Information Theoretic Comparison of MIMO Wireless Communication Receivers in the Presence of Interference

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Abstract  Multiple-input multiple-output (MIMO) wireless communication provides a number of advantages over traditional single-input single-output (SISO) approaches, including increased data rates for a given total transmit power and improved robustness to interference. Many of these advantages depend strongly upon the details of the receiver implementation. For practical communication systems a competition between communication performance and computational complexity exists. To reduce computation complexity, suboptimal receivers are commonly employed. In this paper, the details of a variety of receivers are incorporated into the effects of the channel so that information-theoretic performance bounds can be exploited to evaluate receiver approaches. The performance of these receivers is investigated for a range of environments. Two classes of environments are considered: first, channel complexity, characterized by the shape of the narrowband channel-matrix singular-value distribution, and second, external interference. Receiver approaches include minimum-mean-squared error, minimum interference, and multichannel multiuser detection (MCMUD), given various assumed limitations on channel and interference estimation. Receiver performance implications are also demonstrated using experimental data.
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**Report Number:**

**DISTRIBUTION/AVAILABILITY STATEMENT:**
Approved for public release, distribution unlimited

**Supplementary Notes:**
See also, ADMO01741 Proceedings of the Twelfth Annual Adaptive Sensor Array Processing Workshop, 16-18 March 2004 (ASAP-12, Volume 1), The original document contains color images.

**Security Classification of:**

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**Limitation of Abstract:**
Unlimited

**Number of Pages:**
29
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This work was sponsored by the United States Air Force under Air Force Contract F19628-00-C-0002. Opinions, interpretations, conclusions, and recommendations are those of the authors and are not necessarily endorsed by the United States Government.
Topics

MIMO Communication

- Introduction
- MIMO Phenomenology
- Receiver Approaches
- Receiver Performance Bounds
- Performance Comparison
MIMO Communication

Multiple-Input Multiple-Output

- Single transmitted data stream
- Single received data stream
- Employ multiple modes through environment
- Potential advantages over single-input single-output
  - Diversity
  - Robustness to interference
  - Spectral efficiency
Not All MIMO Receivers Are Equal

• “Standard” MIMO receivers perform badly in difficult environments
  – Ignore the possibility of jamming or external interference
  – Lower computational complexity
• “Optimal” MIMO receiver barely affected by jamming

MIMO Communication
Multiple-Input Multiple-Output

4 x 4 MIMO

Transmit Array
Receive Array
Jammer

Receiver Performance Comparison

Spectral Efficiency (b/s/Hz)

“Optimal”
“Standard”

SNR = 10 dB

Jammer JNR (dB)

x 1000

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MIMO Communication

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- Performance Comparison
The Channel Matrix

- Channel matrix, \( H \), contains complex attenuation between each transmit and receive antenna
  \[ \ddot{z}(t) = H \ddot{x}(t) + \ddot{n}(t) \]

- Large channel matrix singular values are useful

Channel Matrix Singular Values

- Few Useful Modes
- Many Useful Modes
- Scatterers

Sorted by Mode Strength

Relative Power

Low Complexity Channel

High Complexity Channel

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MIMO Capacity Bound(s)

SISO

\[ C_{SISO} = \log_2(1 + \text{SNR}) \]

Informed Transmitter

\[ C_{IT} = \max_{P; \text{tr}P = P_0} \log_2 \left| I + HPH^\dagger \right| \]

Uninformed Transmitter

\[ P \rightarrow \frac{P_0}{nT} \]

\[ C_{UT} = \log_2 \left| I + \frac{P_0}{nT} HH^\dagger \right| \]

\[ = \sum_m \log_2 \left( 1 + \frac{P_0}{nT} \| s_m \|^2 \right) \]

Channel Singular Values

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Channel Complexity Parameterization

- Gaussian channel matrix, $G$
- Simulate more realistic eigenvalue distributions by introducing spatial correlation
  - Parameterized by $\alpha$
- Modified parameterized random channel matrix, $F$

\[
F = a \mathbf{U} \mathbf{A}_\alpha \mathbf{U}^\dagger \mathbf{G}' \mathbf{V} \mathbf{A}_\alpha \mathbf{V}^\dagger
= a \mathbf{U} \mathbf{A}_\alpha \mathbf{G} \mathbf{A}_\alpha \mathbf{V}^\dagger
\]

\[
\mathbf{A}_\alpha = \sqrt{n} \frac{\text{diag}\{\alpha^0, \alpha^1, \ldots, \alpha^{n-1}\}}{\sqrt{\text{tr}\{\text{diag}\{\alpha^0, \alpha^1, \ldots, \alpha^{n-1}\}^2\}}}
\]
Topics
MIMO Communication

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Adaptive Beamforming Receivers

