Detection and Tracking as a Seamless Process

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Abstract Detection and tracking are normally considered separate processes. First the signal processing system associated with a sensor examines the signal to determine whether to call detection. Once detection is called, it is converted to an estimate of one or more of the components of the target's kinematic state, e.g., bearing, position, or velocity. This estimate (contact) is sent to a tracking system that determines whether the contact should be associated with an existing track or used to generate a new one. This process works well in high signal-to-noise ratio (SNR) situations but sacrifices performance in low ones. The tracking community is making progress toward seamless detection and tracking and recovering some of this lost performance.

In this talk, we present a method called likelihood ratio detection and tracking (LRDT) that is a step toward providing an integrated approach to detection and tracking. We provide examples of the application of LRDT to sonar and radar detection and tracking. LRDT is a recursive Bayesian version of track-before-detect. In LRDT one specifies a surveillance region that has a prior probability less than one of containing a target. There is a probabilistic motion model that specifies target motion within the region as well as the possibility of transiting into and out of the region. As sensor information is received, it is converted into a measurement likelihood ratio function and combined with the prior likelihood ratio surface to produce a posterior surface. Peaks in this surface are used to determine whether a target is present and to provide an estimate of its state (track). The process is recursive with the posterior surface from one time period being updated for target motion to become the prior for the next measurement likelihood ratio function.
Detection and Tracking as a Seamless Process

See also, ADM001741 Proceedings of the Twelfth Annual Adaptive Sensor Array Processing Workshop, 16-18 March 2004 (ASAP-12, Volume 1), The original document contains color images.
Detection and Tracking as a Seamless Process

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Outline of Talk

- Tracking
  - Bayesian Tracking
- Bayesian Detection and Tracking
  - Likelihood Ratio Detection and Tracking
- Example
  - Passive Acoustic Array
- Discussion
Bayesian Tracking Framework

- **Prior Distribution**
  - Start with a probability distribution on the state of the target

- **Account for Target Motion**
  - Specify distributions on motion and state transitions.
  - Usually a Markov process

- **Likelihood Functions**
  - Convert observations (contacts) into likelihood functions

- **Posterior Distribution**
  - Combine likelihood with prior to compute the posterior distribution on the target state.

- The posterior distribution is the output of a Bayesian tracker
Likelihood Functions

- The likelihood function converts an observation to a function on the target state space:

\[ L(y|s) = \Pr \{ Y = y \mid X(t) = s \} \text{ for } s \in S \]

- The observation (data) \( Y = y \) is known. The target state \( s \) is not. Thus \( L(y|\bullet) \) is a function on \( S \), the target state space.

- Usually \( L(y|\bullet) \) is not a probability distribution on \( S \).

- Likelihood functions are also used for association and classification.
Example
Bearing and Detection Observations

Bearing Likelihood

Detection Likelihood
Bayes Markov Single Target Recursion

Recursion

Initial Distribution: $p(t_0, s_0)$ for $s_0 \in S$ at time $t_0$

For $k \geq 1$ and $s_k \in S$

Motion Update: $p^*(t_k, s_k) = \int_S q_k(s_k | s_{k-1}) p(t_{k-1}, s_{k-1}) ds_{k-1}$

Calc Likelihood: $L(y_k | s_k) = \Pr\{Y_k = y_k | X(t_k) = s_k\}$

Information Update: $p(t_k, s_k) = \frac{1}{C} L(y_k | s_k) p^-(t_k, s_k)$

Kalman Filtering is a special case of this recursion that holds under linear Gaussian assumptions
Likelihood Ratio Detection and Tracking (LRT) vs Tracking

- **Tracking**
  - Assumes target is present
  - Uses only sensor responses that are above threshold (contacts)
  - Uses these responses to estimate state of target

- **Likelihood Ratio Detection and Tracking (LRT)**
  - Does not assume target present
  - Uses below threshold sensor responses
  - Determines
    - Whether target present
    - Target state if present
  - Is a Bayesian form of Track-Before-Detect (TBD) processing
Mathematical Formalism for LRT

- Same as Bayesian tracking except
  - We extend the state space $S$ by adding the null state $\phi$ to represent the possibility that no target is present in the area of interest.
  - We let $S^+ = S \cup \phi$ be this extended state space.
- We assume there is at most one target in the region so that
  $$p(0,\phi) + \int_S p(0,s)ds = 1$$
- We define the cumulative likelihood ratio as
  $$\Lambda(t,s) = \frac{p(t,s)}{p(t,\phi)} = \frac{\Pr\{X(t) = s \mid \text{Observations to time } t\}}{\Pr\{X(t) = \phi \mid \text{Observations to time } t\}}$$
Mathematical Formalism Continued

- Measurement likelihood ratio for the observation $Y_k = y$

$$
\mathcal{L}_k(y \mid s) = \frac{L_k(y \mid s)}{L_k(y \mid \phi)} = \frac{\Pr \{Y_k = y \mid X(t) = s\}}{\Pr \{Y_k = y \mid X(t) = \phi\}}
$$

- This is the ratio of the likelihood of obtaining the observation $Y_k = y$ given target present at $s$ to the likelihood of obtaining the observation given no target present.
Simplified Likelihood Ratio Recursion:
Probability mass moving into region equals amount moving out

(Initial likelihood ratio) \[ \Lambda(0, s) = \frac{p(0, s)}{p(0, \phi)} \text{ for } s \in S \]

For \( k \geq 1 \) and \( s \in S \),

(Motion Update) \[ \Lambda^-(t_k, s) = q_k(s | \phi) + \int q(s | s_{k-1}) \Lambda(t_{k-1}, s_{k-1}) ds_{k-1} \]

