal will not be attenuated as much by the boundaries and so atmospheric attenuation will play a larger role. Since atmospheric attenuation generally has a much smaller effect than boundary attenuation, the received signal typically has fewer echoes, but if the walls are highly reflective, echoes will be more prominent, making localization more difficult (9).

2.3.2 Source Localization. One of the main assumptions made in this thesis is that the receiver and source locations are known. In the boardroom example, this assumption is usually valid, but in the intelligence gathering example, the source locations will be harder to determine. The process of determining source positions is called source localization.

There are two types of source localization–active localization and passive localization. Active localization introduces some form of energy into the system to gather information. A familiar form of active localization is found in remote sensing–the radar. The radar is the electromagnetic counterpart to sonar, which uses ultrasonic pulses that feed information about the environment back to a receiver. Ultrasonic imaging is widely used during pregnancies to observe a fetus during gestation. It can also be used in the gaseous environment of the atmosphere, but since the atmosphere is much more tenuous than the human body, it can be harder to focus the image (2).

An inherent problem in active sensing is the possibility of detection. Since an active sensing method introduces excess energy into a system, this excess energy, in whatever form it may be, may be detected, and so a more passive method of determining source localization is desired.

If there are two microphones in a room, a signal will take different lengths of time to propagate to microphones at different distances from the source. This duration is called the time difference of arrival (TDOA), and it can be exploited to determine source locations. The set of possible locations of a source is a hyperbola with its receiver as the focus. Using the TDOA method, the intersections of these hyperbolae can be found, which will give the location of the source. Since this method uses the recording as the means of determining source locations, there is no further preparatory effort needed. If the receiver is not detected, then neither is the process of source localization (5).

2.3.3 Time-Reversal. Once the source location has been determined, the information needed to make the necessary transformation to remove the effects of the environment is complete. One method similar to the attempts being made in this thesis is called Time Reversal. This is a method for removing the environmental effects of the propagational medium in ultrasonic images, but the process is very similar to that of filtering an acoustic signal. When ultrasonic images are made, the environment changes the reflected signals making the images blurred. The objective is to remove the environmental effects on the image. When this process is put into the framework of information gathering, time reversal and RTF inversion are virtually equivalent. The information is acted upon by the environment, giving a received signal to which a transformation is applied to retrieve the original information. The time reversed focused image is simply the filtered signal–only the nature of the information is different (6).

III. *Mathematical Analysis*

3.1 Introduction

In this chapter, the mathematics of acoustics lead to solutions for the three problems posed in Chapter I. Initially, a general solution for an arbitrary domain will be explored and then the spatial operator specified to a rectangular domain. The second part of this chapter deals with the dimensional analysis of the wave equation in an attempt to ascertain exactly what physical quantity is being measured. Section four consists of restating of the problem by Duhamel's Principle so that the differential equation can be solved more readily by analytic methods. Sections five, six, and seven deal with Problems 1, 2, and 3, respectively. In each of these sections, it is shown how the three-dimensional modified wave equation is the key to solving the three problems, if applied properly.

3.2 The Eigenfunction Approach

Consider an arbitrary domain in which the sound wave will propagate. Associated with this domain are the boundary conditions that define the behavior of the wave at the boundary of this domain. Let L be the linear differential operator that will define the spatial shape of the wave modes. Then L is the spatial operator of the wave equation and so the homogeneous wave equation in this domain (scaled with respect to the speed of the wave, c) can be written as

$$LU = U_{tt}. \tag{3.1}$$

where $U = U(\vec{x}, t)$ is the displacement of whatever measurement is being taken–in terms of sound, this is usually pressure. Assuming that the operator L has no dependency on t, such as time-dependent coefficients, then $U(\vec{x}, t)$ can be assumed to be separable such that

$$U(\vec{x}, t) = u(\vec{x})\, T(t) \ .$$

Even though Chapter II defined the forcing function driving the wave equation, it is omitted here because of the application of Duhamel's Principle in section four, which reformulates the inhomogeneous initial boundary value problem with quiescent initial conditions into one that is homogeneous with nonquiescent initial conditions. By finding the eigenfunctions u_n of the spatial linear operator L, then the substitution

$$LU_n = -\lambda_n U_n = -\lambda_n u_n(\vec{x})\, T(t) \tag{3.2}$$

where λ_n is the eigenvalue associated with the eigenfunction u_n, eliminates the operator in the wave equation, yielding

$$-\lambda_n u_n T(t) = u_n T''(t) \ . \tag{3.3}$$

The eigenvalues are all positive, and so therefore there is a nontrivial solution to this equation (7:389). Dividing both sides of the equation by u_n eliminates the spatial dependence and yields only a time-dependent ordinary differential equation

$$\begin{aligned} T'' + k_n^2 T &= 0 & (3.4)\\ k_n^2 &= \lambda_n \ , & (3.5)\end{aligned}$$

which has the solution

$$T_n(t) = A_n \sin(k_n t) + B_n \cos(k_n t) \ . \tag{3.6}$$

The solution to the wave equation in terms of the eigenfunctions of the spatial operator L is therefore the superposition of the individual eigenfunctions multiplied by the time-dependent solution:

$$\begin{aligned} U_n(\vec{x}, t) &= u_n(\vec{x})\, T_n(t) = u_n(\vec{x}) \left[A_n \sin(k_n t) + B_n \cos(k_n t)\right] & (3.7)\\ U(\vec{x}, t) &= \sum_n U_n(\vec{x}, t) \ . & (3.8)\end{aligned}$$

From this point, the problem becomes that of finding the eigenfunctions of the spatial operator L. The operator and the associated boundary conditions determine both the shape the eigenfunctions take as well as the values which the eigenvalues λ_n can take on.

The eigenfunctions describe the natural modes of vibration of the fluid and the eigenvalues determine the frequencies at which these mode shapes vibrate (7:389).

Now consider a rectangular domain Ω such that

$$\Omega = \{\vec{x} = (x, y, z) : 0 < x < W,\ 0 < y < L,\ 0 < z < H\}\ . \tag{3.9}$$

This domain is a specific case of the general domain discussed above, and so the eigenfunctions for this domain can be found. For a rectangular domain, the spatial operator in the wave equation is the Laplacian operator in three-dimensional Cartesian coordinates, so for this specific domain, the homogeneous wave equation is

$$\Delta U = U_{xx} + U_{yy} + U_{zz} = U_{tt}\ . \tag{3.10}$$

The following sections go through the process of using this eigenfunction approach by first finding the time component in the same way it was found above and then by finding the eigenfunctions of the Laplacian operator.

3.3 Dimensional Analysis

Dimensional analysis is the process of examining the dimensions of a physical quantity to describe the units of measurement. In Chapter II, it was stated that the pressure is simpler to measure than temperature and density and that is also provides more accurate results. Furthermore, a microphone measures the change in pressure created by a sound wave, so it seems logical to use this measurement in determining the effects of the environment on a signal.

Let $[\alpha]$ denote the dimension, or units, of the quantity α. Let $u(\vec{x}, t)$ denote the pressure measured at location $\vec{x}$ at time t and define

$$[u(\vec{x}, t)] = P = \frac{F}{D^2} = \frac{KD}{T^2 D^2} = \frac{K}{T^2 D} \tag{3.11}$$

where K is the unit for mass, T is the unit for time, and D is the unit for length.

Since the differential operator is a rate of change of a quantity with respect to another quantity, the dimension of the derivative is the original dimension divided by the dimension of the quantity the derivative is taken with respect to. In the case of the Laplacian differential operator, the derivative is taken with respect to spatial variables, and so

$$[\Delta u(\vec{x}, t)] = \frac{P}{D^2} = \frac{K}{T^2 D^3} \ . \tag{3.12}$$

Similarly, the dimension of the quantity $\frac{1}{c^2} u_{tt}$ is

$$\left[\frac{1}{c^2} u_{tt}(\vec{x}, t)\right] = \left[\frac{1}{c^2}\right] [u_{tt}(\vec{x}, t)] = \left(\frac{T^2}{D^2}\right)\left(\frac{P}{T^2}\right) = \frac{P}{D^2} = \frac{K}{T^2 D^3}. \tag{3.13}$$

It is important to note that the terms Δu and $\frac{1}{c^2} u_{tt}$ have the same dimension. Any two physical quantities added or subtracted together must have the same dimension. Keeping this in mind, it is easily seen that

$$\left[\vec{\Gamma} \cdot \nabla u\right] = \left[\vec{\Gamma}\right] [\nabla u] = \left[\vec{\Gamma}\right] \frac{P}{D} = \left[\vec{\Gamma}\right] \frac{K}{T^2 D^2}$$

implies that

$$\left[\vec{\Gamma}\right] = \frac{1}{D},$$

which is consistent with the fact that the atmospheric attenuation occurs as the wave travels through space. As discussed earlier, atmospheric attenuation is very small compared with practical boundary attenuation, and so examination of the dimensionless quantity $W\vec{\Gamma}$, where W is the largest dimension of the domain, can lead to a justifiable approximation of $\vec{\Gamma} = 0$. In a room with absolute temperature 293 K with 30% relative humidity, the atmospheric attenuation is only .025 decibels per one hundred meters. In a typical room, this attenuation is negligible, and so the dampening term discussed in Chapter II will be dropped from the wave equation, since the assumption is being made that the largest dimension of the room is small compared to the speed of sound.

3.4 Duhamel's Principle

3.4.1 Impetus. When solving any differential equation, ordinary or partial, it is always easier to deal with a homogeneous problem than a driven one. As the problem was stated earlier, the differential equation driven by a forcing function with its associated initial and boundary conditions is

$$\begin{array}{rcccl} Lu & = & f & , & \vec{x} \in \Omega,\ t > 0 \\ u(\vec{x},0) & = & 0 & , & \vec{x} \in \Omega,\ t = 0 \\ u_t(\vec{x},0) & = & 0 & , & \vec{x} \in \Omega,\ t = 0 \\ \frac{\partial u}{\partial n} & = & \beta(x)\, u & , & \vec{x} \in \partial\Omega,\ t > 0 \end{array} , \tag{3.14}$$

while what is sought is a differential equation with associated initial and boundary conditions in the form

$$\begin{array}{rcccl} Lu & = & 0 & , & \vec{x} \in \Omega,\ t > 0 \\ u(\vec{x},0) & = & 0 & , & \vec{x} \in \Omega,\ t = 0 \\ u_t(\vec{x},0) & = & g & , & \vec{x} \in \Omega,\ t = 0 \\ \frac{\partial u}{\partial n} & = & \beta(x)\, u & , & \vec{x} \in \partial\Omega,\ t > 0 \end{array} . \tag{3.15}$$

There is a theorem that accomplishes this goal, called Duhamel's Principle (11:298). This theorem allows for the reformulation of the original problem into a homogeneous one. This homogeneous problem has the same linear differential operator, but without quiescent initial conditions.

3.4.2 Restatement of Problem. If $u(\vec{x},t)$ satisfies the original differential equation, then $v(\vec{x},t;\tau)$ satisfies the initial boundary value problem

$$\begin{array}{rcll} \Delta v - \frac{1}{c^2} v_{tt} & = & 0, & \vec{x} \in \Omega,\ t > \tau \\ v(\vec{x},t;\tau) & = & 0, & \vec{x} \in \Omega,\ t = \tau \\ v_t(\vec{x},t;\tau) & = & -s(\tau)\,\delta(\vec{x}-\vec{x}_0), & \vec{x} \in \Omega,\ t = \tau \\ \frac{\partial v}{\partial n} & = & \beta v, & \vec{x} \in \partial\Omega,\ t > \tau \end{array} \tag{3.16}$$

where

$$u(\vec{x},t) = \int_0^t v(\vec{x},t;\tau)\, d\tau \ . \tag{3.17}$$

Since this equation is homogeneous in a rectangular domain, we can use the method of separation of variables to solve for $v(\vec{x}, t; \tau)$, which will yield the solution in terms of $u(\vec{x}, t)$.

3.5 Deriving and Inverting the RTF

3.5.1 Separation of Variables.

The geometry of a domain and its associated boundary is probably the largest driving factor in determining the form that the solution of a differential equation will take. Using a rectangular domain for the wave equation results in a solution in the form of infinite series of trigonometric functions, while solving the same equation in a circular domain introduces Bessel functions into the solution. It is fortuitous that most rooms are roughly rectangular, since this allows the use of this method to solve the equation.