Suboptimal

Beamformer Outputs
\[ \tilde{z}' = W^\dagger (H\tilde{x} + \tilde{n}) \]
\[ W \equiv (\tilde{w}_1 \tilde{w}_2 \cdots \tilde{w}_{n_T}) \]

Minimum Mean Squared Error
\[ \tilde{w}_{n}^{MMSE} \propto \left( I + R + \frac{P_o}{n_T} HH^\dagger \right)^{-1} h_n \]
If Known

Minimum Interference
\[ \tilde{w}_{n}^{MI} \propto P_n^\perp \tilde{h}_n \]
\[ P_n^\perp = I_{n_R} - \overline{H}_n (\overline{H}_n^\dagger \overline{H}_n)^{-1} \overline{H}_n^\dagger \]
\[ \overline{H} \equiv \begin{pmatrix} \tilde{h}_1 & \overline{H}_1 \end{pmatrix} \]

or
\[ \tilde{w}_{n}^{MI} \propto \operatorname{min \ eigenv} \left\{ R + \frac{P_o}{n_T} \overline{H}_n \overline{H}_n^\dagger \right\} \]
Multi-Channel Multi-User Detection (MCMUD)

“Optimal” MIMO Receiver

- Effective in environments with
  - Multiple access interference
  - Challenging multipath
  - Jamming
- Iterative decoder
  - Estimation subtraction (multi-user detection)
  - Spatially adaptive beamformers

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Topics
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- Performance Comparison
Information Theoretic Capacity

**Optimal**

**Signal Model**
\[ \tilde{z} = H \tilde{x} + \tilde{n} \]

**Mutual Information**
\[ \mathcal{I}(\tilde{z}, \tilde{x}|H) = h(\tilde{z}|H) - h(\tilde{z}|\tilde{x}, H) \]
\[ = h(\tilde{z}|H) - h(H\tilde{x} + \tilde{n}|\tilde{x}, H) \]
\[ = h(\tilde{z}|H) - h(\tilde{n}) , \]

- **Receive-Signal Entropy**
  \[ h(\tilde{z}|H) = \log_2 |\pi e \langle \tilde{z} \tilde{z}^\dagger \rangle| \]
  \[ = \log_2 |\pi e \sigma_n^2 \left( I_{n_R} + H \langle \tilde{x} \tilde{x}^\dagger \rangle H^\dagger \right)| \]

- **Noise-Like Entropy**
  \[ h(\tilde{n}) = \log_2 |\pi e \langle \tilde{n} \tilde{n}^\dagger \rangle| \]
  \[ = \log_2 |\pi e \sigma_n^2 I_{n_R}| \]

**In Interference Environment**
\[ h(\tilde{z}|H) \leq \log_2 \left\{ \pi e \sigma_n^2 I + \sigma_n^2 R + H \langle \tilde{x} \tilde{x}^\dagger \rangle H^\dagger \right\} \]
\[ h(\tilde{z}|\tilde{x}, H) \leq \log_2 \left\{ \pi e \sigma_n^2 I + \sigma_n^2 R \right\} \]

**Uninformed Transmitter Capacity**
\[ C_{UT} = \log_2 \left| I_{n_R} + \frac{P_o}{n_T} \tilde{H} \tilde{H}^\dagger \right| \quad ; \quad \tilde{H} = (I + R)^{-1/2} H \]

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Beamformer Receiver Extension to Information Theoretic Bounds

Signal Model
\[ \tilde{z} = \mathbf{H}\tilde{x} + \tilde{n} \quad \Rightarrow \quad \tilde{z}' = \mathbf{W}^\dagger(\mathbf{H}\tilde{x} + \tilde{n}) \]
\[ \mathbf{W} \equiv (\tilde{w}_1 \tilde{w}_2 \cdots \tilde{w}_{n_T}) \]

Noise-Like Entropy
\[ h_{uc}(\tilde{z}'|\tilde{x}, \mathbf{H}) \rightarrow \sum_l h_{uc}(\tilde{z}'|x_l, \mathbf{H}) \]
\[ = \sum_{m} \log_2 \left( \pi e \sigma_n^2 \tilde{w}_m^\dagger \left\{ \mathbf{I}_{n_R} + \mathbf{R} + \frac{P_o}{n_T} \mathbf{H}_m \mathbf{H}_m^\dagger \right\} \tilde{w}_m \right) ; \quad \mathbf{H} \equiv \left( \tilde{h}_1 \mathbf{H}_1 \right) \]