(Information Update) \[ \mathcal{L}(y_k | s_k) = \frac{L_k(y_k | s_k)}{L_k(y_k | \phi)} \]
\[ \Lambda_k(t_k | s_k) = \mathcal{L}(y_k | s_k) \Lambda^-(t_k, s) \]

(Logarithm Form) \[ \ln \Lambda_k(t_k | s_k) = \ln \mathcal{L}(y_k | s_k) + \ln \Lambda^-(t_k, s) \]
LRT Implementation Schematic

1. Position Marginal
   Target Trajectory
   Prior Likelihood Ratio

2. Velocity Sheets
   Motion Updated Likelihood Ratio
   Motion Updated

3. True Target Velocity
   Measurement Likelihood Ratio

4. Position Marginal
   Posterior Likelihood Ratio
   Information Update
Velocity Sheet Example

Kinematic State Space: Velocity Sheets
Passive Acoustic Towed Array Example
Simulated Data – Idealized situation

- Twin line array with 50 phones in each line
  - 75 Hz design frequency
- Targets radiate at 5 frequencies: 50, 55, 60, 65, and 70 Hz
- Background noise is complex Gaussian at phones
  - SNR = -9 dB at each frequency at the phones
- Range focused beam forming
  - 720 bearings and 7 ranges: 1000, 1750, 2500, 3250, 4000, 4750, and 5500m
- Measurements are mod square of output from each beam and frequency every 4 seconds
Measurement Log Likelihood Ratio Function

\[ H_0: \Pr \left\{ |B(k,\omega)|^2 = y \right\} = \frac{1}{\sigma_n^2(k,\omega)} \exp \left( \frac{-y}{\sigma_n^2(k,\omega)} \right) \]

\[ H_1: \Pr \left\{ |B(k,\omega)|^2 = y \right\} = \frac{1}{\sigma_s^2(k,\omega) + \sigma_n^2(k,\omega)} \exp \left( \frac{-y}{\sigma_s^2(k,\omega) + \sigma_n^2(k,\omega)} \right) \]

Noise \( \sigma_n^2 \) estimated for each beam, frequency, and time

Signal \( \sigma_s^2 = .125\sigma_n^2 \) (Alternative: specify prob dist on signal level)

\[ \ln \mathcal{L} \left( |B(k,\omega)|^2 \mid k \right) = \frac{|B(k,\omega)|^2}{\sigma_n^2(k,\omega)} - \frac{|B(k,\omega)|^2}{\sigma_s^2(k,\omega) + \sigma_n^2(k,\omega)} \ln \left( \frac{\sigma_s^2(k,\omega) + \sigma_n^2(k,\omega)}{\sigma_n^2(k,\omega)} \right) \]
Scenario

Simulation Scenario

Array

Target 1

Target 2

Target 3

North-South Position (m)

East-West Position (m)

0

1000

2000

3000

4000

5000

-1000

-2000

-3000

-3000

-2000

-1000

0

1000

2000

3000

4000

5000

-2000
Beam Output: Summed over Frequency - Range 5500m

Beam Power (dB)
range 5500m, frequency 50-70Hz

Relative Bearing (deg)
Measurement Log Likelihood Ratio – Range 5500m

- Time: 00:00:00
  - Log Likelihood Ratio: 0.3
  - Relative Bearing: 50

- Time: 00:05:00
  - Log Likelihood Ratio: 0.2

- Time: 00:10:00
  - Log Likelihood Ratio: 0.1

- Time: 00:15:00
  - Log Likelihood Ratio: -0.05

- Time: 00:20:00
  - Log Likelihood Ratio: -0.1

- Time: 00:25:00
  - Log Likelihood Ratio: -0.15
Log Likelihood Ratio Surfaces in Geographic Space

Motion Updated LLR Surface
00:00:20

Cumulative LLR Marginal at 00:00:20

Measurement LLR Surface
00:00:20

Measurement LLR at 00:00:20
Cumulative Log Likelihood Ratio Surfaces

Before Carve Out at 00:00:20

After Carve Out at 00:00:20
Creation of Nodestar Tracks

1. Detection Data
2. LRT Track-Before-Detect Likelihood Ratio Surface
3. Contacts Declared Target(s) from Peak Detections
4. Contact Manager (Associates Contacts)
5. Associated Contacts
6. Non-associated Contacts
7. Create a New Track
   - New Nodestar Single Target Tracker \(N+1\)
8. Nodestar Single Target Tracker \(N\)
9. Nodestar Single Target Tracker \(m\)
Nodestar Tracks

Nodestar Marginals at 00:00:00

position east (km)

position north (km)
Nodestar Bearing Tracks

NS Tracks on Measurement LLR
range 5500m

Relative Bearing (degrees)
Discussion

- LRT example illustrates process of going from unthresholded sensor output to track initiation without seams
- Process of “carving out” contacts and sending to a tracker is a seam which needs to be addressed to complete seamless process
- The basic methodology shown in the example has been applied to data collected at sea.
  - System (LRT plus Nodestar) was able to track a large number (40+) of targets simultaneously and automatically.
  - Had some mis-association problems when tracks crossed in bearing
    - Adding spectrum to target state may improve this
  - Has been applied to matched field data taken at sea.
  - Good array element localization and estimates of background noise crucial
- Further discussion of Likelihood Ratio Detection and Tracking may be found in *Bayesian Multiple Target Tracking* by Stone, Barlow, and Corwin, Artech 1999