The method of separation of variables is a valid method of solving a partial differential equation in rectangular domains. A rectangular domain is a domain in which the boundaries with respect to all variables are not dependent on any other variables. For example, in Cartesian coordinates, a circular domain is invalid for using separation of variables to derive a solution since the boundary is of the form $y = \pm\sqrt{a^2 - x^2}$. However, performing a coordinate transformation into polar coordinates will result in a rectangular domain in terms of the radius and angle, upon which a separation of variables method can be used. Since the domain in question is already rectangular, the equation is ready to be solved. Begin by assuming that $v(\vec{x}, t; \tau) = Q(x, y, z)\, T(t; \tau)$.

3.5.1.1 The Time Component.

Assuming that v takes the form given above, it is easily seen that $v_{tt} = Q(x, y, z)\, \ddot{T}(t; \tau)$, and so the equation becomes

$$\frac{\Delta Q}{Q(x, y, z)} = \frac{\ddot{T}}{c^2 T(t; \tau)} = a_1 \tag{3.18}$$

where a_1 is some real constant. Focusing only on the second and last parts of this equation leads to the ordinary differential equation

$$\ddot{T} - a_1 c^2 T = 0 \tag{3.19}$$

which can take on three forms, each based on the value of a_1. These three cases are described next.

Case 1: $a_1 = \lambda_1^2 > 0$. In this case, the solution is the linear combination of hyperbolic functions

$$T(t;\tau) = A\sinh(\lambda_1 ct) + B\cosh(\lambda_1 ct) \ . \tag{3.20}$$

If $A+B$ is nonzero, then as t approaches infinity this function is unbounded, and so would violate the law of conservation of energy for the system, since the original signal has finite energy. Case 1 therefore does not yield a nontrivial solution.

Case 2: $a_1 = \lambda_1^2 = 0$. In this case, the solution is

$$T(t;\tau) = At + B \ . \tag{3.21}$$

By applying the first initial condition,

$$v(\vec{x},\tau;\tau) = Q(\vec{x})\,T(\tau;\tau) = 0$$

and assuming that $Q(x)$ is nonzero, then

$$\begin{aligned} T(\tau;\tau) &= A\tau - B = 0 \\ B &= A\tau \\ T(t;\tau) &= A(t-\tau) \ . \end{aligned}$$

And by applying the second initial condition,

$$v_t(\vec{x},\tau;\tau) = -s(\tau)\,\delta(\vec{x}-\vec{x}_0)$$

the implication

$$s(\tau) \propto A\tau,$$

comes up, and since A is assumed to be a constant, this implies that the signal is linear, and so violates the finite-energy law as well. Therefore, Case 2 also leads to only a trivial solution.

Case 3: $a_1 = -\lambda_1^2 < 0$. In the case of the last possibility, the solution is of the form

$$T(t;\tau) = A\sin(\lambda_1 ct) + B\cos(\lambda_1 ct)\ . \tag{3.22}$$

Applying the first initial condition and assuming that $Q(\vec{x}) \neq 0$ for all $\vec{x}$ yields

$$\begin{aligned} v(\vec{x}, t; \tau) &= Q(\vec{x})\, T(\tau;\tau) = 0 &(3.23)\\ T(\tau;\tau) &= A\sin(\lambda_1 c\tau) + B\cos(\lambda_1 c\tau) = 0 &(3.24)\\ B &= -A\tan(\lambda_1 c\tau) &(3.25)\\ T(t;\tau) &= A(\sin(\lambda_1 ct) - \tan(\lambda_1 c\tau)\cos(\lambda_1 c\tau)) &(3.26)\\ T(t;\tau) &= C\sin(\lambda_1 c(t-\tau)) &(3.27) \end{aligned}$$

where $C = A\cos(\lambda_1 c\tau)$. Since this is a constant, this is allowed. Because the second initial condition is not quiescent, it must be applied after the eigenfunctions have been found. As this is a finite function and is not identically equal to zero, then the case is valid, and so

$$T(t;\tau) = A\sin(\lambda_1 c(t-\tau)) \tag{3.28}$$

3.5.1.2 The Z Component. Now, let $Q(x, y, z) = P(x, y)\, Z(z)$. Then

$$\frac{P_{xx} + P_{yy}}{P(x, y)} = -\frac{Z''}{Z} - \lambda_1^2 = a_2 \tag{3.29}$$

where a_2 is some real constant. Examining the Z component first yields the ordinary differential equation

$$Z'' + \left(\lambda_1^2 + a_2\right) Z = 0 \tag{3.30}$$

which has characteristic values

$$\pm\sqrt{-\lambda_1^2 - a_2}\ . \tag{3.31}$$

Again, there are three cases which will describe what value a_2 takes.

Case 1: $a_2 < -\lambda_1^2$. Let $\pm\sqrt{-\lambda_1^2 - a_2} = \pm q^2$. In this case, the quantity under the radical sign is positive, and so

$$Z(z) = Ae^{qz} + Be^{qz} \ . \tag{3.32}$$

On the floor, at $z = 0$, the Robin boundary condition is

$$\begin{aligned} -Z'(0) &= \beta_z Z(0) & (3.33)\\ 0 &= Z'(0) + \beta_z Z(0) & (3.34)\\ 0 &= Aq + Bq + \beta_z A + \beta_z B & (3.35)\\ B &= -A \ . & (3.36) \end{aligned}$$

On the ceiling, at $z = H$, the boundary condition is

$$\begin{aligned} Z'(H) &= \beta_z Z(H) & (3.37)\\ 0 &= Z'(H) - \beta_z Z(H) & (3.38)\\ 0 &= Aqe^{qH} + Bqe^{qH} - \beta_z \left(Ae^{qH} + Be^{qH}\right) & (3.39)\\ 0 &= A(q - \beta_z) + B(q - \beta_z) & (3.40) \end{aligned}$$

which leads to a trivial solution since $A = -B$. Since this solution would lead to the entire function $v(\vec{x}, t; \tau)$ being identically zero, this case is invalid.

Case 2: $a_2 = -\lambda_1^2$. In this case the characteristic root is simply, in which case the solution takes the form

$$Z(z) = Az + B \ , \tag{3.41}$$

and so

$$Z'(z) = A \tag{3.42}$$

The boundary conditions leads to the following two equations:

$$\begin{aligned} Z'(0) + \beta_z Z(0) &= 0 && (3.43) \\ Z'(H) - \beta_z Z(H) &= 0 && (3.44) \end{aligned}$$

but it is clear that this case is trivial as well, and so case 2 is also invalid.

Case 3: $a_2 > -\lambda_1^2$. In this case, the equation has the characteristic values

$$\pm i\sqrt{a_2 + \lambda_1^2}\ , \tag{3.45}$$

the solution takes the form

$$Z(z) = A\sin(qz) + B\cos(qz) \tag{3.46}$$

where $q = \sqrt{a_2 + \lambda_1^2}$. Using the boundary conditions, the coefficients can be solved for:

$$\begin{aligned} Z'(0) + \beta_z Z(0) &= 0 = Aq + B\beta_z && (3.47) \\ B &= -A\frac{q}{\beta_z} \\ Z'(H) - \beta_z Z(H) &= 0 = A\left(q\cos(qH) - \beta_z \sin(qH)\right) - B\left(q\sin(qH) + \beta_z\cos(qH)\right) \\ 0 &= A\sin(qH)\left[-\beta_z + \frac{q^2}{\beta_z}\right] + A\frac{q^2}{\beta_z}\sin(qH) + Aq\cos(qH) && (3.48) \\ \tan(qH) &= \frac{2q\beta_z}{q^2 - \beta_z^2} && (3.49) \end{aligned}$$

So the eigenvalues associated with the Z-component of the equation, call them $q = \lambda_l$, are the intersections of the curves $\tan(qH)$ and $\frac{2q\beta_z}{q^2-\beta_z^2}$, and so

$$a_2 = \lambda_1^2 - \lambda_l^2\ . \tag{3.50}$$

The eigenvalue-eigenfunction solution for the Z component is therefore

$$Z(z) = A_l \left(\sin(\lambda_l z) - \frac{\lambda_l}{\beta_z} \cos(\lambda_l z) \right), \; l = 1, 2, 3, ...$$

3.5.1.3 The Y Component. One the left-hand side of the equation remains the expression

$$\frac{P_{xx} + P_{yy}}{P(x,y)} = -\lambda_l^2 \; . \tag{3.51}$$

Letting $P(x,y) = X(x)\,Y(y)$ results in the elimination of the last partial differential operators and leaves only the ordinary differential equations

$$\frac{X''}{X(x)} = -\frac{Y''}{Y(y)} - \lambda_l^2 = a_3 \tag{3.52}$$

where a_3 is some real constant. Looking at the $Y(y)$ portion of this equation leads to the ordinary differential equation

$$Y'' + \left(a_3 + \lambda_l^2\right) Y = 0 \; . \tag{3.53}$$

Since the basic ordinary linear differential operator is acting upon Y as is being acted upon Z, the same arguments for the value of the constant $\left(a_3 + \lambda_l^2\right)$ are made as in the previous section. So by using the same methodology used for the Z component, the Y component solution is

$$Y_m(y) = B_m \left(\sin(\lambda_m y) - \frac{\lambda_m}{\beta_y} \cos(\lambda_m y) \right), \; m = 1, 2, 3, ... \tag{3.54}$$

where λ_m are the intersections of the curves $\tan(qL)$ and $\frac{2q\beta_y}{q^2 - \beta_y^2}$, and

$$q = \lambda_m = \sqrt{a_3 + \lambda_l} \; . \tag{3.55}$$

3.5.1.4 The X Component. The final component of the equation is the X component. It is of the form

$$X'' - a_3 X = 0 \tag{3.56}$$

Since λ_3^2 is a positive number, the equation has characteristic roots

$$\pm i\sqrt{a_3} \tag{3.57}$$

with the associated solution

$$X_n(x) = C_n\left(\sin(\lambda_n x) - \frac{\lambda_n}{\beta_x}\cos(\lambda_n x)\right),\ n = 1, 2, 3, ... \tag{3.58}$$

where $\lambda_n = -\sqrt{a_3}$. Using all of the relationships between the λ_i's, the expression

$$\lambda_1^2 = \lambda_{l,m,n}^2 = \lambda_l^2 + \lambda_m^2 + \lambda_n^2\ . \tag{3.59}$$

3.5.1.5 Joining the Components. Now that the four components of the solution have been found, they are multiplied together to solve for $v_{l,m,n}(\vec{x}, t; \tau)$, after which the principle of superposition is used to find $v(\vec{x}, t; \tau)$. These four components are

$$\begin{aligned}
T_{l,m,n}(t;\tau) &= A_{l,m,n}\sin(\lambda_{l,m,n}c(t-\tau)) && (3.60)\\
X_n(x) &= C_n\left(\sin(\lambda_n x) - \frac{\lambda_n}{\beta_x}\cos(\lambda_n x)\right) && (3.61)\\
Y_m(y) &= D_m\left(\sin(\lambda_m y) - \frac{\lambda_m}{\beta_y}\cos(\lambda_m y)\right) && (3.62)\\
Z_l(z) &= E_l\left(\sin(\lambda_l z) - \frac{\lambda_l}{\beta_z}\cos(\lambda_l z)\right) && (3.63)\\
l, m, n &= 1, 2, 3, ...
\end{aligned}$$

Let $\rho \in \{x, y, z\}$, $D \in \{W, L, H\}$, and $\sigma \in \{l, m, n\}$. Then the spatial components of the solution, the eigenfunctions with their associated eigenvalues for each component, can be generalized by the function

$$F_\sigma(\rho; D) = \left(\sin(\lambda_\sigma \rho) - \frac{\lambda_\sigma}{\beta_\rho}\cos(\lambda_\sigma \rho)\right)\ . \tag{3.64}$$

Factoring out the constants and multiplying the generalized functions together with the time component, the general solution is

$$v_{l,m,n}\left(\vec{x},t;\tau\right) = K_{l,m,n}F_l\left(z;H\right)F_m\left(y;L\right)F_n\left(x;W\right)\sin\left(\lambda_{l,m,n}c\left(t-\tau\right)\right), \quad t > \tau \tag{3.65}$$

$$v\left(\vec{x},t;\tau\right) = \sum_{l,m,n} K_{l,m,n}F_l\left(z;H\right)F_m\left(y;L\right)F_n\left(x;W\right)\sin\left(\lambda_{l,m,n}c\left(t-\tau\right)\right) \quad t > \tau, \tag{3.66}$$

where $K_{l,m,n} = A_{l,m,n}E_lD_mC_n$.