Receive-Signal Entropy
\[ h_{uc}(\tilde{z}'|\mathbf{H}) = \sum_{m}^{n_T} \log_2 \left( \pi e \sigma_n^2 \left[ \tilde{w}_m^\dagger \left\{ \mathbf{I}_{n_R} + \mathbf{R} + \frac{P_o}{n_T} \mathbf{H}_m \mathbf{H}_m^\dagger \right\} \tilde{w}_m + \frac{P_o}{n_T} \tilde{w}_m^\dagger \tilde{h}_m \tilde{h}_m^\dagger \tilde{w}_m \right] \right) \]

Receiver Beamformer Capacity
\[ C_{uc} = \sum_{m}^{n_T} \log_2 \left[ 1 + \left( \tilde{w}_m^\dagger \left\{ \mathbf{I}_{n_R} + \mathbf{R} + \frac{P_o}{n_T} \mathbf{H}_m \mathbf{H}_m^\dagger \right\} \tilde{w}_m \right)^{-1} \frac{P_o}{n_T} \|	ilde{w}_m^\dagger \tilde{h}_m\|^2 \right] \]
Topics
MIMO Communication

• Introduction
• MIMO Phenomenology
• Receiver Approaches
• Receiver Performance Bounds
• Performance Comparison
  – Benign
  – Channel Complexity
  – MIMO Interference
  – Jamming
  – Experimental
Performance Comparison
Benign Environment (No Interference)

Minimum Mean Squared Error
\[ \hat{w}_n^{MMSE} \propto \left( I + R + \frac{P_0}{nT} HH^\dagger \right)^{-1} \tilde{h}_n \]

or

Minimum Interference
\[ \hat{w}_n^{MI} \propto P_n \tilde{h}_n \]

Versus MCMUD

- MMSE has only slight loss compared to MCMUD
- MI performs badly particularly at lower SNR
Performance Comparison
Function of Channel Complexity

- Study 2 regimes of channel complexity
  - $\alpha = 1$
  - $\alpha = 0.5$
- Significant losses for both MI and MMSE at lower channel complexity
Performance Comparison
Effects of Interference

- Second interfering MIMO transmitter
  - Equal transmit power
- MI performs bad at all SNR
- Both MMSE and MI perform badly compared to MCMUD at high SNR
  - More strong signals than antennas
Performance Comparison
Effects of Jammer

- Significant losses for both MI and MMSE over most SNR
- Terrible performance for receivers that are blind to interference structure
MIMO Experiment
Summer 2002

- Investigate channel phenomenology
- Study space-time coding
- Explore transmitter coherence requirements
- Demonstrate robustness to
  - Jamming
  - Cochannel interference

16-Channel Hi-Fidelity Data Recording System

2 Groups of 4, or 8 Coherent Transmitters Near PCS band

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4x4 MIMO Performance

Motion, Jammers, and LO Errors

- 2 Noise Jammers (25 dB JNR)
- Moving transmitter (25 mph)
- Error-free 2b/s/Hz data-link
- MCMUD near performance of jammer-free environment!
- Interference-blind & MI receivers perform badly

Experimental MIMO Performance

Jammer Spatial Mode Distribution

SISO SINR (dB)

Relative Power (dB)

Mode #

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Summary

• Presented overview of robust MIMO communication

• Introduced bounds for variety of MIMO receivers
  – MMSE
  – MI
  – MCMUD

• MCMUD advantage significant in many environments
  – Spatially correlated channels (rate improvement > 70)
  – Interference (rate improvement > 5)
  – Jamming (rate improvement > 1000)

• Demonstrated experimental MCMUD immunity to jamming
Acknowledgements

- MIT Lincoln Laboratory
  - New Technology Initiative Board
- Experiment team
  - Sean Tobin, Jeff Nowak, Lee Duter, John Mann, Bob Downing, Peter Priestner, Bob Devine, Tony Tavilla, Andy McKellips, Gary Hatke
- Code, algorithm and experiment design
  - Keith Forsythe, Peter Wu, Ali Yegulalp
- Analysis support
  - Amanda Chan
- Students
  - Nick Chang (U. Mich), Naveen Sunkavally (MIT)
Backups
Experimental Results
Successive MCMUD Iterations

Receiver Bit Error Rate

Mean SISO SNR (dB)

Iteration #3

Bit Error Rate

Space-Time-Frequency Filter
With Multiuser Detection

Training-based
Space-Frequency Filter

Data-Directed
Space-Time-Frequency Filter

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Channel Modes
Experimental Results

Transmit Array

Receive Array

Cambridge

Relative Power (dB)

Mode #

1 2 3 4

Relative Power (dB)

Mode #

1 2 3 4

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Adaptive Beamforming in Multipath

Space-Time-Frequency Adaptive Processing

- Delayed and Doppler Shifted Signal
- Receive Array
- Space-Time-Frequency Filter Cube
- Moving Transmitter

- Delay Taps
- Frequency Taps

Adapted Coefficients

Filters weights jointly take into account space-time-frequency correlations

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