3.5.2 Determining Coefficients. Now the second initial condition must be applied to the solution to determine the values of the coefficients $K_{l,m,n}$. Recall that the second initial condition is

$$v_t\left(\vec{x},t;\tau\right) = -s\left(\tau\right)\delta\left(\vec{x}-\vec{x}_0\right) \tag{3.67}$$

and since there is only one term in $v\left(\vec{x},t;\tau\right)$ which is a function of t, the derivative is easy to compute:

$$v_t\left(\vec{x},t;\tau\right) = \sum_{l,m,n} K_{l,m,n}F_l\left(z;H\right)F_m\left(y;L\right)F_n\left(x;W\right)\lambda_{l,m,n}c\cos\left(\lambda_{l,m,n}c\left(t-\tau\right)\right) \tag{3.68}$$

$$v_t\left(\vec{x},\tau;\tau\right) = \sum_{l,m,n} K_{l,m,n}F\left(z;H\right)F_m\left(y;L\right)F_n\left(x;W\right)\lambda_{l,m,n}c = -s\left(\tau\right)\delta\left(\vec{x}-\vec{x}_0\right) \tag{3.69}$$

Now, by taking the weighted inner product over the domain, the coefficients $K_{l,m,n}$ can be solved for algebraically. Since these functions are all linearly independent, then the inner product has no weighting function by the Sturm-Liouville Theorem (7). Then the left-hand side of the inner product equation is

$$\left\langle \sum_{l,m,n} K_{l,m,n}F_l\left(z;H\right)F_m\left(y;L\right)F_n\left(x;W\right)\lambda_{l,m,n}c\ ,\ F_{l'}\left(z;H\right)F_{m'}\left(y;L\right)F_{n'}\left(x;W\right)\right\rangle \tag{3.70}$$

where $\langle u,v\rangle = \int_\Omega u\left(\vec{x}\right)v\left(\vec{x}\right)d\vec{x}$. Now, by taking the inner product over the domain, the inner product is

$$\int_0^D F_\sigma(\rho;D)\,F_{\sigma'}(\rho;D)\,d\rho, \tag{3.71}$$

the value of which, since the eigenvalues are the intersections of two curves, depend on the value of σ and σ'. The important thing is that the eigenfunctions are orthogonal, and so for $\sigma \neq \sigma'$, the value of the inner product is equal to zero, and so the triple sum on the left-hand side collapses to one single term

$$c\lambda_{l',m',n'}K_{l',m',n'}f\left(z,l',H\right)f\left(y,m',L\right)f\left(x,n',W\right) \tag{3.72}$$

$$f(\rho,\sigma,D)=\int_0^D F_\sigma(\rho;D)\,F_{\sigma'}(\rho;D)\,d\rho\ . \tag{3.73}$$

Now, the right-hand side of the inner product equation is

$$\langle -s(\tau)\,\delta(\vec{x}-\vec{x}_0)\ \ ,\ F_{l'}(z;H)\,F_{m'}(y;L)\,F_{n'}(x;W)\rangle \tag{3.74}$$

and since the Dirac delta function acts as an evaluation when it is in the integrand, the right-hand side simplifies to

$$-s(\tau)\,\psi_{l',m',n'}(\vec{x}_0)\ \ , \tag{3.75}$$

where

$$\psi_{l,m,n}(\vec{x})=F_l(z;H)\,F_m(y;L)\,F_n(x;W) \tag{3.76}$$

and defining $f_{l,m,n}$ as

$$f_{l,m,n}=f(x,n,W)\,f(y,m,L)\,f(z,l,H) \tag{3.77}$$

the equation of inner products is now

$$c\lambda_{l',m',n'}K_{l',m',n'}f_{l',m',n'}=-s(\tau)\,\psi_{l',m',n'}(\vec{x}_0)\ \ , \tag{3.78}$$

and so,

$$K_{l,m,n}=\frac{-s(\tau)\,\psi_{l,m,n}(\vec{x}_0)}{c\lambda_{l,m,n}f_{l,m,n}}\ . \tag{3.79}$$

3.5.3 The Solution.

3.5.3.1 The Time Domain. Now that the initial conditions have been utilized to solve for the undetermined coefficients, the solution for $v\left(\vec{x}, t; \tau\right)$ is

$$v\left(\vec{x}, t; \tau\right) = \sum_{l,m,n} \frac{\psi_{l,m,n}\left(\vec{x}\right)\psi_{l,m,n}\left(\vec{x}_0\right)\left[-s\left(\tau\right)\sin\left(\lambda_{l,m,n}c\left(t-\tau\right)\right)\right]}{c\lambda_{l,m,n}f_{l,m,n}} , \tag{3.80}$$

and so, by Duhamel's Principle, the time domain solution is

$$u\left(\vec{x}, t\right) = \sum_{l,m,n} \frac{\psi_{l,m,n}\left(\vec{x}\right)\psi_{l,m,n}\left(\vec{x}_0\right)\int_0^t -s\left(\tau\right)\sin\left(\lambda_{l,m,n}c\left(t-\tau\right)\right)d\tau}{c\lambda_{l,m,n}f_{l,m,n}} . \tag{3.81}$$

3.5.3.2 The Laplace Transform. Now, the orginal signal $s\left(t\right)$ can be computed, and since it is contained within an integral, a transformation must be applied to eliminate this integral. The usual transformation of choice is the Fourier transform, but Fourier transforms deal with doubly infinite convolutions (the limits of integration are positive and negative infinity), whereas this integral has finite limits of integration. The Laplace transform can eliminate this integral most appropriately (10:31), and so taking the Laplace transform of both sides with respect to the time variable yields

$$\mathcal{L}\left\{\int_0^t -s\left(\tau\right)\sin\left(\lambda_{l,m,n}c\left(t-\tau\right)\right)d\tau\right\} = -\mathcal{L}\left\{s\left(t\right)\right\}\mathcal{L}\left\{\sin\left(\lambda_{l,m,n}c\left(t\right)\right)\right\} \tag{3.82}$$

$$\mathcal{L}\left\{f\left(t\right)\right\} = \int_0^\infty e^{-\mu t}f\left(t\right)dt \tag{3.83}$$

and so the Laplace transform of a finite convolution is the product of the Laplace transforms of the elements of the convolution. Coupled with the fact that $\mathcal{L}\left\{\sin\left(\alpha t\right)\right\} = \frac{\alpha}{\mu^2+\alpha^2}$ this result yields

$$\hat{u}\left(\vec{x}, \mu\right) = -\hat{s}\left(\mu\right)\sum_{l,m,n}\frac{\psi_{l,m,n}\left(\vec{x}\right)\psi_{l,m,n}\left(\vec{x}_0\right)}{f_{l,m,n}\left(\mu^2+\lambda_{l,m,n}^2\right)} \tag{3.84}$$

$$\hat{u}\left(\vec{x}, \mu\right) = \mathcal{L}\left\{u\left(\vec{x}, t\right)\right\} \tag{3.85}$$

$$\hat{s}\left(\mu\right) = \mathcal{L}\left\{s\left(t\right)\right\} . \tag{3.86}$$

3.5.4 The Inversion.

3.5.4.1 The Laplace Transform.

The Laplace transform of the original signal, $\hat{s}(\mu)$ can now be solved for algebraically. This simple operation results in the Laplace domain solution of Problem 1:

$$\hat{s}(\mu) = \frac{-\hat{u}(\vec{x}, \mu)}{\sum_{l,m,n} \frac{\psi_{l,m,n}(\vec{x})\psi_{l,m,n}(\vec{x}_0)\lambda_{l,m,n}}{f_{l,m,n}\left(\mu^2+\lambda_{l,m,n}^2\right)}} . \tag{3.87}$$

3.5.4.2 Back To the Time Domain.

Now, by taking the inverse Laplace transform of both sides, the time-dependent and source location independent solution to Problem 1 is

$$s(t) = \mathcal{L}^{-1}\left\{ \frac{-\hat{u}(\vec{x}_r, \mu)}{\sum_{l,m,n} \frac{\psi_{l,m,n}(\vec{x}_r)\psi_{l,m,n}(\vec{x}_0)}{f_{l,m,n}\left(\mu^2+\lambda_{l,m,n}^2\right)}} \right\} \tag{3.88}$$

where $\vec{x}_r$ is the receiver location. Now, this Laplace transform will most likely need to be solved numerically since it is the Laplace transform of a term with a triple infinite sum in the denominator, but once completed, the solution to Problem 1 is complete.

3.6 Filtering Multiple Sources

3.6.1 Introduction.

In this next section Problem 2, the problem of filtering multiple signals, is explored. The goal of this section is to generalize Problem 1 to filter extraneous signals from a recording. In this section, all computations are made in the Laplace transformed time domain, and so all results will be stated in terms of the time-dependent signal's Laplace transform, unless the specified equation explicitly contains the variable t. In this section, $\vec{x}_{r,i}$ denotes the location of the i^{th} receiver while $\vec{x}_{s,i}$ denotes the location of the i^{th} source.

3.6.2 Restating the Problem.

Since this problem concerns multiple signals, the right-hand side of the modified wave equation must reflect this change. Whereas in

Problem 1 the right-hand side is

$$s\left(t\right)\delta\left(\vec{x}-\vec{x}_0\right)\ , \tag{3.89}$$

the right-hand side of the wave equation in Problem 2 has a sum of signals at distinct locations. The appropriate expression for n sources at location i, for $i = 1, 2, ..., n$ is

$$\sum_{i=1}^{n} s_i\left(t\right)\delta\left(\vec{x}-\vec{x}_{s,i}\right)\ . \tag{3.90}$$

If there are also multiple receivers, then this system consists of the convolution equations

$$u\left(\vec{x}_{r,j},t\right)=\sum_{l,m,n}\frac{\psi_{l,m,n}\left(\vec{x}_{r,j}\right)\sum_{i=1}^{n}\psi_{l,m,n}\left(\vec{x}_{s,i}\right)\int_0^t -s_i\left(\tau\right)\sin\left(\lambda_{l,m,n}c\left(t-\tau\right)\right)d\tau}{c\lambda_{l,m,n}f_{l,m,n}} \tag{3.91}$$

$$i = 1, 2, ..., n \qquad\qquad j = 1, 2, ..., m$$

and taking the Laplace transform of both sides results in the system of equations

$$\hat{u}\left(\vec{x}_{r,j},\mu\right)=-\sum_{l,m,n}\frac{\psi_{l,m,n}\left(\vec{x}_{r,j}\right)\sum_{i=1}^{n}\psi_{l,m,n}\left(\vec{x}_{s,i}\right)\hat{s}_i\left(\mu\right)}{f_{l,m,n}\left(\mu^2+\lambda_{l,m,n}^2\right)} \tag{3.92}$$

$$\hat{u}\left(\vec{x}_{r,j},\mu\right)=\mathcal{L}\left\{u\left(\vec{x}_{r,j},t\right)\right\}\ . \tag{3.93}$$

In a more compact form, the system can be written as

$$\hat{u}_j\left(\mu\right) = -\sum_{i=1}^{n}\hat{s}_i\left(\mu\right)H\left(\vec{x}_{r,j},\vec{x}_{s,i},\mu\right) \tag{3.94}$$

$$H\left(\vec{x}_{r,j},\vec{x}_{s,i},\mu\right) = H_{ij}=-\sum_{l,m,n}\frac{\psi_{l,m,n}\left(\vec{x}_{r,j}\right)\psi_{l,m,n}\left(\vec{x}_{s,i}\right)}{f_{l,m,n}\left(\mu^2+\lambda_{l,m,n}^2\right)} \tag{3.95}$$

$$\hat{u}_j\left(\mu\right) = \hat{u}\left(\vec{x}_{r,j},\mu\right)\ . \tag{3.96}$$

3.6.3 A General Solution. Since this system is linear with two separate indices, it can be written as a matrix equation, where m is the number of receivers and n is the number of sources. The $m \times n$ matrix of transfer functions is

$$\mathbb{H} = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1n} \\ H_{21} & H_{22} & \cdots & H_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ H_{m1} & H_{m2} & \cdots & H_{mn} \end{bmatrix} \tag{3.97}$$

and the system of linear equations is the matrix equation

$$\mathbb{H}\hat{\mathbf{s}} = \hat{\mathbf{u}} \tag{3.98}$$

where

$$\hat{\mathbf{s}} = \begin{bmatrix} \hat{s}_1(\mu) \\ \hat{s}_2(\mu) \\ \vdots \\ \hat{s}_n(\mu) \end{bmatrix} \tag{3.99}$$

$$\hat{\mathbf{u}} = \begin{bmatrix} \hat{u}_1(\mu) \\ \hat{u}_2(\mu) \\ \vdots \\ \hat{u}_m(\mu) \end{bmatrix}. \tag{3.100}$$

This matrix equation can be solved to find the vector $\hat{\mathbf{s}}$. As is the case with all linear equations, certain conditions upon $\mathbb{H}$ must be satisfied in order for there to be a solution, if any. $\mathbb{H}$ must be invertible, which is discussed in a later section, to obtain a unique solution. If there is not a unique solution, as in the case where the number of sources outnumbers the number of receivers, then a solution of minimum norm can be found, and if there are more receivers than signals, a least-squares solution can be computed.

3.6.4 A Specific Case–2 Receivers, 2 Sources. To apply this problem to a real-world problem, consider the case of two sources and two receivers. By placing no limitations on the location of the receivers or sources, this example confirms that a single signal can be found by the received signal when the other signal is considered to be $\hat{s}_i(\mu) = 0$. Since this is a linear problem, the system of equations reverts to the original problem, but with linearly dependent equations. If the source and receiver locations are constrained such that no two sources or receivers are collocated, then the matrix equation can be used to solve for the individual original signals. The assumptions from Problem 1 are still needed, i.e. the sources are point sources not located on the boundary and the microphones are omnidirectional. The system of equations for this problem is

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \hat{s}_1(\mu) \\ \hat{s}_2(\mu) \end{bmatrix} = \begin{bmatrix} \hat{u}_1(\mu) \\ \hat{u}_2(\mu) \end{bmatrix} . \tag{3.101}$$

Note that the spatial dependencies of $\hat{s}_i(\mu)$ and $\hat{u}_j(\mu)$ are contained in the subscripts. Now, before a solution to this problem can be found, the nature of the matrix $\mathbb{H}$ must be determined, as it determines the existence of a solution.

3.6.5 Limitations. When dealing with the solvability of a matrix equation, there are three possibilities. The first is that there exist an infinite number of solutions. In this case, the number of sources outnumbers the number of receivers and so the system is underdetermined. When this is the case, a solution which has minimum norm must be sought, most easily by using the Moore-Penrose pseudoinverse of the form where $m < n$ (4:114). The second possibility is that there exists no solution to the equation. If this is the case, then a least-squares solution can be attempted by using the Moore-Penrose pseudoinverse for $m > n$. The final possibility is that there exists a unique solution to the system of equations. In this case, the matrix $\mathbb{H}$ is invertible, and standard matrix algebra can be used to solve for the unknown vector $\hat{\mathbf{s}}$. $\mathbb{H}$ is invertible only if $\mathbb{H}$ is a square matrix, and in the case of two receivers and two sources, this is true, but the existence of a solution is guaranteed if and only if the determinant of the matrix is nonzero.

3.6.5.1 The Determinant. Given that the determinant of a 2×2 matrix is

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc , \tag{3.102}$$

the determinant of the matrix $\mathbb{H}$ can be easily computed to be

$$\det(\mathbb{H}) = \begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix} = H_{11}H_{22} - H_{12}H_{21} \ . \tag{3.103}$$

Multiplying any arbitrary two transfer functions together yields the sum

$$H_{ij}H_{hk} = \sum_{l,m,n} \frac{\psi_{l,m,n}\left(\vec{x}_{r,j}\right)\psi_{l,m,n}\left(\vec{x}_{s,i}\right)}{f_{l,m,n}\left(\mu^2 + \lambda_{l,m,n}^2\right)} \sum_{p,q,r} \frac{\psi_{p,q,r}\left(\vec{x}_{r,k}\right)\psi_{p,q,r}\left(\vec{x}_{s,h}\right)}{f_{p,q,r}\left(\mu^2 + \lambda_{p,q,r}^2\right)} \ , \tag{3.104}$$

and with a bit of algebraic manipulation, the expression for the determinant can be simplified to

$$|\mathbb{H}| = \sum_{l,m,n,pq,r} \frac{\psi_{l,m,n}\left(\vec{x}_{s,1}\right)\psi_{p,q,r}\left(\vec{x}_{s,2}\right)\left[\psi_{l,m,n}\left(\vec{x}_{r,1}\right)\psi_{p,q,r}\left(\vec{x}_{r,2}\right) - \psi_{l,m,n}\left(\vec{x}_{r,2}\right)\psi_{p,q,r}\left(\vec{x}_{r,1}\right)\right]}{f_{l,m,n}f_{p,q,r}\left(\mu^2 + \lambda_{l,m,n}^2\right)\left(\mu^2 + \lambda_{p,q,r}^2\right)} \ . \tag{3.105}$$

Clearly, if

$$\psi_{l,m,n}\left(\vec{x}_{r,1}\right)\psi_{p,q,r}\left(\vec{x}_{r,2}\right) = \psi_{l,m,n}\left(\vec{x}_{r,2}\right)\psi_{p,q,r}\left(\vec{x}_{r,1}\right) \tag{3.106}$$

then the determinant of the matrix is equal to zero. This will happen only if $\vec{x}_{r,1} = \vec{x}_{r,2}$ which implies that if the receivers are located in the same position, then the matrix is not invertible. This makes sense considering that if two microphones are collocated, they will record the same signal. There would be no difference between the transformed signals $\hat{u}_1(\mu)$ and $\hat{u}_2(\mu)$, and so the matrix would have linearly dependent rows. Similarly, if the sources are located at the same position, then the determinant would be

$$H\left(\vec{x}_{r,1}, \vec{x}_{s,i}, \mu\right) H\left(\vec{x}_{r,2}, \vec{x}_{s,i}, \mu\right) - H\left(\vec{x}_{r,2}, \vec{x}_{s,i}, \mu\right) H\left(\vec{x}_{r,1}, \vec{x}_{s,i}, \mu\right) = 0 \ . \tag{3.107}$$

Since $H\left(\vec{x}_{r,j}, \vec{x}_{s,i}, \mu\right)$ has no dependence upon the source signal (only its location), two different signals propagated from the same location will be impossible to separate using this method. Therefore, for the matrix $\mathbb{H}$ to be invertible, the only requirement is that the two sources be located separately and the two receivers be located separately. If a source and receiver were located together, then there is no problem. In fact, this would make it very easy to filter the two signals.

3.6.5.2 The Condition Number. The determinant is not the only factor in the solvability of a system of linear equations. There is also the condition number of the matrix. The condition number is a measure of the stability of the matrix operator in terms of solving a system of linear equations. If a matrix is nearly linearly dependent (meaning that the determinant is close to zero), then the matrix will have a high condition number. The condition number is not unique; it depends upon which norm is chosen to evaluate the matrix under. Therefore, there are a number of different possibilities of norms that can be used. When dealing with matrices, the Frobenius norm is a good norm to use since the evaluation of the norm requires only the squaring and summing of the matrix components. The Frobenius norm of a matrix is defined as

$$\|A\|_F^2 = \sum_{i,j} a_{ij}^2 \ . \tag{3.108}$$

Once the norm is chosen, the condition number of the matrix A is defined as

$$\kappa_\alpha (A) = \|A\|_\alpha \left\|A^{-1}\right\|_\alpha \ . \tag{3.109}$$

where α is the choice of norm used in finding the condition number. The expression can be read as "The condition number κ of the matrix A with respect to the norm α is equal to the α norm of A times the α norm of A^{-1}." As stated before, the Frobenius norm is useful here because it requires minimal calculations and the inverse of a 2×2 matrix has a very simple formula. The Frobenius norm of the matrix $\mathbb{H}$ with two receivers and two sources is given by

$$\|\mathbb{H}\|_F = \left(H_{11}^2 + H_{12}^2 + H_{21}^2 + H_{22}^2\right)^{\frac{1}{2}} \ . \tag{3.110}$$

and the norm of $\mathbb{H}^{-1}$, where

$$\mathbb{H}^{-1} = \frac{1}{H_{11}H_{22} - H_{21}H_{12}} \begin{bmatrix} H_{22} & -H_{12} \\ -H_{21} & H_{11} \end{bmatrix} , \tag{3.111}$$

is given by

$$\left\|\mathbb{H}^{-1}\right\|_F = \frac{1}{\det(\mathbb{H})}\left(H_{11}^2 + H_{12}^2 + H_{21}^2 + H_{22}^2\right)^{\frac{1}{2}} = \frac{1}{\det(\mathbb{H})}\left\|\mathbb{H}\right\|_F \ . \tag{3.112}$$

So the condition number of the matrix $\mathbb{H}$ is

$$\kappa_F(\mathbb{H}) = \frac{\|\mathbb{H}\|_F^2}{\det(\mathbb{H})} \tag{3.113}$$

and it is clear that the condition number of the matrix is inversely related to the determinant of the matrix of transfer functions. Simply put, if the receivers or sources are located closely together, then the condition number will be large, and the matrix will be ill-conditioned, yielding a potentially unstable result. There are ways around using a matrix with a large condition number, such as using a singular value decomposition of the matrix, or a QR decomposition, but these measures will not be discussed here.

3.6.5.3 The Solution. If $\mathbb{H}$ is invertible with a low condition number, then the system of equations can be solved simply by inverting the matrix $\mathbb{H}$. Multiplying both sides of the equation by $\mathbb{H}^{-1}$ yields the solution

$$\begin{bmatrix} \hat{s}_1(\mu) \\ \hat{s}_2(\mu) \end{bmatrix} = \begin{matrix} \frac{H_{22}\hat{u}_1(\mu) - H_{12}\hat{u}_2(\mu)}{\det(\mathbb{H})} \\ \frac{H_{11}\hat{u}_2(\mu) - H_{21}\hat{u}_1(\mu)}{\det(\mathbb{H})} \end{matrix} \ . \tag{3.114}$$

3.7 Filtering Sources of Variable Location

3.7.1 Restating the Problem. In Chapter I, Problem 3 was introduced as the problem of filtering the signal of a mobile source. In Problems 1 and 2, the sources were assumed to have constant location during the recording, and so the position was independent of time. In Problem 3, however, the position of the source is a function of time, and so the problem is somewhat more complicated than the previous problems. The partial differential equation needed to solve this problem is very close to that of Problem 1, with one slight change:

$$\Delta u - \frac{1}{c^2}u_{tt} = s(t)\,\delta(\vec{x} - \vec{x}_0(t)) \ , \tag{3.115}$$

where $\vec{x}_0(t)$ is the location of the source at time t. Applying Duhamel's Principle to this problem yields the same homogeneous differential equation, but with the initial condition

$$v_t(\vec{x},\tau;\tau) = -s(\tau)\,\delta(\vec{x}-\vec{x}_0(\tau)) \ . \tag{3.116}$$

The analysis of the problem is very much the same as it was with Problem 1, but when the weighted inner product is taken over the domain, the right-hand side of the inner product equation is somewhat different:

$$\langle -s(\tau)\,\delta(\vec{x}-\vec{x}_0(\tau)) \ , \ F_{l'}(z;H)\,F_{m'}(y;L)\,F_{n'}(x;W)\rangle \tag{3.117}$$

Whereas in Problem 1, only the time-dependent signal was a function of τ, any term that is dependent upon the source location $\vec{x}_0$ has a time-dependency. The right-hand side of the inner product equation is

$$-s(\tau)\,\psi_{l',m',n'}(\vec{x}_0(\tau)) \ . \tag{3.118}$$

The complications do not manifest until the Laplace transform of the finite convolution is taken. Since the source location is dependent upon time, it cannot be factored out of the integral, and so the Laplace transform is more complicated. The convolution is

$$\int_0^t -s(\tau)\,\psi_{l',m',n'}(\vec{x}_0(\tau))\sin(\lambda_{l,m,n}c(t-\tau))\,d\tau \ . \tag{3.119}$$

To simplify this convolution, define

$$G(t) = -s(t)\,\psi_{l',m',n'}(\vec{x}_0(t)) \tag{3.120}$$

Then the integral becomes

$$\int_0^t G(\tau)\sin(\lambda_{l,m,n}c(t-\tau))\,d\tau \ , \tag{3.121}$$

and by taking the Laplace transform, the expression is similar to that of Problems 1 and 2, but with $\hat{G}(\mu)$ instead of $\hat{s}(\mu)$:

$$\mathcal{L}\left\{\int_0^t G(\tau)\sin(\lambda_{l,m,n}c(\tau - t))\,d\tau\right\} = \hat{G}(\mu)\,\mathcal{L}\{\sin(\lambda_{l,m,n}ct)\} \tag{3.122}$$

$$\hat{G}(\mu) = \mathcal{L}\{G(t)\}\ . \tag{3.123}$$

$\hat{G}(\mu)$ can be expressed in the same way as $\hat{s}(\mu)$ in the previous problems. So,

$$\hat{G}(\mu) = \frac{\hat{u}(\vec{x},\mu)}{\sum_{l,m,n}\frac{\psi_{l,m,n}(\vec{x})\psi_{l,m,n}(\vec{x}_0)}{f_{l,m,n}\left(\mu^2+(\lambda_{l,m,n}c)^2\right)}}\ , \tag{3.124}$$

but a solution for $\hat{s}(\mu)$ is desired, so some analysis on $\hat{G}(\mu)$ must be performed. By defining

$$M(t) = \psi_{l',m',n'}(\vec{x}_0(t))\ , \tag{3.125}$$

then taking the Laplace transform of both sides of

$$G(t) = s(t)\,M(t) \tag{3.126}$$

yields

$$\hat{G}(\mu) = \hat{s}(\mu) * \hat{M}(\mu) = \int_0^\infty \hat{s}(\eta)\,\hat{M}(\mu - \eta)\,d\eta \tag{3.127}$$

since the Laplace transform of a product of two functions is the convolution of their Laplace transforms. By taking a second Laplace transform with respect to the transform variable η, the convolution is converted to a multiplication of the two doubly transformed functions, and $s(t)$ can be solved for by taking the inverse Laplace transforms of both sides, first with respect to η, and second with respect to μ.

There is a simpler way to achieve an approximation of this result, however. Assuming that the velocity of the source is small compared to the speed of sound, that is

$$\left|\frac{d\vec{x}}{dt}\right| \ll c, \tag{3.128}$$

then by sampling the source location over small time intervals and summing the signals over the length of the recording, the problem can be converted to one similar to Problem 1, but with the signal computed on each interval and then summed. In essence, Problem 1 is applied to each interval. To achieve this goal, let

$$\vec{x}_0\left(t_i\right)=\vec{b}_i,\ t_{i-1}\leq t\leq t_i,\ i=1,2,...,N \tag{3.129}$$

where $t_0=0$ and $t_N=T$, the total length of time of the recording. Then, for $t_{i-1}\leq t\leq t_i$,

$$u_i\left(\vec{x},t\right) = \sum_{l,m,n}\frac{\psi_{l,m,n}\left(\vec{x}\right)\psi_{l,m,n}\left(\vec{b}_i\right)\int_0^t -s_i\left(\tau\right)\sin\left(\lambda_{l,m,n}\left(t-\tau\right)\right)d\tau}{c\lambda_{l,m,n}f_{l,m,n}} \tag{3.130}$$

$$u\left(\vec{x},t\right) = \sum_{i=1}^{N}u_i\left(\vec{x},t\right) \tag{3.131}$$

Thus, applying the results of Problem 1 to each interval of the recording yields the original signal for that interval, and so the total signal is simply the sum of the signal over each interval:

$$s\left(t\right)=\sum_{i=1}^{N}s_i\left(t\right) \tag{3.132}$$

For these definitions of $u\left(\vec{x},t\right)$ and $s\left(t\right)$ to make sense, then outside of the interval $[t_{i-1},t_i]$, then $u_i\left(\vec{x},t\right)$ and $s_i\left(t\right)$ must be identically equal to zero. Continuity of the functions and their derivatives at the endpoints of the integral must also be considered so that the signal is smooth after being filtered. While the number of calculations required to perform this is N times the number of calculations required to filter a source with constant location, it is more applicable to the requirements of filtering the signals from a room of people. Efficiency must be considered when deciding how many points to sample, as well as the difficulty in determining the source location at any given time.

3.8 Conclusion

3.8.1 Results. In Problem 1, it is shown that the linear differential operator acting on a function $u\left(\vec{x},t\right)$ with a driving term $s\left(t\right)\delta\left(\vec{x}-\vec{x}_0\right)$ is invertible, and the original signal can be found after it has been acted on by the environment in which it propagates, as long as the geometry of the domain is sufficiently simple. In this case, the solution was found for a rectangular room, but it was shown in Section 2 that the problem can be generalized easily, where the only difference in the procedure is that of finding the eigenfunctions. Rectangular domains are especially good for solving analytically due to the fact that the Laplacian operator for this coordinate system is separable in the spatial domains. Because of this, the eigenfunctions were relatively easy to find. However, with different domains, the eigenvalues and eigenfunctions can be harder to compute, such as the case with a cylindrical domain. The Laplacian operator in a cylindrical Cartesian domain is not separable, except in the z-direction. However, after applying a coordinate transformation from Cartesian coordinates to polar coordinates, the operator is separable with the difference being that the coefficients are no longer constant. Because of this difference, functions containing Bessel functions emerge as eigenfunctions. Furthermore, in some irregular domains, a closed-form solution for the eigenfunctions may not even be possible, and so a numerical method such as a finite element method would be required to find the eigenfunctions. So, it is easy to see how a change in the domain geometry leads to different mode shapes for the room transfer function.

In Problem 2, the circumstances under which two signals can be filtered were explored, and the results were applied to the case of two receivers and two sources. While this is the simplest case conceivable, it is not very practical. It is easy to generalize the solution under two sources and receivers to a larger problem simply by expanding the dimensions of the matrix $\mathbb{H}$, but this leads to more computationally intensive equations. The circumstances under which the $\mathbb{H}$ matrix is ill-conditioned were also discussed, as well as the effects on the solution when $\mathbb{H}$ is ill-conditioned, leading to a potentially unstable result. This instability can be avoided by using singular value decompositions or QR decompositions.

In Problem 3, a source with variable location was discussed. The difficulty in solving this equation comes about from the extra terms left in the convolution integral, and the method of applying a second Laplace transform was offered as a means of extracting the original signal. A sampling method was then devised that yields an approximation of the total signal by sampling the source location during a time interval and summing the solutions on these intervals together, while assuming that the velocity of the source is small with respect to the speed of sound. While more costly in execution than Problem 1, it is far more practical than considering a human source stationary during the time of recording.

3.8.2 Practicality. As discussed before, the largest obstacle to overcome in solving Problem 1 for a particular domain is the geometry of the room. A different domain could produce vastly different results. The possibility of not knowing the parameters of the room must also be considered, such as the coefficients of reflection and whether or not the boundaries transmit sound outside of the domain.

The question of real-time filtering is important since the importance of the information being filtered could be very high. In the case of intelligence gathering, decisions may be based on the information contained in the recordings, and so it is necessary that this information be found as quickly as possible, so real-time filtering is somewhat important here. In the case of using a filter to eliminate noise heard in a phone call from a loud room, real-time filtering is a necessity for the product to be marketable to the public. However, in the case of recording boardroom meetings, the need for real-time filtering is nonexistent, since the filtered signals would be stored for future use anyway. There is no inherent problem in the mathematics of the idea of real-time filtering using this method, since the expression for $u(\vec{x}, t)$ is not dependent upon the total duration of the recording, T. Therefore, a computer program could filter each second recorded in a period of time dependent only upon the number of signals and number of receivers, and of course the speed at which it can perform calculations. The implementation of such a program is developed and discussed in Chapter IV.

The final limitation on the practicality of this method is that of being able to determine source locations. It is entirely possible to determine source locations using an echolocation procedure much the same as bats use ultrasonic sonar to determine the location of their prey (source here). Whether or not this active form of source localization or a passive form using the difference in times at which the receivers actually receive the signal depends on the circumstances under which the recording is being made. If it is covert or would interfere with the conversation being recorded, then a passive form of source localization must be used. If it is a meeting that everyone knows is being recorded or where there is no way that they could know about the active localization, then an active form of source localization can be used.

IV. Application Analysis

4.1 Introduction

This chapter attempts to determine the net effect of the environment on a signal in a computational approach using the results of Chapter III. The lack of experimental data severely hinders the full extent of the application of these results, but the procedure can be applied to known signals to determine how well the assumptions made in solving Problems 1 and 2 fit with the real-world effects of an environment on acoustic signals. Of the effects measured computationally, the net change in signal content in the Laplace transform domain will be measured on single-frequency tones. Without more powerful computing tools, the full analytic value of these approximations is diminished, but it serves to show the number of calculations required to filter the signal accurately. For example, the loop in the program that determines the components of the matrix $\mathbb{H}$ can require millions upon millions of cycles, depending on how accurate the results are needed to be. In the computations performed in this chapter, the loop that found these components cycles approximately two million times to achieve a numerical approximation of the transfer function.

The program used to implement the solutions found in Chapter III is MATLAB, version 7.0.1. The original signal is input symbolically and then quantified numerically to aid in computation speed. The original program used the symbolic algebra toolbox for MATLAB almost exclusively, but the computing power needed to perform this symbolically is very high, and the times for computing the transfer functions alone in this manner took more than 15 minutes per calculation. Since this method was so inefficient, it was dismissed as a possibility for finding the transfer functions. The numerical approach greatly reduces the computational time needed, but still takes approximately 6 minutes to implement for a simple single-frequency tone and so would be very inefficient for computing the transfer function for a highly intricate pattern of a typical human voice speaking for an extended period of time. The actual MATLAB code used in determining these transfer functions is in Appendix A.

Because the computational power needed for computing a single transfer function is so high, the computation of multiple transfer functions is avoided, but is discussed in the Section 3 of this chapter. The MATLAB code to implement the two receiver/two source case is also contained in Appendix A. Suggestions for how to implement this code more efficiently are contained in Chapter V.

Finally, in all calculations, the atmospheric attenuation was ignored since the room is assumed to be sufficiently small to ignore the 0.025 $\frac{dB}{100\ m}$ attenuation due to atmospheric friction. This eliminated some unnecessary complications in the writing of the code. To eliminate further complications in the code, the eigenvalues were approximated by $\frac{\sigma\pi}{D}$ for $\sigma = 1, 2, 3, ...$ It was shown in Chapter III that the eigenvalues were the intersection of a rational function and a tangent function, but for low β and low index, this approximation is fairly valid.

4.2 Single Frequency Tones

The human auditory range in young adults is 20 Hz to 20 kHz. This section uses the MATLAB code for one source and one receiver to determine the effect of an environment on a single-frequency tone. The parameters for the environment that were used are

$$\begin{aligned} W &= 10\ m \\ L &= 10\ m \\ H &= 3\ m \\ \vec{x}_r &= (2\ m,\ 2\ m,\ 2\ m) \\ \vec{x}_s &= (9\ m,\ 9\ m,\ 1.5\ m) \\ \beta_x &= 10 \\ \beta_y &= 10 \\ \beta_z &= 10 \quad . \end{aligned}$$

The room is modeled after a typical medium-sized room with sufficiently small coefficients of reflection to avoid extraneous signal reflection in order to aid in computation.

4.2.1 Low Frequency Range–20 Hz. In the very lowest range of human auditory perception, the change in the Laplace transform of the signal is greatest. The two changes were computed at two distinct source locations–one at a moderate distance from the receiver and the other very close to the receiver. The lack of accuracy in these computations is intuitively seen in the fact that the change in the Laplace transform of the original and received signals is identical. This change in the signal is computed by approximating the integral

$$I = \int_{-20000}^{20000} \left(\hat{s}\left(\mu\right) - \hat{u}\left(\mu\right)\right)^2 d\mu = \int_{-20000}^{20000} \hat{s}\left(\mu\right)\left(1 - H\left(\mu\right)\right)^2 d\mu \tag{4.1}$$

with the rectangular rule of Riemann integration. The width of each rectangle is 10, and so the total number of terms in the Riemann sum is 4001.

4.2.1.1 Source Location 1. Using the source location given above, which places the source at a large distance from the receiver with respect to the size of the room, the original signal was assumed to be

$$s\left(t\right) = \sin\left(2\pi\left(20\right)t\right)$$

which oscillates fairly rapidly–20 cycles per second. The Laplace transform of this signal is shown in Figure 4.1.

The transfer function is determined solely by the environment in which the signal propagates, and so if the parameters of the room remain constant when two different signals are being propagated from the same location, then the two signals are acted upon by the same transfer function. Since we are simply looking at the transformed transfer function along the real line, and since there are poles along the imaginary axis of the transform domain, there is a peak in the value of the transfer function close to $\mu = 0$. The transfer function for the room described above is shown in Figure 4.2. The scale on Figures 4.1 and 4.2 are very different, but when scaled, both are very nearly zero except when close to the origin. The received signal can be found simply by multiplying the transformed signal and the transformed transfer function together. The received signal in the transform domain is shown in Figure 4.3.

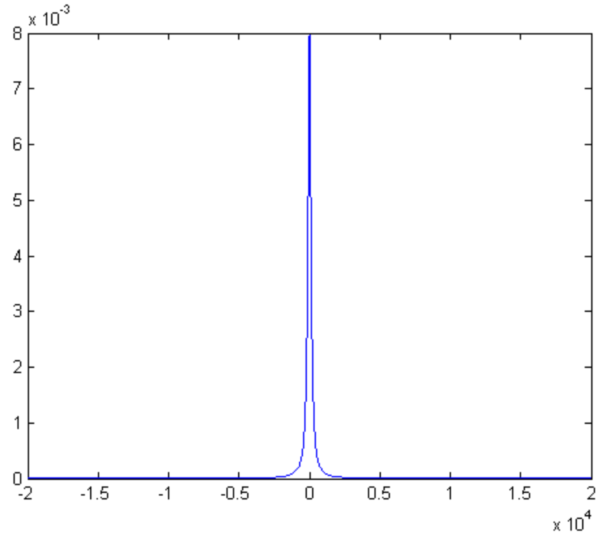

Figure 4.1 Laplace Transform, 20 Hz.

Source Location 2. The second source location is

$$\vec{x}_s = (.1\ m,\ 9\ m,\ 2.9\ m)$$

Since the signal is the same, only the transfer function changes with respect to the transform variable μ from the previous source location. The transfer function for this source location is shown in Figure 4.4.

4.2.1.2 Middle Frequency Range–2 kHz. The middle range of human auditory perception is around 2 kHz, which oscillates much more rapidly than the 20 Hz signal, and so to model it accurately in the time domain requires the use of approximately 100 times more sample points than with the 20 Hz signal. Therefore, it is more useful to perform these calculations completely in the Laplace transform domain.

Source Location 1. In this section, the original signal is assumed to be

$$s(t) = \sin(2\pi(2000)t) \tag{4.2}$$

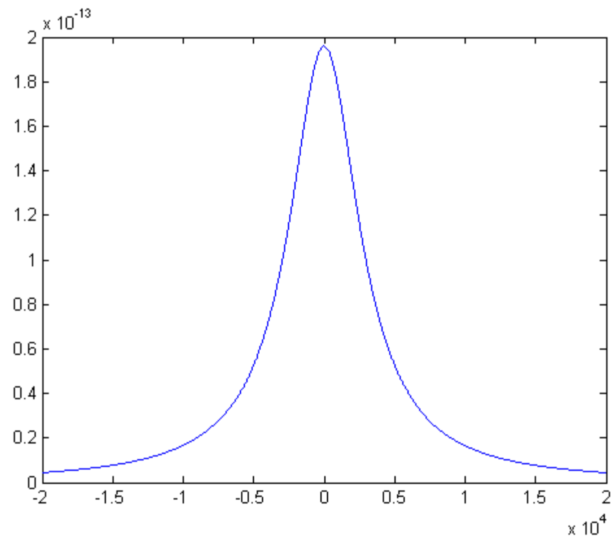

Figure 4.2 Transfer Function, Laplace Domain

which has a Laplace transform of

$$\hat{s}\left(\mu\right) = \frac{4000\pi}{\mu^2 + \left(4000\pi\right)^2} \ . \tag{4.3}$$

the graph of which is contained in Figure 4.6.

Whereas the 20 Hz signal dropped off closer to zero, this signal drops off much farther from the origin. Furthermore, the maximum value of the transform is on the order of hundredths whereas the maximum value of the 20 Hz signal transform was on the order of 1. The transformed received signal is given in Figure 4.7.

Source Location 2. In this location, the received signal from the original 2 kHz signal is much the same as that for source location 1, since the transfer functions are not so different for these two source locations.

4.2.1.3 High Frequency Range–20 kHz. At the upper frequency range of human auditory perception, the transformed signal looks much the same as that of the 2 kHz signal, but with more curvature. As with the 2 kHz signal, the number of sample points for any time-domain computations is prohibitive, and so only the Laplace transform domain will be dealt with. The Laplace transform of the 20 kHz signal is given in Figure

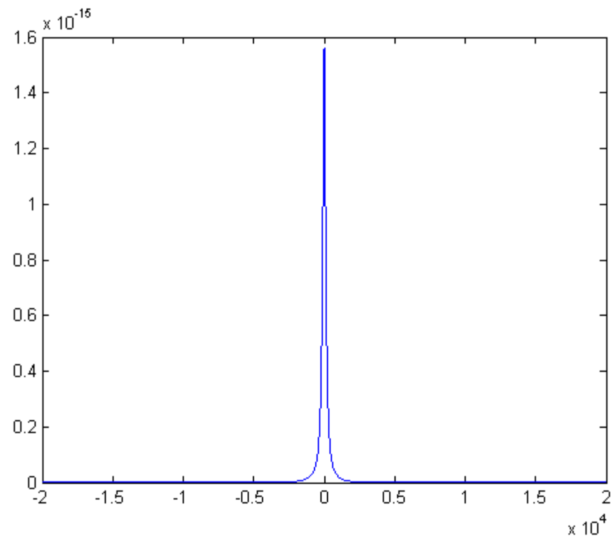

Figure 4.3 Received 20 Hz Signal, Transform Domain.

4.8.By applying the transfer function to this transformed signal, the received signal is attained, which is shown in Figure 4.9.

4.2.2 Multifrequency Signals. Different multifrequency signals can be considered for implementation in the MATLAB program, but the received signals cannot be computed accurately due to their complexity. There are two different types of multifrequency signals that can be implemented easily by modifying the program, chirps and simultaneous tones of different frequency.

4.2.2.1 Chirps. Chirps are signals that either increase or decrease in frequency with time. The rate of increase or decrease affects the transformed signal by its Laplace transform. One example of an increasing tone that ascends in pitch quickly is the time-signal $s(t) = \sin(2\pi t^2)$.MATLAB encounters difficulty in attempting to find a closed-form solution for the Laplace transform of a chirp, and so it must be computed numerically. The graph of the numerically computed Laplace transform is shown in Figure 4.10. It is apparent that there is a pole at the origin, and so when the transfer function is applied to this transformed signal, the pole will be carried through to the received signal by multiplication.

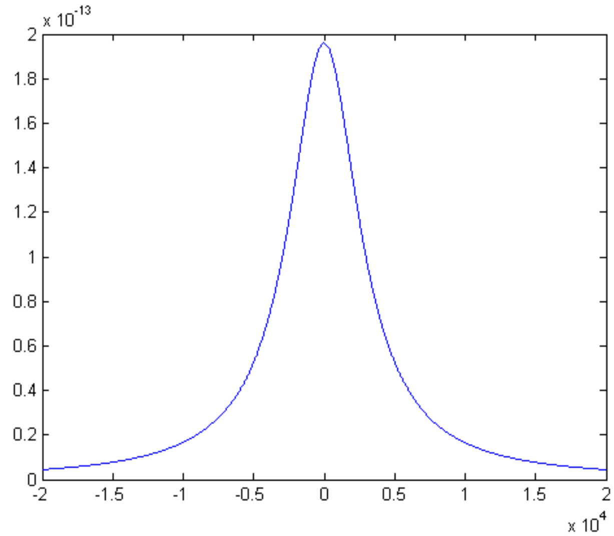

Figure 4.4 Transfer Function, Laplace Domain, Source Location 2.

An example of a slowly ascending chirp is easy to find. Simply decrease the value of the exponent in the rapidly ascending chirp, but without going to or below 1. By lowering the exponent to only 1.5, the chirp ascends in pitch much more slowly, as shown in Figure 4.11.

Since the frequency increases much more slowly than $s(t) = \sin\left(2\pi t^2\right)$, even with only a slight decrease in the exponent, MATLAB has much less difficulty in computing the Laplace transform analytically, and with a program that computes the received signal well, will yield a better result than with a rapid ascension chirp.

Descending chirps behave very similar in MATLAB to their corresponding ascending chirps. By simply replacing the exponent of the time variable with its reciprocal, the chirp will descend in pitch at the same rate that its counterpart ascends. A phase shift must be introduced as well, to avoid a singularity in the derivative at $t = 0$. The rapidly ascending chirp's descending counterpart is $s(t) = \sin\left(2\pi\sqrt{t+1}\right)$, which decreases much more slowly than its counterpart increases. Note that the scale on the t-axis of Figure 4.13 is ten times that of Figure 4.11.

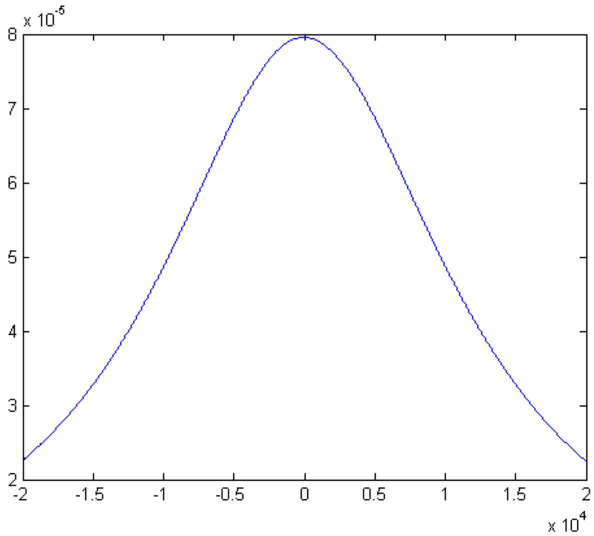

Figure 4.5   Laplace Transform, 2 kHz Signal.

The counterpart to the slowly ascending chirp is the descending chirp

$$s(t) = \sin\left(2\pi\sqrt[3]{(t+1)^2}\right) \tag{4.4}$$

Note again that the descending chirp descends much more slowly in pitch than its counterpart, as Figure 4.14 has a scale ten times that of Figure 4.10. Like the rapidly ascending chirp, MATLAB encounters difficulty in performing the Laplace transform of this signal because of its high initial frequencies.

4.2.2.2 Simultaneous Tones of Different Frequency. The touch-tone phone system uses a signal consisting of two distinct tones overlapped for each button on a telephone. One component of the signals for the buttons in a row all have the same tone and one component of the signals for the buttons in a column share the same tone. This is an example of a use for a ditonal frequency. A human voice is comprised of many tones overlapping and being propagated simultaneously, so these signals are the closest to human voices of those that have been analyzed. An example of a ditonal signal, i.e. a signal comprised of two tones is $s(t) = \sin(2\pi(200t)) + \sin(2\pi(400t))$, whose graph is displayed in Figure 4.15.The Laplace transform of this signal is much easier for MATLAB to compute than that of chirps of any kind, since this is simply the sum of two single-tone

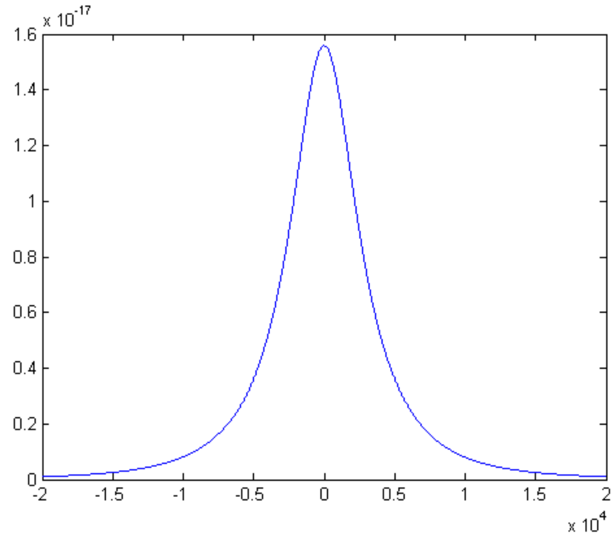

Figure 4.6 Laplace Transform, 2 kHz Signal.

frequencies, and the Laplace transform is a linear transformation. In fact, given the value of the amplitudes at given frequencies, a fair approximation of a human voice could be made using this program. Once again, however, the filtering of such a signal is impossible with the program provided in Appendix A. The graphs of signals with multiple tones, however, becomes much more complicated than that in Figure 4.15. For example, a signal consisting of four frequencies, 200 Hz, 850 Hz, 1700 Hz, and 2500 Hz with amplitudes of 2, 5, 3, and 4, respectively (these numbers are chosen somewhat randomly, only constrained to fall within the low to middle range of human auditory perception) has the signal

$$s(t) = \sum_{i=1}^{4} A_i \sin(2\pi(f_i t)) \tag{4.5}$$

$$A_i = (2, 5, 3, 4) \tag{4.6}$$

$$f_i = (200,\ 850,\ 1700,\ 2500) \tag{4.7}$$

which has a graph displayed in Figure 4.15. The complexity of the signal is very clear, and this is only for four distinct tones. With the continuous range of tones contained in the average human voice, the signal only gets much more complicated, making it very difficult to filter in a computer program.

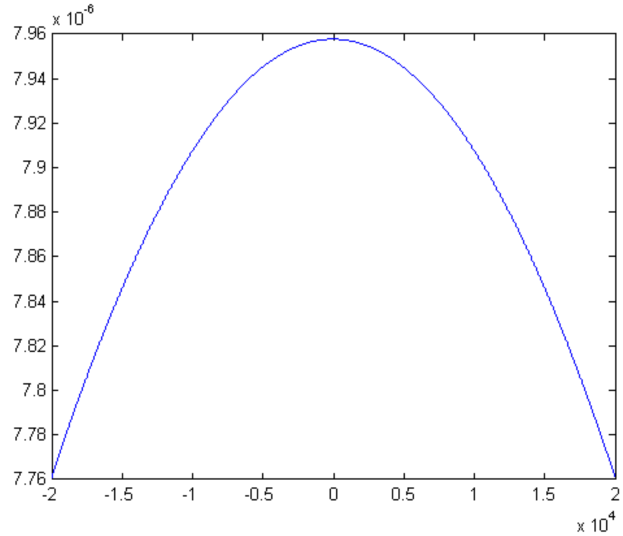

Figure 4.7 Laplace Transform, 20 kHz.

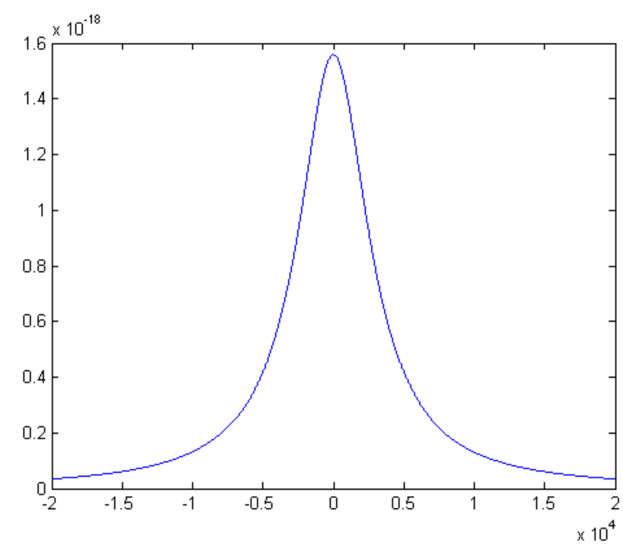

Figure 4.8 Transformed Received Signal, 20 kHz.

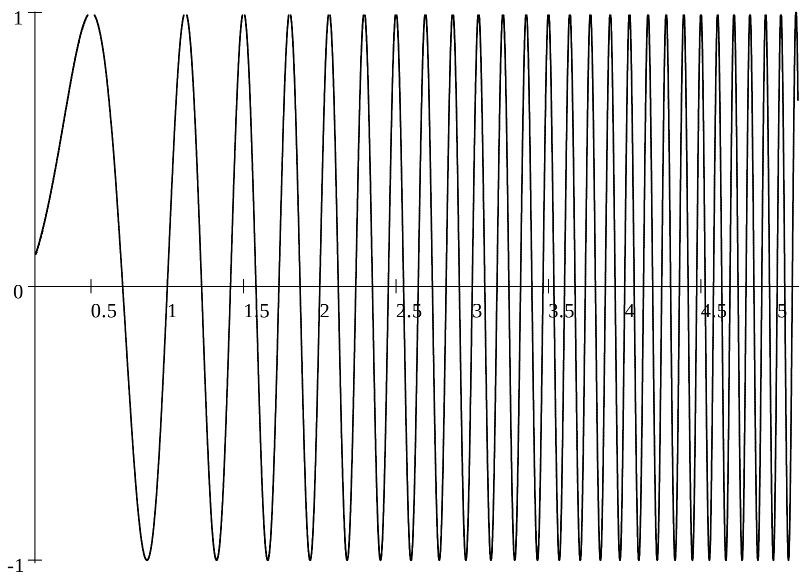

Figure 4.9 Rapid Ascension Chirp, Time Domain.

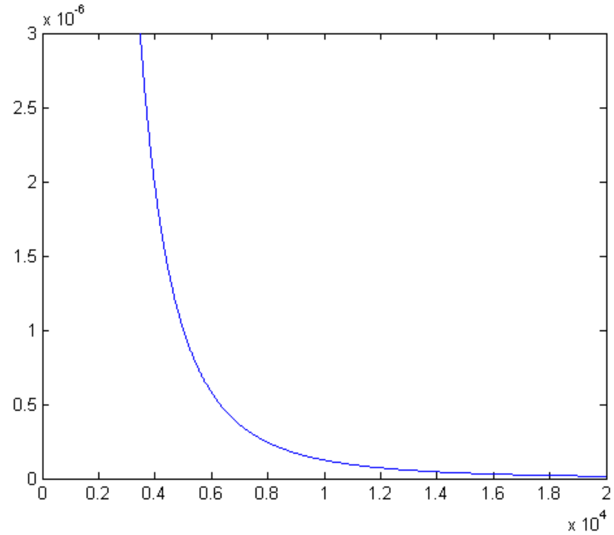

Figure 4.10 Laplace Transform, Rapid Ascension Chirp.

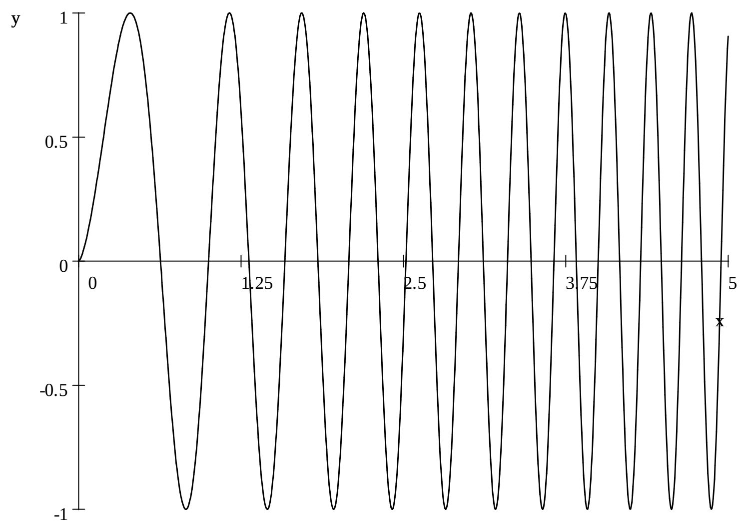

Figure 4.11 Slow Ascension Chirp.

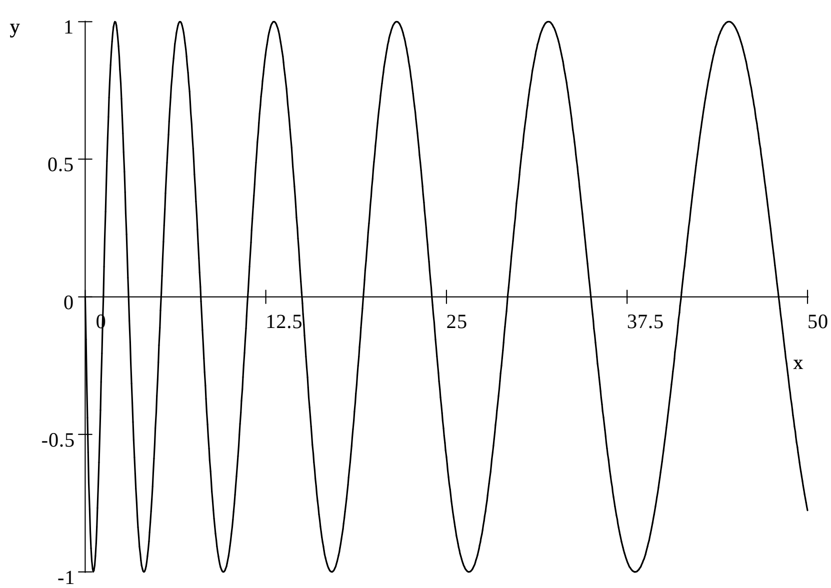

Figure 4.12 Rapid Descension Chirp.

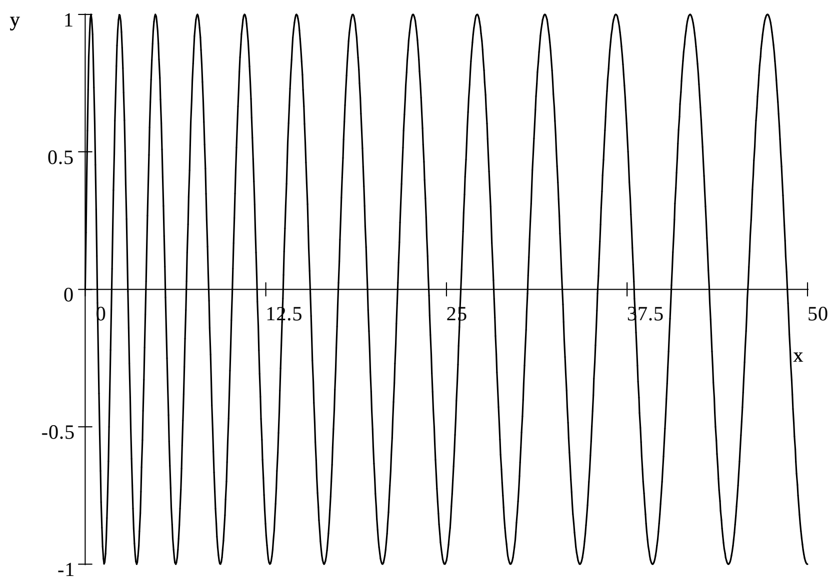

Figure 4.13 Slowly Descending Chirp

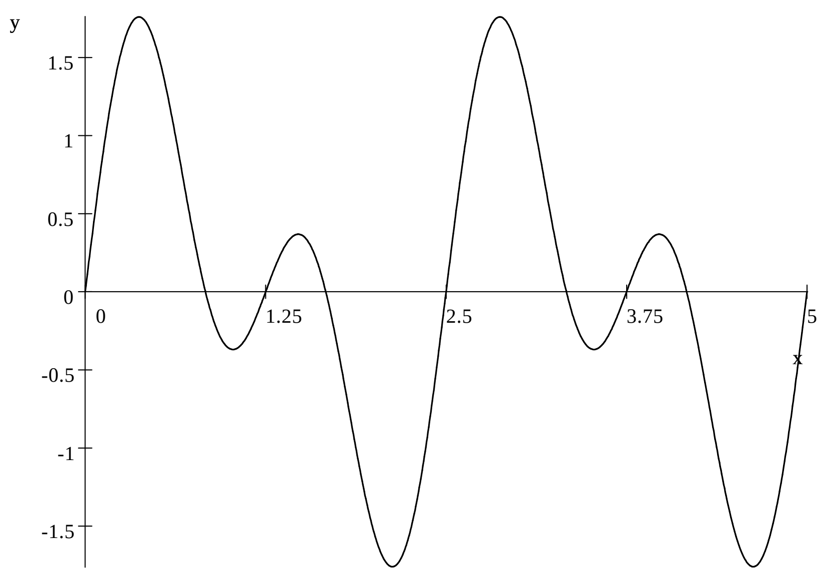

Figure 4.14 Ditonal Signal of 200 Hz and 400 Hz, Equal Amplitude.

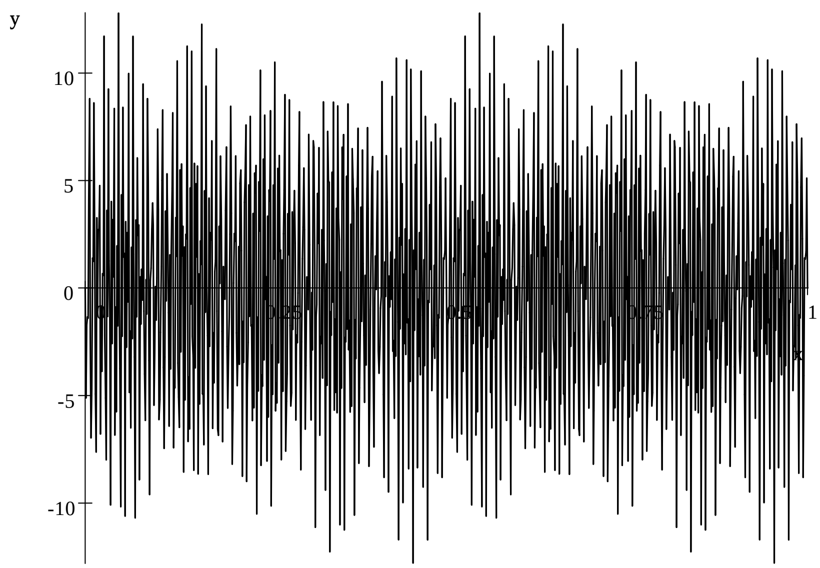

Figure 4.15 Four-Tone Signal of Varying Amplitude.

V. Conclusions and Recommendations

5.1 Summary

The initial problem of finding and inverting a room's acoustic transfer function begins with finding the rule by which acoustic signals propagate and then by finding the rule by which the propagation affects the signal. Once this rule is found, the effects of the environment can be reversed, thereby yielding the original signal. When multiple receivers and multiple sources are involved, the set of effects on the signals is described by a system of linear equations in a transformed domain, which can be expressed in terms of a matrix. Under certain conditions, this matrix can be inverted, and a unique solution for the set of signals can be found, but quite often these conditions are not met, resulting in a least-squares or minimum norm solution. On top of these constraints is the fact that the computations are made with the assumption that the sources are in a fixed, known location throughout the duration of the recording. Once this assumption is false, the problem of inverting even a single transfer function becomes much more difficult, involving multiple Laplace transforms.

With all of these conditions met, it is theoretically possible to filter acoustic signals exactly, but the implementation of this process in a program as advanced as MATLAB can be prohibitive, even for simple signals such as single-frequency tones. When considering the acoustic complexities of the human voice, the implementation could take a prohibitively long period of time. The goal of a real-time filter is one that would benefit both the Department of Defense and private industry. With real-time filtering, information from a bugged room could be processed immediately, yielding potentially life-saving information, and conversations held over a mobile phone in a noisy room would not be as frustrating as they usually are. Products such as the adaptive earpiece Jawbone take advantage of this technology to filter the noise from a loud room (1).

The raw computing power necessary to perform these tasks with any degree of analytic computation is far higher than that available on a typical desktop computer, and so a purely numerical solution would be much easier to compute, if less accurate than an analytic solution. This involves being able to take Laplace transforms numerically as well as improving upon the accuracy of the numeric computations already implemented in the code provided in Appendix A.

5.2 Conclusions

Based on the analysis of the modified wave equation performed in Chapter III, it is possible to filter acoustic signals analytically, yielding a transfer function that can be applied to the Laplace transform of any signal. With this altered signal, the original signal's Laplace transform can be found, which can, in turn be inverted to yield the exact original signal.

By generalizing the number of signals and receivers in a room, the problem of filtering acoustic signals can also be solved exactly, given that certain conditions are met. These conditions relate directly to the determinant of the matrix of transfer functions. If the sources are too close to one another or the receivers are too close to one another, then the determinant of the matrix is close to zero, making the matrix ill-conditioned. If the matrix is not ill-conditioned, however, the computation of its inverse is stable and the unique solution of a vector of the original signals can be produced.

Further generalizing the problem of finding and inverting the room transfer functions to accommodate mobile sources results in a finite convolution with more terms than with a non-mobile source, which in turn requires taking two Laplace transforms to result in an equation for the original signal. This doubly transformed signal can be transformed by taking the inverse Laplace transform with respect to the second transform variable, and then with respect to the first transform variable. The analytic form of this signal is very complex, if it exists at all. The inverse Laplace transform may have to be taken numerically in order to yield a result.

The difficulty in implementing these analytic solutions is quite clear from the graphs of the transformed received signals presented in Chapter IV. Because of the enormous number of calculations required to compute the transfer functions of the room, a reliable solution cannot be found using the code in Appendix A, and so a better program must be developed for finding these transfer functions numerically.

5.3 Recommendations for Future Research

Given the sheer volume of assumptions made in this thesis, there are countless avenues of research opportunity available in this field. The assumption of point sources and receivers is highly invalid in real-life acoustics when dealing with human vocal signals. Since the human head is not best described as a point source, the signals propagated from a human mouth are not omnidirectional. The shape of the throat and mouth acts as an antenna, shaping and directing the signal in the direction the head is facing. Furthermore, microphones are of a definite size, and so they cannot be considered as point receivers, but as directionalized receivers. After all, human ears act as antennae in receiving the signals, making the reception directionalized. Modifying the wave equation in a way that accounts for these changes in signal sources and receivers would yield a more accurate result.

Exploring the nature of the matrix of transfer functions is another area of possible research opportunity. If a matrix is ill-conditioned, or is even noninvertible, other methods than inversion are possible to find the solutions. Using the Moore-Penrose pseudoinverse is discussed in Chapter III as a way of finding these solutions, but if the matrix is ill-conditioned, even these pseudoinverses require other transformations, such as a singular-value decomposition or a QR decomposition. Finding a method of solving problems of this type could be crucial in situations where the location of the sources and receivers cannot be changed, but must be accepted, such as the intelligence gathering scenario.

More importantly, the research in the aforementioned fields is useless without a proper implementation. A more efficient computer program must be developed based on the findings of whatever research is conducted. Perhaps using a different programming language would be beneficial to the efficiency of the method, such as using C, where the programs can be compiled to run faster. Once the programs are running faster, it is easier to implement more accurate computations, where the transfer function can be computed with more than two million cycles, as it is done in the code written for this thesis. Furthermore, the use of experimental data will be very helpful in implementing these codes. Not only would the implementation be used to actually invert the effects of the room on a signal, but the actual data would include the directionalized nature of both the sources and the receivers, giving a more accurate result. The lack of experimental data in this thesis greatly diminishes the applicable value, but the groundwork for future experiments is now laid.

Appendix A. MATLAB Code

This appendix contains the code for finding the transfer function of a room and produces the signal received by a microphone located at the user specified location.. The first section contains the program for a single source and single receiver. The second code is a program that finds the Laplace transform of a signal numerically.

A.1 Single Source, Single Receiver Transfer Function Program

```
clear
clear
H=3;
L=10;
W=10;
xr1=2;
yr1=2;
zr1=2;
xs1=0;
ys1=9;
zs1=2.9;
bx=10;
by=10;
bz=10;
c=374;

v=-20000:10:20000;

for i=1:4001
    sum(i)=0;
    sumplus(i)=0;
end
```

```
for i=1:4001
    for l=1:10
        for m=1:10
            for n=1:10
                num=(sin(n*pi*xr1/W)+(n*pi/bx/W)*cos(n*pi*xr1/W))*
                  (sin(m*pi*yr1/L)+(m*pi/by/L)*cos(m*pi*yr1/L))*
                  (sin(l*pi*zr1/H)+(l*pi/H/bz)*cos(l*pi*zr1/H))*
                  (sin(n*pi*xs1/W)+(n*pi/W/bx)*cos(n*pi*xs1/W))*
                  (sin(m*pi*ys1/L)+(m*pi/L/by)*cos(m*pi*ys1/L))*
                  (sin(l*pi*zs1/H)+(l*pi/bz/H)*cos(l*pi*zs1/H));
                den(i)=(((.5+bx^2*W^3/(2*(n*pi)^2))*
                  (.5+by^2*L^3/(2*(m*pi)^2))*
                  (.5+bz^2*H^3/(2*(l*pi)^2)))*
                  (v(i)^2+c^2*((l*pi/H)^2+(m*pi/L)^2+(n*pi/W)^2)));
                sumplus(i)=sum(i)+num/den(i);
                sum(i)=sumplus(i);
            end
        end
    end
end
```

A.2 Numerical Laplace Transform Program

```
clear
t=0:.0001:5;
for i=1:50001
    chirp(i)=sin(2*pi*t(i)^2);
end

for i=1:4001
    lchirp(i)=0;
```

```
    lchirpplus(i)=0;
end

v=-20000:10:20000;

for i=1:4001
    for j=1:50001
         lchirpplus(i)=lchirp(i)+exp(-v(i)*t(j))*chirp(j);
         lchirp(i)=lchirpplus(i);
    end
end

figure
plot(t,chirp)
figure
plot(v,lchirp)
```

Bibliography

1. *Jawbone–The Adaptive Headset, A Technical Overview*. Technical Report, Aliphcom, 2004.
2. C. Wykes, F. Nagi, P. Webb. "Ultrasound Imaging in Air." *Conference Publication No. 369*. 77–81. IEE, 1993.
3. Choi, S. and A. Cichocki, "Adaptive blind separation of speech signals: Cocktail party problem," 1997.
4. Demmel, James W. *Applied Numerical Linear Algebra*. The Society for Industrial and Applied Mathematics, 1997.
5. Dogancay, Kutluyil and Ahmad Hashemi-Sakhtsari. "Target Tracking by Time Difference of Arrival Using Recursive Smoothing," (2004).
6. Fink, Mathias. "Time Reversal of Ultrasonic Fields–Part I: Basic Principles," *IEEE Transactions On Ultrasonics, Ferroelectrics, and Frequency Control*, *Vol. 39, No. 5* (September 1992).
7. Garabedian, P.R. *Partial Differential Equations*. AMS Chelsea Publishing, 1964.
8. Morse, Philip M. and K. Uno Ingard. *Theoretical Acoustics*. McGraw-Hill Book Company, Inc., 1968.
9. Smith, Brain A. *Binaural Room Simulation*. MS thesis, School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, December 1993.
10. Sneddon, Ian B. *Fourier Transforms*. Dover Publications, 1995.
11. Zachmanoglu, E.C. and Dale W. Thoe. *Introduction to Partial Differential Equations with Applications*. The Williams and Wilkins Company, 1976.

REPORT DOCUMENTATION PAGE			*Form Approved* *OMB No. 074-0188*

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of the collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to an penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.
PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From – To)*
21-03-2005	Master's Thesis	August 2003 – March 2005

4. TITLE AND SUBTITLE	
Environmental Acoustic Transfer Functions and Filtering Acoustic Signals	**5a. CONTRACT NUMBER**
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	**5d. PROJECT NUMBER**
Dias, Brandon P., Second Lieutenant, USAF	**5e. TASK NUMBER**
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(S)	**8. PERFORMING ORGANIZATION REPORT NUMBER**
Air Force Institute of Technology Graduate School of Engineering and Management (AFIT/EN) 2950 Hobson Way, Building 640 WPAFB OH 45433-8865	AFIT/GAM/ENC/05-2
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	**10. SPONSOR/MONITOR'S ACRONYM(S)**
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

When dealing with acoustic signals, the environment in which the signal propagates affects the signal in measurable ways. These effects lead to echoes, changes in amplitude, and sometimes a confluence of other signals which may render the signal useless in terms of information retrieval. When considering acoustics in terms of pressure changes due to the driving signal, we are able to model and measure the effects of the environment on a given signal. Once this model is known, we are able to completely invert the environmental transfer enacted on the signal and filter out other signals, as long as some assumptions are held in the implementation of this procedure.

15. SUBJECT TERMS

Acoustics, Signal Processing, 3-D Sound

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			Mark Oxley, Ph.D. (AFIT/ENC)
U	U	U	UU	75	**19b. TELEPHONE NUMBER** *(Include area code)* (937) 255-3636, ext 4515 (mark.oxley@afit.edu)

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18